ASSESSMENT OF ANALYSIS TECHNIQUES FOR MULTI-PLATE ANCHORS IN SAND

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ABSTRACT

This work focuses on the numerical simulation of multi-plate anchor systems (e.g., helical anchors) in sand subjected to vertical loading. In assessing the stiffness and capacity of these multi-plate anchor systems, full awareness of the abilities and limitations of the various analysis methods must be understood. This work first summarizes studies completed by others and then goes on to assess the failure mechanisms of multi-plate anchors in sand and the influence of (1) plate width-to-depth ratio, (2) number of plates, and (3) relative positioning of plates. The analysis makes use of (1) conventional limit analysis, (2) so-called modified limit analysis that employs reduced strength parameters to account for the influence of soil dilatancy, and (3) the displacement-based finite element method, which considers elastic as well as plastic deformation leading to failure. The work critically reflects on limitations in the current analysis methods for helical ground anchors.

Keywords: plate anchor, helical anchor, sand, numerical analysis, elastoplastic, limit analysis.

INTRODUCTION

Helical anchors are a relatively low cost foundation type, typically consisting of a steel rod and single or multiple plate attachments. In recent decades, they have seen rapid growth, and as outlined by Merifield and Sloan (2006) and Hambleton et al. (2014). Design techniques for helical anchors have previously been predominately empirical, as much of the past research in this field is experimentally based.

It is common practice for helical anchors to be modeled as horizontally oriented plate anchors in both numerical and experimental analysis. This work focuses on analysis based on the finite element method, including so-called finite element limit analysis and the conventional displacement-based finite element method, with some comparisons made to experimental studies. Numerical simulation of anchor pull-out capacity in cohesionless soils is stunted by the many difficulties faced when using numerical analysis with cohesionless soils. Additional variations can be seen when comparing the serviceability based load capacities to the ultimate load. In this work, both the force-displacement response (stiffness) and the ultimate anchor capacity are assessed, with more focus on the latter.

This work focuses only on analysis techniques for multi-plate anchors with vertical loading in cohesionless soils. Many studies have been completed in this area, some of which assume a rectangular plate in a plain strain model (Rowe & Davis, 1982; Merifield & Sloan, 2006; Cerfontaine et al., 2019). Whereas others use axisymmetric conditions (Baker and Kondner, 1966; Saeedy, 1987; Murray & Geddes, 1987; Ghaly & Hanna, 1994; Sakai & Tanaka, 1998; Merifield et al., 2006;) and minimal studies considered both axisymmetric and plane strain in the one study (Vesic, 1971; Tagaya et al., 1988; Murray & Geddes, 1987; Sarac, 1989).

A benchmarking study is first completed to compare the computed results to the break-out factors for circular plate anchors determined numerically by Merifield et al. (2006) and experimentally by Murray & Geddes (1987), Saeedy (1987) and Baker and Kondner (1966). This section of the study also investigates the plate width-to-depth ratio. The influence of the number of plates and the relative positioning of each of

these plates is then further investigated. In this investigation, particular consideration is given to assessing the ability of each analysis method to capture realistic failure mechanisms and well as the influence of some known errors within analysis techniques (e.g., mesh sensitivity).

PREVIOUS STUDIES AND CURRENT LIMITATIONS

Traditionally, theoretical studies in this area have been predominantly in the form of analytical approaches for single plate anchors. Notable studies include those by Meyerhof and Adams (1968), Murray and Geddes (1987), Basudhar and Singh (1994) and Smith (1998). For the case of analytical methods, many assume associated plastic flow, which results in unrealistic volume change and requires failure mechanisms to extend to the surface of the model. For the case of numerical analysis, both axisymmetric and plane strain analysis have been completed, with more detailed studies for plane strain. Rowe and Davis (1982), Vermeer and Sutjiadi (1985), Tagaya et al. (1988), Koutsabeloulis and Griffiths (1994) and Basudhar and Singh (1994) all conducted plane stain analysis on single-plate anchors. Tagaya et al. (1988), Koutsabeloulis and Griffiths (1994) and Merifield et al. (2006) presented the case of axisymmetric analysis. A more detailed summary of a wider range of studies in this area is provided by Merifield and Sloan (2006).

Studies of multiple plate anchor studies on granular material have predominantly been experimental, including publications from Baker and Kondner (1966), Ovesen (1981), Murray and Geddes (1987), Saeedy (1987), Dickin (1988), Tagaya et al. (1988), Ghaly et al. (1991), Ghaly and Clemence (1998), Ilamparuthi et al. (2002), Stanier (2011), Liu et al. (2012) and Tsuha et al. (2012). The major limitation of these studies is the lack of investigation into the interaction of multiple plates, which was experimentally assessed by Hao et al. (2018).

Finally, it should be noted that careful consideration should be given to the use of elastoplastic finite element analysis, and it should only be used by experienced professionals. In this study, the effect of mesh dependency on the elastoplastic finite element approach is addressed. Further analysis into other aspects of the abilities and difficulties of this technique is presented in Toh and Sloan (1980) and Sloan and Randolph (1982).

ANALYSIS METHODS

The three analysis techniques to be used in this work are (1) limit analysis (LA), (2) limit analysis with reduced parameters to account for the effects of a non-associated flow rule (Davis, 1968; Drescher and Detournay, 1993), denoted as modified limit analysis (MLA), and (3) elastoplastic finite element analysis (EP). These techniques are defined in subsequent sections. All three analysis techniques have been considered, as there is no perfect technique for the case of granular material. Assessing all three together highlights the possibilities and limitations of each. While an overview of each technique is provided below, the reader is referred to the detailed discussion and comparison of these methods by Sloan (2013) for more information.

Limit Analysis (LA)

Limit analysis considers a mode of failure expressed either in terms of an admissible stress field (lower bound limit analysis) or displacement field (upper bound limit analysis). Limit analysis cannot readily account for non-associative plastic flow, which is critical for the case of granular materials, nor can it capture the realistic failure mechanisms for multi-plate anchors. For this reason, the results can be expected to be somewhat inaccurate compared to the true results. Additionally, it is expected that the failure mechanism of limit analysis will not be capable of capturing the complex anchor geometry of multiple anchor plates. Both the upper and lower bound limits of a collapse load are utilized in this work to determine the bounds of the collapse load for the given assumptions. All analysis completed here using the finite element limit analysis code OptumG2 incorporates mesh adaptivity with three steps and a target of 1000 elements. It should be noted that the number of elements specified following mesh adaptivity are generally well over this target.

Modified Limit Analysis (MLA)

This analysis incorporates all the same details outlined in Limit Analysis with the addition of material parameter factoring. In order to account for the non-associative behavior of sand, the reduced parameters approach first proposed by Davis (1968) and subsequently documented by Drescher and Detournay (1993) aim to capture the reduction in material strength caused by non-associative plastic flow. The general accuracy of the method is not well understood and cannot be assumed to provide comprehensive method to account for non-associative behavior. The material used in this study is cohesionless, and the friction angle is reduced according to the following equations (Drescher and Detournay, 1993):

$$\tan \varphi^* = \eta \tan \varphi$$
[1]

where φ = friction angle, ψ = dilation angle and η is given by

$$\eta = \frac{\cos\psi\cos\varphi}{1 - \sin\psi\sin\varphi}$$
[2]

Elastoplastic Finite Element Analysis (EP)

Elastoplastic finite element analysis was implemented through the multiplier technique provided in OptumG2, which progressively increases the applied load (in this case an upward orientated force) until a state of failure has occurred. A collapse load evaluated using this analysis technique in OptumG2 is determined using traditional displacement-based finite element analysis.

A major limitation of EP is mesh dependency, meaning that for a variation in mesh the resulting load will vary, with no convergence for an increasingly fine mesh. Although this limitation can be avoided by incorporating viscosity into the numerical model (Perzyna, 1966), this adds further complexities and OptumG2 does not offer this feature. A further assessment into the mesh dependency of this analysis is provided as part of the benchmarking and parametric studies. Finally, the EP has a significantly longer computational time when compared to LA. However, it is able to capture the effects of intricate anchor details LA cannot capture when assessing the failure mechanism.

PROPOSED MODEL AND BENCHMARKING STUDY

This work utilized OptumG2 for the analysis of a dense sand in order to determine the collapse load of different idealized ground anchors. The material properties for the dense sand were taken as the values presented in Table 1, which were selected in order for comparison to previous studies. An axisymmetric model was used, and boundaries were set sufficiently far so as to not influence the failure zones for an overall domain size of 40 m wide by 30 m high. In all cases plate elements were used to model the anchors and plates. These have frictionless interfaces and are treated as rigid elements in OptumG2. The load is applied above the surface to ensure no interaction with the surface material, with the load multiplier used to assess the collapse load applied to the full geometry. All plates are assumed to have zero thickness and sizes specific to each analysis are detailed in the subsequent sections.

In order to verify the analysis techniques used in this work, a benchmarking study was conducted wherein the pullout capacity of a single horizontal anchor was compared to previous axisymmetric studies. The simplified horizontal plate anchor has numerous previous studies on granular material, with results from various numerical and experimental approaches. The axisymmetric results by Merifield et al. (2006), Murray & Geddes (1987), Saeedy (1987) and Baker and Kondner (1966) are compared here.

This analyses for this study were completed to assess the effect of the depth of embedment (*H*) to plate diameter (*B*) ratio (*H*/*B*) on the anchor pullout load, with the dimensions indicated in Fig. 1. The investigation included the three analysis techniques previously outlined in order to assess limitations and compute results for single-plate anchors as a basis for comparison with multi-plate anchors. For the single-plate anchor, the depth was kept at a constant of H = 20 m, and the plate diameter was such that the maximum H/B ratio was 10.

For this study, both upper and lower bounds were computed for LA and MLA, and the EP analysis with consideration of mesh sensitivity was also completed. In order to achieve the mesh sensitivity analysis for each H/B ratio computed, ten meshes were used to determine the collapse load. The meshes ranged from very fine to coarse with both uniform and non-uniform meshes incorporated into the analysis. Note that non-uniform meshes were achieved by specifying different areas of the same material, as shown in Fig. 2. It was determined that the break out factor (see Eq. [3] below) varied by an average of approximately $\pm 5\%$, and this is assumed to be the error caused by mesh dependency.

The results of the benchmarking study are presented in Fig. 3, where the break-out factors are calculated as

$$N_{\gamma} = \frac{q_u}{\gamma H}$$
[3]

where q_u = average normal stress on the plate (kN/m²), γ = soil unit weight (kN/m³) and H = depth to the lowest plate (m). The average normal stress q_u is calculated as the ultimate force Q_u (Fig. 1) divided by the area of a single plate.

One can observe in Fig. 3 that the computed LA bounds produce the highest results, which is to be expected since associated plastic flow is assumed. MLA and EP both attempt to account for the non-associative behavior of sand, and thus produce lower estimates of the break-out factor. The LA data matches very closely with the numerical data reported by Merifield et al. (2006), denoted in the figure by SNAC. Additionally, the MLA and EP presented similar trends and fall within all of the previously published experimental data. From this is can be seen that all three analysis techniques give a reasonable approximation of the resisting loads for single horizontal plate anchors.

RESULTS OF PARAMETRIC STUDY

Following the benchmarking study, the influence of the number of plates and relative spacing of plates on the ultimate load was investigated. The break-out factor is no longer assessed as the normalization of the average stress over the area of a plate, which is less applicable with multi-plate anchors, but rather the normalized ultimate force $Q_{u}/\gamma H^3$. This was completed by assessing cases with varying plate numbers and spacing, where the anchor depth is kept constant at H = 20 m in order to better assess the capabilities and limitations of the analysis techniques. Each of the cases investigated were assessed using upper and lower bounds for both LA and MLA, as well as EP.

Number of plates

The influence of the number of plates on the break-out factor was assessed by using a constant length of B = 2 m and spacing of x = 2 m. From the previously assessed single plate anchor, additional plates were added, and the influence of these plates on the failure load and mechanism was assessed. As displayed in

Fig. 1, the plates are progressively included from the bottom, until a total of eight plates are added to the anchor and analyzed.

Both LA and MLA presented constant break-out factors and therefore the addition of multiple plates resulted in no benefit. This is due to the assumption of associativity embedded into the analysis technique and is a limitation, one which cannot be addressed within limit analysis. Limit analysis and elastoplastic analysis can be compared visually by assessing the failure mechanisms as presented in Fig. 4, where it can be seen that both variations of limit analysis predict similar failure mechanisms whilst the EP analysis predicts a significantly different failure mechanism. In Fig. 4, it can be seen that there are four plates and EP is able to capture the shear failure along the soil encompassing all plates. In contrast, LA selects a single failure surface, which extends from the bottom plate to the surface at an angle of approximately 45° and shows no failure along the upper three plates. Specifically for the case of EP, an increase in the number of plates resulted in a variation of the failure mechanism, as shown in Fig. 5. It can be seen that with a greater number of plates the failure plane is directed in a vertical orientation for an increasing distance towards the surface. This variation in the failure mechanism produces a variable collapse load and resulting break-out factor. As with the benchmarking study, mesh sensitivity is a limitation of EP and its influence on the results was assessed. For four plates, it was found that the error due to mesh sensitivity of this arrangement is approximately $\pm 10\%$.

The load-displacement curves for an increasing number of plates are presented in Fig. 6. These results suggest that increasing the number of plates may reduce the collapse load, whereas increasing the number of plates always results in a stiffer response. The decrease in capacity observed with an increasing number is plates is counterintuitive but nevertheless theoretically possible, and this requires further exploration. Furthermore, the reported displacements are unrealistically large given the expected anchor displacement required to mobilize the full load. Therefore, caution should be used when selecting the collapse load. This is a known limitation of elastoplastic displacement-based finite element methods, which were investigated by Sloan and Randolph (1982).

Relative positioning of plates

The influence of the relative positioning of plates on the break-out factor was assessed by using a constant length of B = 2 m and five anchor plates. The spacing was varied from x = 0.25 m up to x = 3 m. As displayed in Fig. 2, the plate spacing is progressively increased and each stage analyzed in a manner similar to the investigation previously completed.

As with the analysis completed for the influence of the number of plates, both LA and MLA presented constant break-out factors. This was due to the previously identified limitations of limit analysis. The EP analysis also produced very similar results to the previous analysis. The similarities can be seen in the failure mechanisms and load-displacement curves in Figs. 7 and 8, respectively.

CONCLUDING REMARKS

The work compiled in this paper utilized OptumG2 to complete a numerical study assessing analysis techniques for multi-plate anchors such as helical anchors. Three analysis techniques, LA, MLA and EP, were used to assess anchor capacity. A benchmarking study was first completed where break-out factors for single-plate anchors were compared to results from previous studies, hence verifying the numerical analysis techniques used. The benchmarking study confirmed that, for the case of a single horizontal plate anchor, all analysis techniques provided sensible results with explainable differences, with EP and MLA providing results closest to previously reported experimental data. This was followed by a parametric study examining the effects of the number of plates and plate spacing.

The results of the parametric study for multi-plate anchors presented a range of ultimate loads for all assessed limit analysis techniques, which did not encompass the results presented by EP. This difference can be attributed to the limitations of limit analysis—and in particular the assumption of associativity—and the significantly varying failure mechanisms presented by the two analysis techniques. In all cases of the parametric study, both LA and MLA presented constant ultimate loads, and this was determined to be a limitation of the analysis technique and a result of a constant failure surface.

Notably, for the case of EP, an increase in the number of plates resulted in a decrease in the ultimate capacity but an increase in stiffness. This would indicate that for a service load a larger number of plates would be desirable, whereas for ultimate load capacity a single plate is most desirable. While this counterintuitive result is theoretically possible, it requires further exploration in more detailed analyses and experimental investigations.

Both LA and MLA assume an associative flow rule, and as a result of volume change, the failure mechanisms in all cases unrealistically extended to the surface. LA was determined to be the least accurate due to its inability to capture the realistic soil behavior of multiple plates. For single plate anchors, MLA was determined to capture a more reasonable anchor capacity through the use of modified parameters. However, it was still limited in its ability to capture a realistic failure mechanism for multiple plates, which is further indicated by a constant ultimate load with a varying number of plates. Finally, EP was able to capture the intricate details of failure around multiple plates. Due to the assumption of non-associative plastic flow, the loading of anchors was able to cause elastic deformation and produce local failure mechanisms. The effect of mesh dependency was further assessed for elastoplastic analysis and found to produce variability in the predictions of ultimate capacity of up to 10%. There are still other potential sources or error in EP analysis, such as modeling assumptions relating to structural elements and contact, which are not addressed in this work. In all cases, caution should be used with all numerical analysis techniques.

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TABLES

Parameter	Value
Friction Angle, φ (°)	40
Dilation Angle, ψ (°)	10
Cohesion, c (kPa)	0
Dry unit weight, γ (kN/m ³)	18
Young's Modulus, E (MPa)	50
Poisson's Ratio, v	0.3

FIGURES



Fig. 1. Plate Anchor Dimensions



Fig. 2. Examples of meshes used in EP mesh sensitivity analysis



Fig. 3. Benchmarking study comparison (plot best viewed in color)



Fig. 4. Failure mechanisms for (left) upper bound limit analysis (middle) lower bound limit analysis (right) elastoplastic finite element analysis (contours show a representation of the deviatoric strain)



Fig. 5. Development of failure mechanism with increasing number of plates from elastoplastic finite element analysis (contours show a representation of the deviatoric strain)



Fig. 6. Load-displacement curves for collapse loads for increasing number of plates (EP analysis; contours show a representation of the deviatoric strain; plot best viewed in color)



Fig. 7. Development of failure mechanism with increasing plate spacing (EP analysis; contours show a representation of the deviatoric strain)



Fig. 8. Load-displacement curves for increasing plate spacing x (EP analysis; plot best viewed in color)