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ABSTRACT

Recent surveys on the labor-supply responses of men document a divergence in the estimates of substitution and income effects obtained using various estimation approaches. Generally, studies accounting for nonlinear tax schedules in a static setting via a piecewise-linear approach produce estimates that typically imply higher substitution and lower income responses than are suggested by empirical work applying other approaches. This paper demonstrates that maximum likelihood estimation of a consumer-choice problem with nonlinear budget sets implicitly relies on the satisfaction of inequality constraints that translate into behaviorally meaningful restrictions. These constraints arise not as a consequence of economic theory, but instead as a requirement to create a properly defined statistical model. In the analysis of piecewise-linear budget sets, the implicit constraints required by maximum likelihood in estimation amount to imposition of Slutsky conditions at all wage-income combinations associated with kink points. In the analysis of differentiable budget

sets, the tacit constraints invoked by maximum likelihood also involve inequality restrictions on Slutsky terms. The empirical work presented in this study supports the contention that these implicit constraints play a major role in explaining the discrepancies in estimates found in the literature on men's labor supply.

I. Introduction

Measuring the work disincentive effects of taxation has been a major research activity in the empirical literatures of both labor economics and public finance over the past two decades. During this period, there has been a steady expansion in the econometric sophistication applied to measure these effects. Whereas the early literature relied on simple estimation methods (e.g., least-squares and instrumental-variable procedures) to infer the influence of taxation on hours of work, recent studies estimate these effects using maximum likelihood techniques that are designed to account for piecewise-linear budget constraints. The latter techniques produce estimates of key behavioral parameters associated with prime-age males that diverge from the central tendency of estimates obtained from the simpler empirical methodologies. To date, only speculation has been offered to explain the source of this divergence in estimates. This paper goes a long way towards filling this gap.

As documented in the surveys of Pencavel (1986) and Hausman (1985), empirical studies of men's labor supply based on econometric approaches incorporating piecewise-linear constraints produce results that typically imply higher substitution and lower income effects than are suggested by empirical work based on other approaches; whereas piecewise-linear analyses almost always produce estimates indicating positive substitution and negative income responses, other estimation approaches often suggest negative substitution effects and zero or positive income responses. The piecewise-linear results translate into larger estimates of both labor-supply responses and deadweight losses associated with the progressivity of taxation. Such evidence has been cited by many in the recent policy and academic debate over tax reform as support for lower marginal tax rates. Further, the piecewise-linear analyses imply larger estimates of compensated substitution responses that have the sign predicted by economic models of consumer choice, which is in stark contrast to much of the other empirical work on labor supply. This finding of greater consistency with economic theory has been interpreted in the literature as evi-

dence confirming the merits of accounting for taxes using the piecewise-linear approach.

Contrary to this interpretation, this paper shows that the divergence in the estimates obtained from the alternative empirical methods follows directly from features of the econometric models that implicitly restrict parameters to obey certain inequalities. The simple estimation approaches impose no restrictions, but maximum likelihood techniques incorporating piecewise-linear budget constraints require local satisfaction of the Slutsky condition over a wide range of wage-income combinations. Requiring the Slutsky condition to hold at various points in estimation does not come about due to the introduction of restrictions based on economic theory, such as those associated with the assumption of quasiconcavity; instead, this requirement arises purely from properties needed to obtain a properly defined statistical model. It is no surprise that compensated effects estimated by piecewise-linear techniques are typically nonnegative in analyses of labor supply since this nonnegativity constraint is essentially imposed by the procedure. The imposition of this constraint also explains why these techniques compute higher substitution and lower income effects than are obtained using the simpler estimation approaches; the constraint is met by these adjustments in the effects. The degree of the progressivity of the tax schedule dictates the range over which the Slutsky condition must hold in the application of maximum likelihood with piecewise-linear budget constraints. Increasing the number of tax brackets considered in an analysis broadens the range.

This paper goes on to explore the implications of approximating tax schedules (and budget constraints) by smooth differentiable functions in the application of maximum likelihood methods. Such a procedure also presumes that parameters satisfy particular inequality restrictions, but these restrictions are weaker than those implied by the Slutsky condition. As in the analysis of piecewise-linear constraints, the degree of progressivity or nonlinearity in tax schedules affects the nature of the inequality constraints imposed by maximum likelihood methods.

To determine the influence of the parametric restrictions invoked by maximum likelihood procedures on cross-sectional estimates of substitution and income effects, this study explores the consequences of applying alternative procedures in an empirical analysis of men's labor supply. The data set used in this analysis consists of the 1975 cross-section of prime-age married males drawn from the Michigan Panel Study of Income Dynamics (PSID). The empirical work compares estimates obtained using approaches incorporating both piecewise-linear and differential tax schedules. The results from this work highlight the importance of the parametric restrictions implicit in maximum likelihood analyses of taxes and labor supply. These findings offer a powerful explanation for a major

source of the discrepancy found in the literature on the relevant ranges of substitution and income responses associated with men's hours-of-work behavior.

The outline of this paper is as follows. To provide a characterization of the tax schedules recognized in empirical analyses of men's labor supply, Section II describes the basic features of income taxes faced by individuals in the U.S. in 1975. In addition to presenting information on such features as the numbers and the positions of tax brackets relevant for a representative sample of workers, this discussion also examines the accuracy of several procedures for approximating tax schedules that simplify the empirical analysis. Section III describes the application of maximum likelihood methods incorporating piecewise-linear budget constraints, and it identifies the parametric restrictions implicitly imposed by these methods. Section IV presents a parallel discussion of maximum likelihood with differentiable budget constraints. Finally, Section V reports results from an extensive empirical analysis designed to assess the role of the inequality restrictions imposed by maximum likelihood methods on the estimation of substitution and income parameters in the analysis of men's labor supply.

II. The Structure of Taxes

This section outlines the basic features of income taxes in the U.S. as they relate to labor supply. The discussion begins by characterizing the way tax schedules distort the opportunity sets faced by individuals. Rather than offering a purely institutional description with hypothetical examples, the following analysis summarizes the consequences of taxes on a random sample of prime-age males in 1975, which serves as the data set used in our empirical work. Examining the effect of taxes in this context enables one to convey a comprehensive picture of the complexities introduced beyond the simple case of a linear budget constraint, while also providing an opportunity to determine how well various approximation procedures capture the essential features of tax schedules. We consider two such procedures: the convexification of budget constraints and the creation of a differentiable budget constraint.

A. *Features of the Tax Function*

Perhaps the easiest way to convey the complexities introduced by the U.S. tax system is to describe the budget constraint faced by a worker. The overall tax schedule in 1975, the year of concern in this analysis, was composed of four component parts:

(2.1) $T(Y, E) = FEDTX(FX) + STATX(SX)$

$$+ EIC(Y, E) + SSTAX(E)$$

where

$$
\begin{aligned}
T(Y, E) &= \text{overall tax schedule;} \\
Y &= \text{unearned income;} \\
E &= \text{earned income;} \\
FEDTX(FX) &= \text{federal income tax schedule;} \\
FX &= \text{federal taxable income (i.e., } Y + E - \text{ federal deductions);} \\
STATX(SX) &= \text{state income tax schedule;} \\
SX &= \text{state taxable income (i.e., } Y + E - \text{ state deductions);} \\
EIC(Y, E) &= \text{earned income credit schedule; and} \\
SSTAX(E) &= \text{social security tax schedule.}
\end{aligned}
$$

To understand the complexities introduced by the various components of the tax schedule, initially consider only the schedules associated with the federal and state income taxes. In 1975, the federal schedule was progressive with thirteen brackets for couples filing jointly while the state schedules were either proportional or progressive with anywhere from 2 to 24 brackets. Figure 1 shows a hypothetical budget constraint for an individual faced with federal income taxes alone, state income taxes alone, or both. In this diagram, h denotes hours of work, and C measures total after-tax income or the consumption of market goods. The budget constraint is composed of several segments corresponding to the different marginal tax rates that an individual faces. In particular, he faces a tax rate of t_A between $\bar{h}_0$ hours and $\bar{h}_2$ hours (segment 1 of his constraint) and tax rates of t_B and t_C respectively in the intervals $(\bar{h}_2, \bar{h}_4)$ and $(\bar{h}_4, \bar{h}_6)$ (segments 3 and 5 in the figure). Thus, with the variable W denoting the individual's gross wage rate, the net wages associated with each segment are: $w_1 = (1 - t_A)W$ for segment 1; $w_3 = (1 - t_B)W$ for segment 3; and $w_5 = (1 - t_C)W$ for segment 5. Also, each segment has associated with it a virtual income (i.e., income associated with a linear extrapolation of the budget constraint) calculated as: $y_1 = Y - T(Y, 0)$; $y_3 = y_1 + (w_1 - w_3)\bar{h}_2$; and $y_5 = y_3 + (w_3 - w_5)\bar{h}_4$. Changes in tax brackets create the kink points which are designated 0, 2, 4, and 6.

Figure 2 shows a budget constraint affected only by the Earned Income Credit (EIC) schedule,[1] and Figure 3 shows a budget constraint that re-

1. The EIC in 1975 was a negative income tax scheme which could induce, in the simplest case, two kinks in a person's constraint: one where the proportional credit reached its maximum ($\bar{h}_2$ in Figure 2), and one at the breakeven point where the credit was fully taxed away ($\bar{h}_4$ in the figure). The tax rates associated with the first two segments are t_A, which is negative, and t_B, which is positive. Thereafter, the EIC imposed no further tax.
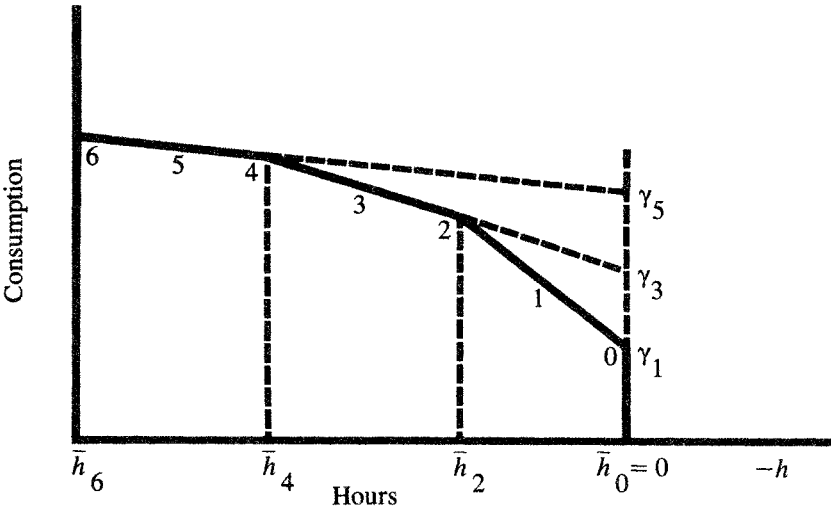
Figure 1
Budget Constraint with Income Taxes
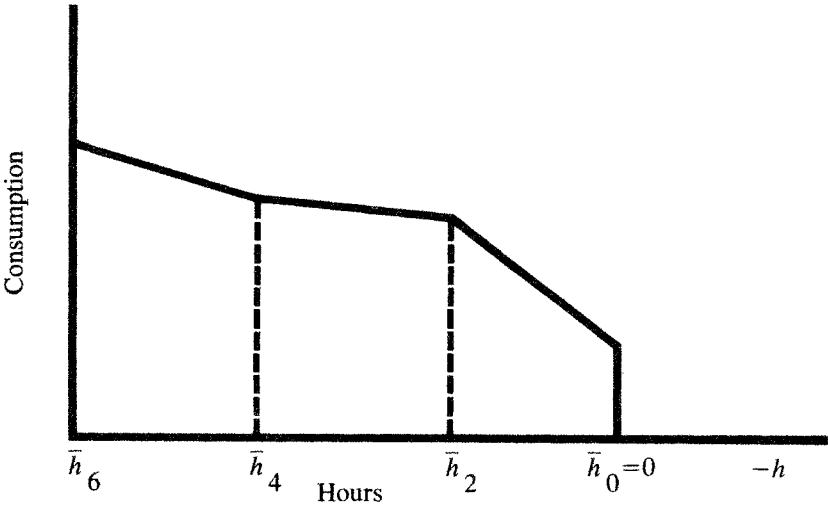
Figure 2
Budget Constraint with EIC
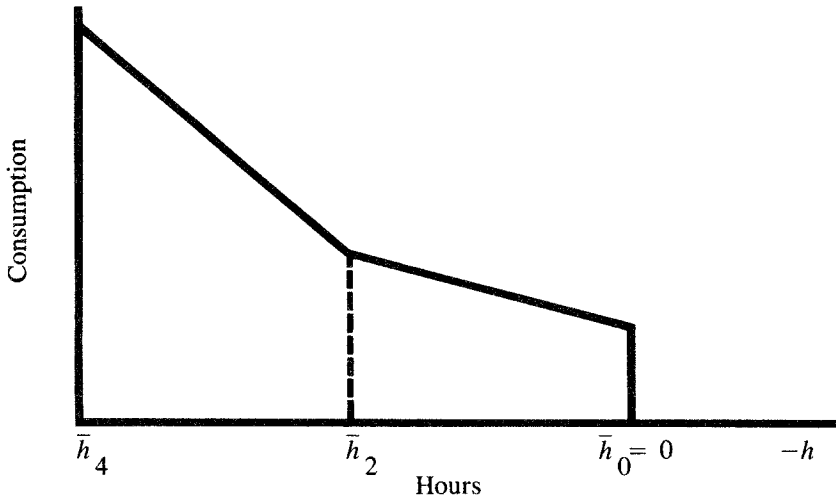
Figure 3
Budget Constraint with Social Security Tax

flects the effects of the social security tax alone.[2] Both of these taxes induce nonconvexities in opportunity sets.

Summing the four components of taxes creates an overall tax schedule with two noteworthy features. First, the tax schedule faced by a typical individual includes a large number of different rates. Translated into the hours-consumption space, this implies a large number of kink points in the budget constraint. Second, for most individuals the tax schedule contains nonconvex portions, which arise from three potential sources. The first source arises from a fall in the EIC tax rate at the breakeven point for the EIC. In Figure 2 that point occurs at $\bar{h}_4$ where the tax rate falls from a positive value to zero. The second source occurs when the social security tax hits its maximum (at $\bar{h}_2$ in Figure 3), at which point the corresponding tax rate goes from positive to zero. Finally, the third source is due to a nonconvexity introduced because of the structure of the standard deduction.[3]

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2. The 1975 social security tax was a proportional tax on earnings up to a specified earnings level, after which the amount of tax paid was the same regardless of earnings. As a result, Figure 3 shows a constraint with a single interior kink (given by $\bar{h}$ in the figure) corresponding to the maximum proportionally taxed earnings level. The tax rate on the segment leading up to that kink is t_A, switching to zero on the second segment.
3. Below $11,875 gross income the standard allowable deduction was constant at $1,900, but between $11,875 and $16,250 the deduction switched to being proportional to income. As a result, the tax rate a person faced dropped at the $11,875 total income point.

B. Features of Constructed Budget Constraints

Consider the actual impact of these various taxes on the budget constraints of a representative sample of prime-age males in 1975.The sample consists of 1,017 working men drawn from the PSID. For each member of the sample we form a gross hourly wage rate, W, by dividing his total labor income in 1975 by his total hours worked in that year, and we calculate a pre-tax level of unearned income, Y, by subtracting his total labor income from the sum of his and his wife's taxable income.[4] With these two values as inputs, we construct a budget constraint for each individual in the sample using the tax schedule given by (2.1) with a few modifications. First, the federal and state tax schedules are combined for each person; the brackets of each state schedule are adjusted to make them the same as those for the federal schedule, and only federal income tax deductions are allowed (i.e., $SX = FX$ in Equation (2.1)). Second, the schedules presented in the figures in Section A are of the most basic form; those faced by most individuals in our sample have complicating features.[5]

As expected, the constructed budget constraints generally contain large numbers of interior kink points. The first part of Table 1 presents a variety of information on the characteristics of these points in the sample: it presents statistics describing the distribution of the number of kinks across individuals in row 1; the distribution of the location of these kinks in terms of hours (i.e., the h's) in the sample in row 2; and the distribution of the number of nonconvex sections occurring in these constraints in row 3.[6] Seventy-five percent of the individuals making up our sample have budget constraints with at least eleven interior kinks; 90 percent of the sample face more than eight kinks. The maximum and the minimum number of hours at which someone has an interior kink point are about five hours and 5,839 hours, respectively, which reveals that there are kinks located in virtually the entire range of allowable hours. Further, the quartiles for the distribution of the h's indicate that kink points cover that

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4. More information on the selection of the sample and the formation of variables is given in Appendix A.

5. An appendix is available upon request from Tom MaCurdy that provides a detailed account of the tax schedules used to construct the budget constraints analyzed in this discussion and in the following empirical work. In the version of this paper distributed in working-paper series, this appendix is designated as "Appendix F," which is not included in the published version of the paper.

6. Whereas the distributions of the number of kinks and the number of nonconvex sections is calculated using a single observation per individual to form the sample, the distribution of kink locations takes each h as a separate observation. The maximum hours allowed on constructed budget constraints is 5,840.

range fairly evenly. According to row 3, three-quarters of the sample faced at least two nonconvex sections of their budget set and most faced three. Despite the large numbers of kinks spread widely across the allowable range, only one person in our sample is observed to work a number of hours that places him at a kink point on his budget constraint.

C. Convexification of the Budget Constraint

The analysis of this paper does not deal directly with the nonconvexities in the tax schedule, choosing instead to use a convexified approximation to each person's schedule. Figure 4 shows the method of approximation applied in this analysis. The solid lines represent the actual budget constraint, while the dotted line shows the approximation over the relevant region. Thus, at a nonconvex part of the tax schedule, the net wages on the corresponding portions of the budget constraint are replaced with a single wage rate, constructed as the slope of the line joining the kink points that frame the relevant section. The result is a convex budget set. Rows 4 and 5 of Table 1 present statistics describing the characteristics of the constructed convexified constraints analogous to the statistics provided in rows 1 and 2 associated with the original budget sets.

To judge the accuracy of this approximation for individuals in our sample, Table 1 also presents information on the greatest vertical difference between the convexified budget set and the original nonconvexified bud-
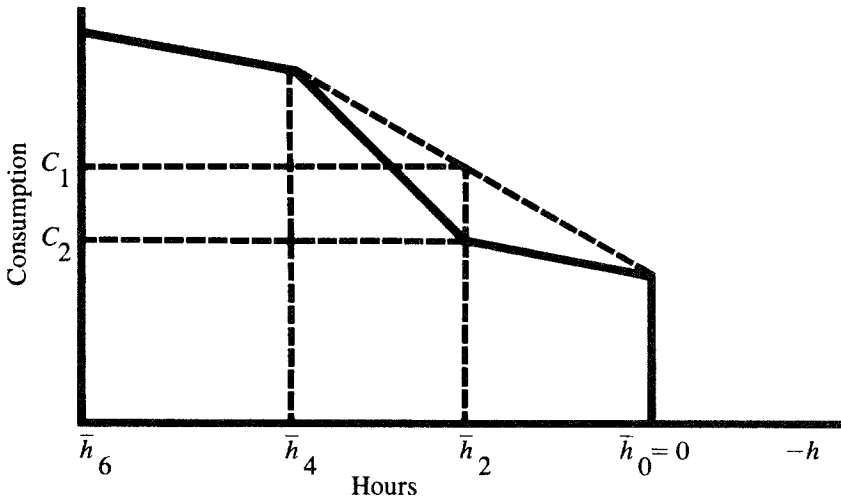
Figure 4
Convexification of a Budget Constraint

Table 1
Characteristics of Budget Constraints

Variable	Min	Quartiles[a]			Max
		25%	50%	75%	
Nonconvexified Constraints					
(1) Number of Interior Kinks per Individual[b]	0	11	14	17	19
(2) Hours Location ($\bar{h}$) of Interior Kinks[c]	4.6	1,007.0	2,027.0	3,480.3	5,839.0
(3) Number of Nonconvex Sections per Individual[d]	0	2	3	3	3
Convexified Constraints					
(4) Number of Kinks per Individual[b]	0	7	9	11	16
(5) Hours Location ($\bar{h}$) of Interior Kinks[c]	4.6	918.1	2,560.5	4,046.3	5,839.0

Differences Between Constraints

(6) Maximum Difference Between Convexified and Nonconvexified Constraints per Individual (Dollars)	0	55.7	75.4	137.7	313.7
(7) Relative Difference Between Convexified and Nonconvexified Constraints per Individual (percentage)[e]	0	0	1	2	4

a. Minima, maxima, and quartiles of the distributions associated with each variable.

b. This variable is formed by counting the number of interior kink points on each person's budget constraint. The variable has the same number of observations as there are people in the sample.

c. This variable contains the sets of all interior kink points for all individuals in the sample.

d. This variable is formed by counting the number of nonconvex sections, or the number of times a segment of the budget constraint is followed directly by a segment associated with a higher net wage rate, incorporated in each individual's budget constraint. The variable has the same number of observations as there are individuals in the sample.

e. Calculated as the absolute difference divided by the height of the nonconvexified constraint $\times$ 100.

get set for each person, as well as the percentage increase over the original constraint that this difference represents at the point where the maximum difference occurs. In Figure 4 the largest difference is given by $(C_1 - C_2)$ dollars, and it represents a $(C_1 - C_2)/C_2) \times 100$ percent increase over the nonconvexified constraint. These measures show how much extra income is attributed to each person using the convexified constraint. Rows 7 and 8 of Table 1 demonstrate that the differences between the constraints are quite small in both absolute and relative terms.[7] The maximum absolute difference is about \$314, and the largest percentage increase is 4 percent; the majority of constraints are shifted by about 1 percent.

D. Constructing a Differentiable Constraint

Besides using convexified constraints, the following analysis also considers the application of a differentiable approximation to tax schedules; a feat carried out by first fitting a function to the marginal tax rate and then integrating. To approximate the marginal tax rate schedule the function must fit a step function closely and still maintain differentiability at the step points. In addition, it must be easily integrable to obtain a simple closed form for the tax function.

To understand the nature of the approximation applied to this analysis, return to Figure 1 and consider the tax schedule determining the budget constraint presented there. One can represent the underlying schedule as follows:

$$(2.2) \quad MTR(X(h)) = t_A \text{ from } X(\bar{h}_0) \text{ to } X(\bar{h}_2)$$
$$= t_B \text{ from } X(\bar{h}_2) \text{ to } X(\bar{h}_4)$$
$$= t_C \text{ above } X(\bar{h}_4),$$

where

$$MTR(X(h)) = \text{marginal tax rate,}$$
$$X(h) = \text{taxable income at } h \text{ hours of work, and}$$
$$t_i = \text{marginal tax rate, } i = A, B, C.$$

For expositional simplicity, suppose that $t_A = 0$.

Consider the following approximation of this schedule which uses three

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7. Of course, a small difference in the convexified and the nonconvexified constraints does not necessarily imply that convexification has unimportant consequences on labor-supply estimates. If hours and consumption are near perfect substitutes, then even a small shift in the budget constraint could induce a large change in hours of work.

flat lines at the heights $t_A (= 0)$, t_B and t_C and normal distribution functions parameterized to switch the three lines on and off at appropriate points:

$$(2.3) \quad M\hat{T}R(X(h)) = t_B(\Phi_1(X(h)) - \Phi_2(X(h))) + t_C\Phi_2(X(h))$$

where

$$\Phi_i(X(h)) = \text{cumulative distribution function with mean } \mu_i$$
$$\text{and variance } \sigma_i^2, i = 1, 2.$$

The middle segment of the tax schedule has height t_B and runs from taxable income $X(\bar{h}_2)$ to $X(\bar{h}_4)$. To capture this feature, parameterize $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$ with means $\mu_1 = X(\bar{h}_2)$ and $\mu_2 = X(\bar{h}_4)$, respectively, with both variances set small. The first distribution function, $\Phi_1(\cdot)$, takes a value close to zero for taxable income levels below $X(\bar{h}_2)$ and then switches quickly to take a value of one for higher values. Similarly, $\Phi_2(\cdot)$ takes a value of zero until near $X(\bar{h}_4)$ and one thereafter. The difference between the two equals zero until $X(\bar{h}_2)$, one from $X(\bar{h}_2)$ to $X(\bar{h}_4)$, and zero thereafter. Thus, the difference takes a value of one just over the range where t_B is relevant. Notice that we can control when that value of one begins and ends by adjusting the values μ_1 and μ_2. Also, we can control how quickly this branch of the estimated schedule turns on and off by adjusting the variances of the cumulative distribution functions, trading off a more gradual, smoother transition against more precision. In general, adjusting the mean and variance parameters allows one to fit each segment of a schedule virtually exactly, switch quickly between segments, and still have differentiability at the switch points.

A generalization of this approximation takes the form

$$(2.4) \quad M\hat{T}R(X(h)) = \sum_{i=1}^{K} [\Phi_i(X(h)) - \Phi_{i+1}(X(h))]b_i(X(h))$$

where the functions $b_i(X(h))$ are polynomials in income. To approximate the federal income taxes with its thirteen brackets using this formulation in a manner analogous to the example considered above, one would set $K = 13$ and $b_i =$ the marginal tax rate t_i associated with the ith bracket. With the Φ_i denoting normal cdf's, Function (2.4) yields closed form solutions when it is either integrated or differentiated.

The subsequent empirical analysis dealing with differentiable constraints presents results considering federal taxes alone, with the tax schedule approximated using (2.4) where $K = 2$ and the function b_1 represents a third-order polynomial in taxable income which increases monotonically over the range \$0 to \$44,000 (the range over which marginal tax rates varied in 1975). We use a single polynomial b_1 to capture changing

marginal tax rates rather than expanding K to allow for more accurate and abrupt shifts between brackets in order to explore the empirical consequences of substantial smoothing of the budget constraint. In our particular formulation, the function $\Phi_1(x)$ is parameterized to switch from zero to one near $0 taxable income; $\Phi_2(x)$ switches from zero to one near the $44,000 mark; and $\Phi_3(x)$ is zero everywhere. At negative values of taxable income the marginal tax rate is zero, and at income levels above $44,000 the right-hand side of (2.4) becomes $b_2 = 0.5$ which represents the highest marginal tax rate applicable in 1975.[8] The discrepancy between the actual tax schedule and this differentiable approximation never exceeds $36.19, and even this occurs when total taxes equal the high value of $2,735. All the available evidence indicates that this methodology yields a very close approximation to the federal tax schedule.

III. Econometric Analysis of Piecewise-Linear Budget Constraints

Beginning with the work of Burtless and Hausman (1978), there has been a steady expansion in the use of statistical models that characterize the distributions of discrete-continuous variables as a framework to infer the influence of taxes on hours of work. Considered at the forefront of research in this area, these models offer a natural mechanism for capturing the bracket features of tax schedules. This section briefly describes the application of such models, and it identifies the restrictions imposed by this type of approach on estimates of labor supply functions. To simplify the exposition, the discussion focuses on the case in which budget constraints are convex. Analysis of this case conveys all of the essential ideas.

A. Describing the Economic Problem

Suppose that Figure 1 depicts the budget constraint faced by an individual. According to the basic economic premise underlying the theory of labor supply, consumers adjust their hours of work to maximize utility subject to the expenditure constraint $C = Y + E - T(Y, E)$, where C denotes total after-tax income or the consumption of market goods, Y

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8. The appendix mentioned in footnote 5, which is available upon request from Tom MaCurdy, describes the details concerning the specification and the integration of the differentiable tax function used in this analysis.

represents gross income excluding own earnings, $E = Wh$ designates gross earnings, and the function T determines tax payments. Summarize the preferences of consumers by the function $h^s(w, y, v)$ which specifies the labor supply of an individual who possesses characteristic v and who faces a standard linear budget constraint defined by the wage rate w and nonlabor income y. Utility maximization with quasi-concave preferences determines the functional form of h^s. One may think of v as a "taste shifter" variable that accounts for individual heterogeneity. The density function $f_v(v)$ describes this heterogeneity across consumers in the population, with the sample space of v defined by the interval $(\underline{v}, \bar{v})$. Assume that h^s is monotonically increasing in v, so that $\partial h^s / \partial v > 0$. Let the function $v^s(h, w, y)$ designate the inverse of $h^s(w, y, v)$ solved in terms of v with $h = h^s$.

Given this characterization of behavior in the presence of convex budget sets, one can readily infer an individual's hours of work from knowledge of h^s, W, Y, and v. The value of h lies on segment $i = 1, 3, 5$ in Figure 1 if a person's characteristics v is an element of the set determined by

(3.1) $h^s(w_i, y_i, v) \in (\bar{h}_{i-1}, \bar{h}_{i+1})$,

where the variables w_i and y_i represent the marginal wage rate and the virtual income applicable on segment i, and the quantities $\bar{h}_i$ denote the hours associated with kink points i. (See Section II.A for further explanation of this notation.) Occurrence of this event implies that hours of work are given by

(3.2) $h = h^s(w_i, y_i, v)$.

Alternatively, the value of h resides at an inferior kink i if v is an element of the set defined by

(3.3) $h^s(w_{i-1}, y_{i-1}, v) \geq \bar{h}_i$ and $h^s(w_{i+1}, y_{i+1}, v) \leq \bar{h}_i$.

Occurrence of this event means that the equation

(3.4) $h = \bar{h}_i$

determines hours of work.

B. Specifying the Likelihood Function of Hours

Relationships (3.1)–(3.4) along with the density function f_v directly induce a distribution on hours of work. The introduction of several definitions simplifies the derivation of this distribution. As a partition of the sample space of h, consider the sets

(3.5) $\Lambda_i = \{h : \bar{h}_{i-1} < h < \bar{h}_{i+1}\}$ $i = 1, 3, 5$

$\Lambda_i = \{h : h = \bar{h}_i\}$ $i = 0, 2, 4, 6.$

Define the quantities v_i^L and v_i^U by the equations

(3.6) $h^s(w_{i-1}, y_{i-1}, v_i^L) = \bar{h}_i$ $i = 2, 4, 6$

$h^s(w_{i+1}, y_{i+1}, v_i^U) = \bar{h}_i$ $i = 0, 2, 4$

with $v_0^L = \underline{v}$ and $v_6^U = \bar{v}$. Utility maximization in the presence of convex constraints ensures that

(3.7) $v_i^U - v_i^L \geq 0,$

which motivates the superscript notation with "U" and "L" signifying the "upper bound" and the "lower bound" associated with a kink. As a partition of the sample space of v, consider the sets

(3.8) $\Theta_i = \{v : v_{i-1}^U < v < v_{i+1}^L\}$ $i = 1, 3, 5$

$\Theta_i = \{v : v_i^L \leq v \leq v_i^U\}$ $i = 0, 2, 4, 6.$

Finally, given the inverse function $v^s(h, w, y)$, and the fact that the Jacobian term $\partial v^s / \partial h$ is positive as a consequence of the assumption $\partial h^s / \partial v > 0$, a standard change of variables from v to h implies the density

(3.9) $f_h = f_h(h | w, y) = \dfrac{\partial v^s}{\partial h} f_v(v^s(h, w, y))$

for hours of work with w and y held fixed.

Relations (3.1)–(3.4) combined with the definitions (3.5)–(3.9) imply that an individual's hours of work fall in "state of the world" $i = 0, \ldots, 6$ if $h \in \Lambda_i$, or equivalently if $v \in \Theta_i$. So, the probability that hours are at kink $i = 0, 2, 4, 6$ is

(3.10) $\Pr(h \in \Lambda_i) = \Pr(v \in \Theta_i) = \displaystyle\int_{v_i^L}^{v_i^U} f_v(v) \, dv.$

The probability that hours fall on segment $i = 1, 3, 5$ is

(3.11) $\Pr(h \in \Lambda_i) = \Pr(v \in \Theta_i) = \displaystyle\int_{v_{i-1}^U}^{v_{i+1}^L} f_v(v) \, dv.$

The distribution of hours conditional on falling into the region associated with segment i is

(3.12) $\dfrac{f_h(h | w_i, y_i)}{\Pr(v \in \Theta_i)}.$

Defining δ_i as indicator variables signifying the realization of state i (i.e., $\delta_i = 1$ if state i occurs and $= 0$ otherwise), the following function describes the distribution of labor supply:

$$(3.13) \quad l_h(h) = \prod_{i=0,2,4,6} [\mathrm{Pr}(\delta_i = 1)]^{\delta_i} \prod_{i=1,3,5} [f_h(h|w_i, y_i)]^{\delta_i}$$

with

$$(3.14) \quad \mathrm{Pr}(\delta_i = 1) = \int_{v_i^L}^{v_i^U} f_v(v) \, dv \quad \text{for } i = 0, 2, 4, 6.$$

This function incorporates both continuous and discrete components, with densities describing hours on segments and mass points determining the probability of occupancy at kinks.

C. Accounting for Unobserved Wages

A critical assumption maintained in the above analysis concerns the observability of the gross wage W for all individuals. Knowledge of the exact budget constraint is needed to construct the bounds v_i^L and v_i^U [see Equations (3.6)] which, of course, requires complete information on the gross wage rate. While this information is available for individuals who work, such is not the case for nonworkers. Inspection of (3.13) and (3.14) reveals that the implied contribution to the likelihood for nonemployed individuals is

$$(3.15) \quad \mathrm{Pr}(\delta_0 = 1) = \int_{\underline{v}}^{v_0^U} f_v(v) \, dv \quad \text{with } v_0^U = v^s(0, (1 - t_A)W, y_1).$$

Clearly, this component cannot be evaluated without data on W.

The solution to this problem requires an expression for the probability $\mathrm{Pr}(\delta_0 = 1)$ that can be calculated without specific knowledge of W. One can obtain such an expression by introducing a distribution for W. Denote the joint distribution of the variables v and W by the bivariate density $f_{vw}(v, W)$. The above analysis implicitly assumes independence of v and W.[9] According to the economic model outlined above, an individual does not work if $h^s((1 - t_A)W, Y, v) < 0$ which obtains for all combinations of v and W such that $v < v_0^U = v^s(0, (1 - t_A)W, Y)$. Thus

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9. If this is not true, the function f_v must be interpreted as the density of v conditional on W.

$$(3.16) \quad \Pr(\delta_0 = 1) = \int_0^\infty \int_{\underline{v}}^{v_0^U} f_{vw}(v, W) \, dv \, dW.$$

Replacing (3.15) by (3.16) in the specification of l_h given by (3.13) involving that component associated with $i = 0$ yields a formulation of the likelihood function that recognizes the absence of information on wages for nonworkers.

Proposed remedies found in the literature that purport to deal with the problem of missing wages do not follow the above approach and lead to improper specifications of likelihood functions. One of these approaches, proposed by Hausman (1981b), constructs a fitted value $\hat{W}$ for the wage rate using familiar censored regression techniques and interprets $\hat{W}$ as the wage faced by nonworkers. This procedure has the effect of introducing two distinct distributions for wages (one for W and another for $\hat{W}$), with individuals presumably able to earn either W or $\hat{W}$ depending on whether they are actually observed to be working. Reference to (3.15) indicates that it is the distribution of W that is relevant for specifying the nonwork choice, and knowledge of $\hat{W}$ is inadequate to characterize this choice. Partial substitution of $\hat{W}$ for W in formulating a likelihood function produces nonsensical statistical specifications.[10] A second procedure utilized by Arrufat and Zabalza (1986) for handling the problem of missing wages replaces W by $\hat{W}$ for all observations, not just nonworkers. This technique leads to misspecified budget constraints for everyone in the sample; individuals earn W, not $\hat{W}$.

D. Introducing Measurement Error

Virtually all empirical studies implementing this type of econometric approach allow for measurement error in hours of work. There are several sound reasons for introducing this generalization. One reason relates to the widespread suspicion that reporting error contaminates data on hours of work and that measured hours can deviate significantly from true hours. A more compelling reason arises from practical considerations. The statistical model outlined above implies that bunching in hours of work should occur at kink points if hours directly measure h. However, only a trivial number of individuals, if indeed there are any at all, report hours of work at interior kink points. Section II notes that only a single person is observed at a kink for the data set considered here; similar situations invariably apply for any data source currently used in the literature. Such evidence provides the basis for immediately rejecting the dis-

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10. An earlier discussion of this point appears in Heckman and MaCurdy (1981).

tributional implications of the specifications presented above, unless one admits that the data on hours does not directly represent h. This is precisely the role played by introducing a measurement-error component in the data.

To incorporate this feature into the above analysis, suppose that H denotes measured hours of work and that the function $H^m(h, \varepsilon)$ relates H to true hours h and to an error component ε. In particular, $H = H^m$ when $h > 0$ and $H^m > 0$, and $H = 0$ when $h = 0$ or $H^m \leq 0$. In contrast to information on h, knowledge of H does not suffice to allocate individuals to the correct branches of the budget constraints or to the marginal tax rate faced by individuals, other than at zero hours of work. The state of the world a consumer occupies can no longer be directly observed, and one confronts a discrete-data version of an errors-in-the-variable problem.

In specifying the likelihood function of H, assume that the measurement function H^m increases monotonically in ε; so $\partial H^m/\partial \varepsilon > 0$. Denote the inverse of this function solved in terms of ε as $\varepsilon^m(H, h)$. Suppose that the density function $f_\varepsilon(\varepsilon|h)$ describes the distribution of ε conditional on h. Setting $H = H^m$ and ignoring any restrictions on the value of H, a standard transformation from ε to H produces the conditional density function

$$(3.17) \quad f_H(H|h) = \frac{\partial \varepsilon^m}{\partial H} f_\varepsilon(\varepsilon^m(h, H)|h).$$

Defining δ_H as an indicator variable with $\delta_H = 1$ if $H > 0$ and $\delta_H = 0$ if $H = 0$, the likelihood function of H takes the form

$$(3.18) \quad l_H(H) = [1 - \Pr(\delta_H = 1)]^{1-\delta_H} [l_H(H|\delta_H = 1)\Pr(\delta_H = 1)]^{\delta_H}$$

with

$$(3.19) \quad l_H(H|\delta_H = 1)\Pr(\delta_H = 1) = \sum_{i=2,4,6} \Pr(\delta_i = 1) f_H(H|\bar{h}_i)$$

$$+ \sum_{i=1,3,5} \int_{\Lambda_i} f_H(H|h) f_h(h|w_i, y_i) \, dh$$

and

$$(3.20) \quad \Pr(\delta_H = 1) = \sum_{i=2,4,6} \Pr(\delta_i = 1) \int_0^\infty f_H(H|\bar{h}_i) \, dH$$

$$+ \sum_{i=1,3,5} \int_0^\infty \int_{\Lambda_i} f_H(H|h) f_h(h|w_i, y_i) \, dh \, dH,$$

where the notation $\int_{\Lambda_i}$ indicates integration over the set Λ_i. All empirical work analyzing labor supply with piecewise-linear budget constraints relies on a variant of (3.17)–(3.20) when carrying out maximum likelihood estimation. According to these expressions, the variable H follows a continuous distribution with no bunching implied at any point, other than at zero.[11]

There are two strategies for interpreting and modeling the error term ε. The first involves specifying ε to represent reporting error which contaminates the observation on h for individuals who work. Persons correctly report when they don't work; so $H = 0$ when $h = 0$. But when they work h hours, one observes $H = H^m(h, \varepsilon)$ hours. The above expression for $l_H(H)$ simplifies in this case. By construction, one must choose the density function $f_\varepsilon(\varepsilon|h)$ to ensure that reported hours of work are always positive, which requires $H^m > 0$ for all $h > 0$ and feasible ε. As a result, one can replace (3.20) by

$$(3.21) \quad \Pr(\delta_H = 1) = 1 - \Pr(\delta_H = 0) = 1 - \Pr(h = 0) = 1 - \Pr(\delta_0 = 1),$$

where either (3.15) or (3.16) provides the expression for $\Pr(\delta_0 = 1)$.

The second interpretation of ε involves considering it as an "optimization" error that reflects the degree to which individuals' actual hours of work (H) deviate from their desired hours (h). Accordingly, one may observe that a person is not working even though $h > 0$ because a low realization of ε causes $H^m < 0$. Such an occurrence presumably captures the notion of involuntary unemployment. Formally, $H = 0$ and $\delta_H = 0$ when either $h = 0$ or $H^m \leq 0$ and $H = H^m$ and $\delta_H = 1$ otherwise. The main consequence of this modification is that the condition $h = 0$ or equivalently $\delta_0 = 1$ no longer determines whether $\delta_H = 0$. Thus, relation (3.21) does not apply and a simplification of expression (3.20) is not available. Most of the empirical studies in this area that introduce ε do so in a way that is consistent with the optimization error interpretation.

E. Restrictions Imposed on Estimated Labor Supply Functions

We are now in a position to develop a central point of this paper, which answers the following questions. If one estimates a labor supply function by maximum likelihood methods using the above specification of either l_h or l_H, is the estimated function restricted to satisfy certain properties? If so, what are these properties?

While utility maximization served as the basis for developing the above

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11. The term bunching here refers to point masses or spikes in the distribution. While spikes are not present, clustering can occur at kinks in the sense that the density function can rise or fall rapidly at these points.

formulas for l_h and l_H, the current analysis completely ignores the implications of this behavioral hypothesis. Instead, this discussion interprets l_h and l_H as if they were merely distributional assumptions and considers whether estimation using these specifications imposes parametric constraints on the labor supply function $h^s(w, y, v)$.

Not surprisingly, no constraints are imposed in the simplest cases; say, when taxes are strictly proportional and all individuals work. In such an instance the budget constraint reduces to a single linear segment. The likelihood function l_h becomes $f_h(h \mid w_1, y_1)$ and correspondingly, l_H simplifies to the expression

$$\int_0^\infty f_H(H \mid h) f_h(h \mid w_1, y_1)\, dh$$

where w_1 and y_1 are the marginal wage and virtual income defining the budget set. Maximizing this specification to estimate the parameters of h^s will not typically impose any properties on h^s. Stated more precisely, both l_h and l_H constitute well-defined likelihood functions without imposing any restrictions on h^s.

This situation changes, however, when one considers cases in which budget sets contain kinks. Inspection of (3.13)–(3.20) reveals that the expressions for the likelihood functions l_h and l_H involve the components $\Pr(\delta_i = 1)$ and that these probabilities are negative unless

(3.22) $v_i^U - v_i^L \geq 0$ $i = 0, 2, 4, 6$

where the bounds v_i^L and v_i^U are defined by (3.6).[12] The issue of concern here is whether these inequalities restrict the parameter estimates of the labor supply function h^s.

To investigate this issue, define the functions: $y^*(\omega) = y_{i-1} + (w_{i-1} - \omega)\bar{h}_i$ which corresponds to virtual income with $y^*(w_{i+1}) = y_{i+1}$; and $v^*(\omega) = v^s(\bar{h}_i, \omega, y^*(\omega))$ where v^s continues to denote the inverse of the labor supply function h^s solved in terms of v. Differentiating the identity $h^s(\omega, y^*(\omega), v^*(\omega)) = \bar{h}_i$ with respect to ω and exploiting the fact that $dy^*/d\omega = -\bar{h}_i$ yields the relationship

(3.23) $\dfrac{\partial h^s}{\partial \omega} - \dfrac{\partial h^s}{\partial y}\,\bar{h}_i + \dfrac{\partial h^s}{\partial v}\,\dfrac{dv^*}{d\omega} = 0.$

Consider a value of ω in the interval $[w_{i+1}, w_{i-1}]$. Given that the function

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12. These inequalities ensure that the probabilities $\Pr(\delta_i = 1)$ are nonnegative for $i = 0, 2, 4, 6$. The probabilities $P(\delta_i = 1)$ for $i = 1, 3, 5$ are nonnegative by construction; the inequalities $v_{i+1}^L - v_{i-1}^U \geq 0$ follow immediately from Definitions (3.6) and the maintained assumption that h^s monotonically increases in v.

$v^*(\omega)$ is differentiable on this interval, the Mean Value Theorem ensures that there exists an $\bar{\omega}$ in this interval such that

$$(3.24) \quad \frac{dv^*}{d\omega}\bigg|_{\bar{\omega}} = \frac{v^*(w_{i-1}) - v^*(w_{i+1})}{w_{i-1} - w_{i+1}} = \frac{v_i^L - v_i^U}{w_{i-1} - w_{i+1}},$$

where the latter expression follows from (3.6) which implies that $v^*(w_{i-1}) = v^s(\bar{h}_i, w_{i-1}, y_{i-1}) = v_i^L$ and $v^*(w_{i+1}) = v_i^U$. Combining (3.24) with (3.23) yields

$$(3.25) \quad \frac{\partial h^s}{\partial w}\bigg|_{\bar{\omega}} - \frac{\partial h^s}{\partial y}\bigg|_{\bar{\omega}} \bar{h}_i = \frac{\partial h^s}{\partial v}\bigg|_{\bar{\omega}} \frac{v_i^U - v_i^L}{w_{i-1} - w_{i+1}} \geq 0$$

where the inequality follows from the information that $\partial h^s/\partial v > 0$, $w_{i-1} - w_{i+1} > 0$ and restriction (3.22). The expression on the left of (3.25) represents the Slutsky term associated with the labor supply function evaluated at some wage between w_{i+1} and w_{i-1} and a corresponding virtual income which places an individual at $\bar{h}_i$ hours of work.

Thus, we find that the restrictions incorporated in the requirement $\Pr(\delta_i = 1) \geq 0$ at interior kink points translate into important parametric constraints on the labor supply function. By construction, compensated substitution effects will exhibit the appropriate sign at these points, and the implied underlying preferences will satisfy quasi-concavity. In contrast to the interior kinks (i.e., points 2 and 4 in Figure 1), inequalities (3.22) need not impose constraints on the labor supply function at exterior kinks (i.e., points 0 and 6 in Figure 1). As long as the lower boundary $\underline{v}$ determining the minimum value of v is sufficiently small (say $\underline{v} = -\infty$), the condition $v_0^U \geq v_0^L = \underline{v}$ associated with nonparticipation will be satisfied without any parametric restrictions on h^s. Similarly, at the extreme upper kink point, the condition $\bar{v} = v_6^U \geq v_6^L$ creates no restrictions when the upper boundary $\bar{v}$ of the sample space of v is sufficiently large (say, $\bar{v} = \infty$).

Any estimation approach involving the evaluation of $\Pr(\delta_i = 1)$ at interior kink points invokes the constraint $\Pr(\delta_i = 1) \geq 0$, which imposes the parametric restrictions developed above. While these restrictions involve economically meaningful quantities, it is important to recognize that the imposition of these inequalities in estimation does not stem from direct attempts to introduce constraints implied by economic theory, such as those associated with "global" quasi-concavity in preferences. Instead, the parametric restrictions invoked here come purely from properties needed to obtain a properly defined statistical model.

In the application of maximum likelihood without measurement error, estimation requires the likelihood function l_h given by (3.13) to be positive when evaluated at all observations in the sample. Thus, it must be the case that $\Pr(\delta_i = 1) > 0$ for any individual who locates at kink point i.

Consequently, maximum likelihood methods using l_h will produce an estimated h^s that necessarily satisfies Slutsky negativity (positivity in the case of labor supply) at all points in a set composed of those combinations of marginal wages and virtual incomes associated with all observations occurring at interior kink points in the sample. As one increases the number of individuals in the sample who are observed to locate at interior kinks of their respective budget constraints, one expands the number of combinations of wages and incomes at which estimated compensated substitution effects are constrained to be positive. Such an event typically broadens the range over which an estimated labor supply function possesses the properties implied by consumer demand theory.

Use of the likelihood function l_H which accounts for measurement error in estimation leads to an even wider combination of wages and income at which estimates are constrained to satisfy Slutsky conditions. Examination of specifications (3.18)–(3.20) indicates that the probabilities $\Pr(\delta_i = 1)$, $i = 2, 4$, are a part of the likelihood function for all observations in the sample who work. Consequently, the construction of l_H imposes the restriction that $\Pr(\delta_i = 1) \geq 0$ for all the feasible interior kink points of a working individual's budget constraint, regardless of whether his hours of work are near these points.[13] Thus, obtaining estimates of h^s by maximum likelihood methods applied to l_H will produce coefficients that necessarily imply positive compensated substitution effects for all wage-income combinations associated with any interior kink point of an employed individual's budget set which represents a feasible solution for h.

Feasibility in this context means that a person can be at that level of hours with positive probability. With regard to estimation using the likelihood function l_H, limiting the admissible range of the heterogeneity component v will invariably lead to a smaller span of the budget constraint over which the Slutsky condition is imposed at kinks. Truncation of the distribution f_v is the most common mechanism that limits the range or the support of v; the most extreme case arises when one assumes a degenerate distribution for v (i.e., when v is a constant and there is no heterogeneity). With the support of f_v truncated, the probabilities $\Pr(\delta_i = 1)$ corresponding to values of v falling outside this support are equated to zero when computing l_H given by (3.18). The assignment of these probabilities

13. It is, of course, possible to use l_H in estimation and not require it to be defined over the entire range of the data. This results in a nonsensical statistical model, but one can in principle permit some probabilities to be negative as long as the functions l remain positive over a sufficient range such that $\ln(l)$ is defined for each observation. Under such circumstances, the labor supply function need not satisfy the Slutsky condition at those kink points associated with negative probabilities.

to zero does not impose inequality (3.22), which in turn avoids invoking the Slutsky condition at the kink points associated with these probabilities. Consequently, maximum likelihood estimation using l_H does not necessarily impose nonnegativity of compensated substitution effects at all interior kink points on all budget constraints; instead, it forces satisfaction of such inequality restrictions only at those kink points that are attainable (i.e., have a nonzero probability) for at least one individual in the sample.[14]

One way to think of the main result developed here involves a reverse form of inference from the form most economists are used to following. Economists are comfortable in accepting the following proposition: if preferences are quasi-concave in the presence of piecewise-linear constraints, then the implied likelihood function describing consumer choices satisfies conventional distributional properties. The result developed above implies a converse form of this proposition. Namely, if a likelihood function of the sort used in the analysis of piecewise-linear constraints obeys familiar distributional properties, then the implied specification for preferences must exhibit quasi-concavity at particular points. These points consist of wage-income combinations associated with all feasible interior kinks included in the sample.[15] Expanding the set of these kinks either by increasing the number of individuals making up a sample or by enlarging the number of kinks attainable for particular sample observations creates more wage-income combinations at which Slutsky conditions must hold. Requiring satisfaction of Slutsky conditions to obtain a

14. Of course, when the distribution of v is degenerate, most individuals have no attainable kink points on their budget constraints and the remaining individuals have only one attainable point. In the degenerate case, satisfaction of the Slutsky condition still plays a role in ensuring that the underlying statistical model is well-defined. If this condition fails, then relationships (3.1)–(3.4) no longer necessarily determine a unique value for an individual's hours of work. If such an event occurs, one must introduce parametric restrictions to avoid the problem of nonuniqueness which renders the statistical model either as an incomplete or as a nonsensical description of the data. The nature of these restrictions will vary depending on the particular locations of kinks, along with the values of wages and virtual incomes associated with these kinks. Generally speaking, one would expect a greater diversity in kinks to require restrictions that come closer to invoking satisfaction of the Slutsky condition.

15. This main result also holds in the case of nonconvex budget constraints. In such instances kink points at nonconvex portions of the constraints are typically not feasible (i.e., the probability is usually zero that equilibrium occurs at these points). Some interior kinks in convex regions of the constraint may also not be feasible. The Slutsky condition need not hold at these two categories of interior kinks for distributional functions to be well-defined. However, at those interior kinks that are feasible the Slutsky condition must apply, and it is even possible for this condition to apply for more than one wage-income combination associated with any particular point.

properly defined statistical model not only arises in the application of maximum likelihood methods; it is needed whenever an estimation method uses a statistical formulation that involves evaluations of probabilities associated with occupancy at interior kinks.[16]

IV. Econometric Analysis of Differentiable Budget Constraints

Approximating the tax schedule by a differentiable function as described in Section II leads to a much simpler approach for developing an empirical model of labor supply that recognizes the influence of taxes. This section outlines the principal features of such an approach, closely paralleling the analysis of the piecewise-linear case for easy comparison. The discussion assumes throughout that budget sets are convex.

A. Describing the Economic Problem

The introduction of a nonlinear tax schedule into a model of labor supply poses few analytical difficulties when the schedule generates a strictly convex constraint set with a twice-differentiable boundary. Maintaining the same notation as above, summarize the preferences of consumers by the labor supply function $h^s(w, y, v)$; and recall that the inverse of this function is $v^s(h, w, y)$ and the density function $f_v(v)$ describes heterogeneity in preferences across the population. In the presence of taxes, an individual maximizes utility subject to the budget constraint $C = E + Y - T(Y, E)$, where the tax function T is now twice-differentiable in the argument E.

The labor supply literature offers a simple characterization of the hours-of-work choice implied by this utility maximization problem. Define the following three functions:

$$(4.1) \quad \tau(Y, E) = \tau(Y, Wh) = \frac{\partial T}{\partial E};$$

$$w(h) = [1 - \tau]W = [1 - \tau(Y, Wh)]W;$$

$$y(h) = Y + E - [1 - \tau]E - T$$

$$= Y + [\tau(Y, Wh)]Wh - T(Y, Wh).$$

The quantity τ denotes the marginal tax rate on earnings; $w(h)$ represents the marginal wage at h hours of work with τ evaluated at Y and E; and $y(h)$ corresponds to virtual income evaluated at the same combination of Y and E. An individual does not work if

(4.2) $h^s(w(0), y(0), v) \leq 0$.

When this condition fails, then $h > 0$ and hours satisfy the implicit equation

(4.3) $h = h^s(w(h), y(h), v)$.

This characterization follows from work on taxes and labor supply (e.g., Hall 1973) that represents a consumer as facing a linear budget constraint in the presence of nonlinear tax programs. This linear constraint is constructed in a way to make it tangent to the actual nonlinear opportunity set at the optimal solution for hours of work. The implied slope of this linearized constraint is $w(h)$ and the corresponding value of virtual income is $y(h)$. Equation (4.3) constitutes a structural relationship that determines hours of work using a form of the labor supply function that applies in the familiar linear case.

Analytically solving this implicit equation for h in terms of W, Y, v and parameters of the functions h^s and T yields the labor supply function that applies in the nonlinear tax case. One obtains an expression for this labor supply function by applying the Implicit Function Theorem to carry out a transformation from the variable v to the variable h using the equation $v = v^s(h, w(h), y(h))$. With a convex budget set and quasi-concave preferences, economic theory predicts a unique solution for h for any fixed specification of preferences indexed by v. This uniqueness combined with the assumption $\partial h^s/\partial v > 0$ implies that a monotonically increasing relationship links h and v (given W, Y and the values of parameters). This in turn implies that the Jacobian, dv^s/dh, associated with this transformation is positive throughout the relevant range. To verify this proposition and to derive an explicit expression for this Jacobian, total differentiation of (4.3) with v replaced by $v^s(h, w(h), y(h))$ yields

$$
\begin{aligned}
1 &= \frac{\partial h^s}{\partial w}\frac{dw}{dh} + \frac{\partial h^s}{\partial y}\frac{dy}{dh} + \frac{\partial h^s}{\partial v}\frac{dv^s}{dh} \\
&= \frac{\partial h^s}{\partial w}\left(-\frac{\partial \tau}{\partial E} W^2\right) + \frac{\partial h^s}{\partial y}\left(\frac{\partial \tau}{\partial E} W^2 h\right) + \frac{\partial h^s}{\partial v}\frac{dv^s}{dh}.
\end{aligned}
$$

Rearranging this relationship produces

$$(4.4) \quad \frac{dv^s}{dh} = \frac{1 + \left(\frac{\partial h^s}{\partial w} - \frac{\partial h^s}{\partial y} h\right) \frac{\partial \tau}{\partial E} W^2}{\frac{\partial h^s}{\partial v}}.$$

Convexity of the function (i.e., $\partial \tau / \partial E > 0$) and quasi-concavity of preferences ensure that the Jacobian dv^s/dh possesses the same sign as the partial $\partial h^s / \partial v$, which is positive by assumption.

B. Specifying the Likelihood Function of Hours

Relationships (4.1)–(4.3) combined with the density function f_v directly induce a distribution on hours of work. There are only two states of the world in this formulation. A person either does not work which sets the indicator variable $\delta_0 = 1$, or a person works which implies $\delta_0 = 0$. According to condition (4.2), $h = 0$ when $v \leq v_0^U = v^s(0, w(0), y(0))$. Thus, the specification of the probability $\Pr(\delta_0 = 1) = \Pr(h = 0)$ is given by either (3.15) or (3.16).

The simplifications achieved with a differentiable tax function arise in the formulation of that part of the likelihood function associated with the state $h > 0$. In contrast to the case involving a piecewise-linear budget constraint, a purely continuous distribution describes the hours worked by employed individuals. Carrying out the transformation from the variables v to h described above creates a density for h of the form

$$(4.5) \quad g_h(h) = \frac{dv^s}{dh} f_v(v^s(h, w(h), y(h))).$$

This formulation exploits the implications of the economic model by presuming that the Jacobian term dv^s/dh is positive, which avoids the need for introducing the absolute value of this term in (4.5). Conditional on working, the likelihood function of hours is given by

$$(4.6) \quad l_h(h | \delta_0 = 0) = \frac{g_h(h)}{\Pr(\delta_0 = 0)}.$$

This result confirms that h follows a continuous distribution over the range $h > 0$.

Combining this density with the probability of not working creates the unconditional distribution of hours. Analogous to a conventional Tobit analysis, the formula

$$(4.7) \quad l_h(h) = [\Pr(\delta_0 = 1)]^{\delta_0} [g_h(h)]^{1 - \delta_0}$$

represents the likelihood function appropriate for a random sample including both workers and nonworkers.

C. Introducing Measurement Error

The modifications needed to admit measurement error as a source of variation in hours of work in the case of a differentiable budget constraint are quite similar to those described in Section III.D. Of course, a major motivation for incorporating measurement error in the piecewise-linear case does not exist when constraints are differentiable. A continuous distribution describes the pattern of h which avoids the stringent implications concerning the bunching of hours at particular values (i.e., at those values associated with interior kink points).[17] Consequently, one cannot immediately reject the use of (4.7) as a distributional assumption with just a casual inspection of the data.

To specify the likelihood function of observed hours of work in the presence of measurement error, recall the notation presented in Section III.D. The variable H denotes measured hours: $H = 0$ and $\delta_H = 0$ when either $h = 0$ or $H^m(h, \varepsilon) \leq 0$; and $H = H^m$ and $\delta_H = 1$ otherwise. Defining the conditional density function $f_H(H|h)$ by (3.17), the likelihood function of H takes the form

(4.8) $l_H(H) = [1 - \Pr(\delta_H = 1)]^{1 - \delta_H} [l_H(H|\delta_H = 1)\Pr(\delta_H = 1)]^{\delta_H}$

with

(4.9) $l_H(H|\delta_H = 1)\Pr(\delta_H = 1) = \displaystyle\int_0^\infty f_H(H|h)g_h(h)\, dh$

and

(4.10) $\Pr(\delta_H = 1) = \displaystyle\int_0^\infty \int_0^\infty f_H(H|h)g_h(h)\, dh\, dH,$

where $g_h(h)$ is given by (4.5).

The considerations noted in Section III.D concerning the two strategies for interpreting and modeling measurement error apply fully in this discussion as well. Specifying ε as reporting error permits a simplification of the expression of the probability $\Pr(\delta_H = 1)$ as indicated by (3.21).[18] Such

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17. Of course, this distributional function can imply clustering around points where marginal tax rates change rapidly and $\partial \tau / \partial E$ becomes large, but no spikes occur at particular values of hours.

18. In such a case, $\int_0^\infty f_H(H|h)\, dH = 1$.

a simplification of formula (4.10) is not available if one interprets ε as optimization error.

D. Restrictions Imposed on Estimated Labor Supply Functions

We now return to the questions asked at the beginning of Section II.C. The application of maximum likelihood methods to estimate the parameters of h^s imposes different restrictions in the differentiable tax case due to the new specifications obtained for l_h and l_H. Instead of using formulas (3.13) and (3.18) for these likelihood functions which are appropriate for analysis of the piecewise-linear case, estimation now uses specifications (4.7) and (4.8) for l_h and l_H.

Examination of these specifications reveals that both l_h and l_H are well-defined as likelihood functions as long as the Jacobian dv^s/dh is nonnegative. Violation of this condition implies that the density $g_h(h)$ is negative, which obviously cannot occur if l_h and l_H are valid descriptions of distributional assumptions. Relation (4.4) indicates that this nonnegativity condition translates into the property

$$(4.11) \quad \frac{\partial h^s}{\partial w} - \frac{\partial h^s}{\partial y} h \geq - \left(\frac{\partial \tau}{\partial E} W^2 \right)^{-1} \leq 0.$$

The left-hand-side of this inequality is the Slutsky term. This inequality result does not require compensated substitution effects to be positive as quasi-concave preferences mandate, only that these effects cannot become too negative.

One can readily relate this inequality condition to the results derived above assuming piecewise-linear budget constraints. As noted in Section II.D an approximation of a tax schedule around a kink point using a differentiable function can be made arbitrarily close by letting the marginal tax rate shift more sharply between the levels associated with the segments adjacent to the kink. A more rapid shift implies a larger value of the derivative $\partial \tau / \partial E$. At the extreme, $(\partial \tau / \partial E)^{-1} = 0$ and condition (4.11) requires the Slutsky term to be nonnegative, which corresponds directly to the result obtained in the piecewise-linear case.

Maximum likelihood procedures implicitly impose parametric restrictions to ensure that (4.11) holds at various combinations of hours, marginal wages and virtual incomes. If one uses l_h in such a procedure, then the estimated parameter values cannot imply a violation of (4.11) at any of the data combinations $(h, w(h), y(h))$ actually observed in the sample. If a violation occurs, then the evaluation of l_h for the observation associated with this combination would result in a nonpositive value which causes the overall log likelihood function to approach minus infinity—which clearly cannot represent a maximum.

Maximum likelihood estimation applied to l_H incorporating measurement error broadens the range over which the Slutsky term must satisfy inequality (4.11). The definition of l_H as a likelihood function requires (4.11) not only to hold at those combinations of hours, marginal wages, and virtual income directly observed in the data, but also at any combination that lies along the feasible portion of the budget constraint of any individual who is a member of the sample. Since maximum likelihood procedures will assume the validity of such restrictions when calculating estimates of the coefficients of h^s, the resulting estimated labor supply function can be expected to exhibit compensated substitution effects that obey inequality (4.11) over a very wide range of hours, wages, and incomes.[19]

V. Empirical Results

We now turn to the important issue of whether the parametric restrictions associated with the above econometric approaches actually influence the estimates obtained for substitution and income effects in an analysis of labor supply. The following discussion explores this issue using annual data on a random sample of prime-age male workers in the year 1975—the same data set described in Section II.B which is drawn from the PSID. Given the high employment rates for this group of workers, this analysis ignores the complications associated with nonparticipation and zero hours of work. In addition to examining the changes that arise from applying alternative approaches, the subsequent analysis also investigates the consequences of altering assumptions concerning the nature of heterogeneity and measurement error.

A. Specifications of Labor Supply, Heterogeneity, and Measurement Error

Many empirical studies of labor supply report estimates based on a simple linear characterization given by

$$(5.1) \quad h_L^s = \theta + \gamma z + \alpha w + \beta y,$$

where the quantities θ, γ, α, and β are coefficients, the vector z includes individual characteristics, and the variables w and y represent the mar-

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19. As noted in the discussion of the piecewise-linear case, it is computationally feasible to use l_H in estimation and not require l_h to be defined over the entire range of its support. Computationally one merely requires l_h to be nonnegative over a sufficiently large region to ensure $l_H > 0$. Of course, not requiring $l_h \geq 0$ over its relevant range produces a nonsensical statistical model.

ginal wage and virtual income. If one directly interprets h_L^s as a labor supply function, then α corresponds to the uncompensated wage effect, while β determines the income effect.

The following empirical analysis considers three specifications of labor supply derived by introducing a source of population heterogeneity in h_L^s. The first assumes

(5.2) $h^s(w, y, v) = h_L^s$ with $v = \theta$;

or, equivalently,

$$h^s = \mu_v + \gamma z + \alpha w + \beta y + (v - \mu_v)$$

where the intercept μ_v is the expected value of v and the error term $v - \mu_v$ has zero mean. Thus, (5.2) implies a linear specification with a homoskedastic error and substitution and income effects that are constant across individuals. The second specification assumes

(5.3) $h^s(w, y, v) = h_L^s$ with $v = \alpha$,

which permits the substitution coefficient in h_L^s to differ across individuals. The third specification considered below sets

(5.4) $h^s(w, y, v) = h_L^s$ with $v = \beta$

which allows income effects to vary over the population. While the random intercept specification given by (5.2) represents the most widely used model in early empirical work on labor supply, the random income-effect specification given by (5.4) has been a prominent choice in recent work (e.g., Hausman (1981a, 1981b) and Blomquist (1983)) that applies econometrically sophisticated methods to analyze piecewise-linear budget sets.

The admissible distributions for the heterogeneity component v vary according to which of these three specifications one considers. As noted in Sections III and IV, Slutsky effects must satisfy certain inequality restrictions to obtain properly defined likelihood functions. When considering convex piecewise-linear budget sets along with measurement error, these inequality restrictions must hold at each interior kink point associated with any individual in the sample. Section II shows that the range of kinks for the sample under consideration here essentially spans the space of hours, wages, and income, which effectively requires the Slutsky condition to be met globally. Satisfying this requirement imposes a non-negativity constraint on compensated substitution effects which implies the restriction

(5.5) $\alpha - \beta h \geq 0$

for the specifications of labor supply introduced above.

In the case of the random intercept specification, condition (5.5) does not limit the choice of the admissible distributions for $v(= \theta)$. The subsequent analysis presents estimates assuming that the density of v associated with this formulation is given by

$$(5.6) \quad f_v(v) = \phi\left(\frac{v - \mu_v}{\sigma_v}\right)\Big/ \sigma_v$$

where $\phi(\cdot)$ denotes the density function of a standard normal; or $v \sim N(\mu_v, \sigma_v^2)$.[20]

Condition (5.5) does restrict the choice of f_v in the case of the random substitution-effect specification. For h arbitrarily close to zero, α must be positive. To ensure satisfaction of this restriction, the following analysis assumes that $v = \alpha$ possesses a truncated normal distribution over positive values. This implies

$$(5.7) \quad f_v(v) = \frac{\phi\left(\frac{v - \mu_v}{\sigma_v}\right)\Big/ \sigma_v}{\Phi\left(\frac{\mu_v}{\sigma_v}\right)} \quad \text{for } v \geq 0$$

where $\Phi(\cdot)$ is the cumulative distribution function associated with a standard normal.

Finally, condition (5.5) also limits the admissible functional forms for f_v in the random income-effect specification. In this case, assuming that α is relatively small, β must be negative for large h. Choosing f_v to be a truncated normal distribution over negative values imposes this property. This implies

$$(5.8) \quad f_v(v) = \frac{\phi\left(\frac{v - \mu_v}{\sigma_v}\right)\Big/ \sigma_v}{\Phi\left(-\frac{\mu_v}{\sigma_v}\right)} \quad \text{for } v \leq 0$$

with $v = \beta$ in this case. Previous work using the random income-effect formulation entertains this latter distributional assumption.

The last items needed to construct the likelihood functions associated with the above specifications of labor supply involve designating the characteristics of measurement error. This analysis considers two distinct formulations. The first assumes that

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20. This distributional assumption formally, of course, does not ensure that $\Pr(h > 0) = 1$ which is taken for granted in this analysis. However, if the mean of h is sufficiently large and σ_v is sufficiently small, then the probability $\Pr(h = 0)$ is negligible. This describes the situation for prime-age male workers.

(5.9) $H = H^m(h, \varepsilon) = h + \varepsilon$

with

(5.10) $f_\varepsilon(\varepsilon) = \phi\left(\dfrac{\varepsilon}{\sigma_\varepsilon}\right)\Big/ \sigma_\varepsilon;$

that is, the measurement function H^m is linear and measurement error ε is normally distributed with zero mean.[21] This formulation is the one used in the recent empirical work on taxes and labor supply.

The second characterization of measurement error considered in the following empirical analysis goes beyond presuming that only hours are contaminated by reporting error. It also recognizes that using average hourly earnings as the gross wage variable—as is done in this analysis—leads to a measurement error problem in wages as well. In particular, this formulation assumes that

(5.11) $H = H^m(h, \varepsilon) = he^\varepsilon$ and $W = \dfrac{E}{h} = \dfrac{E}{H}\, e^\varepsilon$

with

(5.12) $f_\varepsilon(\varepsilon) = \phi\left(\dfrac{\varepsilon + \sigma_\varepsilon^2/2}{\sigma_\varepsilon}\right)\Big/ \sigma_\varepsilon,$

where the variable W denotes the gross wage rate appearing in the above derivations of the likelihood function l_h. This multiplicative structure specifies that observed hours (H) deviate from true hour $s(h)$ by a factor of proportionality (e^ε), and that the inverse of this factor of proportionality relates observed average hourly earnings (E/H) to the true gross wage rate (W). Observed earnings (E) are measured without error. The unusual form for the mean of the normal distribution generating the error ε given by (5.12) ensures that the expected value of the quantity e^ε equals one (so, h is an unbiased estimate of H).

B. A Brief Description of the Data

The following empirical analysis estimates various combinations of the above labor-supply specifications and measurement-error formulations using a data set consisting of 1,017 prime-age males drawn from the 1976 wave of the PSID. Appendix A presents the details concerning the con-

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21. Similar to the qualification presented in a previous footnote, this distributional assumption does not imply that $\Pr(H^m > 0) = 1$ when $h > 0$ which is presumed in this discussion. This presumption, however, is a reasonable approximation as long as h has a very low probability of being close to zero.

struction and the characteristics of this data set. The gross wage variable W is calculated as the individual's 1975 yearly earnings divided by his hours worked for that year. This variable has a median value of $6.39, a mean of $6.89, and an inter-quartile range of $3.63. The unearned income variable Y is calculated as the husband and wife's combined taxable income for 1975 minus the husband's earned income; it has a median value of $2,000, a mean of $3,714 and an inter-quartile range of $5,900. The lowest value of Y is $-$7,900 and the highest is $57,640. The hours-of-work variable is the individual's total annual hours in 1975. This variable has a median value of 2,114 hours, a mean of 2,236 hours and an inter-quartile range of 506 hours. After imposing various sample selection criteria designed to admit prime-age men who are married heads and not self-employed (see Appendix A for further details), all remaining observations worked in 1975 which implies an employment rate for our sample of literally 100 percent.

The subsequent estimation also makes use of a number of individual characteristics included in the vector z in (5.1). The variables included in z are: an age variable that takes a value of zero if the person is between 25 and 45 and a value of (age $-$ 45) for ages beyond 45; the number of children less than six years old; the number of people in the individual's immediate family; the amount of equity a person has in his house; and a health variable that indicates whether an individual has a condition limiting the type of work that he can do.

C. Estimation Using the Piecewise-Linear Methodology

This discussion presents results obtained by applying the econometric approach described in Section III to estimate all three of the specifications of labor supply introduced above. The analysis considers only the linear model of measurement error. The multiplicative structure given by (5.11) is an unattractive option in the applications examined here due to its implication that earnings are observed without error. Such an implication combined with a segmented budget constraint means that observations on earnings in the sample should be bunched at kink points, which is grossly inconsistent with the evidence as noted in Section II.[22]

22. As mentioned previously, one can readily reject the distributional assumptions implied by the piecewise-linear approach if one can determine that an insufficient number of individuals occupy kinks on their budget constraints as is implied by distribution function (3.13). The multiplicative measurement-error model implies that each individual's earnings, E, are perfectly observed. We know from the discussion of Section II that only a single individual in our sample has a value of earnings that places him exactly at a tax-bracket threshold or, equivalently, at a kink. For the introduction of measurement error in the piecewise-linear approach to avoid the problem of a lack of observations at kinks, it is earnings and not necessarily hours that must be measured with error and not be directly observed.

The likelihood functions used to compute the estimates of the various specifications are explicit parameterizations of the function l_H given by (3.18)–(3.20). Appendix B presents the parameterizations appropriate for analyzing models that incorporate the linear measurement-error formulation described by (5.9) and (5.10) into the labor-supply specifications (5.2), (5.3), and (5.4) with their corresponding distributions (5.6), (5.7), and (5.8). These parameterizations follow directly from the framework outlined in Section III, except for variants of the random income-coefficient specification in which unearned income Y is zero or negative for some observations in the sample. (Out of 1,017 total observations, $Y = 0$ for 215 and $Y < 0$ for 4). Appendix B outlines the implied parameterizations of l_H for all variants of the random income-coefficient model; the parameterizations associated with the random intercept and the random substitution-effect specifications are readily inferred from this appendix.

1. Estimates of Piecewise-Linear Specifications

Table 2 presents maximum likelihood estimates of the main coefficients associated with the three alternative specifications of labor supply. In those instances where a distribution is estimated for a coefficient, the implied percentiles are reported at the 1 percent, 25 percent, 50 percent, 75 percent, and the 99 percent levels. When a single estimate for a coefficient is obtained, it is listed in the row corresponding to the median with its standard error in parentheses just below it. Instead of presenting separate estimates for the intercept and the parameters determining the effects of the variables included in z, the table lists only the median of the predicted value of the quantity $\theta + \gamma z$ in the sample. Appendix C reports an expanded set of estimates, along with a brief discussion of the extensive estimation carried out to develop the findings summarized in Table 2.

The results presented in Table 2 generally convey a similar picture across the alternative specifications: the implied estimates of both substitution and income effects take values near zero and heterogeneity distributions are tightly concentrated. Beginning with the results for the random intercept model, the first two columns of the table present results both for "constrained" estimation which imposes the restriction that $\Pr(\delta_i = 1) \geq 0$ at each interior kink point and for "unconstrained" estimation which only requires the overall log likelihood function to be positive for each observation.[23] In the case of the constrained results, the estima-

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23. As described in footnote 14, unconstrained estimation permits the valuations of probabilities to go below zero. These probabilities correspond to the quantities given in (3.14). Operationally, we admitted the possibility that $v_i^U < v_i^L$ which generates a negative value for

Table 2

Estimates of Specifications Using the Piecewise-Linear Approach[a]

Coefficients		Models: Source of Heterogeneity			
		$\nu = \theta$		$\nu = \alpha$	$\nu = \beta$
		Constrained[b]	Unconstrained[b]		
α	99%			0.0000502	
	75%			0.0000501	
	50%	0.00003	−79.2	0.0000500	0.00001
		(—)	(16.8)		(—)
	25%			0.0000499	
	1%			0.0000498	
β	99%				−0.00008
	75%				−0.002
	50%	−0.0071	−0.0080	−0.0072	−0.006
		(0.0075)	(0.0072)	(0.0075)	
	25%				−0.011
	1%				−0.035
$z'\gamma + \theta$	50%	2,267	2,640	2,270	2,340
μ_ν		2,313.8	2,610.2	0.00005	0.296
		(107.4)	(112.8)	(0.023)	(0.321)
σ_ν		1.0	1.0	0.0000002	0.049
		(—)	(—)	(0.032)	(—)
σ_ε		538.2	524.2	538.3	536.4
		(16.4)	(17.2)	(30.5)	(35.7)

a. Standard errors in parentheses.
b. In the constrained estimations the condition $\Pr(\delta_i = 1) \geq 0$ is imposed at all relevant interior kink points for every individual. In the unconstrained estimations this condition is not imposed.

$\Pr(\delta_i = 1)$. There is some question as to whether such an unconstrained estimation method produces consistent parameter estimates. For example, Van Soest et al. (1988) presents a situation in which maximum likelihood procedures applied to a simultaneous probit model generates inconsistent estimates when this model is not guaranteed to satisfy internal coherency (i.e., the property that probabilities are individually between zero and one and sum to one). This situation arises even if the model is internally coherent for the true parameter values.

tion procedure forces the substitution coefficient as close to zero as possible; letting α become negative violates the Slutsky condition on at least one feasible kink in the sample which renders the assignment of a negative probability associated with the occupancy of this kink. Because the estimate of α directly encounters a binding inequality restriction, the table reports no standard error for this coefficient. The estimate for β does not hit a constraint, even though it is very small in size and is statistically insignificant. The near-zero values obtained for both the estimates of α and β means that taxes along with their induced nonlinearities are an inconsequential factor determining observed hours of work. Without the nonlinearities created by tax effects, one cannot identify the distinct influences of the two random elements ν and ε because both terms enter linearly in the labor-supply specification.[24] To resolve this identification issue, we fix the standard deviation σ_ν to a small number (i.e., to 1.0) and then estimate the standard deviation σ_ε; the table lists a dash in place of the standard error below the value of the parameter σ_ν to signify that it arises from the imposition of a constraint. Fixing σ_ν at different values induces only slight changes in the estimates obtained for the other coefficients except, of course, for σ_ε.[25]

Inspection of the results obtained for "unconstrained" estimation of the random intercept specification confirms that the implicit imposition of the Slutsky condition at interior kink points creates a binding constraint on the estimate of α. With removal of the nonnegativity restrictions on probabilities, the estimated value of α moves negative while the estimate of β changes only slightly.

Turning to the empirical results obtained for the other specifications presented in Table 2, in the random substitution-effect specification the estimate of β again lies in the immediate neighborhood of zero and the distribution of α spikes at a very small positive value. In the random income-effect specification, the estimate of α encounters the zero constraint in the estimation and the distribution of β bunches up near zero. More particularly in this latter specification, serious problems arose in using a truncated normal distribution as a description of the variation in β across the population. Inspection of the estimates reported for the param-

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24. Besides having a direct linear effect on labor supply as does the error term ε, the random component ν shifts hours by changing the value of h which in turn moves the marginal tax rate faced by an individual. Because the estimates of α and β are so small, changes in marginal wages and in virtual incomes essentially have no influence on hours worked. Thus, for all practical purposes, shifts in ν do not alter observations in hours in any way different than shifts in ε.

25. We investigated a very wide range of values for σ_ν, and only trivial changes in estimates occurred. Attempts to estimate σ_ν as a free parameter yielded a statistical model that was not identified numerically; the standard errors of both σ_ν and σ_ε became very large.

eters μ_ν and σ_ν for the case $\nu = \beta$ reveals that truncation of the normal distribution occurs very far in the tail of the untruncated distribution. Indeed, far enough in the tail that difficulties arose in evaluating the relevant probabilities.[26] To avoid computational difficulties, we had to impose the restrictions that μ_ν be no more than six times σ_ν which became a binding constraint. This is the reason why no standard error is reported for $\sigma_\nu(= \mu_\nu/6)$ in the random β case. In an attempt to get around this problem we estimated distributions of β that admitted a spike just to the left of zero using a two-branch truncated compound-normal distribution. This attempt failed, however, as we either ended up in the tails of both of the untruncated normals or ended up in our original situation with all the weight on one of the branches and in the tail of that branch.

2. Implications of Piecewise-Linear Results

The near-zero estimates for substitution and income effects imply, not surprisingly, very small compensated wage elasticities. In the constrained random intercept case, where both coefficients are essentially zero, the compensated wage effect and the associated elasticity for a representative individual are 15.0 and 0.031, respectively—that is, for an individual with average wages, income, characteristics and preferences.[27] In the random income-effect case, with α being zero and the distribution of β being packed up against zero, the compensated effect is 12.7 and the compensated elasticity is 0.026 for a representative individual. In the random wage coefficient case, the compensated effect and elasticity are 15.2 and 0.032, respectively.[28]

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26. This problem has been encountered by another study using the same specification considered here. Blomquist (1983) reports estimates corresponding to truncation occurring 17 standard deviations in the tail. Conventional computer algorithms are incapable of evaluating with this degree of truncation. In personal conversation, Blomquist stated that he switched to a computer algorithm that was accurate at very extreme truncation points. We did not investigate the use of this alternative algorithm.

27. We construct a compensated effect, $(\alpha - \beta h)$, for a representative individual by calculating the value implied for this effect at the median values of h and ν. The median value of h is 2,114, and the median values of the ν distribution are: -0.006 when $\nu = \beta$; 0.00005 when $\nu = \alpha$; and 0 when $\nu = \theta - \mu_\nu$. In constructing compensated elasticities, we set $w = \$4.38$ which is the median marginal wage evaluated at 2,114 hours.

28. In the random income-coefficient case, the compensated effect for a representative individual with a low wage (i.e., treated exactly as in footnote 27 except he is given a wage of $3.25) is 12.7 with an associated elasticity of 0.019. For a high wage representative individual (i.e., $w = \$5.62$) the same model produces a compensated effect of 12.7 and an elasticity of 0.033. In the random wage coefficient case a low wage individual has a compensated effect of 15.2 and an elasticity of 0.024, while a high wage individual has compensated effect of 15.2 and an elasticity of 0.041. For the linear heterogeneity case a low wage individual has a compensated effect of 15.0 and an elasticity of 0.023 and a high wage individual has a compensated effect of 15.0 and an elasticity of 0.040.

Another interesting implication of the results in Table 2 concerns the characteristics of the distribution of true hours h implied by the various specifications of labor supply. The estimates in this table provide the information needed to construct the function l_h given by (3.13) for any individual in the sample, which directly determines the distribution of h for this individual. Table 3 describes features of l_h corresponding to the three specifications.

Initially considering the random income-effect model, the third column of Table 3 presents statistics describing the locations and the dispersions of the distributions l_h estimated for individuals making up our sample, with the median of l_h measuring location and with two measures of range characterizing dispersion. Looking at the first set of rows of this column indicates that the locations of l_h across individuals are fairly concentrated; 50 percent of the individuals have medians of h that are no more than 104 hours apart. According to the results listed in the second and the third group of rows, the estimated l_h's possess narrow dispersions. The interquartile range associated with l_h is less than or equal to 68 hours for 50 percent of the individuals in the sample, and only 1 percent have an interquartile range that exceeds 338 hours. The results describing the sizes of the 98-percent ranges of the estimated l_h's also indicate a relatively small degree of dispersion.

To characterize a typical l_h implied by the estimates of this specification, Figure 5 presents a plot of l_h for a representative individual.[29] In this figure the variable h_a denotes the maximum value that true hours can take for this representative person (i.e., h_a is the number of hours associated with $\beta = 0$). The fact that the distribution of β stacks up against zero shows up in the narrow range between h_a and h_b, where h_b marks the 10th percentile of l_h for this person. The difference between h_a and h_b is only about 65 hours.

Column 2 of Table 3 demonstrates that the implied distributions l_h are even more concentrated for the random substitution-effect specification. This arises not because the distribution of α becomes stacked up against zero—a glance at the relevant μ_ν and σ_ν in Table 2 shows that the truncation of the α distribution is of little importance—but because the distribution becomes tightly concentrated around a small positive mean. As in the case of the previous specification, the central locations of the l_h's across individuals is very concentrated; 50 percent of the individuals in the sam-

29. One creates this distribution by first constructing a budget constraint for a typical sample member (in the sense of footnote 27). Then the programmed l_h function is evaluated, using that constructed constraint and the median value of $z'\gamma + \theta$, at values of ν responding to 5 percent intervals of the estimated distribution (i.e., at ν values corresponding to the 5th percentile, 10th percentile, etc. of the estimated $f_\nu(\nu)$ density).

Table 3
Distribution of Medians and Ranges Associated with l_h *Across Individuals Implied by Piecewise-Linear Estimates*[a]

Variable	Summary Statistics Percentiles of Distrib.	Models: Source of Heterogeneity $\nu = \theta$	$\nu = \alpha$	$\nu = \beta$
Median[b]	99%	2,354	2,355	2,461
	75%	2,284	2,285	2,388
	50%	2,258	2,258	2,353
	25%	2,210	2,210	2,284
	1%	1,950	1,950	2,031
Interquartile Range[c]	99%	1.4	0	338
	75%	1.4	0	127
	50%	1.4	0	68
	25%	1.3	0	33
	1%	1.3	0	12
98 Percent Range[d]	99%	4.7	0.1	1,334
	75%	4.7	0	490
	50%	4.6	0	261
	25%	4.6	0	129
	1%	3.1	0	51

a. One can impute the distribution of desired hours, l_h, for each individual in the sample and then calculate the median and the ranges associated with that distribution. To describe how the l_h's vary across people, this table presents percentiles corresponding to distributions of the medians and the ranges of the individual l_h's in the sample.
b. These variables summarize the sample distribution of the median number of hours associated with each individual's l_h.
c. These variables summarize the sample distribution of the interquartile range (i.e., the number of hours at the 75th percentile of the l_h distribution − the number of hours at the 25th percentile) of each individual's l_h.
d. These variables summarize the sample distribution of the 98 percent range (i.e., the number of hours at the 99th percentile of the l_h distribution − the number of hours at the first percentile) of each individual's l_h.

ple have medians of h that lie within 75 hours of one another. If one were to plot a representative l_h implied by this specification, it would essentially appear as a mass point with unit probability in Figure 5 located at median hours.

Due to our inability to identify the standard deviation of θ as noted in the above discussion, we cannot determine the scale of l_h associated
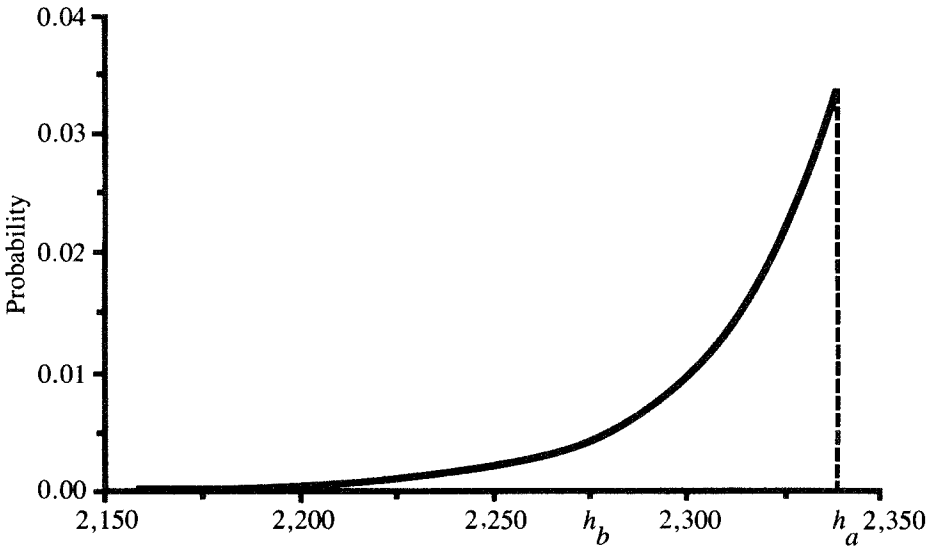
Figure 5
Representative Hours Distribution Implied by the Random Beta Case

with the random intercept specification. The first column of Table 3 presents the summary statistics associated with the implied estimates of the l_h's, assuming the standard deviation of v listed in this table is valid. If one were to plot a representative l_h for this specification, it too would essentially appear as a spike in Figure 5.

None of the three estimated distributions characterized above offer a reasonable description of hours worked in our sample, and we anticipate that rigorous tests would reject each of the distributional assumptions considered. The empirical findings indicate that the implicit imposition of Slutsky inequalities introduces binding constraints in the maximum likelihood estimation of the linear labor supply model, at least for the sample of U.S. males analyzed in this empirical work. Our consideration of other distributional assumptions for heterogeneity did not change this basic finding. Further, we entertained alternative specifications for preferences in our empirical analysis to allow for backward bending behavior in the labor supply function and still did not discover an adequate description of our data. In particular, we devoted considerable effort to the estimation of models incorporating a labor-supply specification derived from a fully interacted quadratic in the after-tax wage and in income, with heterogeneity entering through random intercepts. In estimations involving this quadratic specification, we were never able to ascertain the attainment of a

global optimum;[30] each application of the estimation procedure ended at parameter values where the Slutsky condition formed a binding constraint at one or more feasible kinks included in our sample. Our investigations of alternative distributional assumptions and preference specifications suggest that providing an acceptable description of our data using the piecewise-linear approach will likely require quite sophisticated formulations.

D. Estimation Using the Differentiable Constraint Methodology

Following a presentation that parallels the previous subsection, this discussion describes results obtained using the differentiable budget constraint outlined in Section II.D and the econometric approach in Section IV. This empirical analysis considers only the random intercept specification given by (5.2) and (5.6), but it admits a wider variety of measurement error formulations. In particular, since the bunching of hours around interior kink points is no longer an issue, one can entertain models incorporating no measurement error, additive measurement error, or multiplicative measurement error.

Three varieties of likelihood functions are used to estimate the parameters of the random intercept specification. For the formulation with no measurement error assumed, the analysis uses l_h given by (4.4)–(4.7), with (5.2) and (5.6) inserted into formula (4.4). In the cases of additive and multiplicative measurement error, the specifications of the likelihood functions are variants of l_H given by (4.8)–(4.10), with either (5.9)–(5.10) or (5.11)–(5.12) assumed as the formulation for measurement error. Appendix D presents the implied parameterizations for all three likelihood functions.

1. Estimates of Differentiable-Tax Specifications

Table 4 presents maximum likelihood estimates of the random intercept specifications of labor supply in conjunction with no measurement error, additive measurement error and multiplicative measurement error, respectively. This table reports three categories of results: "unconstrained" which does not require the density function l_h to be nonnegative over its relevant range;[31] "density-constrained" which restricts l_h to be

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30. No matter where we started initial parameter values, the estimation procedure encountered binding Slutsky conditions for various feasible kinks and eventually was unable to find a direction to continue optimization. Further, the stopping point of each run depended on the positions of the starting values.

31. See footnote 19 for further discussion of this case.

Table 4
Differentiable Budget Constraint Estimates with Linear Heterogeneity

Measurement Error Formulation	Coefficients					
	α	β	$z'\gamma + \theta$	μ_ν	σ_ν	σ_ε
None						
Slutsky	0.001	−0.000001	2,260	2,262.6	539.2	—
Constrained[a]	(—)	(—)		(89.6)	(30.3)	
Unconstrained	−86.8	0.0138	2,560	2,519.9	493.6	—
	(20.0)	(0.0061)		(112.7)	(33.6)	
Additive						
Slutsky	0.001	−0.000001	2,250	2,257.2	160.0	509.7
Constrained[a]	(—)	(—)		(88.7)	(—)	(16.8)
Density	−2.2	0.0045	2,230	2,226.3	160.0	520.2
Constrained[b]	(—)	(0.0058)		(87.9)	(—)	(17.3)
Unconstrained	−105.6	0.0209	2,610	2,531.3	158.8	508.8
	(14.1)	(0.0045)		(108.1)	(78.9)	(37.7)
Multiplicative						
Density	−65.2	0.0105	2,560	2,406.7	508.9	162.4
Constrained[b]	(—)	(0.0019)		(115.8)	(63.2)	(71.0)
Unconstrained	−102.1	0.0207	2,540	2,455.7	361.9	206.2
	(15.2)	(0.0060)		(126.4)	(104.8)	(138.1)

a. In this estimation the constraints $\alpha > 0$ and $\beta < 0$ are imposed.
b. In this estimation the constraints $l_h(h) > 0$ are imposed for all values of h used as points of evaluation in numerical integration routines applied to calculate l_H.

nonnegative;[32] and "Slutsky-constrained" which imposes the restrictions $\alpha > 0$ and $\beta < 0$. Analogous to the previous discussion, the table reports the median of the predicted value of $\theta + z\delta$ for each model. When inequality constraints become binding in estimation for any parameter, the table reports a dash in place of a standard error under the coefficient. Also, analogous to the approach adopted in Section V.C.1 when considering the random intercept model with linear measurement error, we fix the value of the parameter σ_ν to identify the distinct standard deviations σ_ν and σ_ε when the nonlinearities induced by tax effects become an inconsequential factor in estimation. The requirement to fix σ_ν occurs only when estimating the constrained versions of the "additive" models listed in rows 3 and 4; and for these specifications we set σ_ν equal to the value 160 which is comparable to the estimate of 159 obtained for the unconstrained

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32. We impose this constraint by requiring the Jacobian term which is a part of l_h to be nonnegative at all points of evaluation encountered in the implementation of numerical integration routines.

version of this model listed in row 5.[33] An expanded set of estimates appears in Appendix E.

The conclusions implied by these results agree with those reached in Section V.C. The Slutsky condition creates a binding constraint when it is imposed. In contrast to the piecewise-linear approach one does not need to impose the Slutsky condition to obtain a well-defined likelihood function for this analysis. What is required, as indicated by (4.11), involves restricting the Jacobian term to be positive which invokes an inequality constraint that is weaker than the Slutsky condition. A comparison of the results in Table 4 reveals that the imposition of no constraints yields a negative sign for the substitution coefficient α and a positive sign for the income coefficient β. Imposing the inequality constraint of a positive Jacobian term increases the estimate of α and decreases the estimate of β, although the Slutsky condition remains violated over some regions.[34] Requiring global satisfaction of the Slutsky restriction further moves the estimated values of α and β in a predictable direction. Comparing the results in Table 4 with the findings in Table 2 reveals that there is no perceptible difference in the estimates obtained assuming differentiable and piecewise-linear tax functions when one entertains comparable parametric restrictions in the two cases.[35]

2. Implications of Differentiable-Tax Results

These estimates translate into compensated substitution effects and elasticities that vary in sign. All three unconstrained cases imply negative

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33. When we fixed σ_v to a value comparable to the piecewise-linear results presented in Table 2, the application of numerical integration routines produced instabilities due to high concentration of the heterogeneity distribution.

34. The difference between the Slutsky-constrained and the density-constrained results arises from the use of a smooth polynomial (i.e., the third-order polynomial b_1 in (2.4)) to capture the changes in marginal tax rates across the different income levels. Had we instead chosen a specification of (2.4) that expanded K to mirror the abrupt shifts in marginal taxes very accurately, then the implied value of $\partial \tau / \partial E$ would be very large at the income levels defining brackets. As a consequence, relationship (4.11) shows that density-constrained estimation would require the Slutsky term to be nonnegative at such points. Thus, there would be no difference between the Slutsky-constraint and the density-constrained estimates under such circumstances. This supposition is exactly what we found when we considered such specifications for differentiable constraints.

35. Recall from Section II that the piecewise-linear construction of budget constraints differs from the differentiable construction in that piecewise-linear constraints account for federal, state, EIC and social security taxes, whereas differential constraints account for only federal taxes. Consequently, besides varying degrees of smoothing in the formulation of constraints, nonidentical treatment of tax sources offers another reason why the piecewise-linear and the differentiable estimates need not be the same even with comparable constraints imposed.

compensated effects. In the no measurement error unconstrained case a representative individual has a compensated effect of -116.0 and an associated elasticity of -0.240.[36] In the case with additive measurement error and the Jacobian constrained to be positive, the estimates also yield a compensated effect of the wrong sign; the compensated effect in this instance for a representative individual is -11.7 and the associated elasticity is -0.024. In the density-constrained estimation of the multiplicative measurement error case, the implied compensated effect and corresponding elasticity for a representative individual are -87.4 and -0.18, respectively. As in Section V.D, the cases in which one imposes $\alpha > 0$ and $\beta < 0$ essentially imply zero compensated effects and elasticities.

Inferring the characteristics of the distributions of true hours h from these findings involves constructing the function l_h given by (4.7) for each individual in the sample. Table 5 presents statistics describing these distributions for various specifications in the differentiable tax case. We do not report results for any specifications incorporating linear measurement error because all estimated variants either violate the properties of a distribution or involve the imposition of an arbitrary identification restriction on the variance of heterogeneity which determines the dispersion of l_h. The rows in Table 5 describing the medians of the l_h's show that the distributions for various individuals are grouped quite closely, which conforms to the findings in the piecewise-linear cases. However, as conveyed by the statistics on the interquartile and the 98-percent ranges, the dispersions of the distributions estimated with differentiable taxes are more spread out than in either the random substitution-effect or random income-effect cases with piecewise-linear constraints. Of course, there is no reason to expect similarity in these dispersion measures because the stochastic specification of both heterogeneity and measurement error differ quite substantially in these various models.

E. Reconciling Various Findings in the Literature

As noted in the Introduction, the surveys of Pencavel (1986) and Hausman (1985) on men's labor supply document that studies applying the piecewise-linear econometric methodology tend to find estimates indicating more positive substitution and more negative income effects than studies implementing simpler econometric methods based on least-squares or instrumental-variable procedures. Focusing on results produced by the simpler methods, Pencavel (1986) argues that estimates of uncompensated substitution elasticities tend to lie between -0.17 and

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36. A representative individual is defined exactly as in the random θ case in footnote 27.

Table 5

Distribution of Medians and Ranges Associated with l_h Across Individuals Implied by Differentiable-Constraint Estimates[a]

Summary Statistics		Models: Form of Measurement Error		
Variable	Percentiles of Distrib.	None: Slutsky Constrained	None: Unconstrained	Multiplicative: Density Constrained
Median[b]	99%	2,350	2,575	2,580
	75%	2,275	2,325	2,300
	50%	2,250	2,250	2,200
	25%	2,225	2,150	2,000
	1%	1,950	1,875	1,700
Interquartile Range[c]	99%	750	975	1,200
	75%	725	750	900
	50%	725	700	800
	25%	725	700	800
	1%	725	675	500
98 Percent Range[d]	99%	2,525	3,050	3,300
	75%	2,525	2,550	2,800
	50%	2,500	2,450	2,600
	25%	2,500	2,375	2,500
	1%	2,475	2,325	2,100

a. One can impute the distribution of desired hours, l_h, for each individual in the sample and then calculate the median and the ranges associated with that distribution. To describe how the l_h's vary across people, this table presents percentiles corresponding to distributions of the medians and the ranges of the individual l_h's in the sample.

b. These variables summarize the sample distribution of the median number of hours associated with each individual l_h.

c. These variables summarize the sample distribution of the interquartile range (i.e., the number of hours at the 75th percentile of the l_h distribution — the number of hours at the 25th percentile) of each individual's l_h.

d. These variables summarize the sample distribution of the 98 percent range (i.e., the number of hours at the 99th percentile of the l_h distribution — the number of hours at the first percentile) of each individual's l_h.

-0.08 and income effects fall in a range that overlaps zero.[37] Concentrating on estimates obtained using the piecewise-linear methodology, Hausman (1985) reports a pattern with the span of uncompensated substitution elasticities shifted higher than that of Pencavel's to positive values (as high as 0.09), and with the range of income elasticities shifted lower than Pencavel's to consist of the values -0.17 to -0.04.[38] Whereas the im-

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37. See Table 1.1a and page 69 in Pencavel (1986).
38. See Table 5.1 and page 240 in Hausman (1985).

plied compensated substitution effects are commonly negative according to Pencavel's survey, they are always positive according to Hausman's.

The empirical results of this study offer a simple explanation for a major source of the discrepancy in Pencavel's and Hausman's conclusions about the relevant ranges of substitution and income effects associated with men's hours of work behavior. Contrary to suggestions in the literature that the piecewise-linear approach produces different estimates as a consequence of its recognition of taxes, the above results indicate that the occurrence of divergent estimates arise from an implicit enforcement of Slutsky conditions at various points along budget constraints. The findings summarized by Pencavel (1986) primarily represent parameter estimates computed using methods that impose no implicit restrictions, and the Slutsky condition rarely holds when evaluated at the estimates obtained by such methods. In contrast, Hausman (1985) reports estimates derived solely from maximum likelihood procedures which require the satisfaction of constraints on Slutsky terms. The imposition of these constraints necessitates a shift in estimates to produce higher values of compensated substitution effects, which in turn leads to higher estimates for uncompensated effects and lower values for income effects. This observation surely contributes much to explaining the divergent conclusions reached by Hausman (1985) and Pencavel (1986).

One should not, of course, misconstrue this argument to conclude that maximum likelihood methods induce some sort of "upward bias" in estimates or that the restrictions implicit in these methods are undesirable to account for in estimation.[39] After all, given a valid model specification, maximum likelihood produces estimates that are both consistent and asymptotically efficient. It achieves efficiency gains in part by relying on satisfaction of the Slutsky condition which represents a structural feature of the economic model. In essence, one can interpret the application of maximum likelihood incorporating nonlinearities of budget sets—such as those induced by taxes—as a procedure that introduces "penalty functions" favoring estimates that imply larger compensated substitution effects. When considering piecewise-linear budget sets, these penalty functions take the form of nonnegativity constraints on probabilities; and in the analysis of differentiable constraints these functions enter as a part of the Jacobian term. The presence of such penalty functions encourages larger estimates of compensated substitution effects. The problem at issue

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39. Quite the contrary, the application of an estimation procedure that produces results violating the inequality restrictions on Slutsky terms implicit in maximum likelihood would be deemed as unacceptable as an economic description of behavior. For the underlying structural model to be economically meaningful, the Slutsky terms must satisfy these inequalities as well as more stringent restrictions.

in the empirical analysis of men's labor supply relates to the finding that the imposition of the Slutsky condition appears to represent a binding constraint for many of the specifications and data sources considered in the literature. In such applications, the penalty functions incorporated in maximum likelihood constitute influential factors on the determination of estimates, factors that need not produce more reliable results.[40]

The results of this study also raise serious questions about the reliability of evidence cited by much of the literature to support recent tax reform proposals aimed at lowering marginal tax rates. In the recent policy and academic debate over tax reform, much of the controversy revolved around the presumed size of the work disincentive effects of income taxes. Whereas previous tax policy has been primarily motivated by the view that these work disincentive effects are small,[41] the recent flat-rate tax proposals have as their basic premise that labor supply is strongly affected by the after-tax real wage rate and that a highly progressive tax schedule leads to substantial deadweight losses. The evidence offered to support this latter premise is invariably taken from empirical analyses applying the piecewise-linear methodology, with the work of Hausman (1981a) easily constituting the mostly cited source.[42]

The empirical results presented above sharply contradict the view that the piecewise-linear approach produces large estimates of labor-supply responses and of deadweight losses; and the results of this study are especially relevant for evaluating the findings of Hausman (1981a) because these results are obtained using exactly the same data source and specifications. According to the estimates reported in Table 2, all substitution and income effects are essentially zero when calculated via the piecewise-linear methodology. While the estimates in Table 2 considered

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40. Of course, if the Slutsky condition is truly violated for the specification of labor supply under consideration in an empirical analysis, then no estimation procedures provide reliable results from an economic perspective. Note, however, that if such misspecifications exist, there is no reason to expect the inconsistency arising in maximum likelihood to show up in a way that places estimates at values at which the Slutsky condition becomes a binding restriction along budget constraints. Depending on the source of misspecification, an optimum can in principle occur at parameter values implying satisfaction of the Slutsky condition at all points. One is only guaranteed in the application of maximum likelihood that an optimum cannot occur where the Slutsky term violates the inequalities (3.25) or (4.11).
41. See, for example, Pechman (1977), pp. 68–69, for an expression of this view.
42. For example, in their recent book on tax reform, Hall and Rabushka (1983, p. 55) use Hausman's estimates of income and substitution effects in calculating the large benefits they expect to accrue if their proposal for a flat tax is adopted. Also, many standard textbooks— such as the intermediate macroeconomic text by Dornbusch and Fischer (1984, p. 587) and the public finance text by Stiglitz (1988, Chapter 19)—rely on Hausman's results in their discussions of tax policy to conclude that the elimination of the progressivity of taxes would induce a significant growth in labor supply.

individually are not outside the range implied by other studies applying this methodology, the finding of simultaneously small substitution and income responses has not been obtained in other work. In an empirical study that should in principle be replicated by the above analysis of the random income-coefficient case, Hausman (1981a) reports 0.2 as his estimate of the uncompensated substitution coefficient α and -0.12 as the median of the distribution of the income coefficient β.[43] A comparison of Hausman's estimates with those of Table 2 reveals substantial differences in the values obtained for both the substitution and the income effects.[44] While the difference in the estimates of the uncompensated substitution response is inconsequential in the sense that both Hausman's estimate and the results of this study imply tiny responses to wage changes,[45] the difference in the estimates of the income effects imply sharply dissimilar responses to income changes. The measure of the income effect (i.e., the median of β) listed in Table 2 is more than an order of magnitude lower than the value reported by Hausman. Thus, considering an individual with median preferences, a $1,000 increase in after-tax nonwage income induces this individual to decrease his annual hours of work by 120 hours according to Hausman's estimate and by only six hours according to the corresponding estimate obtained in this study. This lower value for the income effect obtained in the analysis presented above implies much smaller estimates of compensated substitution elasticities than are used by studies relying on Hausman's results, and these smaller estimates directly yield considerably lower valuations of welfare losses induced by progressive taxation.[46]

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43. These estimates are taken from the results for the convex case reported in Table 2 of Hausman (1981a), with the estimates translated into units that are comparable to the coefficient values presented in Table 2 of this paper.

44. This nonreplication of Hausman's estimates may in part be due to differences in sample selection criteria or in the construction of wage and income variables; the measures of wages and income in the sample used in the above estimation are not the same as Hausman's measures. However, we were unable to replicate Hausman's estimates for any other constructions of a data set on prime-age men's labor supply formulated from the 1976 Wave of the PSID, and we considered many such data sets in an effort to determine the source of the nonreplicability of Hausman's estimates. All the results obtained from analyses of these different samples closely resembled those reported in Table 2. Appendix C discusses this issue further and presents a set of estimates based on an alternative sample construction, one that is designed to come as close as possible to the data set used by Hausman (1981a).

45. A dollar increase in the after-tax wage leads to a 0.2 hour increase in annual hours of work using Hausman's estimate of α and to a 0.00001 hour increase using the corresponding estimate of α obtained in this study.

46. Using his estimates for men for the 1975 tax case, Hausman calculates the welfare cost of labor supply distortions to be a significant 29 percent of tax revenues. The estimates of this study suggest that the 2.5 percent figure calculated for 1961 by Harberger (1964) comes

Of course, there are many potential problems ignored in the empirical analysis presented in this study, and accounting for these problems may very well lead to estimates that suggest larger labor-supply responses and welfare costs. Among these shortcomings are the restrictive linearity features of the labor-supply function and the potential endogeneity of income and wages.[47] No doubt, a far more problematic aspect of this analysis concerns its reliance on a naive economic model of labor supply that is purely static in character. Clearly, much more work needs to be done before one can assess the importance of any of these factors on our evaluations of the influence of taxes on hours-of-work behavior.

VI. Conclusion

The application of maximum likelihood techniques to estimate models of labor supply incorporating nonlinear tax schedules assumes the validity of particular inequality restrictions involving key behavioral parameters. The characteristics of these restrictions vary according to whether one considers piecewise-linear or differentiable schedules. In an analysis of piecewise-linear budget constraints, Section III shows that the likelihood function used for estimation is not formally defined unless the Slutsky condition is locally satisfied at all feasible interior kink points of budget sets—that is, at all interior kinks which represent a feasible option for some individual in the sample. Violation of this condition at one of these kinks directly translates into a negative

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closer to measuring the cost, although Harberger arrives at this number using a very different set of estimates than those obtained here for the piecewise-linear case. Whereas Harberger (1964, p. 48) presumes an uncompensated substitution elasticity equal to $-.25$ and a "total income elasticity" (i.e., the wage rate times the income derivative) equal to $-.375$, the results here essentially imply values of zero for both of these terms. Of course, the linear labor supply function entertained in this paper does not allow for a negative uncompensated substitution effect of the sort assumed by Harberger given the constraints imposed by maximum likelihood that requires satisfaction of the Slutsky condition near zero hours of work. Other studies applying the piecewise-linear approach support Hausman's finding of a substantial welfare cost associated with income taxation. Using data on married men in Sweden, Blomquist (1983) presents an estimate of deadweight on the order of 20 percent of tax revenues. While Blomquist's value for the income effect implies small responses which agrees with the results presented in this study, his value of the uncompensated substitution elasticity is about 0.08 which exceeds the estimates obtained here. Without further investigation, it is not clear whether Blomquist's large valuation of deadweight loss arises from the size of coefficient estimates, the particular circumstances faced by Swedish males, or the higher degree of progressivity in the Swedish income tax.

47. Note that the endogeneity of wages due to measurement error is not ignored in the multiplicative measurement error case considered above.

probability associated with locating at this point which, of course, presents a nonsensical implication. In an analysis of differentiable budget constraints, Section IV demonstrates that an inequality restriction involving the Slutsky term is needed to ensure nonnegativity of the likelihood function used for estimation in this case. This restriction is weaker than the Slutsky condition in that it allows compensated substitution effects to be the wrong sign by a small amount. Thus, behavioral parameters must satisfy less stringent inequality restrictions when one converts piecewise-linear constraints into differentiable counterparts.

The characteristics of the inequalities imposed on parameters also depend on the properties of tax schedules. The inequality restrictions vanish if taxes are strictly proportional. Increasing the curvature of the tax function enhances the restrictions. In the piecewise-linear case, greater curvature means more interior kinks and, correspondingly, more points at which the Slutsky condition must hold. In the differentiable case, the larger value of the derivative of the marginal tax rate with respect to earnings (i.e., the greater the curvature) forces a less negative value for the Slutsky term.

The results presented in Section II indicate that workers in the U.S. face an overall tax schedule that exhibits a considerable amount of curvature. In an empirical examination of the impact of taxes on the budget constraints of a representative sample of male workers in 1975 drawn from the PSID, Section II shows that the opportunity set of an average individual incorporates numerous kinks which are spread evenly over a wide range of possible hours of work. The differentiable approximation outlined in that section exhibits similar properties. On the basis of these findings, one would expect the inequality restrictions implicit in the use of maximum likelihood techniques to be a potentially important factor in estimation.

Section V confirms this suspicion. That section summarizes results from maximum likelihood estimation of several models of men's labor supply involving both piecewise-linear and differentiable budget constraints using the random sample of workers examined in Section II. Starting with a familiar baseline function of labor supply which is linear in wages and income, the analysis considers specifications that allow for heterogeneity from one of three sources: differences in income effects across people; differences in substitution effects; or differences in intercepts. In addition, this empirical analysis admits measurement error of more than one variety in hours of work. The empirical results obtained for all models strongly reflect the influence of the inequality restrictions invoked by the application of maximum likelihood in the respective context. From all appearances for the data set considered here, these restrictions become strictly binding constraints that essentially determine the

estimates of all effects. Accordingly, all substitution and income effects are effectively zero when estimated by maximum likelihood methods incorporating piecewise-linear tax schedules. One obtains precisely the same result in analyses incorporating differential schedules if one imposes parametric restrictions analogous to those inherent in the piecewise-linear approach. Not imposing the restrictions yields slightly different results in the differentiable case due to its less stringent inequality constraints. Ignoring the constraints implicit in all these methods produces estimates of uncompensated substitution effects that are small and often negative, and estimates of income effects that are always tiny and sometimes the wrong sign (i.e., positive). Such unrestricted estimates resemble those obtained in the empirical literature on men's labor supply that applies unsophisticated estimation approaches, including those approaches that ignore the presence of taxes altogether.

Consequently, these findings go a long way toward explaining the divergence in the estimates obtained by the various empirical methodologies. The evidence of Section V strongly suggests that applying econometric approaches incorporating piecewise-linear constraints to analyze men's labor supply produces more positive estimates of substitution effects and more negative estimates of income effects because of computational features inherent in the econometric procedure. In essence, these features come about because the piecewise-linear approaches rely on an underlying structural economic model in their statistical formulation and assume the validity of this model in the computation of estimates. This creates an empirical framework that requires the satisfaction of particular economic assumptions for its interpretation as a meaningful statistical description of the data. Unfortunately, the necessary economic assumptions appear not to hold for the types of empirical specifications entertained in the literature. In estimation, the piecewise-linear approaches impose these assumptions without regard to their validity, and this induces a predictable shift in parameter values that readily explains the discrepancies in estimates obtained by the various empirical methodologies.

Regardless of the estimation approach one decides to implement, obtaining reliable estimates of substitution and income effects associated with labor supply requires consideration of an empirical specification that obeys the Slutsky condition over a broad region of the wage-income space. Further, this specification must capture relevant behavioral features, such as a backward-bending property which evidence suggests is a factor in determining men's hours of work. No doubt, a major source of the restrictions operative in the empirical analysis presented in this paper and in other work arises from the simplicity of the functional form assumed in estimation. Linearity in substitution and income effects offers

little flexibility to describe labor supply behavior; it rules out negative uncompensated substitution effects when an estimation procedure imposes the Slutsky condition at a large number of kinks which map out a wide range of wage-income combinations. Introducing a more flexible specification of labor supply that admits both a backward bending property and broad satisfaction of the Slutsky condition offers a possible avenue for avoiding the sources of computational restrictions on estimates identified in this study. Unfortunately, our experiences in following this avenue left us somewhat pessimistic about the prospects of its success. In all of our attempts to estimate richer empirical specifications using maximum likelihood methods, we always encountered the imposition of binding parametric restrictions attributable to the computational features of these methods. In each instance, we could trace the constraint to preventing the violation of the Slutsky condition at one or more kink points in our sample. This finding probably comes as no surprise to those familiar with the empirical literature on men's hours of work. In empirical analyses assuming conventional static economic models of labor supply, "global" satisfaction of the Slutsky condition typically fails unless it is imposed.

There are, of course, many options available for enriching the empirical framework considered in this study. At the less ambitious end of the spectrum, one could expand the search for an empirical specification of labor supply that overcomes the problems cited above; or one could simply impose global quasi-concavity on preferences and estimate behavioral responses conditional on this assumption. At the more ambitious and promising end, one could advance beyond the static model of taxes and labor supply and incorporate such elements as life-cycle factors, multiperson households and uncertainty into the economic characterization. Incorporating such factors into an estimation framework with taxes will undoubtedly be a difficult task. Among the challenges, the issues addressed in this study will potentially increase in complexity in a more general approach.

Appendix

Description of the Main Data Set

This appendix presents details of the data set used in the body of this paper. The first section details the selection criteria used for choosing the sample, while the second describes the variables used in the estimation.

A. *The Sample*

The data used in the sample are from the University of Michigan Panel Study of Income Dynamics (PSID) for 1975 (interview year 1976). It should be noted that some questions in the PSID ask about current status rather than asking the respondent to recall his or her status in the previous year. In such cases, we considered responses in both the 1975 and 1976 interviews to capture changes that might have occurred in 1975. Recall from the main text that this investigation concentrates on prime-age married males. To construct a random sample of such individuals, the following rules were applied to the Wave IX tape to determine omission from the sample:

1) Observations with identification numbers greater than 4,999 since these observations are part of a nonrandom, low income sample;
2) Observations where the head of household was single, widowed, divorced or separated in 1975 (in Table A1 this is termed an invalid marriage variable);
3) Observations for which family composition had changed, except if the changes were in members other than head or wife;
4) Observations where the head's age was less than 25 or greater than 55 at the time of the 1975 interview;
5) Observations which had a female head of household.

Further criteria were applied to eliminate men with nonstandard work status:

6) Observations where the PSID classified the head as retired, permanently disabled, housewife, student, or other at either the 1975 or 1976 interview;
7) Observations from outside the U.S.;
8) Observations where the head was self-employed at either the 1975 or the 1976 interview;
9) Observations where the head was a farmer at either the 1975 or 1976 interview.

In addition one observation was deleted because the observed hours of work was 8,850 hours per year, which implied working more than 24 hours per day.

The order in which the criteria were applied and their effects in terms of the number of observations deleted using each criterion is given in Table A1. Row 15 of the table emphasizes the fact that once all the above criteria were applied the resulting sample contained only workers.

Table A1

Selection Criteria	Number of Observations Deleted
1) Observations from the non-random low income sample	2,544
2) Observations with an invalid marriage variable	1,071
3) Observations with a change in family composition	185
4) Observations with head's age less than 25	229
5) Observations with head's age greater than 55	464
6) Observations where head is retired, permanently disabled, housewife, student or other at 1976 interview	44
7) Observations from outside the U.S.	4
8) Observations with female head of household	7
9) Observations where head was retired, permanently disabled, housewife, student or other at 1975 interview	15
10) An observation with an unreasonably high number of hours worked	1
11) Observations where head was self-employed at 1975 interview	202
12) Observations where head was self-employed at 1976 interview	68
13) Observations where head was a farmer at 1975 interview	5
14) Observations where head was a farmer at 1976 interview	6
15) Observations with zero hours worked	0
Total Observations Deleted	4,845
Total Number of Candidate Observations	5,862
Remaining Sample	1,017

B. The Variables

Table A2 presents summary statistics for the variables used in the empirical analysis in this study. The three central variables in this analysis are the 1975 wage rate, hours of work and non-earned income for each person

Table A2
Characteristics of the Main Data Set

| | First Data Set | |
	Mean	Standard Deviation
Wage	6.89	3.34
Income	3,713.78	4,687.84
Hours	2,236	544
KIDSU6	0.49	0.73
FAMSIZ	3.78	1.47
AGE45	1.39	2.72
HOUSEQ	17,892	20,592
BHLTH	0.6	0.22
Number of Observations	1,017	

in the sample. The wage variable is the average hourly earnings of the head reported for 1975 in the PSID; it is calculated there by dividing total labor earnings in 1975 by total hours worked for money in 1975. Total hours worked for money in 1975 also serves as the hours of work variable used in all estimations. The nonearned income variable is formed by subtracting the total labor income of the head from the total 1975 taxable money income of the head and his spouse. Both income and wages are measured in dollars for purposes of the estimation.

There are also five "taste shifter" variables used in estimation which, apart from a constant, form the z vector in the text. The variables are: the number of children less than six years old (KIDSU6), a variable taken from the 1975 interview; the number of people in the family unit (FAM-SIZ), also taken from the 1975 interview; the amount of equity the family had in their house measured in dollars (HOUSEQ), obtained by subtracting the remaining mortgage principal in 1976 from the value of the house in 1976; an age variable taking a value of zero if the man was less than 45 years of age in 1975 and (age − 45) if he was older than 45 (AGE45); and a health variable which takes a value of one if, at the 1976 interview, the person reported having a physical or nervous condition that limited the amount of work he could do, and a value of zero otherwise (BHLTH).

Appendix B

Likelihood Functions with Piecewise-Linear Constraints

This appendix presents specifications of the likelihood functions used in our empirical analysis of labor supply in the presence of piecewise-linear budget sets. The main focus of this discussion is on the random income-coefficient model which involves features that do not completely conform to the simple framework outlined in Section III. Specifically, the model under consideration here consists of Relation (5.4) for the labor supply function with the densities f_ν and f_ε representing the distributional properties of heterogeneity and measurement error. For concreteness, suppose that Figure 1 describes the budget set faced by an individual with a modification introduced so that initial virtual income (i.e., y_1), need not be positive.

The derivation of the likelihood function associated with the random income-coefficient model requires the consideration of three distinct cases: $y_1 > 0$; $y_1 = 0$; and $y_1 < 0$. While the first of these cases falls entirely within the framework of Section III, the latter two cases do not, primarily due to the fact that the labor supply function $h^s(w, y, \nu)$ is not monotonically increasing in the heterogeneity component ν (i.e., $\partial h^s/\partial \nu = 0$ for some ν when $y_1 = 0$, and $\partial h^s/\partial \nu < 0$ is possible when $y_1 < 0$). The appendix begins by considering the simpler case $y_1 > 0$, and then proceeds sequentially to consider the more complex cases $y_1 = 0$ and $y_1 < 0$. The analysis of the case $y_1 > 0$ encompasses all of the essential ideas needed to specify the likelihood functions associated with either the random intercept or the random substitution-effect model; so, these models are not discussed here.

A. Positive Initial Virtual Income

Specifying the likelihood function for the random income-coefficient model for the case $y_1 > 0$ follows directly from a straightforward translation of the analysis presented in Section III with $\nu = \beta$. Solving for the boundary values v_i^L and v_i^U, one obtains

(B.1) $v_i^L = (\bar{h}_i - \theta - z\gamma - \alpha w_{i-1})/y_{i-1}$ $i = 2, 4, 6$

$v_i^U = (\bar{h}_i - \theta - z\gamma - \alpha w_{i+1})/y_{i+1}$ $i = 0, 2, 4$

with $v_0^L = \underline{v} = -\infty$ and $v_6^U = \bar{v} = 0$. If $v_{j-1}^U < 0$ and $v_{j+1}^L > 0$ for any j, then hours are truncated from above on the jth face at the point $\nu = 0$, and faces and kinks beyond j (i.e., to the left of j in Figure 1) are ignored. If $v_j^L < 0$ and $v_j^U > 0$ at any kink j, then v_j^U is set equal to zero, and all faces and kinks beyond j are ignored.

Introducing linear measurement error given by (5.9) with $f_\varepsilon(\varepsilon) = f_\varepsilon(\varepsilon\,|\,h)$ denoting the density of ε, the density $f_H(H\,|\,h)$ appearing in (3.17) equals $f_\varepsilon(H - \theta - z\gamma - \alpha w_i - v y_i)$ on faces and $f_\varepsilon(H - \bar{h})$ at kinks. The implied likelihood function corresponding to (3.18) is given by

$$(\text{B.2}) \quad l_H(H) = \sum_{i=2,4,6} f_\varepsilon(H - \bar{h}_i)[F_v(v_i^U) - F_v(v_i^L)]$$

$$+ \sum_{i=1,3,5} \int_{v_{i-1}^U}^{v_{i+1}^L} f_\varepsilon(H - \theta - z\gamma - \alpha w_i - v y_i) f_v(v) \, dv,$$

presuming that $\Pr(\delta_H = 0) = \Pr(H = 0)$ equals zero—which implies that the probability of not working is negligible. The notation F_v in (B.2) denotes the cumulative distribution function associated with f_v.

If the densities f_v and f_ε are members of the normal or the truncated-normal families, then one can readily evaluate the integrals

$$(\text{B.3}) \quad \int_{v_{i-1}^U}^{v_{i+1}^L} f_\varepsilon(\varepsilon\,|\,v) f_v(v) \, dv$$

appearing in (B.2) by completing the square which decomposes these integrals into products of univariate cumulative normal distribution functions and a normal density function (e.g., see Zellner (1971, Chapter 3) or Blomquist (1983, Appendix B)). To consider alternative distributional assumptions and avoid the need for numerical integration of (B.3), we also estimated variants of f_v falling into the class of mixtures of truncated normals. Members of this class take the form

$$f_v(v) = \sum_{m=1}^{M} \lambda_m f_{vm}(v)$$

where the λ_m's are weights between zero and one that add up to one, and the f_{vm} are variants of truncated normal distributions. In this case, the integral

$$\int_{v_{i-1}^U}^{v_{i+1}^L} \left\{ \sum_{m=1}^{M} \lambda_m f_\varepsilon(\varepsilon\,|\,v) f_{vm}(v) \right\} dv$$

reduces to

$$\sum_{m=1}^{M} \lambda_m \left\{ \int_{v_{i-1}^U}^{v_{i+1}^L} f_\varepsilon(\varepsilon\,|\,v) f_{vm}(v) \, dv \right\},$$

which can be readily computed using the procedure outlined above.

B. Zero Initial Virtual Income

Now consider the specification of the likelihood function for the random income-coefficient model for the case $y_1 = 0$ in Figure 1 with $y_5 > y_3 > 0$. Analysis of this case depends critically on whether the following condition holds:

(B.4) $\bar{h}_2 - \theta - z\gamma - \alpha w_1 < 0.$

If (B.4) holds, then an individual will not locate on segment 1 or at kink 0. In other words, the preferences implied by the random income-coefficient model dictate that $h \geq \bar{h}_2$ which signifies an optimum on the budget constraint in Figure 1 on or to the left of kink 2. Under these circumstances, the boundary values v_i^L, $i = 4, 6$, and v_i^U, $i = 2, 4$, are given by formula (B.1) with $v_2^L = \underline{v} = -\infty$. Specification (B.2) represents the likelihood function implied in this instance with segment 1 eliminated from consideration.

If (B.4) does not hold, then an individual sets hours equal to the value $\theta + z\gamma + \alpha w_1$ (≥ 0) with probability one. Under these circumstances, the appropriate specification for the likelihood function becomes simply

(B.5) $l_H(H) = f_\varepsilon(H - \theta - z\gamma - \alpha w_1).$

According to the implied preference structure, true hours h are set deterministically equal to the quantity $\theta + z\gamma + \alpha w_1$, and the only source of variability in H is due to measurement error.

C. Negative Initial Virtual Income

Finally, consider the case in which $y_1 < 0$; in particular, assume that $y_1 < y_3 < 0$ and $y_5 > 0$ in Figure 1. Analysis of this case is the most complex of all, for it varies depending on which of the following three sets of conditions apply:

(B.6)   (a) $\bar{h}_4 - \theta - z\gamma - \alpha w_5 < 0$

   (b) $\bar{h}_4 - \theta - z\gamma - \alpha w_3 > 0$

   (c) $\bar{h}_4 - \theta - z\gamma - \alpha w_5 > 0$

     $\bar{h}_4 - \theta - z\gamma - \alpha w_3 < 0.$

Observing that $\bar{h}_4 - \theta - z\gamma - \alpha w_5 > \bar{h}_4 - \theta - z\gamma - \alpha w_3$, one sees that (a), (b), (c) are mutually exclusive and exhaustive. The conditions listed in (B.6) involve evaluations around kink 4 which corresponds to the right-side border of the first budget segment associated a positive virtual income (i.e., $y_5 > 0$ and $y_3 < 0$).

If (B.6.a) holds, then the maximization of utility implied by the random income-coefficient model leads to a selection of hours on or to the left of kink 4; so $h \geq \bar{h}_4$. Specification (B.2) gives the likelihood function under these circumstances with segments 1 and 3 and kinks 0 and 2 eliminated. In this revised specification of (B.2), the boundary values v_6^L and v_4^U are determined by formula (B.1), and the remaining boundary values are given by $v_4^L = \underline{v} = -\infty$ and $v_6^U = \bar{v} = 0$.

If, on the other hand, condition (B.6.b) holds, then the selection of hours occurs on or to the right of kink 4; so $h \leq \bar{h}_4$. Further, due to the nonnegativity constraint imposed on consumption goods C, the utility-maximized choice of hours will not fall below the value $\bar{h}_{\min}$ which corresponds to that level of hours on the budget constraint at which $C = 0$. For concreteness, suppose that $h_{\min}$ lies on segment 1 of the budget set.

Interpreting the heterogeneity component as $u \equiv -\beta$ rather than as β under these circumstances permits the direct application of the analysis in Section III with hours restricted in the range $\bar{h}_{\min} \leq h \leq \bar{h}_4$. To maintain consistency with the notation of this appendix, we continue to interpret $v = \beta$; so $u = -v$. The variable u plays the role of the heterogeneity component introduced in Section III rather than v because it satisfies the properties assumed in that discussion (i.e., the relevant budget set is convex under these circumstances and $\partial h^s / \partial u > 0$ over the implied range of hours). Define $-u^* = v^* = (\bar{h}_{\min} - \theta - z\gamma - \alpha w_1)/y_1$ as the boundary value associated with the new kink $h_{\min}$ representing the minimal admissible hours.

Further define the bounds

$$(\text{B.7}) \quad u_i^L = (\bar{h}_i - \theta - z\gamma - \alpha w_{i-1})/(-y_{i-1}) \quad i = 2, 4$$

$$u_i^U = (\bar{h}_i - \theta - z\gamma - \alpha w_{i+1})/(-y_{i+1}) \quad i = 2$$

with $u_4^U = -\underline{v} = \infty$ and $\bar{v} = 0$. (Note that the sample space of u is $(-\bar{v}, -\underline{v} = (0, \infty).)$ Referring to (B.1), one sees that $v_i^L = -u_i^L$, $i = 2, 4$, and $v_i^U = -u_i^U$, $i = 2$. Further, note that

$$(\text{B.8}) \quad \int_a^b f_\varepsilon(\varepsilon \mid u) f_u(u) \, du = \int_{-b}^{-a} f_\varepsilon(\varepsilon \mid v) f_v(v) \, dv.$$

Depending on the values of parameters and variables, an individual's hours can be truncated anywhere between $\bar{h}_{\min}$ and $\bar{h}_4$. When $v^* < 0$, the admissible range of hours is $(\bar{h}_{\min}, \bar{h}_4)$. Under these circumstances, the implied likelihood function corresponding to (3.18) is given by

(B.9) $l_H(H) = f_\varepsilon(H - \bar{h}_{\min})[F_\nu(\bar{\nu}) - F_\nu(\nu^*)]$

$$+ \sum_{i=2,4} f_\varepsilon(H - \bar{h}_i)[F_\nu(\nu_i^L) - F_\nu(\nu_i^U)]$$

$$+ \sum_{i=1,3} \int_{\nu_{i+1}^L}^{\nu_{i-1}^U} f_\varepsilon(H - \theta - z\gamma - \alpha w_i - \nu y_i) f_\nu(\nu) \, d\nu,$$

where $\nu_4^U = \underline{\nu}$ and $\nu_0^U = \nu^*$ assuming $\nu^* < 0$. When $\nu^* > 0$ and $\nu_2^L < 0$, the admissible range of hours is $(\theta + z\gamma + \alpha w_1, \bar{h}_4)$, with truncation occurring on segment 1. Eliminating consideration of the mass point $\bar{h}_{\min}$ in (B.9) and replacing ν_0^U by $\bar{\nu}$ gives the formula for the likelihood function appropriate for estimation under these circumstances. Other instances of truncation (i.e., at kink 2 or on segment 3) can be handled with analogous modifications.

Finally, consider the third case in which conditions (B.6.c) hold. When this situation arises, utility maximization with the random income-coefficient model implies that an individual locates only at kink point 4 in the modified version of Figure 1 with $y_1 < y_3 < 0 < y_5$. Thus, $h = \bar{h}_4$ with probability one, and the likelihood function for this case becomes simply

(B.10) $l_H = f_\varepsilon(H - \bar{h}_4)$.

Thus, in summary, if condition (B.6.a) holds, the specification of the likelihood function is given by a variant of (B.2). If condition (B.6.b) holds a variant of (B.9) provides the appropriate specification. If conditions (B.6.c) obtain, relation (B.10) describes the implied likelihood function.

Appendix C

Estimation and Results Using the Piecewise Linear Approach

This appendix describes the estimations of specifications associated with the piecewise-linear methodology and presents complete sets of results. The following discussion considers five cases: the constrained and unconstrained random intercept cases; the random substitution-effect case; and two random income-effect cases, with the second case estimated using an alternative construction of the data set which is designed to provide a more favorable basis for replicating some of the prominent results in the literature.

In the results presented here, constrained estimation means that the condition $\Pr(\delta_i = 1) \geq 0$ is imposed at all feasible interior kink points. Satisfaction of this condition often requires the imposition of a constraint

on at least one parameter. When constraints are encountered in carrying out estimation, the method used here fixes the appropriate parameter(s) at the constraint and optimizes the function over the remaining parameters. The condition is checked before each iteration and the value of constraints recalculated so that the constraints may be turned on and off or altered. Such repeated checking is necessary due to the fact that the exact value of a constraint can change according to the values of other parameters and according to the specific kinks where the nonnegativity constraint binds.[48]

The shifting nature of constraints, however, also means that the path to an optimum can be highly idiosyncratic. Changing the initial parameter values imply different points at which constraints first become binding and, given that successive optimization depends on the locations of these points, somewhat different optima can obtain. Adding in the fact that some parameters moved away from constraints and then back to new constrained values, it is easy to see why optimization paths could be tough to duplicate. In addition, Monte Carlo experiments performed to verify accurate programming of likelihood functions suggest that the sample size used in this empirical work (i.e., 1,017 observations) is too small to produce reliable estimates of parameters for some specifications, especially those involving random income effects with truncation occurring far in the tails.[49] This does not mean that results presented in this paper ought

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48. As demonstrated in the text, imposing the condition is equivalent to checking at each kink the ordering of the bounds on the set of values of v that can generate that kink. In practice, the likelihood function is programmed so that if the condition is violated, the function routine returns a low function value to discourage continuing in that direction. Despite this penalty, some parameter values continued to move in a direction that implied violation of the condition at some kink for some person. To handle that problem, when a parameter ran into a constraint, the direction vector determining the next step in the optimization path was purged of the effect of the offending parameter so that the next step in the optimization was carried out as though that parameter was fixed at the constraint.

To see how this was done, note that the optimization program solves for the direction vector, DX, using the system

$$HX\ DX\ =\ GX,$$

where HX is a negative definite matrix chosen according to the algorithm used, and GX is the first partial vector. The DX vector is then used to update the parameter vector, X_0, according to the formula: $X_1 = X_0 + STP \cdot DX$, where STP is the chosen step size. If a parameter hits a constraint, then the corresponding element of GX is set to zero and the corresponding row and column of HX are set to zero, except for a negative one placed on the diagonal. As a result, the DX vector has the same form that it would take if that parameter were fixed at the constraint.

49. In particular, in those cases with $v = \beta$, our Monte Carlo results indicate that maximum likelihood estimates of μ_v and σ_v tend to be relatively precise for sample sizes of 1,000 when

to be discounted, only that perfect replication of them might be difficult. Repeated estimations of each of the cases presented below were started at a variety of initial parameter values and each produced essentially the same findings reported in Table 2.

A. Random Intercept Model

The likelihood function estimated in the random intercept cases takes a form similar to the specifications given in Appendix B. In the constrained estimation case, the wage coefficient moved quickly to a constraint and remained there throughout estimation. Given the binding nature of this constraint, a dash instead of a standard error is reported below the estimate of α in column one of Table C1. The estimate of β encounters no constraint.

Table C1 includes estimated values for coefficients associated with the individual "taste shifter" variables: something not included in Table 2. According to Column 1 of Table C1, which reports results for the constrained linear heterogeneity case, the estimates of these coefficients are generally of the expected sign but are not very precisely estimated. Specifically, the number of children under six has a positive effect, being older has a negative effect, and bad health has a negative effect on hours. While all of these are as expected, each implies a relatively small absolute effect; for example, an extra child under six years old causes a man to work only 11 extra hours in a year. The negative sign on family size and the positive sign on house equity are in the opposite direction from what one would predict on the basis of a naive static model of labor supply, but neither is significantly different from zero.

The likelihood function for the unconstrained case is the same as the one used in the constrained case, except now the condition ($\Pr(\delta_i = 1) \geq 0$) is not imposed at interior kink points; only $l_H > 0$ is required. The estimation did not induce any binding constraints determining parameters. The results, given in the second column of Table C1, are quite similar to those in the constrained case with one important exception: the wage coefficient shifts from zero to a statistically significant negative value.

$\mu_v/\sigma_v = 2$, but not when $\mu_v/\sigma_v \geq 4$. To obtain reliable estimates when $\mu_v/\sigma_v = 4$, we required sample sizes around 4,000 observations. Since the estimates for μ_v and σ_v reported in the paper encounter the constraint $\mu_v/\sigma_v = 6$ (i.e., truncation cannot occur more than 6 standard deviations in the tail), there is reason to suspect that the sample of 1,017 observations used in this study is too small to place much credence in the estimates.

Table C1
Complete Results From Estimations Using the Piecewise-Linear Approach[a]

| | Models: Source of Heterogeneity | | | | |
| | $\nu = \theta$ | | $\nu = \alpha$ | $\nu = \beta$ | |
Coefficients	Constrained	Unconstrained		Main Data Set	Alternative Data Set
CONSTANT	—	—	2,314.1	2,319.1	2,225.0
			(107.4)	(111.9)	(100.7)
KIDSU6	11.3	−2.6	11.8	11.6	−5.8
	(49.7)	(47.4)	(49.7)	(49.9)	(50.6)
FAMSIZ	−10.7	−1.0	−10.7	−11.2	6.1
	(20.6)	(15.4)	(20.6)	(20.6)	(21.7)
AGE45	−22.7	−19.2	−22.6	−22.5	−19.3
	(14.7)	(13.8)	(14.8)	(14.8)	(17.4)

HOUSEQ	0.0018 (0.0013)	0.004 (0.0008)	0.0018 (0.0013)	0.0018 (0.0013)	0.0031 (0.0012)
BHLTH	−150.8 (172.5)	−192.5 (161.1)	−151.1 (172.5)	−153.2 (173.4)	−208.3 (199.8)
α	0.00003 (—)	−79.2 (16.8)	—	0.00001 (—)	0.0001 (—)
β	−0.0071 (0.0075)	−0.0080 (0.0072)	−0.0072 (0.0075)	—	—
μ_ν	2,313.8 (107.4)	2,610.2 (112.8)	0.00005 (0.0231)	0.296 (0.321)	0.796 (0.613)
σ_ν	1.0 (—)	1.0 (—)	0.0000002 (0.0231)	0.0493 (—)	0.133 (—)
σ_ε	538.2 (16.4)	524.2 (17.2)	538.3 (30.5)	536.4 (35.7)	587.7 (32.6)
Average Value of Log-Likelihood Function	−0.7996	−0.7725	−0.7995	−0.7998	−0.8946

a. Standard errors in parentheses.

B. Random Substitution Effect Model

Estimation of this model turned out to be no more complicated than the models incorporating random intercepts. The specification for the distribution of α becomes a virtual spike at a small positive value; almost no truncation occurs.

Column 3 of Table C1 presents complete results for this case. The estimates associated with the effects of "taste shifter" variables, wages and income are essentially identical to those obtained in the constrained random intercept case (column 1).

C. Random Income Effect Case

The first estimation of the random income-coefficient specification uses the same sample and set of variables as in the cases described above. (See Appendix A for a description of this data set.) There are several noteworthy points concerning experiences involved in the estimation of this specification. First, the wage coefficient moved quickly to the nonnegative constraint and stayed there throughout the optimization. Second, in the course of estimation the mean of the untruncated normal distribution associated with truncated distribution of β became steadily larger, while the variance of the untruncated distribution shrank. This resulted in evaluations far out in the tail of the untruncated normal. Eventually, the process had to be stopped because the normal cumulative and density routines on the computer could no longer provide meaningful evaluations that far in the tail. To avoid such difficulties, we restricted the ratio of the mean and the standard deviation of the untruncated distribution not to exceed six. This constraint was encountered early in the estimation and remained binding throughout.

This behavior of the truncated normal suggests the appropriate density of β is one that stacks up just below zero. To explore this possibility, a mixture of truncated normals was tried in an attempt to allow for a concentration near zero without imposing constraints of the sort just mentioned. The likelihood function associated with a mixture of truncated normals is set out in Appendix B. Estimation was carried out with the weights on the various distributions both fixed and estimated, and with various combinations of spiked distributions and degrees of truncation in normal densities. Invariably, either all the weight went to a single distribution which replicated the problem described above, or all the distributions moved such that the mean came to be six times larger than the standard deviation.

Column 4 of Table C1 presents the full results for the random income coefficient case estimated using the data set analyzed above. As in the

linear heterogeneity cases, the coefficients on the children-under-6, age and health variables are of the expected sign while those on family size and house equity are not. As in those cases, the "taste shifter" effects are not precisely estimated, nor are they large in absolute value. No standard errors are reported for the wage coefficient or for the standard deviation of the untruncated income-coefficient distribution because both ended at constrained values.

D. Random Income Effect Case—Use of An Alternative Data Set

A comparison of the results obtained above for the random β specification with those found in Hausman (1981a) shows a large discrepancy in the size of the estimated income effect.[50] This difference seems somewhat perplexing in light of the facts that both data sets are drawn from the same source and that the estimation approaches are the same. While there may be several possible explanations for the difference, one that seems likely stems from noncomparable sample selection criteria and variable definitions.[51] To examine such a possibility, the model was re-estimated using several data sets. In all instances the results closely resembled those presented in this paper.

As one example, consider the results using a data set that we designed to come closer to the data used in Hausman (1981a). This alternative data construction differs from the one used above in two ways. First, the sample selection criteria are different from those set out in Appendix A, primarily in that any criteria taken from the 1975 interview are not used. The resulting sample, which contains 1,100 observations, is closer in size to the 1,085 observations in Hausman's sample. One notable difference between the alternative data set constructed here and the one described in Hausman (1981a) is that 0.5 percent of Hausman's sample are nonworkers, while our sample contains only workers. This is a conscious choice on our part: we deleted three individuals in constructing our alternative data set for having zero hours of work.

The second difference between our main data set and our alternative data set involves the calculation of the wage and nonlabor-income variables. Our inference from Hausman (1981a) is that the wage variable used there is not average hourly earnings. The other possibilities in the PSID

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50. Hausman (1981a) reports a median of the β distribution equal to -0.12, while Column 4 of Table D1 reports estimates implying a median of -0.006.

51. The data set used in Hausman (1981a) contained 1,085 observations and the mean of the wage variable and nonlabor income variable were $6.18 and $1,266, respectively. The data set used here had 1,017 observations, with the mean of the wage being $6.89 and the mean of non-labor income being $3,714.

are two reported wage values: one from the 1975 interview and the other from the 1976 interview. The 1975 wage value suffers from the fact that it only pertains to wage earners, leaving 52.5 percent of the sample without data on wages. Thus, in the alternative data set we use the 1976 value, which is constructed as a combination of the directly reported wages of hourly-wage workers and the imputed hourly earnings of salaried workers. The mean of this 1976 wage variable is $6.57, which translates into $6.21 when converted into 1975 values using the CPI; Hausman (1981a) reports an average of $6.18 for wages. The use of this wage variable in our labor supply analysis is somewhat troubling because the relevant question in the PSID interview asks about wages at the time of the interview in 1976, rather than asking about the respondent's wages in 1975 which represents the year in which hours of work are observed.

There is also a difference in the nonlabor income variable used in the alternative data set. Hausman (1981a) reports using an imputed income variable based on assigning an 8 percent return to financial assets. We found no financial asset information in the PSID. We calculated an income variable as the taxable income of the head and his wife in 1975 minus the labor income of both the head and his wife in 1975. The resulting quantity represents the sum of the five categories of asset income in the PSID. The mean of this new income variable is $736. This value falls well below the $3,714 mean for the nonlabor income variable in our main data set, but is still not as close as we might wish to the $1,266 reported in Hausman (1981a) as the mean of his nonlabor income variable.

The "taste shifter" variables used in earlier specifications were formulated exactly as in Hausman (1981a), so they were not changed in the construction of the alternative data set.

The estimation results, set out in Column 6 of Table C1, are remarkably similar to the random income-coefficient results described in Section C in terms of their implications for substitution of income effects. The wage coefficient hits a constraint at zero, and the ratio of the mean to the standard deviation of the untruncated income coefficient distribution hits the constraint at six. The medians of the β distributions implied by the estimates obtained using our main data set and our alternative data set are -0.006 and -0.015, respectively. Both medians imply trivial income effects and are at least one order of magnitude below the estimate reported in Hausman (1981a).

Appendix D

Likelihood Functions with Differentiable Constraints

This appendix presents the likelihood functions for the specifications that involve a differentiable budget constraint. There are three such functions; all involve the random intercept heterogeneity specification, but differ in the form of measurement error.

A. No Measurement Error Case

There are several preliminary steps in formulating the likelihood function value for the case without measurement error. Taxable income at the observed level of hours, (H), is calculated using the person's pretax wage, (W), and nonearned income, (Y), along with standard deductions and exemptions. Taxable income is used in conjunction with a differentiable tax function to arrive at values for total taxes paid, $(T(Y, W, H))$, the marginal tax rate, $(\tau(Y, W, H))$, and the second derivative of taxes, $(\partial\tau/\partial E)$, which is also evaluated at Y and WH. With these, one can form the net wage, $w(H)$, and virtual income, $y(H)$.

Combining these elements allows one to calculate the likelihood function

$$(D.1) \quad g_h(H) = \frac{dv^s}{dH} \cdot \phi((H - \mu_v - z\gamma - \alpha w - \beta y)/\sigma_v)/\sigma_v$$

where $\phi(\cdot)$ is the standardized normal density, and the Jacobian is given by

$$(D.2) \quad \frac{dv^s}{dH} = 1.0 + (\alpha - \beta H)\frac{\partial\tau}{\partial E} \cdot W^2.$$

Note that w, y, and $\partial\tau/\partial E$ are all functions of H. Assuming the probability of working is one, specifications (D.1) and (D.2) represent the likelihood function $l_H(H)$ used in estimation with no measurement error.

B. Additive Measurement Error Case

To obtain the specification of the likelihood function appropriate for analyzing the case with additive measurement error, the procedure set out above is repeated for each level of desired hours for each person. Thus, H in (D.1) and (D.2) is replaced everywhere by h. With the additive measurement-error model given by (5.9), specification (4.8) becomes

(D.3) $l_H(H) = \int_0^{5,840} (\phi((H - h)/\sigma_\varepsilon)/\sigma_\varepsilon)g_h(h) \, dh,$

where $g_h(\cdot)$ is given by (D.1) and (D.2). Actual calculation of each individual's $l_H(H)$ is carried out using numerical integration programs.

C. Multiplicative Measurement Error Case

The specification of $l_H(H)$ for the multiplicative measurement error case differs from the one for the additive error case in two ways. First, g_h must be reformulated to take account of error in the wages. The appropriate formulation is now

(D.4) $g_h(h) = \dfrac{dv^s}{dh} \cdot \phi((h - \mu_v - z\gamma - \alpha w - \beta y)/\sigma_v)/\sigma_v,$

where $w = (1 - \tau)(E/h)$, $y = Y + \tau E - T$ and the Jacobian is

(D.5) $\dfrac{dv^s}{dh} = 1 + (\alpha - \beta h) \dfrac{\partial \tau}{\partial E} \cdot (E/h)^2.$

The functions τ, T and $\partial\tau/\partial E$ are evaluated at Y and E. Second, the measurement error density is now:

(D.6) $f_\varepsilon(\varepsilon) = \phi\left(\dfrac{(\ln H - \ln h) + \sigma_\varepsilon^2/2}{\sigma_\varepsilon}\right)\Big/ \sigma_\varepsilon.$

Based on (D.4)–(D.6), an individual's $l_H(H)$ for the multiplicative-measurement-error case becomes:

(D.7) $l_H(H) = \int_0^{5,840} \left(\phi\left(\dfrac{(\ln H - \ln h) + \sigma_\varepsilon^2/2}{\sigma_\varepsilon}\right)\Big/ \sigma_\varepsilon\right)gh(h) \, dh.$

Appendix E

Estimation and Results Using the Differentiable Budget Constraint Approach

This appendix presents results and details of estimation for the cases involving differentiable budget constraints. There are eight cases in this category; all use the random intercept specification of heterogeneity, but each differs according to the form of measurement error and the constraints imposed on parameters. All the subsequent results are derived using the data set described in Appendix A.

The budget constraints used in all eight cases are built by first fitting a differentiable function to the federal tax schedule, as described in the text.[52] Combining these fitted functions with each person's before-tax wage and nonlabor income produces a differentiable budget constraint which accounts for federal income taxes. Each person is given a deduction of $1,900 regardless of income and allowed the nonrefundable $750 exemption for each member of his family. Given these deductions and exemptions along with pretax wages and income, it is a simple matter to form taxable income for any number of hours worked and use that in conjunction with the differentiable tax function to calculate after-tax wages and incomes. Similar functions could be fitted to the state income tax, EIC and FICA schedules individually and then added together with the federal income tax function to form a complete differentiable tax function.[53] For reasons of time and simplicity, this was not done. The budget constraints formulated with the federal schedule alone are sufficient for investigating the main points of the paper.

The eight cases can be broken down according to the type of constraints imposed in estimation. Cases termed "Slutsky-constrained" involve the imposition of the constraints: $\alpha > 0$, $\beta < 0$. These constraints are invoked to aid in comparison with the piecewise-linear constrained cases. "Density-constrained" cases are estimated with the requirement that $l_h(h)$ must be positive over the relevant range of desired hours for each person. Operationally, the Jacobian described in Equation (4.4) is forced to be positive at all points of evaluation in the application of numerical integration routines. This, in turn, implies constraints on the substitution and income effects. Finally, in "unconstrained" cases only the requirement $l_H(H) > 0$ applies in estimation. Appendix D presents the specifications of $l_h(h)$ and $l_H(H)$ for the various cases.

A. No-Measurement-Error Cases

There are two cases considered that do not admit measurement error: unconstrained and Slutsky-constrained. For these models, $l_H(H) = l_h(h)$ and $h = H$, so there is no distinction between density-constrained and unconstrained estimation.

Column 1 of Table E1 presents results for the unconstrained case.

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52. An appendix is available upon request from Tom MaCurdy if one desires detailed information concerning the formulation of the tax function which is used in this empirical analysis. This material is contained in "Appendix F" of the version of this paper distributed in working-paper series.

53. While it is a straightforward task to create a differentiable function that accounts for all taxes in this manner, the best way to ensure convexity of this function is an open question.

Table E1
Complete Results From Estimations Involving Differentiable Budget Constraints[a]

Coefficients	Measurement Error Formulation						
	None		Additive			Multiplicative	
	Unconstrained	Slutsky Constrained	Unconstrained	Slutsky Constrained	Density Constrained	Unconstrained	Density Constrained
KIDSU6	23.4	21.1	25.4	20.5	27.9	18.0	−36.9
	(45.0)	(48.8)	(49.0)	(47.9)	(48.3)	(56.1)	(70.6)
FAMSIZ	0.8	−7.1	10.0	−6.8	−2.0	10.9	44.9
	(18.2)	(20.3)	(19.7)	(20.4)	(20.1)	(21.7)	(25.8)
AGE45	−14.1	−20.7	−10.8	−20.8	−19.0	−8.1	−0.9
	(13.3)	(14.4)	(12.8)	(14.3)	(14.3)	(13.2)	(17.4)
HOUSEQ	0.0027	0.0015	0.0027	0.0017	0.0011	0.0030	0.0013
	(0.0009)	(0.0013)	(0.0009)	(0.0013)	(0.0012)	(0.0013)	(0.0019)

BHLTH	−183.8	−151.1	−193.5	−152.2	−150.9	−188.5	−233.0
	(154.6)	(174.6)	(162.1)	(171.6)	(175.8)	(148.6)	(208.8)
α	−86.8	0.001	−105.6	0.001	−2.2	−102.1	−65.2
	(20.0)	(—)	(14.1)	(—)	(—)	(15.2)	(—)
β	0.0138	−0.000001	0.0209	−0.000001	0.0045	0.0207	0.0105
	(0.0061)	(—)	(0.0045)	(—)	(0.0058)	(0.0060)	(0.0019)
μ_ν	2,519.9	2,262.6	2,531.3	2,257.2	2,226.3	2,455.7	2,406.7
	(112.7)	(89.6)	(108.1)	(88.7)	(87.9)	(126.4)	(115.8)
σ_ν	493.6	539.2	158.8	160.0	160.0	361.9	508.9
	(33.6)	(30.3)	(78.9)	(—)	(—)	(104.8)	(63.2)
σ_ε			508.8	509.7	520.2	206.2	162.4
			(37.7)	(16.8)	(17.3)	(138.1)	(71.0)
Average Value of Log-Likelihood Function	−0.772	−.801	6.159	6.003	6.108	−0.726	−0.784

a. Standard errors in parentheses.

The wage effect is negative and the income effect is positive, implying that the Slutsky condition is violated at all points. The estimated effects of the "taste shifter" variables are of the expected sign except for HOUSEQ, which suggests that more wealth in the form of equity in a house has a positive effect on hours worked. As in the piecewise-linear cases, the taste-shifter effects are generally imprecisely estimated and, with the exception of the poor health measure, are small in size.

Column 2 of Table E1 reports findings for the Slutsky-constrained case. Substitution and income effect estimates both run into the zero constraint and stay there. This and the change of sign of the FAMSIZ effect are the only notable differences in a comparison with the unconstrained case.

B. Additive-Measurement-Error Cases

The specifications associated with additive measurement error are the most comparable to the random intercept models incorporating piece-wise-linear budget sets; all share similar structure and forms of measurement error. The Slutsky-constrained specification imposes restrictions analogous to those invoked in the constrained piecewise-linear case. Comparing the estimates obtained for this specification reported in column 3 of Table E1 to those presented in column 1 of Table C1 corresponding to the constrained piecewise-linear case reveals similar findings. Results obtained for the unconstrained specification in Table E1, on the other hand, differ from their counterparts obtained using the piecewise-linear methodology in the sign on the income effect.

The density-constrained case has no direct counterparts among the piecewise-linear results. Inspection of the estimates given in Column 5 of Table E1 reveals that imposition of the constraint on the Jacobian creates a binding constraint on the substitution effect. The estimated income effect has a positive sign but is not constrained. The results for the "taste shifter" variables are very similar to those obtained in other differentiable budget constraint estimations.

C. Multiplicative-Measurement-Error Cases

The first set of results involving multiplicative measurement error listed in column 6 of Table E1 invokes no constraints. Changing the structure of measurement error from the additive to the multiplicative form does not change the sign or size of the estimated wage, income or "taste shifter" effects. The estimates of the standard deviations obtained for the two structures show substantial discrepancies, but these parameters capture different effects in the two models. The density-constrained results are much less compatible across the additive and the multiplicative struc-

tures, although this is not so surprising since the inequality restrictions on parameters implied by positivity of the Jacobian terms associated with these two specifications are different. While the substitution effect encounters a binding constraint according to the density-constrained results, it is negative in sign. The income effect is positive. Consequently, the Slutsky condition fails to hold at all points.

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