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ASPLUND SPACES AND DECOMPOSABLE NONSEPARABLE BANACH SPACES

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ABSTRACT. We show that an Asplund space of density character $\aleph_1$ is weakly compactly generated if and only if it has a projectional resolution of identity for each equivalent norm. We show that every nonseparable Asplund space has a nonseparable subspace which has an equivalent strictly convex norm. We give an example of a non-Asplund space such that every bounded weakly closed subset is an intersection of finite union of balls. We show the existence of an Eberlein compact K such that $(\mathcal{C}(K), ||.||_\infty)$ has no λ-norming Markushevich basis if $\lambda < 2$.

0. Introduction. In this note we investigate some properties of the nonseparable Banach spaces which admit a "decomposition" into separable subspaces. We show, for instance, that there exists a weakly compactly generated (wcg) Banach space X with no λ-norming Markushevich basis for $\lambda < 2$, and in fact that there exists an Eberlein compact K such that $(\mathcal{C}(K), ||.||_\infty)$ has this property. This improves some results from [**18**]. We also answer a question from [**8**].

Let us recall some notation. Let X be a Banach space of density character $\operatorname{dens}(X) = \mu$. A "decomposition" of X is a well-ordered collection $\{P_\alpha; \omega_0 \le \alpha \le \mu\}$ of projections such that $P_\alpha P_\beta = P_\beta P_\alpha = P_\alpha$ if $\alpha \le \beta$, $P_\mu = \operatorname{Id}_X$, $P_\beta(x) \in \overline{\{P_{\alpha+1}(x); \alpha < \beta\}}$ for all $x \in X$ and β, and $\operatorname{dens}(P_\alpha(X)) \le |\alpha|$ for all α. The decomposition $\{P_\alpha; \omega_0 \le \alpha \le \mu\}$ is called a *projectional resolution of identity* (PRI) if $||P_\alpha|| \le 1$ for all α. It is called a *separable decomposition* if $(P_{\alpha+1} - P_\alpha)(X)$ is separable for all $\alpha < \mu$.

Jayne-Rogers selectors were shown to exist in [**13**] (see [**2**, Chapter I.4]). They are multivalued maps from Asplund spaces X to the set $(X^*)^{\mathbf{N}}$ of countable subsets of X^*. We denote them by Δ. A subset $Y \subset X^*$ is called (λ)-norming if there exists $\lambda < \infty$ such that

$$||x|| \le \lambda \sup\{|f(x)|; f \in Y, ||f|| \le 1\}$$

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for all $x \in X$. The infimum of such λ's, denoted by $r(Y)^{-1}$, is the inverse of the Dixmier characteristic $r(Y)$ of Y.

A Markushevich basis (in short M-basis) of X is a subset $\{(x_\alpha, f_\alpha); \alpha \in A\}$ of $X \times X^*$ such that

$$\overline{\text{span}}^{||\cdot||}\{x_\alpha; \alpha \in A\} = X$$

and

$$\bigcap_{\alpha \in A} \ker(f_\alpha) = \{0\}.$$

An M-basis $\{(x_\alpha, f_\alpha); \alpha \in A\}$ is said to be λ-norming if

$$r(\text{span}\{f_\alpha; \alpha \in A\}) \geq \lambda^{-1}.$$

We refer, e.g., to [**6, 9, 18, 19, 20**] for recent results on M-basis.

A compact set K is called a Corson compact if there exists a set I that is homeomorphic to a subset of

$$\sum(I) = \{x : I \to \mathbf{R}; |\{i; x(i) \neq 0\} \leq \aleph_0\}$$

where $\sum(I)$ is equipped with the pointwise topology.

1. Asplund spaces and application of Jayne-Rogers selectors. Our first statement should be compared to the main result of [**3**]. Its proof actually uses techniques from [**3**].

Proposition 1.1. *Let X be an Asplund space with* $\text{dens}(X) = \aleph_1$. *Then X is weakly compactly generated if and only if* $(X, ||.||)$ *has a* PRI *for each equivalent norm* $||.||$.

Proof. Any wcg space has a PRI [**1**], hence the "only if" part is clear. Let us show the converse.

Let $|.|$ be a given norm on X. By assumption, $(X, |.|)$ has a PRI $\{P_\alpha, 0 \leq \alpha \leq \omega_1\}$. Clearly, $(P_{\alpha+1} - P_\alpha)(X)$ is separable for all $\alpha < \omega_1$, hence we can apply [**5**, Proposition 1.2] which states in particular the existence of an equivalent norm $||.||$ on X such that

$$B_{X^*} = B_{X^*}(||.||) = \overline{\text{conv}}^{||\cdot||}(\mathcal{E}) \tag{1}$$

where $\mathcal{E}$ denotes the set of all points in B_{X^*} which are w^*-strongly exposed in B_{X^*}. Clearly, we have for all $f \in X^*$

$$f = w^* - \lim_{\alpha \to \omega_1} P^*_\alpha(f).$$

It follows that for all $f \in \mathcal{E}$, we have

$$\lim_{\alpha \to \omega_1} ||f - P^*_\alpha(f)|| = 0,$$

and thus there exists $\alpha < \omega_1$ such that $f = P^*_\alpha(f)$. Now (1) implies that

$$X^* = \bigcup_{\alpha < \omega_1} P^*_\alpha(X^*). \tag{2}$$

We may now conclude the proof as in [**3**]. Let $\Delta = X \to (X^*)^{\mathbf{N}}$ be a Jayne-Rogers selector. We let $\alpha_0 = \beta_0 = 0$. The set $\Delta(P_{\alpha_0}(X))$ is norm-separable, hence by (2) there is $\alpha_0 < \alpha_1 < \omega_1$ such that

$$\Delta(P_{\alpha_0}(X)) \subset P^*_{\alpha_1}(X^*).$$

We find similarly $\alpha_1 < \alpha_2 < \omega_1$ such that

$$\Delta(P_{\alpha_1}(X)) \subset P^*_{\alpha_2}(X^*).$$

Continuing in this way, we obtain an increasing sequence (α_n) of countable ordinals. Letting $\beta_1 = \sup\{\alpha_n\}$, we have (see [**2**, Lemma VI.3.1])

$$\Delta(P_{\beta_1}(X)) \subset P^*_{\beta_1}(X^*),$$

and in fact ([**4**]; see [**2**, Lemma VI.3.2])

$$\overline{\text{span}}^{||\cdot||}(\Delta(P_{\beta_1}(X)) = P^*_{\beta_1}(X^*). \tag{3}$$

We now construct by induction an increasing sequence of ordinals $\{\beta_n; n \geq 1\}$ such that (3) is satisfied with β_n. If we now let $\gamma = \sup\{\beta_n\}$, we have (again by [**2**, Lemma VI.3.1 and 3.2])

$$P^*_\gamma(X^*) = \overline{\text{span}}^{||\cdot||}\left(\bigcup\{\Delta(P_{\beta_n}(X)); n \geq 1\}\right)$$

and therefore

$$P_\gamma^*(X^*) = \overline{\bigcup P_{\beta_n}^*(X^*)}^{||.||}. \tag{4}$$

We may now let $\gamma = \beta_{\omega_0}$ and proceed by transfinite induction to construct a "shrinking" PRI on X, that is, a PRI $\{P_\beta; \beta \leq \omega_1\}$ such that (4) is satisfied for any sequence $\{\beta_n\}$ increasing to γ. It is easy to conclude that X is wcg, as in [**2**, Corollary VI.4.4]. □

Note that the assumption "X Asplund" is necessary in Proposition 1.1 since, for instance, there exist wcd spaces of density $\aleph_1$ which are not wcg [**15**] and wcd spaces have PRI's in each norm (see [**2**, Theorem VI.2.5]).

Since R. Haydon's fundamental work [**10, 11**], it is known that there is an Asplund space X with $\operatorname{dens} X = \aleph_1$, on which no "good" renorming can be completed. Our next statement will imply that this cannot take place hereditarily.

Proposition 1.2. *Let X be an Asplund space. The following are equivalent.*

1) *There exists a countable subset D of X^* which separates X.*
2) *Every weakly compact subset W of X is weakly metrizable.*

Proof. 1) ⇒ 2). By compactness, the topology $\sigma(X, D)$ of pointwise convergence on D agrees on W with the weak topology, and $\sigma(X, D)$ is metrizable since D is countable.

2) ⇒ 1). We assume now that no countable subset of X^* separates X.

We will construct by transfinite induction a weakly compact non-metrizable subset W of X. We proceed as follows: pick any $x_1 \in X$ with $||x_1|| = 1$. If the x_α's are constructed for all $\alpha < \beta < \omega_1$, set

$$D_\beta = \overline{\operatorname{span}}^{||.||}\{\Delta(\overline{\operatorname{span}}^{||.||}\{x_\alpha; \alpha < \beta\})\}$$

where Δ denotes as before a Jayne-Rogers selector. Clearly D_β is norm-separable, hence according to our assumption we may pick $x_\beta \in X$ with $||x_\beta|| = 1$ and $f(x_\beta) = 0$ for all $f \in D_\beta$.

We claim that the set $W = \{x_\alpha; \alpha < \omega_1\} \cup \{0\}$ is weakly compact in X. To prove this, it clearly suffices to show that if $\{\alpha_n; n \geq 1\}$ is a strictly increasing sequence of countable ordinals then $\{x_{\alpha_n}\}$ weakly converges to 0. Let $\beta = \sup(\alpha_n)$. We let for all γ, $X_\gamma = \overline{\mathrm{span}}^{||\cdot||}\{x_\alpha; \alpha < \gamma\}$. By ([**4**]; see [**2**, Lemma VI.3.2]), we have

$$X_\beta^* = i_\beta^*(\overline{\mathrm{span}}^{||\cdot||}\{\Delta(X_\beta)\})$$

where i_β^* denotes the canonical quotient map from X^* onto X_β^*. For any $f \in X_\beta^*$ and any $\varepsilon > 0$ there exists $\alpha_1' < \alpha_2' < \cdots < \alpha_k' < \beta$ and $r_1, \ldots, r_k \in \mathbf{R}$ such that

$$\left\| f - i_\beta^* \left(\sum_{i=1}^{k} r_i y_i \right) \right\| < \varepsilon$$

with $y_i \in \Delta(X_{\alpha_i'})$. If $\alpha_n > \alpha_k'$ we now have by construction of the x_α's that $|f(x_{\alpha_n})| < \varepsilon$, and this shows our claim.

The set (W, w) is homeomorphic to the one-point compactification of a discrete set of cardinality $\aleph_1$. Hence it is not metrizable, and this concludes the proof. $\square$

Note that one cannot dispense with the assumption "X Asplund" in Proposition 1.2, since for any set $\Gamma, l_1(\Gamma)$ has the Schur property and therefore any weakly compact subset of $l_1(\Gamma)$ is norm-compact and thus is metrizable.

Corollary 1.3. *Let X be a nonseparable Asplund space. Then X contains a closed nonseparable subspace Y, which has an equivalent strictly convex norm.*

Proof. If there exists $D = \{f_n; n \geq 1\}$ which separates X, we let

$$|||x|||^2 = ||x||^2 + \sum_{n=1}^{\infty} 2^{-n} ||f_n||^{-2} (f_n(x))^2$$

and $|||.|||$ is an equivalent strictly convex norm on X. If such a set D fails to exist, X contains by Proposition I.2 a weakly compact norm metrizable subset W, and thus X contains a wcg nonseparable subspace Y. Since any wcg space has a strictly convex norm (and even an lur norm; see [**2**, Theorem VII.2.1]) the conclusion follows. □

It is not clear to me whether the assumption "X Asplund" is necessary in Corollary 1.3. Note that Corollary 1.3 implies that an Asplund space with no strictly convex norm (such as Haydon's example in [**10**]) contains a nonseparable WCG space (which in particular admits a Fréchet-differentiable norm).

We conclude this section with an observation which answers [**8**, Question E,2]. We refer to [**8**] for basic results about the ball topology.

Proposition 1.4. *There is an equivalent norm on the non-Asplund space $X = l_1(\mathbf{N}) \oplus l_2(c)$ such that the ball topology coincides on bounded subsets of X with the weak topology; that is, such that any weakly closed bounded set is an intersection of finite unions of balls. In particular, X^* contains no proper norming subspace.*

Proof. The last statement is in fact a special case of [**5**, Corollary 2.8]. Indeed, since $\operatorname{dens}(X^*) = \operatorname{dens}(X) = c$, we may apply [**5**] as in the proof of Proposition 1.1 to obtain an equivalent norm $||.||$ on X such that

$$B_{X^*}(||.||) = \overline{\operatorname{conv}}^{||.||}(\mathcal{E}) \tag{1}$$

and clearly (1) implies that X^* contains no proper norming subspace. Now observe that all $f \in \mathcal{E}$ are points of w^*-to-norm continuity of the identity map on B_{X^*}. Then [**7**, Theorem 2.6] and (1) show that every $g \in X^*$ is ball-continuous on the ball of X, and the conclusion follows. □

It is still unknown whether a space such that every closed convex bounded set is an intersection of balls is an Asplund space. Proposition 1.4 and some results from [**21**] support the conjecture that the above problem has a negative answer.

Remark 1.5. If X satisfies the conclusion of Proposition 1.4, then there is no $z \in X^{**}\backslash\{0\}$ such that $||z - x|| = ||z + x||$ for all $x \in X$.

Indeed, the space $\ker(z)$ would then be a norming subspace of X^*. Proposition 1.4 is therefore related to an example produced in [**15**]; see also [**12**, p. 489].

2. Norming Markushevich bases in WCG spaces.

The following statement is the main result of this note.

Theorem 2.1. *There exists a Banach space X which is a direct sum $X = S \oplus R$, with S separable and R reflexive, and which has no λ-norming Markushevich basis for $\lambda < 2$.*

Proof. 1) We let $Z = l_1(\mathbf{N}) \oplus l_2(c)$. The space Z has a separable decomposition and clearly we have $\text{dens}\,(Z) = \text{dens}\,(Z^*) = c$. It follows now from [**5**, Corollary 2.8] and the computations made in [**7**, Proof of Theorem 9] that for all $n \geq 1$, there exists an equivalent norm $||.||_n$ on Z such that

$$r(Y) \leq \frac{1}{2} + \frac{1}{n}$$

for all closed proper subspaces Y of $(Z^*, ||.||_n^*)$. It follows that if $\{(z_\gamma, z_\gamma^*); \gamma \in \Gamma\}$ is a λ-norming M-basis of $(Z, ||.||_n)$ with $\lambda < (1/2 + 1/n)^{-1}$, then $\overline{\text{span}}^{||.||}(z_\gamma^*) = Z^*$. We may and do assume that $||z_\gamma^*|| \leq 1$ for all γ. The operator $T : Z \to l_\infty(\Gamma)$ defined by $T(z) = (z_\gamma^*(z))$ takes its values into $c_0(\Gamma)$, and $T^*(l_1(\Gamma))$ contains $\text{span}\,\{z_\gamma^*; \gamma \in \Gamma\}$ and is therefore norm-dense. But then T^{**} is one-to-one and thus by [**2**, Corollary VI.5.4] Z is Asplund. But since Z contains $l_1(\mathbf{N})$, this is a contradiction. Hence $(Z, ||.||_n)$ has no λ-norming M-basis for $\lambda < (1/2 + 1/n)^{-1}$.

We now set

$$X = \left(\sum \oplus (Z, ||.||_n) \right)_2.$$

The following lemma is a straightforward consequence of [**20**, Proposition 4.6], and [**19**, Proposition 2.6] is a stronger statement. Yet we outline the proof for completeness.

Lemma 2.2. *Let V be a Banach space and Z be a subspace of V such that (B_{Z^*}, w^*) is a Corson compact. If V has a λ-norming M-basis,* *then so does Z.*

Proof. Let $\{(v_i, f_i), i \in I\}$ be a λ-norming M-basis of $(V, ||.||)$. If we let

$$|||v||| = \sup\{|f(v)|; f \in \text{span}\,\{f_i\}, ||f|| \leq 1\}$$

then $|||.|||$ is an equivalent norm on V such that for all $v \in V$,

$$\lambda^{-1}||v|| \leq |||v||| \leq ||v||. \tag{5}$$

We equip V, and its subspace Z as well, with $|||.|||$. We let $g_i = f_{iIZ}$ and

$$Y = \overline{\text{span}}^{||.||}(\{g_i; i \in I\}).$$

The space Y is a one-norming subspace of $(Z^*, ||.||^*)$ and since $\{(v_i, f_i)\}$ is an M-basis, we have for all $z \in Z$,

$$|\{i \in I; g_i(z) \neq 0\}| \leq \aleph_0. \tag{6}$$

Since (B_{Z^*,w^*}) is a Corson compact, Z has an M-basis [**16**, Proposition 4.1] $\{(z_\gamma, h_\gamma); \gamma \in \Gamma\}$. Any Corson compact is angelic, and it follows easily from the Banach-Dieudonné theorem that, for all $h \in Z^*$,

$$|\{\gamma \in \Gamma; h(z_\gamma) \neq 0\}| \leq \aleph_0. \tag{7}$$

This applies in particular to $h = g_i$ for any $i \in I$. Now (6) and (7), together with the fact that span $\{g_i; i \in I\}$ is $(|||.|||) - 1$-norming, imply that $(Z, |||.|||)$ has a one-norming M-basis. This latter fact is shown in [**20**, Theorem 2.3] by the techniques used in [**2**, Chapter VI] and in particular in the proof of [**2**, Lemma VI.7.5].

Hence $(Z, |||.|||)$ has a one-norming M-basis $\{(u_\alpha, t_\alpha); \alpha \in A\}$. It follows that $(Z, ||.||)$ has a λ-norming M-basis. Indeed, for all $x \in X$ with $||x|| = 1$ and all $\varepsilon > 0$, we have by (5) $|||x||| \geq \lambda^{-1}$ and thus there exists $f \in \text{span}\,\{t_\alpha; \alpha \in A\}$ with $|||f|||^* \leq 1$ and $f(x) > \lambda^{-1} - \varepsilon$. Since $||f||^* \leq |||f|||^*$, the conclusion follows. □

Since X is wcg, B_{Z^*} is Eberlein and thus Corson compact for all $Z \subset X$. Hence, by Lemma 2.2, X contains no λ-norming M-basis if $\lambda < 2$. We now recall

Fact 2.3. *Let $X = S \oplus R$, with S separable and R reflexive. Then X is isometric to a direct* sum $X = S_0 \oplus R$, *with S_0 separable, R_0 reflexive and S_0 is one-complemented in X.*

Proof. The space X is clearly wcg, and thus (see [**2**, Lemma VI.2.4]) S is contained into a one-complemented separable subspace S_0 of X. Let $\pi = X \to S_0$ be a norm-one projection. We have

$$R_0 = \ker \pi \simeq X/S_0,$$

but there is a canonical quotient map from $X/S \simeq R$ onto X/S_0, and thus R_0 is reflexive. □

By Fact 2.3, we have for all n,

$$(Z, ||.||_n) = S_n \oplus R_n$$

with S_n separable and $(||.||_n) - 1$-complemented and R_n reflexive. It follows that

$$X \simeq \left(\sum \oplus S_n\right)_2 \oplus \left(\sum \oplus R_n\right)_2$$

is of the prescribed form. This concludes the proof of Theorem 2.1. □

It is still unknown whether there exists a wcg space with no norming M-basis. However, the present approach will not suffice for answering this question. Indeed, we have

Proposition 2.4. *Let X be a Banach space which is a direct sum $X = S \oplus R$, where S is separable and R is reflexive. Then X has a four-norming Markushevich basis.*

Proof. By Fact 2.3 we may assume without loss of generality that S is one-complemented in X, with a projection π of kernel R.

Since S is separable, it has a one-norming Markushevich basis $\{(x_n, f_n); n \geq 1\}$ (see [**14**, p. 44]). The space R is reflexive and thus it has a Markushevich basis $\{(y_\alpha, g_\alpha); \alpha \in A\}$ which is of course one-norming since $\overline{\text{span}}^{||.||}(g_\alpha) = R^*$, by reflexivity. We set

$$\mathcal{B} = \{\{x_n; n \geq 1\} \cup \{y_\alpha; \alpha \in A\}\}$$

and

$$\mathcal{B}^* = \{\{\pi^*(f_n); n \geq 1\} \cup \{(I - \pi^*)(g_\alpha); \alpha \in A\}\}.$$

We claim that $(\mathcal{B}, \mathcal{B}^*)$ is a four-norming M-basis. Indeed, pick $x = x_1 + x_2$ with $x_1 \in S$, $x_2 \in R$. We have

$$\sup\{||x_0||, ||x_2||\} \geq \frac{||x||}{2}.$$

Let us assume that $||x_1|| \geq ||x||/2$. Pick any $\varepsilon > 0$. By the above, there exists $f \in \text{span}\,(f_n)$ such that $||f|| \leq 1$ and $\langle f, x_1 \rangle > ||x||/2 - \varepsilon$.

Since $x_1 = \pi(x)$, we have $\pi^*(f)(x) = f(x_1) > ||x||/2 - \varepsilon$ and $||\pi^* f|| \leq 1$.

Thus, in this case, x is $(1/2)$-normed by $\text{span}\,(\mathcal{B}^*)$. If now $||x_2|| \geq ||x||/2$, we can proceed along the same lines and find $g \in \text{span}\,(g_\alpha)$ with $||g|| \leq 1$ and $\langle g, x_2 \rangle > ||x||/2 - \varepsilon$.

If we set $h = (I - \pi^*)(g)$, we have $\langle h, x \rangle > ||x||/2 - \varepsilon$ and $h \in \text{span}\,(\mathcal{B}^*)$, but this time we only have $||h|| \leq 2$ since $||I - \pi|| \leq 2$. Therefore, in that case, x is $(1/4)$-normed by $\text{span}\,(\mathcal{B}^*)$. This concludes the proof. □

I don't know how to fill the gap between $\lambda = 2$ and $\lambda = 4$ (see Theorem 2.1 and Proposition 2.4). We conclude with

Corollary 2.5. *There exists an Eberlein compact K such that $(\mathcal{C}(K), ||.||_\infty)$ has no λ-norming Markushevich basis if $\lambda < 2$.*

Proof. Let $(X, ||.||)$ be the space provided by Theorem 2.1. Since X is wcg, $K = (B_{X^*}, w^*)$ is Eberlein compact.

Since X is isometric to a subspace of $(\mathcal{C}(K), ||.||_\infty)$, it follows from Lemma 2.2 that $(C(K), ||.||_\infty)$ has no λ-norming M-basis if $\lambda < 2$. □

Note that it follows from Lemma 2.2 that $(\mathcal{C}(K), ||.||_\infty)$ is not isometrically contained into a space which has λ-norming M-basis with $\lambda < 2$.

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