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Aspects of Quadratic Gravity

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Abstract

We discuss quadratic gravity where terms quadratic in the curvature tensor are included in the action. After reviewing the corresponding field equations, we analyze in detail the physical propagating modes in some specific backgrounds. First we confirm that the pure R^2 theory is indeed ghost free. Then we point out that for flat backgrounds the pure R^2 theory propagates only a scalar massless mode and no spin-two tensor mode. However, the latter emerges either by expanding the theory around curved backgrounds like de Sitter or anti-de Sitter, or by changing the long-distance dynamics by introducing the standard Einstein term. In both cases, the theory is modified in the infrared and a propagating graviton is recovered. Hence we recognize a subtle interplay between the UV and IR properties of higher order gravity. We also calculate the corresponding Newton's law for general quadratic curvature theories. Finally, we discuss how quadratic actions may be obtained from a fundamental theory like string- or M-theory. We demonstrate that string theory on non-compact CY_3 manifolds, like a line bundle over $\mathbb{CP}^2$, may indeed lead to gravity dynamics determined by a higher curvature action.

Contents

1 Introduction

There is recently a renewed interest in higher curvature theories. These are theories of the general form $R+\mathcal{R}^n$, where R is the standard Einstein gravity and $\mathcal{R}^n$ denotes collectively n^{th} power of the Riemann, Ricci, Weyl tensors or the curvature scalar [1–7]. Of course, in string theory we are familiar with such a structure as the string effective action contains an infinite, well organized and ghost-free series of higher curvature corrections to the leading Einstein gravity. In particular, as has been noted in [1], the inclusion of quadratic curvature terms in the action makes the theory renormalizable. Contrary to string theory, in the truncated, effective field theory there is a price to be paid namely the appearance of a massive ghost state. This ghost state originates from the square of the Riemann, the Ricci or the Weyl tensor. Note that, as we will discuss, if only the scalar curvature and its square are included in the gravity action, then, contrary to some people's belief, these theories are indeed physical and do not contain ghost like modes. In fact, the $R+R^2$ theory, known as the Starobinsky model [8], propagates besides the usual massless graviton, an additional massive spin-0 state, known as the "scalaron field" or the so called "no-scale field". After a field redefinition [9], one obtains a scalar field minimally coupled to standard Einstein gravity with a potential making the $R+R^2$ theory particularly appealing for cosmological inflation [10]. The $R+R^2$ theory can also be embedded in supergravity [11], and there is a large amount of recent work on the inflationary predictions of the supersymmetric $R+R^2$ theory [12,13]. In addition, quadratic gravity theories have also been discussed in particle physics on the basis of their properties under scale transformations. [14].

Although in higher curvature gravity and its solutions [15], a linear Einstein term was assumed to be always present in the action, the case of pure quadratic curvature theories has also been considered [5,16]. In particular, as was recently pointed out [13], the pure R^2 is interesting for the following reasons: it is the only pure quadratic theory that it is free of ghosts and scale invariant. The latter is a classical statements, which however is expected to be violated quantum mechanically leading to the emergence among others of the Einstein term. The pure R^2 theory is conformally equivalent to Einstein gravity with a minimally coupled scalar field and a cosmological constant.

In this paper we discuss the spectrum of quadratic gravity theories, and in particular, the physical spectrum of the pure R^2 theory. Confirming that this theory is free of ghosts, we find that the spectrum of this theory nevertheless depends on the background: we show that on a flat background the theory propagates only a scalar mode and no spin-2 field, whereas on a curved space it propagates a massless spin-2 graviton and a spin-0 scalar state. This result is in agreement with the observation that the pure R^2 theory is only conformally equivalent to Einstein gravity with a minimally coupled scalar field and a cosmological constant, when formulated on a curved background with $R \neq 0$. On the other hand, for a background with $R = 0$, this conformal transformation is singular.

So a sensible theory of gravity with a propagating spin-2 gravity field can only be obtained if one puts the R^2 theory, which is relevant for the short distance physics in the UV, on a background, which still possesses some curvature at long distances in the IR. Alternatively the theory starts to gravitate, if one adds to the UV R^2 theory the standard Einstein term, which is responsible for the long range gravity interactions in the IR. Hence there is an interesting interplay between UV and IR physics in theories of quadratic gravity.

In the paper we also discuss the Newtonian limit of these theories and we calculate the deviation from Newton's law. Finally, we will show how pure quadratic gravity theories may be obtained from a string-theory set up. In particular, string theory on a non-compact CY_3 of relatively small Euler number and large non-vanishing 4-cycles may lead in a certain limit to a pure Riemann square gravity. We investigate in detail the case of a line bundle over $\mathbb{CP}^2$ and we show explicitly how a pure Riemann square theory is obtained, showing that the associated ghost in this case is due to a truncation of the string effective action.

The structure of this paper is as follows: In section 2 we present the relation between the various quadratic curvature terms. In section 3 we investigate the physical spectrum of quadratic actions by considering fluctuations on flat and curved backgrounds. In section 4 we discuss the Newtonian limit of these theories. In section 5 we explore the possibility for obtaining quadratic gravity actions as a particular limit of the string effective theory. We conclude in section 6.

There is a large literature on the study of the so-called $f(R)$ theories of gravity (see for instance [17] and references therein). More recently, the cosmological properties of R^2-like theories and some of their black hole properties have been studied in [18–21]. The absence of propagating spin 2 modes around flat backgrounds was also encountered in studies of three-dimensional massive gravity [22,23].

2 The Higher Curvature Action and Field equations

The most general formulation of scale invariant gravity is described by an action containing the following three terms being second order in the curvature tensor:

$$S = \int_{\mathcal{M}} d^4x\sqrt{-g}\Big(c_1 C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + c_2 R^2 + c_3 \hat{R}^2_{\mu\nu}\Big). \tag{2.1}$$

The first term with $C_{\mu\nu\rho\sigma}$ being the Weyl tensor is conformally invariant, whereas the R^2 term and the $\hat{R}^2_{\mu\nu}$ are only scale invariant. $\hat{R}_{\mu\nu} = R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R$ is the traceless part of the Ricci tensor. All three couplings c_i are dimensionless.

At the classical level one can reduce the number of independent quadratic curvature terms in the action (2.1) from three to two. This is possible since one can replace each of three quadratic terms by the other two plus the Gauss-Bonnet term GB, which takes in four dimensions the following form:

$$GB = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2. \tag{2.2}$$

The integral of the Gauss-Bonnet term over the space $\mathcal{M}$ is given by the topological Euler number $\chi(\mathcal{M})$,

$$\chi(M) = \frac{1}{32\pi^2}\int_{\mathcal{M}} d^4x\sqrt{g}\ GB + \int_{\partial M} d^4x\sqrt{g}\Phi, \tag{2.3}$$

where Φ is constructed from the second fundamental form of the boundary and the Riemann curvature and represents the boundary contribution. Clearly, for compact manifolds only the Gauss-Bonnet integral defines the Euler number, and it does not contribute to the classical equations of motion in this case. However for topologically non-trivial solutions like gravitational instantons, the GB term will lead to a non-trivial, topological contribution to the path integral of the higher curvature action.

Let us first eliminate the square of the Weyl tensor from the action (2.1) using the following well-known relation:

$$\begin{aligned} C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} &= R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2\hat{R}^2_{\mu\nu} - \frac{1}{6}R^2 \\ &= GB + 2\hat{R}^2_{\mu\nu} - \frac{1}{6}R^2. \end{aligned} \tag{2.4}$$

Using this substitution we can rewrite the action (2.1) in the following form:

$$S = \int_{\mathcal{M}} d^4x\sqrt{-g}\Big(a(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2) + bR^2 + c\ GB\Big), \tag{2.5}$$

with the following relations among the coupling constants:

$$a = 2c_1 + c_3, \quad b = c_2 + \frac{c_3}{12}, \quad c = c_1. \tag{2.6}$$

We will use the action (2.5) in the next section when we discuss the propagation modes of the theory.

Alternatively we can eliminate the $\hat{R}^2_{\mu\nu}$ term from (2.1), leading to:

$$S = \int d^4x\sqrt{-g}\Big(\frac{a}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + bR^2 + g\ GB\Big), \tag{2.7}$$

with $g = -c_3/2$. This form of the action is convenient for deriving the classical field equations. Specifically the latter can be written as [16, 24]

$$W_{\mu\nu} = bJ_{\mu\nu} - aB_{\mu\nu} = 0, \tag{2.8}$$

where

$$J_{\mu\nu} = R\,R_{\mu\nu} - \frac{1}{4}R^2\,g_{\mu\nu} - \nabla_\mu\nabla_\nu R + g_{\mu\nu}\nabla^2 R, \tag{2.9}$$

and $B_{\mu\nu}$ is the Bach tensor:

$$B_{\mu\nu} = \left(\nabla^\rho\nabla^\sigma + \frac{1}{2}R^{\rho\sigma}\right)C_{\mu\rho\nu\sigma}. \tag{2.10}$$

As recently discussed in [16], these field equations are solved in particular by two distinct classes of solutions:

(i) Spaces with vanishing scalar curvature:

$$R = 0. \tag{2.11}$$

This class of solutions of the R^2 theory contains in particular non-Ricci flat spaces, $R_{\mu\nu} \neq 0$. In section 3.1 we will represent the pure R^2 theory in terms of an Einstein frame. These solutions will not be described by that theory. Therefore they deserve special attention, in particular in the description of the fluctuating modes around the scalar flat solutions and the question about the absence of ghost when expanding around vacua with $R = 0$.

(ii) Einstein spaces satisfying:

$$R_{\mu\nu} = 3\lambda\,g_{\mu\nu}, \tag{2.12}$$

where λ is an arbitrary constant. In particular this class of solution contains (anti-) de Sitter space. We will also discuss below the propagating modes in such vacua.

3 Physical modes and ghosts

In this chapter we want to discuss the physical propagating modes of the higher curvature action (2.1). In particular we are interested in the question of the possible ghost modes in the pure R^2 theory. This can be done by expanding the action (2.1) in terms of the fluctuations around some particular curved or flat background with $\bar{R} \neq 0$ or $\bar{R} = 0$ respectively. Alternatively, for the case of the pure R^2 theory and for backgrounds with $R \neq 0$, one can investigate this problem by transforming the action to conventional Einstein gravity with a cosmological constant [13, 16].

3.1 Conformal transformation to Einstein frame

Let us write the pure R^2 action as

$$S = \int d^4x\sqrt{-g}\;b\,R^2. \tag{3.1}$$

Here b is the dimensionless coupling constant that can, as we will see, can be either positive or negative. This action can equivalently be written as

$$S = \int d^4x\sqrt{-g}\left(\Phi R - \frac{1}{4b}\Phi^2\right). \tag{3.2}$$

The dimension 2 scalar field Φ plays the role of a Lagrange multiplier and arises in this conformal (Jordan) frame without space-time derivatives. Through its equation of motion Φ is proportional to the background scalar curvature $\bar{R}$:

$$\Phi = 2b\bar{R}. \tag{3.3}$$

Performing a conformal transformation

$$g_{\mu\nu} = \frac{1}{2}M_P^2\Phi^{-1}\tilde{g}_{\mu\nu} \tag{3.4}$$

the action (3.2) can be written as

$$S = \int d^4x\sqrt{-\tilde{g}}\left(\frac{M_P^2}{2}\tilde{R} - \frac{3M_P^2}{4}\frac{\partial_\mu\Phi\partial_\nu\Phi}{\Phi^2} - \frac{M_P^4}{16b}\right). \tag{3.5}$$

Needless to say, most of the manipulations so far, and those that will follow require the scalar curvature to be different from zero. If that is the case, we have in particular:

(i) de Sitter backgrounds:

$$\bar{R} > 0, \quad b > 0. \tag{3.6}$$

In this case Φ is positive and hence also $M_P^2 > 0$, if we require that the conformal transformation (3.4) does not change the signature of the metric. This class of solutions of the R^2 theory describes de Sitter like backgrounds with positive cosmological constant $\Lambda = M_P^4/16b$ in the Einstein frame. Both the signs of the Einstein term as well as of the scalar kinetic term in the Einstein action (3.5) are such that the spin-2 graviton as well as the scalar field Φ are physical, ghost-free degrees of freedom. Hence gravity acts as an attractive force. This will be further discussed in the Newtonian limit in section 4.

(ii) anti-de Sitter backgrounds:

$$\bar{R} < 0, \quad b < 0. \tag{3.7}$$

Φ and $M_P^2 > 0$ are again positive. Now this class of solutions of the R^2 theory describes anti-de Sitter like backgrounds with negative cosmological constant $\Lambda = M_P^4/16b$ in the Einstein frame. Again the spin two graviton as well as the scalar field Φ are physical, ghost-free degrees of freedom.

In addition to these two cases there are also two further choices R and b leading to unphysical, ghost-like propagating modes:

(iii,iv) de Sitter backgrounds:

$$\bar{R} < 0, \quad b > 0 \quad \text{and} \quad \bar{R} > 0, \quad b < 0. \tag{3.8}$$

In these two cases Φ is negative. Hence the scalar mode in (3.5) is ghost-like and the sign of the Einstein term is such that gravity acts as a repulsive force.

This analysis shows that with the correct identifications of the parameters, the pure R^2 is indeed ghost-free around (anti-) de Sitter backgrounds, and the propagating modes correspond to two spin-2 graviton degrees of freedom plus one scalar with physical kinetic energy. However the above simple analysis relies on the existence of the conformal transformation (3.4), that necessitates backgrounds with $R \neq 0$ throughout space-time. For backgrounds with $\bar{R} = 0$ this transformation is singular. As argued in [16], this limit is similar to the tensionless string limit of string theory in six dimensions. In any case, for flat background with $\bar{R} = 0$ the conclusion about the ghost-freedom of R^2 gravity cannot be drawn immediately, and it is more reliable to directly analyze the propagating modes of the higher curvature action.

3.2 Propagating modes in higher curvature actions

Let us write the action in the following form, where we also include the standard Einstein term plus a cosmological constant:

$$S = \int d^4x\sqrt{-g}\Big(a(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2) + bR^2 + \kappa^2 R + \lambda\Big). \tag{3.9}$$

The first two terms with coefficients a and b are the same as in the action (2.5).

Now we study the propagating modes corresponding to this action. For this, we analyze the poles in the propagators generated by its quadratic part. This was already done in [25], and we are just stating the results of this paper. Specifically, there are three kinds of propagating modes:

(i) A massless spin 2 graviton: this mode is independent of a, b, κ^2. It is the standard massless spin 2 graviton.

(ii) A massive spin two particle with mass $\kappa^2/(-a)$. It is related to the Weyl^2 term in the action. In fact, this spin two state is either a tachyonic ($a > 0$), or a massive ghost ($a < 0$). So one should get rid of it.

(iii) A massive scalar with mass proportional to $\kappa^2/6b$. It is related to the R^2 term in the action.

The pure R^2 is recovered in the $a, \kappa^2, \lambda \to 0$ limit. But this limit is rather delicate and has to be taken with care. So we will refine the above discussion and will set $a = \lambda = 0$, *i.e.* the action is

$$S = \int d^4x\sqrt{-g}\Big(bR^2 + \kappa^2 R\Big). \tag{3.10}$$

This action includes the Starobinski model for $\kappa \neq 0$, $b \neq 0$ as well as the pure R^2 theory in case $\kappa = 0$.

3.2.1 Flat spaces solutions

First we expand the action (3.10) around Minkowski space $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. The scalar fluctuation is denoted by $\varphi = h_\mu{}^\mu$, and the action (3.10) takes the form

$$S_0 = \int d^4x\sqrt{-g}\left\{\frac{\kappa^2}{2}\left(\frac{1}{2}h_{\mu\nu}\Box h^{\mu\nu} - \frac{1}{2}\varphi\Box\varphi - D_\mu\varphi D_\nu h^{\mu\nu} + D^\mu h_{\mu\rho}D_\nu h^{\nu\rho}\right) + b(\partial^\mu\partial^\nu h_{\mu\nu} - \Box\varphi)^2\right\} \tag{3.11}$$

As usual, we can decomposed the symmetric tensor $h_{\mu\nu}$ as

$$h_{\mu\nu} = h^\perp_{\mu\nu} + \partial_\mu a^\perp_\nu + \partial_\nu a^\perp_\mu + (\partial_\mu\partial_\nu - \frac{1}{4}\eta_{\mu\nu}\Box)a + \frac{1}{4}\eta_{\mu\nu}\varphi, \tag{3.12}$$

where $h^{\perp}_{\mu\nu}$ is transverse traceless

$$\partial^{\mu} h^{\perp}_{\mu\nu} = \eta^{\mu\nu} h^{\perp}_{\mu\nu} = 0, \tag{3.13}$$

and $a^{\perp}_{\mu}$ is transverse, *i.e.*, divergenceless

$$\partial^{\mu} a^{\perp}_{\mu} = 0. \tag{3.14}$$

Gauge transformations are the infinitesimal diffeomorphisms $x^{\mu} \to x^{\mu} + \xi^{\mu}(x)$ under which the metric transforms as:

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_{\mu}\xi_{\nu} + \partial_{\mu}\xi_{\nu}. \tag{3.15}$$

Note that we may also decompose ξ_{μ} in transverse and longitudinal parts as

$$\xi_{\mu} = \xi^{\perp}_{\mu} + \partial_{\mu}\xi \tag{3.16}$$

with

$$\partial^{\mu}\xi^{\perp}_{\mu} = 0, \quad \Box\xi = \partial^{\mu}\xi_{\mu}. \tag{3.17}$$

With this decomposition, under the gauge transformation (3.15), we get that

$$h^{\perp}_{\mu\nu} \to h^{\perp}_{\mu\nu}, \tag{3.18}$$
$$a^{\perp}_{\mu} \to a^{\perp}_{\mu} + \xi^{\perp}_{\mu}, \tag{3.19}$$
$$a \to a + 2\xi, \tag{3.20}$$
$$\varphi \to \varphi + 2\Box\xi. \tag{3.21}$$

Therefore, the field

$$\Phi = \varphi - \Box a \tag{3.22}$$

is invariant under (3.15), i.e.

$$\Phi \to \Phi. \tag{3.23}$$

Then, it is easy to verify that

$$\partial^{\mu}\partial^{\nu} h_{\mu\nu} - \Box\varphi = \frac{3}{4}\left(\Box^2 a - \Box\varphi\right) = -\frac{3}{4}\Box\Phi \tag{3.24}$$

and, hence the quadratic action (3.11) becomes:

$$S_0 = \int d^4x\sqrt{-g}\left\{\frac{\kappa^2}{4}\left(h^{\perp}_{\mu\nu}\Box h^{\perp\mu\nu} + \partial_{\mu}\Phi\partial^{\mu}\Phi\right) + \frac{9b}{16}(\Box\Phi)^2\right\}. \tag{3.25}$$

It should be noted that that the Fourier modes $\tilde{h}_{\mu\nu}$ of $h_{\mu\nu}$ have a decomposition similar to (3.12). In particular we have that

$$\tilde{h}^{\perp}_{\mu\nu} = P^{(2)}_{\mu\nu,\rho\sigma}\tilde{h}_{\rho\sigma} \tag{3.26}$$
$$\tilde{\Phi} = P^{(0)}_{\mu\nu,\rho\sigma}\tilde{\Phi} \tag{3.27}$$

where $P^{(2)}_{\mu\nu,\rho\sigma}$, $P^{(0)}_{\mu\nu,\rho\sigma}$ are the projectors for the spin-2 and spin-0 parts of $h_{\mu\nu}$ in momentum space. They are explicitly given by

$$\begin{aligned} P^{(2)}_{\mu\nu,\rho\sigma} &= \frac{1}{2}\left(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\nu\rho}\theta_{\mu\sigma}\right) - \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}, \\ P^{(0)}_{\mu\nu,\rho\sigma} &= \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}, \quad \theta_{\mu\nu} = \eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}. \end{aligned} \tag{3.28}$$

In the Starobinski case around flat space there is then a massless spin-2 fluctuation with propagator

$$\Delta^{(2)}_{\mu\nu,\rho\sigma} = -\frac{2}{\kappa^2}\frac{1}{q^2}P^{(2)}_{\mu\nu,\rho\sigma}. \tag{3.29}$$

It disappears in the limit $\kappa = 0$, so in the pure R^2 theory around flat space there is no propagating spin-2 graviton.

Next, the spin zero propagator reads

$$\begin{aligned} \Delta^{(0)}_{\mu\nu,\rho\sigma} &= -\frac{1}{6b}\frac{1}{q^2(q^2+\kappa^2/6b)}P^{(0)}_{\mu\nu,\rho\sigma} \\ &= -\frac{1}{\kappa^2}\left(\frac{1}{q^2+\kappa^2/6b - i\epsilon} - \frac{1}{q^2+i\epsilon}\right)P^{(0)}_{\mu\nu,\rho\sigma}, \end{aligned} \tag{3.30}$$

where we have also implemented an $i\epsilon$ prescription for the spin-zero propagator. In the Starobinski case with $\kappa, b \neq 0$, $\Delta^{(0)}_{\mu\nu,\rho\sigma}$ describes two scalar modes; the first term in (3.30) corresponds to a massive spin zero particle with mass

$$m^2 = \frac{\kappa^2}{6b}. \tag{3.31}$$

It describes the massive scalar excitation around the flat Starobinski vacuum. The second term in (3.30) describes a massless spin zero ghost. Note that the propagators for the normal massive state and the ghost have opposite imaginary parts. Choosing the same prescription for the normal and the ghost state would result in a violation of the optical theorem and a corresponding violation of unitarity. However, with the above $+i\epsilon$ prescription for the ghost, unitarity is maintained at the cost of propagation of negative energy forward in time [26].

In should be noted that the ghost state encountered above, is the standard ghost emerging also in general relativity. Indeed, by setting $b = 0$, i.e in the general relativity limit, we see that Φ is gauge invariant and more surprisingly it is a ghost. Thus, it seems that general relativity propagates an additional state besides the two helicity ± 2 graviton states. However this ghost is harmless as it is not a propagating physical mode. To see this, let us recall that Φ is written in terms of $h_{\mu\nu}$ as

$$\Phi = P^{\mu\nu}h_{\mu\nu} \tag{3.32}$$

where

$$P^{\mu\nu} = \eta^{\mu\nu} - \frac{4}{3}\Box^{-1}\left(\partial^\mu\partial^\nu - \frac{1}{4}\eta^{\mu\nu}\Box\right). \tag{3.33}$$

As a result, Φ is non-local in time so that the initial data for the metric perturbations $h_{\mu\nu}$ at a given time, are not enough to determine Φ at a later time. Therefore, there is no extra degree of freedom in general relativity as a naive counting indicates. This can also be seen from the fact that although $h^{\perp}_{\mu\nu}$ and Φ are gauge invariant (3.21,3.23), there is still a residual gauge symmetry. Indeed, we may still transform $h^{\perp}_{\mu\nu}$ and Φ as

$$h^{\perp}_{\mu\nu} \to h^{\perp}_{\mu\nu} + D_\mu \xi_\nu + D_\nu \xi_\mu, \tag{3.34}$$

$$\Phi \to \Phi + 2D^\mu k_\mu, \tag{3.35}$$

provided $\xi_\mu,\ k_\mu$ satisfy

$$D^\mu \xi_\mu = 0, \qquad \left(\Box + \frac{R}{4}\right)\xi_\mu = 0, \quad D_\mu k_\nu + 4D_\nu k_\mu = \frac{1}{2} g_{\mu\nu} D^\sigma k_\sigma, \tag{3.36}$$

k_μ being just conformal Killing vectors. Therefore, ξ_μ eliminates four of the six components of $h^{\perp}_{\mu\nu}$ leaving two propagating components which correspond to the ± 2 helicity states of the graviton, whereas, the divergence of k_μ eliminates the would-be spin-0 ghost state Φ.

As a result, in the Starobinsky theory, the massless ghost state appearing in the propagator (3.30) is nothing else than the usual harmless non-propagating state already encounter in general relativity, and can be eliminated either due to its non-local relation to the metric perturbation or by gauge symmetry arguments. Alternatively, we may write the spin-0 part of (3.25) as

$$\begin{aligned} S_0 &= \int d^4x \left(\frac{3\kappa^2}{32}\partial_\mu\Phi\partial^\mu\Phi + \frac{9b}{16}(\Box\Phi)^2 + \Phi T + \cdots\right) \\ &= \int d^4x \left(\frac{3\kappa^2}{32}\partial_\mu\Phi\partial^\mu\Phi + \Psi\Box\Phi - \frac{4}{9b}\Psi^2 + \Phi T + \cdots\right), \end{aligned} \tag{3.37}$$

where the coupling of Φ to the trace $T^\mu_\mu = 8T$ of the energy-momentum tensor has also been included. Then the field equations for Ψ and Φ are

$$-\frac{3\kappa^2}{16}\Box\Phi + \Box\Psi = -T, \tag{3.38}$$

$$\Box\Phi = \frac{8}{9b}\Psi, \tag{3.39}$$

or equivalently:

$$-\frac{\kappa^2}{6b}\Psi + \Box\Psi = -T, \tag{3.40}$$

$$\Box\Phi = \frac{8}{9b}\Psi. \tag{3.41}$$

Again, we see that the theory propagates only a massive scalar degree of freedom $\Psi = 9b/8\,\Box\Phi$ with mass $m^2_\Psi = \kappa^2/6b$, whereas Φ is a not propagating field.

Next we discuss the scalar modes in the pure R^2 theory around flat space. In the limit $\kappa = 0$, the scalar propagator in (3.30) becomes

$$\Delta^{(0)}_{\mu\nu,\rho\sigma} = -\frac{1}{6b}\frac{1}{q^4}P^{(0)}_{\mu\nu,\rho\sigma}. \tag{3.42}$$

It describes a massless dipole, *i.e.* a pair of massless scalars; however to make the propagator well-defined an $i\epsilon$ description should be provided. We will follow a slightly different approach to analyze the spectrum. For the pure R^2 theory, the action (3.37) is written as

$$S_0 = \int d^4x\sqrt{-g}\bigg(\frac{9b}{16}(\Box\Phi)^2 + \Phi T + \cdots\bigg) = \int d^4x\sqrt{-g}\left(\Psi\Box\Phi - \frac{4}{9b}\Psi^2 + \Phi T + \cdots\right) \tag{3.43}$$

and the equations for Φ, Ψ in the presence of a source are now

$$\Box\Psi = -T, \tag{3.44}$$

$$\Box\Phi = \frac{8}{9b}\Psi. \tag{3.45}$$

Thus, Ψ is a normal propagating massless degree of freedom, whereas, Φ is not propagating as it is a non-local function of the metric perturbations or differently put, it can be fixed by the residual gauge symmetry (3.35). As a result, the pure R^2 theory propagates just a single spin-0 state around flat Minkowski background. In particular, no tensor mode can be excited for this theory on flat spacetime. So the pure R^2 theory on flat spaces does not gravitate.

Let us also note at this point that for the action (3.9), although the propagator of the spin-0 part of $h_{\mu\nu}$ is still given by (3.30), the spin-2 propagator turns out to be

$$\Delta^{(2)}_{\mu\nu,\rho\sigma} = -\frac{2}{q^2(q^2a+\kappa^2)}P^{(2)}_{\mu\nu,\rho\sigma}. \tag{3.46}$$

Therefore, it reduces to (3.29) for $a = 0$ as expected and there is an additional pole at $q^2 = -\kappa^2/a$ corresponding to a massive spin-2 ghost.

3.2.2 Curved spaces solutions

We now consider perturbations around the de Sitter background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad \text{with} \quad \bar{R}_{\mu\nu} = \lambda\bar{g}_{\mu\nu}, \quad \lambda = \frac{\bar{R}}{4}. \tag{3.47}$$

Then we find to second order:

$$\begin{aligned} S_2 = b\int d^4x \bigg\{ &\Big[D_\mu D_\nu h^{\mu\nu} - \Box h - \frac{1}{4}\bar{R}\varphi\Big]^2 - \bar{R}\left(-\frac{1}{2}h_{\mu\nu}\Box h^{\mu\nu} + \frac{1}{2}\varphi\Box\varphi + D_\mu\varphi D_\nu h^{\mu\nu}\right. \\ &\left. - D^\mu h_{\mu\rho}D_\nu h^{\nu\rho}\right) + \frac{\bar{R}^2}{6}\left[h_{\mu\nu}h^{\mu\nu} + \frac{1}{4}\varphi^2\right]\bigg\}, \end{aligned} \tag{3.48}$$

where D_μ is the covariant derivative with respect to the de Sitter metric $\bar{g}_{\mu\nu}$. We may again decompose $h_{\mu\nu}$ as

$$h_{\mu\nu} = h^\perp_{\mu\nu} + D_\mu a^\perp_\nu + D_\nu a^\perp_\mu + (D_\mu D_\nu - \frac{1}{4}\eta_{\mu\nu}\Box)a + \frac{1}{4}\bar{g}_{\mu\nu}\varphi, \tag{3.49}$$

where

$$D^\mu h^\perp_{\mu\nu} = \eta^{\mu\nu}h^\perp_{\mu\nu} = D^\mu a^\perp_\mu = 0, \tag{3.50}$$

so that, the quadratic action turns out to be

$$S_2 = b\int d^4x \left\{ \frac{9}{16}\left[\Box\Phi + \frac{\bar R}{3}\Phi\right]^2 + \bar R\left[\frac{1}{2}h^{\perp}_{\mu\nu}\left(\Box - \frac{\bar R}{6}\right)h^{\mu\nu\perp} - \frac{3}{16}\Phi\left(\Box + \frac{\bar R}{3}\right)\Phi\right]\right\}$$
$$= b\int d^4x \left\{ \frac{\bar R}{2}h^{\perp}_{\mu\nu}\left(\Box - \frac{\bar R}{6}\right)h^{\mu\nu\perp} + \frac{9}{16}\Phi\left(\Box^2 + \frac{\bar R}{3}\Box\right)\Phi.\right\} \quad (3.51)$$

This structure of the quadratic action can in fact be obtained by recalling that (with δ_1, δ_2 first order and second order variations respectively)

$$S_2 = b\int d^4x\sqrt{-g}\Big\{(\delta_1 R)^2\sqrt{-g} + 2\bar R(\delta_1 R)(\delta_1\sqrt{-g}) + \bar R^2(\delta_2\sqrt{-g}) + 2\bar R(\delta_2 R)\sqrt{-g}\Big\}$$
$$= b\int d^4x\sqrt{-g}\Big\{(\delta_1 R)^2\sqrt{-g} + 2\bar R\delta_2(R\sqrt{-g}) - \bar R^2(\delta_2\sqrt{-g})\Big\}. \quad (3.52)$$

The last two terms of the above action, giving rise to the last two terms in (3.51), are exactly the quadratic part $\delta_2 S_{EH}$ of the Einstein-Hilbert action

$$S_{EH} = \int d^4x\sqrt{-g}\frac{M_P^2}{2}\Big(R - 2\Lambda\Big) \quad (3.53)$$

with Planck mass M_P and a cosmological constant given by

$$\Lambda = \frac{M_P^4}{16b} \quad (3.54)$$

In addition, the first term in (3.52) gives rise to the first term in (3.51). This is a higher derivative term that gives rise to the kinetic energy of Φ. In other words: pure R^2 theory on a de Sitter background propagates a massless spin-2 graviton and an additional massless spin-0 scalar excitation.

So there is no ghost in the pure R^2 in a background with constant scalar curvature $R \neq 0$. We see, that going from the flat space background to the (anti)-de Sitter background, the situation with respect to the gravity spectrum improves substantially. This situation is similar to higher spin theories of the Vasiliev type (also a kind of $M_P \to 0$ limit of gravity), which are only consistent in (anti)-de Sitter background but not in flat space. The fact that the curvature of space plays a stabilizing role of the UV theory in the IR, by providing propagating gravitons and also a good large energy behaviour, is indeed remarkable.

Summarizing this section, we stress again that pure R^2 theory propagates just a single scalar degree of freedom on flat spacetime and no spin-2 tensor mode. The latter can be recovered by IR regulating the theory by considering non-flat backgrounds (de Sitter or anti-de Sitter) or by adding the Einstein term as in section (3.2.1). Both ways introduce back the graviton spin-2 state, while keeping the massless spin-0 state.

4 Newtonian Limit

The equations of motions in the R^2 theory for a test particle following the trajectory $x^\mu(s)$ with $u^\mu = dx^\mu/ds$ are as usual

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\nu\rho}u^\mu u^\nu = 0. \quad (4.1)$$

We may express the theory in the Einstein frame by using the conformally transformed metric $\bar{g}_{\mu\nu} = g^E_{\mu\nu}$ for $R(g) \neq 0$

$$\bar{g}_{\mu\nu} = \frac{8\mu}{R} g_{\mu\nu}. \tag{4.2}$$

Therefore, in the Einstein frame, the geodesic equation (4.1) followed by a particle will be

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\nu\rho} u^\mu u^\nu = f^\mu, \tag{4.3}$$

where

$$f^\mu = \frac{1}{2R}\left(\delta^\mu_\nu \partial_\rho R + \delta^\mu_\rho \partial_\nu R - g_{\nu\rho}\partial^\mu R\right) u^\nu u^\rho. \tag{4.4}$$

Let us now calculate the Newtonian force between two massive scalars ϕ of mass m in quadratic gravity [1]. The Newtonian potential will be

$$V = -\frac{1}{4m^2}\frac{1}{(2\pi)^3}\int d^3\vec{k}\mathcal{M}_{nr} e^{-i\vec{k}\cdot\vec{r}}, \tag{4.5}$$

where $\mathcal{M}_{nr}$ is the non-relativistic amplitude for the $\phi + \phi \to \phi + \phi$ process. The interaction Lagrangian is

$$\mathcal{L} = \frac{1}{2} h^{\mu\nu}\tau_{\mu\nu}, \tag{4.6}$$

where

$$\tau_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu}\left((\partial\phi)^2 + m^2\phi^2\right). \tag{4.7}$$

Then with incoming momenta p_1, p_2 and outgoing p_3, p_4, as in Fig. 1,

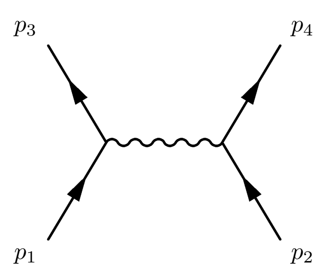

Figure 1: Digram for the calculation of the Newtonian potential

we have that [27]

$$\mathcal{M} = \tilde{\tau}^{\mu\nu}(p_1, p_2)\Delta_{\mu\nu,\rho\sigma}\tilde{\tau}^{\rho\sigma}(p_3, p_4), \tag{4.8}$$

where

$$\tilde{\tau}_{\mu\nu}(p_i, p_j) = \frac{1}{2}\left(p_{i\mu}p_{j\nu} + p_{j\mu}p_{i\nu} - \eta_{\mu\nu}(p_i\cdot p_j + m^2)\right), \tag{4.9}$$

and $\Delta_{\mu\nu,\rho\sigma}$ is the graviton propagator.

Newtonial potential for the $\mathbf{R+R^2+Weyl^2}$ theory: It is straightforward to calculate the Newtonian potential for the general action

$$S=\int d^4\sqrt{-g}\left(\kappa^2 R+b\,R^2+\frac{a}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right). \tag{4.10}$$

In this case, by using Eqs.(3.30) and (3.46), we find that the amplitude (4.8) is given by

$$\begin{aligned}\mathcal{M}=&\frac{1}{k^2(ak^2+\kappa^2)}\Big\{(p_1\cdot p_2)(p_3\cdot p_4)+(p_1\cdot p_3)(p_2\cdot p_4)-(p_1\cdot p_2)(p_3\cdot p_4-m^2)\\&+2(p_1\cdot p_2-m^2)(p_3\cdot p_4-m^2)-\frac{2}{3}\Big[(p_1\cdot p_2-2m^2)(p_3\cdot p_4-2m^2)\Big]\Big\}\\&+\frac{1}{12}\frac{1}{k^2(6bk^2+\kappa^2)}\left(2m^2-p_1\cdot p_3\right)\left(2m^2-p_3\cdot p_4\right).\end{aligned} \tag{4.11}$$

Then, in the non-relativistic limit we get

$$\mathcal{M}_{nr}=-\frac{4}{3}\frac{m^4}{|\vec{k}|^2(a\,|\vec{k}|^2+\kappa^2)}+\frac{1}{3}\frac{m^4}{|\vec{k}|^2(6b\,|\vec{k}|^2-\kappa^2)}. \tag{4.12}$$

It is clear that the first term in the rhs of (4.12) is due to the massless spin-2 graviton, where the second term in due to the spin-0 state. We can express (4.12) as

$$\mathcal{M}_{nr}=-\frac{m^4}{\kappa^2}\left(\frac{1}{|\vec{k}|^2}-\frac{4}{3}\frac{a}{a|\vec{k}|^2+\kappa^2}-\frac{2b}{6b\,|\vec{k}|^2-\kappa^2}\right). \tag{4.13}$$

After Fourier transforming and using (4.5), we find that the Newtonian potential for the theory (4.10) is given by

$$V=-\frac{Gm}{r}+\frac{4Gm}{3}\frac{e^{-\alpha M_p r}}{r}-\frac{Gm}{3}\frac{e^{-\beta M_p r}}{r} \tag{4.14}$$

where

$$\alpha=1/\sqrt{2a},\quad \beta=1/2\sqrt{3b},\quad \kappa^2=1/16\pi G=M_p^2/2. \tag{4.15}$$

Interestingly, for $r\gg r_0$ where $r_0=1/min(\alpha,\beta)M_p$, we get the usual Newton law. However, for $r\ll r_0$ we get that

$$V\approx V_0-\frac{1}{48\pi}(\beta^2-4\alpha^2)mr, \tag{4.16}$$

where $V_0=Gm(\beta-4\alpha)M_p/3$. We see that the potential is finite at $r=0$ and it is confining for

$$\beta^2<4\alpha^2, \tag{4.17}$$

(it is also confining for $\beta=2\alpha$). Therefore, for $\beta^2\leq 4\alpha^2$, the potential is a monotonic function of r, whereas, for $\beta^2>4\alpha^2$ it necessarily develops a minimum as shown in Fig. 2.

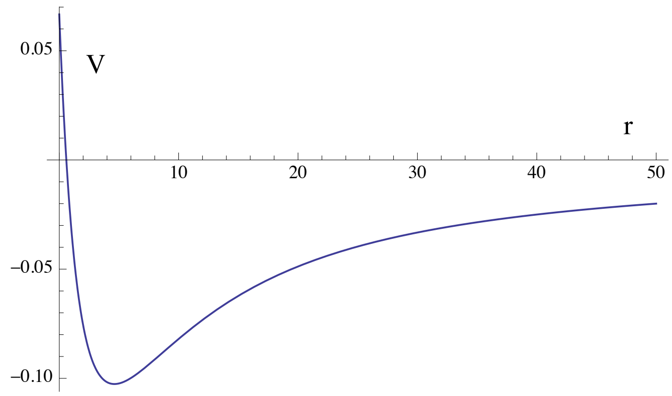

Figure 2: Newtonian potential for $\alpha = .2, \beta = 1$.

We may also employ the general expression (4.13) to find the potential in some particular limits.

Newtonial potential for the $\mathbf{R} + \mathbf{R}^2$ theory: By setting $a = 0$ in (4.13) we get that

$$\mathcal{M}_{nr} = -\frac{m^4}{\kappa^2}\left(\frac{1}{|\vec{k}|^2} - \frac{2b}{6b\,|\vec{k}|^2 - \kappa^2}\right), \tag{4.18}$$

and the Newtonian potential for the $R + R^2$ theory turns out to be

$$V = -\frac{Gm}{r} - \frac{Gm}{3}\frac{e^{-\beta M_p r}}{r}. \tag{4.19}$$

Therefore, the usual Newtonian potential acquires an additional Yukawa contribution from the massive spin-0 state.

Newtonial potential for the $\mathbf{R}^2$ theory: In the $\alpha = 0,\ \kappa = 0$ limit, the first term in the rhs of (4.12) is absent and the we get that

$$\mathcal{M}_{nr} = \frac{1}{18b}\frac{m^4}{|\vec{k}|^4}. \tag{4.20}$$

The Newtonian potential is then given by

$$V = -\frac{1}{72b}m^2\frac{1}{(2\pi)^3}\int d^3\vec{k}\frac{e^{-i\vec{k}\cdot\vec{r}}}{|\vec{k}|^4} \tag{4.21}$$

which, after Fourier transforming the generalized function $1/|\vec{k}|^4$, we find that

$$V = \frac{1}{9\cdot 2^6\,\pi b}m^2 r, \tag{4.22}$$

i.e. it is a confining potential without a Newtonian tail.

5 The quadratic action in String and M-Theory

In string and M-theory higher curvature actions appear naturally in the effective field theory description of the light modes. They are also generated by compactification and decoupling of the massive Kaluza-Klein states. The internal manifold can in fact be chosen to be compact or non-compact. We will explore the origin of higher curvature terms using various compactifications.

5.1 Compact internal spaces

We start with a generic higher curvature expansion in the 11-dimensional M-theory:

$$S_M = \int d^{11}x\sqrt{-G_{11}}\frac{1}{l_{11}^9}\biggl(\sum_{n=1}^{\infty} l_{11}^{2(n-1)}\mathcal{R}_{11}^n\biggr). \tag{5.1}$$

Here l_{11} denotes the 11-dimensional Planck length and the higher curvature terms $\mathcal{R}_{11}^n$ comprise all possible n$^{\text{th}}$ powers of 11-dimensional Riemann tensors, Ricci tensors and Ricci scalars. Actually in 11-dimensional supergravity some of these terms are absent due to supersymmetry conditions.

In the first step, let us reduce the 11-dimensional M-theory on a circle of radius R_{11}, deriving in this way the 10-dimensional type IIA theory. This is done by the well-known ansatz

$$l_{11}^2 = \alpha', \quad R_{11} = (\alpha')^{1/2}, \tag{5.2}$$

where $e^\phi = g_s$ is the string coupling constant $(\alpha')^{-2}$ the string tension. After further rescaling of the 10-dimensional metric, $G_{10} = e^{-\frac{2\phi}{3}}g_{10}$, the generic 10-dimensional type II action takes the form:

$$S_{10,IIA} = \int d^{10}x\sqrt{-g_{10}}\biggl((\alpha')^{-4}\sum_{n=1}^{\infty}(\alpha')^{(n-1)}e^{\frac{2}{3}(n-4)\phi}\mathcal{R}_{10}^n\biggr) \tag{5.3}$$

Actually one has to remark that the terms in the 11-dimensional action, written down so far, are not the only terms in the M-theory R^n expansion after compactification. Namely, upon compactification on a circle, there are additional M-theory loop effects, which contain additional powers of $(l_{11}/R_{11})^l$, where l is basically the loop order in M-theory. They vanish in the limit $R_{11}\to\infty$, *i.e.* they are invisible in the 11-dimensional decompactification limit. In the 10-dimensional $\mathcal{R}_{10}^n$ expansion they lead to terms which scale like $(\alpha')^{n-5}$, however due to the additional R_{11}^{-1}, they scale differently with respect to the string coupling constant e^ϕ.

Furthermore, the 11-dimensional terms written in (5.1), correspond not only to tree-level 10-dimensional IIA string theory, but contain also loop contributions in the string genus expansion. The precise relation is $2n-8=3(h-1)$, where n is the power of $\mathcal{R}_{10}^n$ and h is the genus, i.e. the loop order in IIA. Therefore an arbitrary value of n seems to give rise to non-integer values of h, which are not easily exmmmmplained in string perturbation theory. However, certain terms are not present in 10-dimensional IIA action, like $\mathcal{R}_{10}^2$, $(n=2)$ because they are BPS protected.

Now, in the second step, let us further compactify the IIA action down to four dimensions on a generic six-dimensional space with volume V_6. This leads to the following higher derivative action in four dimensions:

$$S_{4,IIA} = \int d^4x\sqrt{-g_4}\biggl(\sum_{n=1}^{\infty}M_P^{\frac{1}{2}(5-n)}e^{\frac{1}{6}(n-1)\phi}V_6^{\frac{1}{4}(n-1)}\mathcal{R}^n\biggr). \tag{5.4}$$

The four-dimensional Planck mass M_P is defines as usual:

$$M_P^2 = (\alpha')^{-4}e^{-2\phi}V_6. \tag{5.5}$$

Now it is easy to convince oneself that there exist no scaling limit of the parameters M_P, e^ϕ and V_6, in which only the first two terms or even only the $\mathcal{R}^2$ term survive in this expansion. However this conclusion is only true for a generic six-dimensional compact space. In the following we like to demonstrate that for certain non-compact Calabi-Yau spaces there indeed exists a limit in which the 4-dimensional action has the form of $R+\mathcal{R}^2$ or even $\mathcal{R}^2$, with all the higher order terms being absent in a particular scaling limit.

5.2 String theory on non-compact CY_3

We now consider string theory on a non-compact Calabi-Yau three-fold [28, 29]. As before, the string effective action depends on two parameters, e^{ϕ} defining the loop expansion[1], and the string length $l_s \sim \sqrt{\alpha'}$. When we consider type IIA,B theories on a background $M_{10} = M_4 \times X_6$, as shown in [28], the effective action becomes:

$$\mathcal{S}_{eff} = \frac{1}{(2\pi)^7 l_s^8} \int_{M_4 \times X_6} d^{10}x \sqrt{-g} e^{-2\phi} R_{(10)} + \\ \frac{\bar{\chi}}{3(4\pi)^7 l_s^2} \int_{M_4} d^4x \sqrt{-g_{(4)}} \left(-2\zeta(3)e^{-2\phi} \pm 4\zeta(2)\right) R_{(4)} + \mathcal{S}_4 + \cdots \tag{5.6}$$

where the $\pm$-signs are related to whether we consider type IIA or B theories. The parameter $\bar{\chi}$ is given by $\bar{\chi} = 64 \cdot 96\pi^3 \chi$, where

$$\chi = \int d^6x \sqrt{g_6} E_6, \tag{5.7}$$

with

$$E_6 = \frac{1}{96\pi^3} \Big(R_{mnrs} R^{rspq} R_{pq}{}^{mn} - 2R^m{}_{nrs} R^{rpsq} R_{pmq}{}^n \Big). \tag{5.8}$$

For a compact Ricci-flat X_6, χ is just the Euler number, whereas for a non-compact X_6, this term is not the Euler number. Boundary contributions are needed to generate the topological invariant.

In addition to the Einstein terms, there are higher order curvature corrections to (5.6) [30–33], which are important for our discussion. We have in particular at one-loop order:

$$\mathcal{S}_4 = \frac{\zeta(3)\,\omega}{3(4\pi)^7 l_s^2} \int_{M_4} d^4x \sqrt{-g_{(4)}} \Big(4R_{\mu\nu}R^{\mu\nu} - R^2 \Big), \tag{5.9}$$

where

$$\omega = 48 \int_{X_6} d^6x \sqrt{g_6}\, R_{mnpq} R^{mnpq}, \tag{5.10}$$

and $(\mu, \nu, \cdots = 0, \ldots, 3,\ m, n, \cdots = 4, \ldots, 9)$. There is also a quadratic term at tree-level but it turns out to be just the Gauss-Bonnet combination, consistent with the fact that tree-level quadratic curvature terms should be independed of the CY moduli. Note that no cosmological constant is generated because X_6 is a CY manifold [30].

Now, taking the limits

$$\alpha' \to 0, \quad g_s \to 0, \tag{5.11}$$

in order to suppress α' and string-loop corrections, we arrive at

$$\mathcal{S}_{eff} = \frac{2\zeta(3)}{(2\pi)^7 l_s^8} \int_{M_4 \times X_6} d^{10}x \sqrt{-g} e^{-2\phi} R_{(10)} - \frac{\zeta(3)}{3(4\pi)^7 l_s^2} \int_{M_4} d^4x \sqrt{g_{(4)}} \Big\{ e^{-2\phi}\, \bar{\chi}\ R_{(4)} - \omega\big(4R_{\mu\nu}R^{\mu\nu} - R^2\big) \Big\}. \tag{5.12}$$

[1] Note that the 10-dimensional string coupling (resp. dilaton) appears in the effective action, since we are considering a non-compact Calabi-Yau space.

Therefore, string theory on a non-compact CY_3 induces localized 4D terms in addition to the bulk 10D one. This is similar to the DGP theory [35], where a localized 4D and a bulk 5D Einstein term are simultaneously considered, and a crossover parameter controls the regime where the effective gravity is four- or five-dimensional. Here the localized 4D term dominates the bulk 10D one, as long as the 4D Planck mass M_s/g_s ($M_s = 1/l_s$) is much larger than the 10D Planck mass $M_s/g_s^{1/4}$ [29]. But this is exactly the weak string coupling limit (5.11) $g_s \to 0$. Let us note that the non-compact CY_3 is of infinite volume and therefore, the bulk dynamics is always ten-dimensional.

To explicitly demonstrate that finite and no-vanishing χ and ω are possible, we work out a particular case of a non-compact CY_3, namely a complex line bundle over $\mathbb{CP}^2$. The metric for this space can be written as [33, 34]

$$ds^2 = \frac{1}{c^2}d\rho^2 + 6a^2 d\sigma^2 + c^2(dz + A)^2, \tag{5.13}$$

where

$$a^2 = \rho + l^2 \quad c^2 = \frac{2\rho(\rho^2 + 3\rho l^2 + 3l^4)}{3(\rho + l^2)^2} \tag{5.14}$$

$d\sigma^2$ is the metric of $\mathbb{CP}^2$ and A is related to the Kähler form on $\mathbb{CP}^2$:

$$d\sigma^2 = dw^2 + \frac{1}{4}\sin^2 w(\sigma_1^2 + \sigma_2^2) + \frac{1}{4}\sin^2 w \cos^2 w \sigma_3^2, \tag{5.15}$$

$$A = -\frac{3}{2}\sin^2 w\, \sigma_3\, . \tag{5.16}$$

The coordinates ρ, z parametrize the **C**-fiber, and to avoid conical singularities, z takes values from $(0, 2\pi)$. The range of $\rho \in (0, \infty)$ and the range of $w \in (0, \pi/2)$. The 1-forms $\sigma_{1,2,3}$ are the standard left-invariant one forms in $SU(2)$. This Ricci-flat metric contains a free parameter l that determines the minimal size of a four cycle diffeomorphic to CP^2. In that l represents the minimal radius of this 4-cycle at $\rho = 0$.

Using this metric we can evaluate some of the quantities that appear in the effective actions. In particular:

$$\begin{aligned} R_{mnqp}R^{mnpq} &= \frac{40(2a^2 - 3c^2)^2}{3a^8} = \frac{160\, l^{12}}{3(\rho + l^2)^8}, \\ E_6 &= \frac{1}{12\pi^3}\frac{(2a^2 - 3c^2)^3}{a^{12}} = \frac{2}{3\pi^3}\frac{l^{18}}{(\rho + l^2)^{12}}, \end{aligned} \tag{5.17}$$

so that, ω, χ defined in eqs.(5.10,5.7) turn out to be

$$\omega = 6(4\pi)^3 l^2, \tag{5.18}$$

$$\chi = \frac{8}{3}. \tag{5.19}$$

Note that we have considered above only terms up to R^4. Clearly, higher order curvature terms in the bulk will be suppressed in the limit (5.11). Namely, supergravity corrections will be suppressed in the $\alpha' \to 0$ limit, whereas $g_s \to 0$ will suppress string loop effects. Thus, the 10D theory will be described just by the 10D Einstein term. It is expected however, that these 10D higher curvature terms will generate also localized 4D terms, which will contribute to localized R^2 or even to localized terms of higher order like R^4, that might dominate. However, a simple inspection shows that all localized R^2 terms arising from higher curvature terms will be suppressed in the $l_s \to 0$ limit. Indeed, let us consider for example a bulk R^6 term. The latter will give a contribution to (5.12) proportional to

$$\omega_2 \sim l_s^2 \int_{X_6} d^6x\sqrt{g_6}\,(R_{mnpq}R^{mnpq})^2. \tag{5.20}$$

Simply on dimensional grounds, $\omega_2 \sim l_s^2/l^2$ as l is the only scale in the non-compact CY_3. Therefore, such contributions to (5.12) vanish in the $l_s \to 0$ limit. How about localized higher order terms? For example R^6 will also generate a localized R^4 term proportional to ω, *i.e.*,

$$l_s^2 \int_{X_6} d^6x\sqrt{g_6}\,R_{mnpq}R^{mnpq} \int d^4x\sqrt{g_4}\ \mathcal{R}^4 \sim \omega\, l_s^2 \int d^4x\sqrt{g_4}\,\mathcal{R}^4, \tag{5.21}$$

so that, we will have a term

$$(l_s l)^2 \int d^4x\sqrt{g_4}\ \mathcal{R}^4. \tag{5.22}$$

Clearly this terms is subleading for finite l and $l_s \to 0$ and the theory is described by (5.12) in the limit (5.11).

5.3 The quadratic $\mathcal{R}^2$ action

It is clear then that for weak string coupling, the bulk Planck mass is less than the 4D one and the effective theory in the limit (5.11) with ω/l_s^2 finite is described by the action

$$\mathcal{S}_{eff} = \int_{M_4} d^4x\sqrt{g}\left(\frac{M_P^2}{2}R + \frac{1}{g_0^2}R_{\mu\nu}R^{\mu\nu} - \frac{1}{4g_0^2}R^2\right), \tag{5.23}$$

where

$$M_P^2 = -\frac{2\zeta(3)\bar{\chi}}{3(4\pi)^7 l_s^2 g_s^2}, \quad g_0^2 = \frac{3(4\pi)^7 l_s^2}{4\omega\zeta(3)} \tag{5.24}$$

We have ignored the Gauss-Bonnet contribution as it is a total derivative for constant g_s. Note that $\bar{\chi}$ should be negative (as in the case for example of the deformed conifold). By using the identities (2.4) we find that

$$4R_{\mu\nu}R^{\mu\nu} - R^2 = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - GB = 4\hat{R}^{\mu\nu}\hat{R}_{\mu\nu} = 2C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} - 2GB + \frac{1}{3}R^2, \tag{5.25}$$

and therefore

$$\begin{aligned}\mathcal{S}_{eff} &= \int_{M_4} d^4x\sqrt{g}\left(\frac{M_p^2}{2}R + \frac{1}{g_0^2}\hat{R}^{\mu\nu}\hat{R}_{\mu\nu}\right)\\ &= \int_{M_4} d^4x\sqrt{g}\left(\frac{M_p^2}{2}R + \frac{1}{2g_0^2}C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + \frac{1}{12g_0^2}R^2\right).\end{aligned} \tag{5.26}$$

We can follow now a similar procedure as described in section (3.1) to bring the theory in the Einstein frame, where, due to the conformal invariance of the Weyl tensor, the action (5.26) turns out to be

$$\mathcal{S}_{eff} \;=\; \int_{M_4} d^4x\sqrt{g}\,\left\{\frac{M_P^2}{2}R-\frac{1}{2}(\partial\sigma)^2-\frac{3}{4}g_0^2M_P^4\left(1-e^{-\sqrt{\frac{2}{3}}\sigma/M_P}\right)+\frac{1}{2g_0^2}C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}\right\}. \quad (5.27)$$

This is nothing else than the Starobinsky model augmented by the square of the Weyl tensor. The value of g_0 in this case is $g_0^2 \approx 10^{-10}$.

Let us note that when the Einstein term is missing from (5.23), we have that

$$\mathcal{S}_{eff} = \int_{M_4} d^4x\sqrt{g}\,\left(\frac{1}{2g_0^2}C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}+\frac{1}{12g_0^2}R^2\right), \quad (5.28)$$

which, in the Einstein frame takes the form

$$\mathcal{S}_{eff} \;=\; \int_{M_4} d^4x\sqrt{g}\,\left(\frac{1}{2g_0^2}C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}+\frac{M_P^2}{2}R-\frac{1}{2}(\partial\sigma)^2-\frac{3}{4}M_P^4\,g_0^2\right). \quad (5.29)$$

This is Einstein-Weyl gravity with a massless scalar and a cosmological constant.

The spectrum of the theories (5.26) and (5.29) contains the usual massless graviton, a massive spin-2 ghost and a scalar (the scalaron mode). However, the ghost state should be harmless as its appearance can be traced here to the truncation of the string effective action in the particular framework we are discussing here.

6 Conclusions

We have studied here quadratic curvature gravity theories. Such theories have attracted considerable attention, among others things, due to their renormalization properties. In particular, they offer the interesting possibility of yielding a well defined quantum theory, which in addition, reduces to general relativity at long distances. We have elaborate in particular on the pure R^2 theory. We found that it propagates a single massless scalar and no tensor mode on a flat background. The latter appears by infrared regulating the UV theory by considering curved backgrounds, like de Sitter or anti-de Sitter, or by introducing back the Einstein term, which dominates at long distances. Hence we observe a subtle UV/IR correspondence in higher curvature gravity. We also discussed Newton's law in the present framework. Here we find that the extra states in the spectrum of quadratic gravity give additional contributions to the usual Newton's law. In particular, we find that there are two contributions from the spin-2 and spin-0 state leading to an attractive force and an additional contribution from the ghost massive spin-2 state which produces a repulsive Yukawa force. Finally, we explore the possibility that quadratic gravity emerges as some particular limit of a more fundamental theory like string theory. We showed that indeed, string theory on a non-compact CY_3 besides the bulk 10D gravity, it also induces a 4D localized gravity much the same way that DGP model does [35]. In certain region of the parameter space, the localized gravity is dominating leading to a 4D effective description for the gravitational dynamics. In particular, there is a limit in which localized quadratic curvature terms are dominating leading to an effective 4D higher curvature gravity.

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