Progress In Electromagnetics Research, PIER 56, 67–80, 2006

ASPECTS ON THE PHASE DELAY AND PHASE VELOCITY IN THE ELECTROMAGNETIC NEAR-FIELD

J. C.-E. Sten and A. Hujanen

Technical Research Centre of Finland
VTT Information Technology
P.O.Box 1202, 02044 VTT-Espoo, Finland

Abstract—The phase of a complex field and its speed of propagation are fundamental concepts of electromagnetic wave motion. Although it seems to be well-known that faster than light propagation of the phase may occur in, e.g., waveguides and certain dispersive media, it is often ignored that a similar phenomenon, in fact a very marked one, presents itself in the near-field of an arbitrary oscillating current in vacuum. Connected herewith is the observation that the phases of the transverse field components of a dipole approach $kr - \pi/2$, and not kr, in the radiation zone. This article illustrates these phenomena by theoretical and numerical examples as well as indicates their consequences for broad-band wireless communication over short distances.

1. INTRODUCTION

The process of creation of electromagnetic waves by oscillating dipoles was studied in great detail by Heinrich Hertz over a century ago [1], and it has remained the key subject of antenna theory ever since. It may thus come as a surprise (at least for the authors it did) that the wave speed in the reactive near-field, when determined by means of the phase, departs from the speed of light and, in fact, exceeds it considerably [2, 3]. This seems *prima facie* to be in conflict with the notion that, according to the law of causality, unbounded electromagnetic waves in a homogeneous medium should under no circumstances travel faster than light in vacuum.

Of course, there is no mystery involved here. The pitfall, if not embarrassing at least instructive, is that ordinary plane-wave thinking is applied to a mixture of travelling and reactive fields. In this article we seek to demonstrate, by means of an elementary theoretical exercise, that the phase velocity near sinusoidally oscillating point dipoles does indeed exceed the speed of light, without endangering the law of causality. The effect is merely a result of the transition from the quasi-static near-field, where the fields are "in phase" with the source, to the far-field, where the field phases depart from kr by a phase angle of $\pi/2$. In the time-domain the phenomenon manifests itself as a gradual deformation, or a step by step differentiation, of the signal waveform.

The behaviour of the phase in the near-field is also studied by electromagnetic simulations with dipoles of physical extension. Finally, some aspects of the "phase delay" in antenna theory are adduced, the most notable being that the near-field for electrically small antennas extends much farther than the classical far-field limit, $2D^2/\lambda$, D being the overall size of the antenna and λ, the wavelength. As correctly pointed out for instance in [4, p. 33] this limit is relevant only for structures having a size comparable to λ.

Besides being of theoretical interest the authors believe that the issue also has some bearing on certain electromagnetic applications in science and technology, where the distance between the source and the point of observation is very small in terms of the free-space wavelength. The phenomenon is relevant, in particular, in applications involving very large bandwidths, where the distortion of the signal waveform causes problems. Some recent papers, e.g., [5–7], deal with this subject, although from the point of view of the time-domain response of finite length wire structures in the radiation zone. In the present paper, however, the attention is focused on the simplest of radiators, the infinitesimal dipole, and especially the near-field it produces.

2. DESCRIPTION OF VELOCITY CONCEPTS

Let us begin by recapitulating the basic velocity concepts associated with electromagnetic wave motion.

An electromagnetic field in a homogeneous, isotropic and lossless medium is governed by the pair of equations

$$\nabla^2\mathbf{E} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = 0, \qquad \nabla^2\mathbf{H} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\mathbf{H} = 0 \tag{1}$$

where v — the speed at which electromagnetic fields propagate — is a parameter depending entirely of the constitutive parameters of the medium. In vacuum, $v = c_0$, the speed of light.

For harmonic waves of angular frequency ω a local phase velocity $v^{(p)}$ can be defined as [8, Sect. 1.3.3]

$$v^{(p)}(\mathbf{r}) = \left[\mathbf{u} \cdot \nabla\left(\frac{\Phi}{\omega}\right)\right]^{-1} \tag{2}$$

where $\Phi(\mathbf{r})$ expresses the spatial dependence of the wave phase, $\mathbf{u}$ being the unit vector of the direction into which the wave is travelling (in the case of outgoing spherical waves, $\mathbf{u} = \mathbf{u}_r$). The phase velocity of Eq. (2) is *verbatim* the expression for the speed at which a co-phasal surface of the wave propagates. For instance, in the case of a homogeneous monochromatic plane-wave of the form $e^{j(\omega t - \Phi(z))}$ travelling in the positive z-direction, $\Phi(z) = kz$, where $k = \omega/v^{(p)}$ is the wavenumber. Here, and in the sequel, it is worth noticing that $\Phi(z)$ is chosen to be positive when moving in the direction of propagation.

The local group velocity $v^{(g)}$, on the other hand, is given by [8, Sect. 1.3.4]

$$v^{(g)}(\mathbf{r}) = \left[\mathbf{u} \cdot \nabla\left(\frac{\partial \Phi}{\partial \omega}\right)\right]^{-1} \tag{3}$$

the ω-derivative being taken at the mean frequency of the wave spectrum. It expresses the speed at which a surface of constant amplitude of polychromatic wave propagates, being thus a measure for the speed of the envelope of a modulated wave. As a historical point it may be noted that the belief that the group velocity is a measure for the speed at which signals are propagated seems to be due to Lord Rayleigh, who was the first to analyse the concept in acoustics [9, Ch. 1]. When, in the beginnings of the relativity era, it was realised that the electromagnetic group velocity may in some circumstances exceed the speed of light, this view was soon to be rectified by Sommerfeld and Brillouin, who independently showed (by considering the motion of a step-front) that energy and information can be transmitted with no higher speed than that of light.

The phenomenon of superluminal phase and group velocities manifests itself in various circumstances. In waveguides, for instance, it is well-known that the group velocity is greater than c_0, approaching an infinite speed at the "cut-off" frequency. For waves in free space, on the other hand, the effect may be observed in regions of anomalous dispersion, that is, in a medium whose refractive index decreases with increasing frequency [8, Sect. 1.3.3],[9, Ch. I.2] (contrary to so-called normal dispersion, when the index increases). In recent years, large efforts have been directed towards an artificial realisation of this extraordinary property (see e.g., [10, 11]). In particular, we might mention that certain periodic structures called photonic crystals have opened the way to studies of superluminal RF-signal transmission at the laboratory scale [12].

The goal of this paper is to demonstrate and discuss the consequences of the fact that in the near-field of an oscillating dipole, or an electrically small antenna in general, the fields propagate with a phase velocity which is greater than the speed of light.

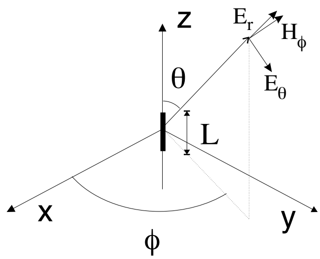

Figure 1. A short dipole, the co-ordinates and the field components.

3. DIPOLE FIELDS

3.1. General Time Dependence

Let us first consider an infinitesimal time-varying electric dipole with the dipole moment $\mathbf{p}(\mathbf{r}, t) = \mathbf{u}_z\delta(\mathbf{r})p(t)$ in vacuum, shown in Fig. 1. The changing moment p of the dipole can be pictured as a current flow

$$I(t) = \frac{1}{L}\frac{dp(t)}{dt} \tag{4}$$

L being the infinitesimal "length" of the dipole. Now, a time-varying dipole moment (or equivalently, a time-varying current) provokes an electromagnetic field in the surrounding space, expressed in terms of the usual spherical co-ordinates (r, θ, φ) [8, Ch. 2.2.3], [13, Sect. 3.5]

$$\begin{aligned} \mathbf{E}(\mathbf{r}, t) &= \frac{1}{4\pi\varepsilon_0}\left[\mathbf{u}_\theta\frac{\sin\theta}{r^2}\left(\frac{p(\hat{t})}{r} + \frac{1}{c_0}\frac{dp(\hat{t})}{dt} + \frac{r}{c_0^2}\frac{d^2p(\hat{t})}{dt^2}\right)\right. \\ &\qquad \left. +2\mathbf{u}_r\frac{\cos\theta}{r^2}\left(\frac{p(\hat{t})}{r} + \frac{1}{c_0}\frac{dp(\hat{t})}{dt}\right)\right] \qquad (5) \\ \mathbf{H}(\mathbf{r}, t) &= \frac{\sin\theta}{4\pi r^2}\mathbf{u}_\varphi\left(\frac{dp(\hat{t})}{dt} + \frac{r}{c_0}\frac{d^2p(\hat{t})}{dt^2}\right) \qquad (6) \end{aligned}$$

where $\hat{t} = t - r/c_0$ is the retarded time variable. Thus, the supposition that the field travels at the speed of light, c_0, indifferent of the waveform chosen for $p(t)$, is implicitly embodied, as provided by the law of causality. Expressions (5), (6) are usually derived by means of the time retarded Hertz vector in a manner given in, for instance [13, Ch. 3.5].

At a sufficiently large distance from the dipole, where terms in the order $1/r$ prevail, the Poynting vector integrated over an arbitrary

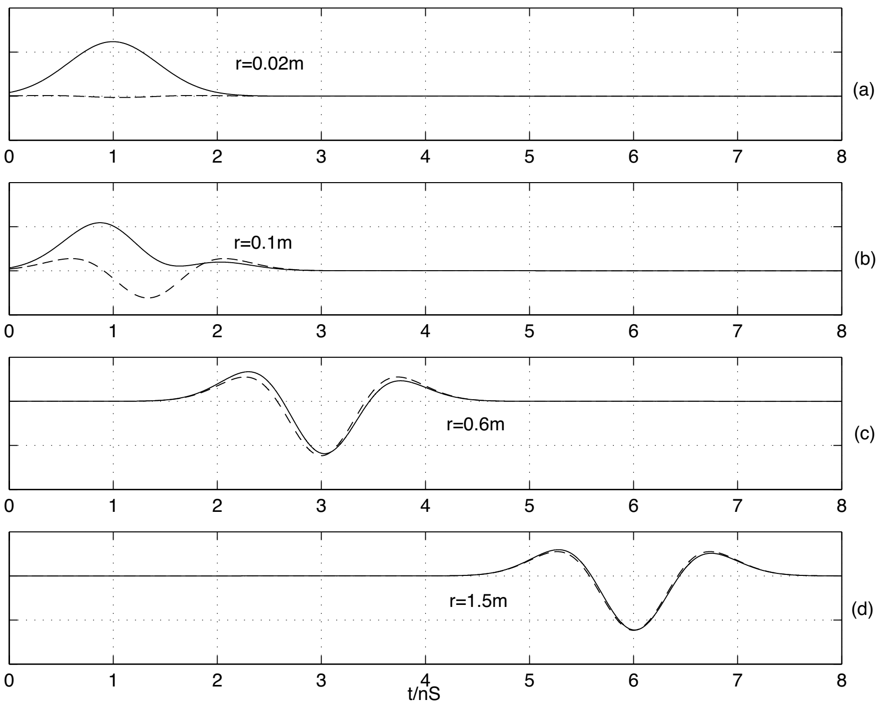

Figure 2. Illustrating the distortion of the E_θ-field waveform shown at four different positions away from the dipole. Magnitudes are normalised.

spherical surface drawn around the dipole gives

$$\lim_{r\to\infty}\int_S (\mathbf{E}\times\mathbf{H})\cdot\mathbf{u}_r dS = \frac{1}{6\pi\varepsilon_0 c_0^3}\left(\frac{d^2p(\hat{t})}{dt^2}\right)^2 \tag{7}$$

which is the total power outflow from the dipole. Thus, in view of the double differentiation of $p(\hat{t})$ in time, it is evident that the waveform at large distances will become distorted, except for strictly harmonic excitation.

To illustrate, let us consider the waveform distortion of the E_θ-field component as a function of distance when the excitation is a Gaussian pulse; $p(t) \sim \exp(-\alpha(t-\tau)^2)$. In the example of Figs. 2(a)–2(d) the waveform (magnitudes normalised) is represented as a function of time at four distances when $\alpha = 1/0.36$ (ns)^2 and $\tau = 1$ ns. The solid line is the total E_θ-field, while the dashed line is the $1/r$-dependent term (the far-field component) relative to the E_θ-field. Interestingly, at a distance of 0.1 m (Fig. 2(b)) the actual peak of the signal occurs near 0.8 ns, that is, *before* the signal has achieved its maximum at the source point. At first sight this might appear counter intuitive. However, it is logically explained by the fact that the wave gradually deforms during the propagation.

3.2. Harmonic Time Dependence

We now turn our attention to the time-harmonic excitation, initially described by Hertz in his admirable work [1]. In the usual complex-vector notation, the electromagnetic field excited by a sinusoidal current of moment $IL = j\omega p$ can be written as (see, e.g., [13, Sect. 3.7])

$$\begin{aligned}\mathbf{E}(\mathbf{r}) &= j\omega\mu IL\frac{e^{-jkr}}{4\pi r}\left[\mathbf{u}_\theta \sin\theta\left(1-\frac{j}{kr}-\frac{1}{k^2r^2}\right)\right.\\ &\quad\left.-2\mathbf{u}_r\cos\theta\left(\frac{j}{kr}+\frac{1}{k^2r^2}\right)\right] \qquad (8)\\ \mathbf{H}(\mathbf{r}) &= jkIL\frac{e^{-jkr}}{4\pi r}\mathbf{u}_\varphi \sin\theta\left(1-\frac{j}{kr}\right) \qquad (9)\end{aligned}$$

Comparing these expressions with those of the corresponding time-domain field, one sees that the individual "terms" of (5) and (6), with their peculiar dependence of time, now intermingle as phasors.

By applying Euler's formula, $e^{jx} = \cos x + j\sin x$, the phase-functions of the three field components are determined as [1, pp. 162–165]

$$\tan\Phi_{E_\theta} = -\frac{kr\sin kr + (1-k^2r^2)\cos kr}{(1-k^2r^2)\sin kr - kr\cos kr} \qquad (10)$$

$$\tan\Phi_{E_r} = -\cot\Phi_{H_\varphi} = -\frac{\cos kr + kr\sin kr}{\sin kr - kr\cos kr} \qquad (11)$$

Hence, by the aid of the well-known relationship

$$\tan(z_1+z_2) = \frac{\tan z_1 + \tan z_2}{1-\tan z_1\tan z_2} \qquad (12)$$

one deduces the explicit formulae for the phase-functions of the field components as

$$\Phi_{E_\theta} = kr - \arctan\left[\frac{kr}{1-k^2r^2}\right] + \begin{cases}\pi/2 & kr \le 1\\ -\pi/2 & kr > 1\end{cases} \qquad (13)$$

$$\Phi_{E_r} = kr - \arctan kr + \pi/2 \qquad (14)$$

$$\Phi_{H_\varphi} = kr - \arctan kr \qquad (15)$$

with appropriate constants included. We wish to emphasise that Φ_{E_θ} is continuous also at $kr = 1$, where the arctan-function switches from $-\pi/2$ to $\pi/2$. One should also point out that another way of reproducing Eqs. (13)–(15) is by always keeping the e^{-jkr}-factor in (8), (9) separate.

From expressions (13)–(15), first derived by Hertz [1, pp. 162–165], as well as from the illustration in Fig. 3, each phase-function can be seen to approach the "normal" $kr + \pi$ times a constant, when $r \to \infty$ as expected. But in the near-field, that is, within the radius of a half a wavelength ($kr = \pi$), say, the situation is more complicated.

The positions at which Φ_{E_θ} is $N\pi/2$, where N is an integer, may be found numerically from Eq. (13). For $N = 1$ one thus finds the distances $r = 0$ and 0.4367λ, as can be verified from Fig. 3, for $N = 2$ correspondingly 0.7133λ, for $N = 3$, 0.9735λ, and so on, the subsequent distances approaching step by step an integer times a quarter of a wavelength. This means, among other things, that E_θ at the origin and on a sphere of radius 0.4367λ are "in phase", while, intuitively one would expect the phase of E_θ to have changed by 180° over a distance of $\lambda/2$. In a sense, the radius 0.4367λ around the source point can be interpreted as the limit outside which Φ_{E_θ} starts to settle itself to regular far-field behaviour.

The phase velocities, obtained using Eq. (2), are

$$v_{E_\theta}^{(p)}/c_0 = \frac{1 - k^2r^2 + k^4r^4}{k^2r^2(k^2r^2 - 2)} \tag{16}$$

$$v_{E_r}^{(p)}/c_0 = v_{H_\varphi}^{(p)}/c_0 = 1 + \frac{1}{k^2r^2} \tag{17}$$

It is easy to see that, when $r \to 0$, $v_{E_\theta}^{(p)}$ approaches asymptotically $-c_0/(2k^2r^2)$, while $v_{E_r}^{(p)} = v_{H_\varphi}^{(p)} \to c_0/(kr)^2$. Moreover, $v_{E_\theta}^{(p)}$ is negative until $kr = \sqrt{2}$, when it changes sign at infinity, where after it rapidly decelerates towards c_0. $v_{E_r}^{(p)}$ and $v_{H_\varphi}^{(p)}$, on the other hand, asymptotically decelerate towards c_0 monotonically as $r \to \infty$. Using Eq. (3) the corresponding group velocities are

$$v_{E_\theta}^{(g)}/c_0 = \frac{(1 - k^2r^2 + k^4r^4)^2}{-6k^2r^2 + 7k^4r^4 - k^6r^6 + k^8r^8} \tag{18}$$

$$v_{E_r}^{(g)}/c_0 = v_{H_\varphi}^{(g)}/c_0 = \frac{(1 + k^2r^2)^2}{3k^2r^2 + k^4r^4} \tag{19}$$

where it can be seen that, at a sufficiently large distances from the source, the group velocities approach c_0 asymptotically from below.

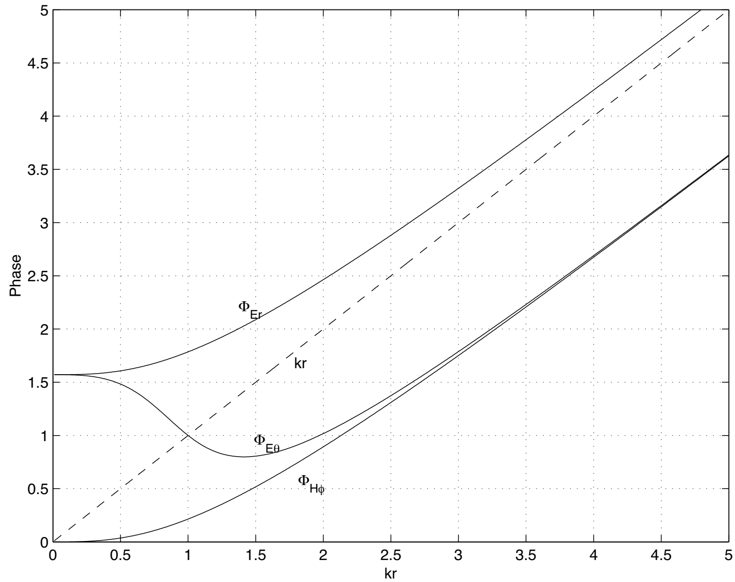

Figure 3. Displaying the phase-functions Φ_{E_θ}, Φ_{E_r} and Φ_{H_φ} (in radians).

In sum, from the point of view of the phase, the three components of the field evolve from the point of origin with a highly variable speed, gradually approaching c_0 as the wave propagates towards infinity.

3.3. Near- and Far-Field Approximations

It is also instructive to compare approximations of the field expressions (8), (9) at different distances from the source. In the near-field of the dipole ($kr \ll 1$) the expressions for the electromagnetic field are:

$$\mathbf{E}^{(\text{near})}(\mathbf{r}) \approx -j\frac{\omega\mu IL}{4\pi k^2 r^3}\left[\mathbf{u}_\theta \sin\theta + 2\mathbf{u}_r \cos\theta\right] \tag{20}$$

$$\mathbf{H}(\mathbf{r})^{(\text{near})} \approx \frac{IL}{4\pi r^2}\mathbf{u}_\varphi \sin\theta \tag{21}$$

They are, of course, the quasi-static Coulomb-field of an electric dipole with the moment, $-jIL/\omega$, and the Ampère-Biot-Savart-field of a differential current element, respectively. It is clearly seen that the magnetic field is virtually "in phase" with the current, while the electric field in "phase quadrature".

In the far-field region ($kr \gg 1$), on the other hand, the field components are given by

$$\mathbf{E}^{(\text{far})}(\mathbf{r}) \approx j\omega\mu IL \frac{e^{-jkr}}{4\pi r}\left[\mathbf{u}_\theta \sin\theta - 2\frac{j}{kr}\mathbf{u}_r \cos\theta\right] \tag{22}$$

$$\mathbf{H}^{(\text{far})}(\mathbf{r}) \approx jkIL \frac{e^{-jkr}}{4\pi r}\mathbf{u}_\varphi \sin\theta \tag{23}$$

taking into account the leading terms of each component. Comparing (20) with (22) and disregarding the kr-dependence, the phase of the θ-field component is seen to be reversed 180°, while the r-component is in a 90° phase lag. The phase of the magnetic field (23) is similarly delayed by 90°, when compared to the quasi-static field (21). It is because of these lags that the phases must be moving with a considerable speed in the near-field so as to catch up with their respective values in the radiation-zone, where they evolve linearly with kr.

4. SIMULATED EXAMPLE

Now that the effect has been demonstrated theoretically one may ask for its significance. Does it manifest itself in practice or is it merely a curiosity?

It is possible to verify the effect of superluminal phase-velocity either through measurements [2, 3] or electromagnetic simulations. Accordingly, let us evaluate using the "Numerical Electromagnetics Code" (NEC) the near-field of an ordinary z-directed thin-wire dipole and consider the phases of E_θ and H_φ as a function of distance from the feed point in the transverse direction, i.e. for $\theta = \pi/2$ (note that E_r is zero in this direction). The total length of the antenna is 0.5 m, the wire diameter is 2 mm, and the frequencies 60 MHz and 300 MHz are used. At these frequencies the wire is $\lambda/10$ and $\lambda/2$ long, respectively. It is also useful to keep in mind that the classical far-field limit [4], $2D^2/\lambda$, is 0.1 m and 0.5 m, at the respective frequency.

Fig. 4(a), 4(b) shows the computed phase functions for the two field components E_θ and H_φ as a function of distance along the transverse direction. Case (a) represents simulations at 60 MHz, case (b) those at 300 MHz. In Fig. 4(a), the overall behaviour of the phases of E_θ and H_φ is quite similar to that of the elementary dipole in Fig. 3, as can be expected due to the small size of the antenna. The phases very close to the antenna are different, however, which is because of the "voltage excitation" used in the simulations, which forces the phase of E_θ to be zero at the feed, while the phase of H_φ is essentially

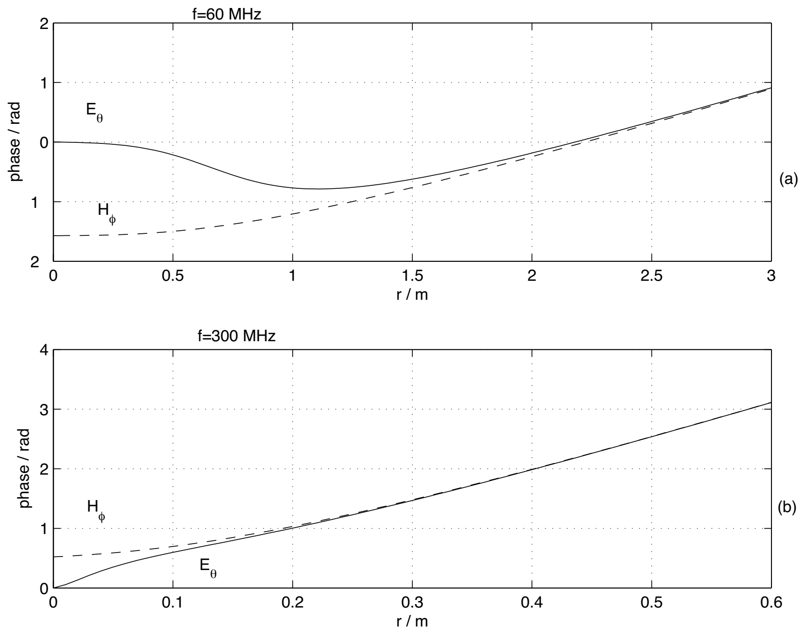

Figure 4. The computed phases of E_θ and H_φ near a 0.5 m long wire dipole at 60 MHz (above) and at 300 MHz (below).

determined by the input impedance of the antenna (if desired, the initial level of the phases could be adjusted by means of impedance matching). In contrast, the phases of Fig. 4b are completely different: both E_θ and H_φ take a linear course almost from the start, and at $r = 0.5$ m ($= \lambda/2$ at 300 MHz) the phases are practically the same.

In Fig. 5(a), (b), showing the local phase velocities corresponding to Fig. 4(a), (b), the region of superluminal propagation is seen to be much more pronounced and to extend much farther away for the lower frequency. The same effect is seen in Fig. 6(a), (b), where the apparent mean phase velocity, defined as

$$v^{(p),\mathrm{app}} = \frac{\omega r}{\Phi(r) - \Phi(0)} \tag{24}$$

is displayed. The apparent phase velocity represents the speed at which the phase appears to have propagated from the phase reference point (in this case the origin) to r. In practical measurements the apparent velocity given by Eq. (24) may be easier to evaluate than the local phase velocity of Eq. (2), because it does not involve numerical differentiation. Of course, $v^{(p),\mathrm{app}}$ approaches c_0 when $r \to \infty$, but not

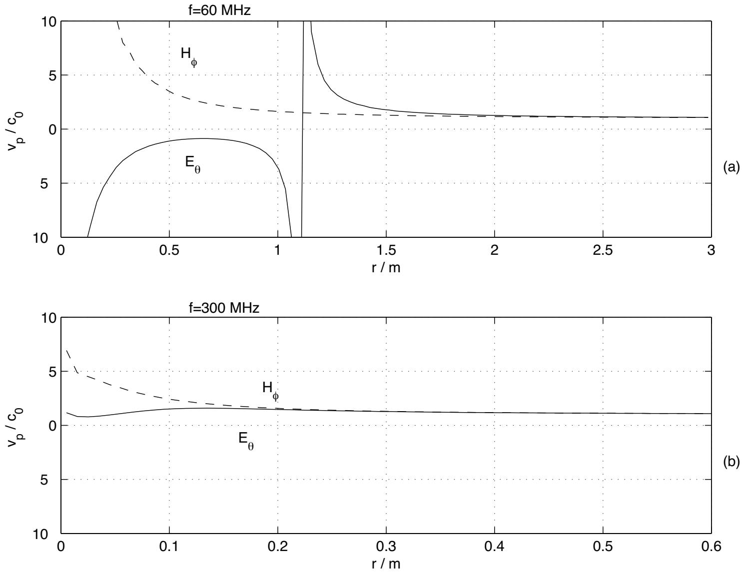

Figure 5. The local phase velocity corresponding to Fig. 4(a), (b).

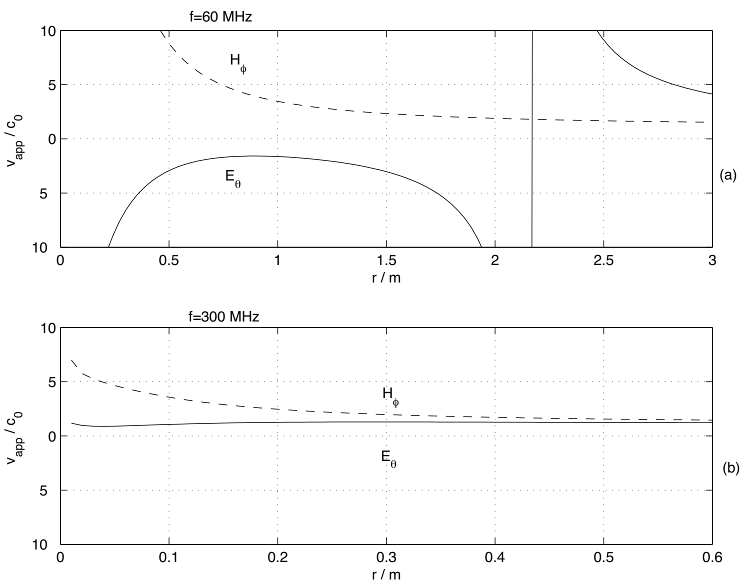

Figure 6. The apparent mean phase velocity corresponding to Fig. 4(a), (b).

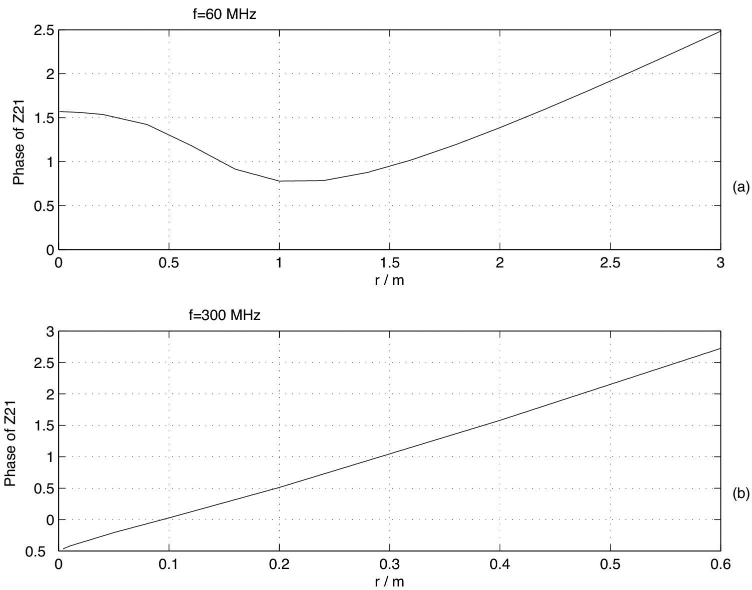

Figure 7. The phase of the mutual impedance of two linear dipoles in parallel as a function of distance at 60 MHz (above) and at 300 MHz (below).

in the same asymptotic sense as $v^{(p)}$.

Fig. 7(a), (b), finally, displays the phase of the coupling or mutual impedance Z_{21} between two identical dipoles, side by side in parallel, when the separating distance is varied. Both dipoles are 0.5 m long, 2 mm in diameter, and the same frequencies are considered as earlier. Since the mutual impedance is essentially the ratio between E_θ (integrated over the length of the receiving antenna) and H_φ at the excitation, it is not surprising that the features of the phase of the E_θ-field, seen in Fig. 4(a), (b), are transferred to the mutual impedance almost identically. Fig. 7(a), for example, shows that the phase of the mutual impedance becomes $\pi/2$ at a distance of circa 0.436λ, which is almost the same distance as was found numerically for the elementary dipole of Fig. 3.

5. DISCUSSION

Finally, one may ask why the behaviour of the phase in the near field appears so little known? So little, in fact, that today's antenna handbooks scarcely mention them.

The answer, we believe, is to be sought in the traditional demarcation between circuit-theory and electromagnetic field-theory. It is well-known that in the domain of circuit theory, where physical distance plays no part, the reaction of each "component" starts instantaneously, at least seemingly. For harmonic signals this creates an impression that the field somehow anticipated the behaviour of the source in time. Although the quasi-static zone around a current carrying region (a sphere of radius of a half wavelength, say) can be considerable at low frequencies, the propagation time is nevertheless so short that a smaller delay than expected is likely to escape the attention. In antenna design, on the other hand, the attention is usually concentrated on the far-field patterns, knowing that antennas are traditionally located beyond each other's reactive zones. Moreover, as shown by the simulated experiment in Section 4, the superluminal effect is predominantly a low-frequency (alternatively, near-field) phenomenon, while antenna engineering is traditionally more concerned with resonant structures.

Notwithstanding, in recent times we have witnessed a narrowing of the gap between the "circuit-domain" and the "antenna-domain", with electrically small antennas and probes being increasingly integrated in various short-range wireless applications of science, medicine and consumer electronics. Additionally, there is a tendency of using wider and wider signal bandwidths in such applications. In view of the illustrations of this article, however, the success of such applications may be facing serious problems. Thus, we feel that it is only a matter of time when the peculiar features and implications of the electromagnetic near-field must be properly taken into account.

REFERENCES

1. Hertz, H. R., *Untersuchungen ueber die Ausbreitung der elektrischen Kraft,* Johann Ambrosius Barth, Leipzig, 1892. Fulltext available on-line at http://www.ubka.uni-karlsruhe.de/ (English transl. in *Electric Waves*, MacMillan, London, 1893).
2. Gough, W., "Wave propagation faster than light," *Eur. J. Phys.*, Vol. 23, 17–19, 2002.
3. Griffiths, H., "Magnetic induction tomography," *Meas. Sci. Technol.,* Vol. 12, 1126–1131, 2001.
4. Balanis, C., *Antenna Theory, Analysis and Design*, Wiley, New York, 1997.
5. Li, H. F., Z. N. Chen, and L.-W. Li, "Investigation of time-domain

characteristics of thin-wire antennas," *Microw. Opt. Techn. Lett.*, Vol. 43, No. 3, 253–258, 2004.

6. Martin, R. G., A. Rubio Bretones, and S. G. Garcia, "Some thoughts about transient radiation by straight thin wires," *IEEE Ant. Prop. Mag.*, Vol. 41, No. 3, 24–33, 1999.
7. Smith, G. S., "On the interpretation for radiation from simple current distributions," *IEEE Ant. Prop. Mag.*, Vol. 40, No. 3, 9–14, 1998.
8. Born, M. and E. Wolf, *Principles of Optics*, 6th ed., Cambr. Univ. Press, Cambridge, 1993.
9. Brillouin, L., *Wave Propagation and Group Velocity*, Ch. 1, Academic Press, New York, 1960.
10. Wang, L. J., A. Kutzmich, and A. Dogariu, "Gain assisted superluminal light propagation," *Nature*, Vol. 406, 277–279, 2000.
11. Haché, A., Dépasser la vitesse de la lumière, *La Recherche*, N° 360, 52–55, Jan. 2003.
12. Haché, A. and L. Poirier, "Anomalous dispersion and superluminal group velocity in a coaxial photonic crystal: theory and experiment," *Phys. Rev. E*, Vol. 65, 036608, 2002.
13. Jones, D. S., *The Theory of Electromagnetism*, Pergamon Press, London, 1964.