Assessment of artificial intelligence models for calculating optimum properties of lined channels

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ABSTRACT

Lined channels with trapezoidal, rectangular and triangular sections are the most common manmade canals in practice. Since the construction cost plays a key role in water conveyance projects, it has been considered as the prominent factor in optimum channel designs. In this study, artificial neural networks (ANN) and genetic programming (GP) are used to determine optimum channel geometries for trapezoidal-family cross sections. For this purpose, the problem statement is treated as an optimization problem whose objective function and constraint are earthwork and lining costs and Manning's equation, respectively. The comparison remarkably demonstrates that the applied artificial intelligence (AI) models achieved much closer results to the numerical benchmark solutions than the available explicit equations for optimum design of lined channels with trapezoidal, rectangular and triangular sections. Also, investigating the average of absolute relative errors obtained for determination of dimensionless geometries of trapezoidal-family channels using AI models shows that this criterion will not be more than 0.0013 for the worst case, which indicates the high accuracy of AI models in optimum design of trapezoidal channels.

Key words | artificial neural network, genetic programming, optimum design, rectangular channels, trapezoidal channels, triangular channels

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HIGHLIGHTS

- In this study, ANN and GP has been applied to optimum design of lined channels for the first time.
- Canals with three common shapes including trapezoidal, rectangular and triangular were designed.
- Also, three new regression-based models were proposed for calculating optimum channel properties.
- The obtained results were compared with that of available models in the literature.
- The comparison indicates the superiority of the two AI models for this purpose.

INTRODUCTION

Water conveyance projects including construction of man-made canals are generally inevitable as water resources are not necessarily close to consumers' locations. The main concern in most of these projects is the budget required for construction cost. In this perspective, optimal design of canals may be interpreted as the most cost-beneficial one. This optimum design not only takes into account a hydraulically feasible condition for flow passing through the channel but also minimizes the construction

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cost. This reality-based interpretation of optimum design of manmade canals has provided an active research field in water resource management (Reddy & Adarsh 2010; Tabari *et al.* 2014; Swamee & Chahar 2015; Roushangar *et al.* 2017).

These studies may be classified based on the shape of canals under investigation (Easa 2018): (1) linear (trapezoidal-family) sections; (2) curved (circular, parabolic and power-law) sections; and (3) linear-curved sections like horizontal bottom and parabolic sides (Das 2010). Among these different canal shapes, the most common cross sections in practice are trapezoidal-family (trapezoidal, rectangular and triangular) and circular sections (Niazkar & Afzali 2015), while the former is the focus of this study.

For optimum design of trapezoidal channels, many studies have been conducted in the literature. These studies can be reviewed based on defining the problem statement including objective functions and constraints, optimization algorithms used for solving the problem of optimum channel design, and their recommendations for calculating channel geometries. Regarding the objective function, Swamee *et al.* (2000) presented a general construction cost including earthwork and lining costs, which has been used in several studies (Aksoy & Altan-Sakarya 2006; Niazkar & Afzali 2015). Furthermore, considered hydraulic constraints of the design problem are Swamee's resistance equation (Swamee *et al.* 2000), Manning's equation (Aksoy & Altan-Sakarya 2006; Bhattacharjya 2006; Bhattacharjya & Satish 2008; Easa *et al.* 2011; Vatankhah & Easa 2011; Niazkar & Afzali 2015), a flooding probability (Das 2007), a minimum value of the freeboard (Bhattacharjya & Satish 2008), and a minimum safety factor (Easa *et al.* 2011). Moreover, various optimization algorithms have been applied to optimum channel designs, and they include a grid search optimization algorithm (Swamee *et al.* 2000), the Lagrange multiplier method (Aksoy & Altan-Sakarya 2006; Das 2007; Han *et al.* 2017, 2019), the sequential quadratic programming method (Bhattacharjya 2006), a hybrid optimization technique (Bhattacharjya & Satish 2007), nondominated sorting genetic algorithm (Bhattacharjya & Satish 2008), ant colony optimization (Nourani *et al.* 2009), Genetic Algorithm and Particle Swarm Optimization (Reddy & Adarsh 2010), the generalized reduced gradient algorithm (Easa *et al.* 2011; Froehlich 2011), the Modified Honey Bee Mating Optimization (MHBMO) algorithm (Niazkar & Afzali 2015), the shuffled frog-leaping algorithm (Orouji *et al.* 2016), and the cat swarm optimization (Liu *et al.* 2016). Finally, equations were proposed for direct (Swamee *et al.* 2000; Aksoy & Altan-Sakarya 2006; Niazkar & Afzali 2015) and iterative (Han *et al.* 2019) computation of optimum geometries of trapezoidal channels.

Based on the literature review conducted, the optimal design of trapezoidal channels has been treated as an optimization problem consisting of a construction-cost function and a hydraulic constraint. Furthermore, different types of solutions including design curves and explicit or implicit equations have been recommended based on the results obtained for the optimization-based design problems. Although artificial intelligence (AI) models have been successfully applied to solving various problems in water resources management (Babovic & Keijzer 2000; Giustolisi 2004; Xu *et al.* 2011; Rodríguez-Vázquez *et al.* 2012; Pourzangbar *et al.* 2017), they have not been implemented for optimum design of trapezoidal-family canals.

In this study, the generalized construction cost including earthwork and lining costs, which has been previously considered in several studies (Swamee *et al.* 2000; Aksoy & Altan-Sakarya 2006; Niazkar & Afzali 2015), was minimized using the MHBMO algorithm. This design problem was solved for a variety of values of parameters involved in the design problem of trapezoidal-family channels. Based on the large database provided, two AI models including artificial neural networks (ANN) and genetic programming (GP) were applied to optimize the design of lined channels with trapezoidal, rectangular and triangular sections. To the author's knowledge, it is the first time that these two AI models have been implemented for optimum design of lined trapezoidal-family channels. Finally, the performances of these AI models were compared with those of the explicit design equations, which are present in the current literature (Swamee *et al.* 2000; Aksoy & Altan-Sakarya 2006; Niazkar & Afzali 2015).

METHODS AND MATERIALS

Problem statement of optimum channel design

Design of a man-made canal is to determine its geometries. In essence, channel geometries play the role of objectives of the problem statement when a canal shape is assumed. Nevertheless, it is not practically possible to take into account all factors involved in the optimal design of an open channel while the most prominent ones need to be considered. Generally, an optimum channel design not only supposes to convey an expected amount of water but also is required to be cost beneficial, particularly when the canal is constructed over a large distance. Hence, the problem of optimum design of open channels may be treated as an optimization problem while the objective function defines a construction cost.

One of the most generalized forms of construction cost of a typical lined channel (C) is comprised of three components (Swamee *et al.* 2000; Aksoy & Altan-Sakarya 2006; Niazkar & Afzali 2015; Niazkar *et al.* 2018). The first cost is the earthwork cost per unit area (β_E) while the second one is the lining cost (β_L). An additional earthwork cost that takes into account different costs of earthwork in different depths (β_A) is the third component. The algebraic summation of these three cost components consists of the objective function of the design problem while a resistance equation is required to be set as a constraint. For the latter, Manning's coefficient, which is the most resistance equation widely used in open channel hydraulics (Niazkar *et al.* 2019a), may be used to preserve a hydraulically viable flow passing through the channel:

$$Q = \frac{1}{n} A R^{2/3} \sqrt{S} \tag{1}$$

where Q is discharge, n is Manning's coefficient, A is channel cross section area, $R = \frac{A}{P}$ is hydraulic radius, P is wetted perimeter and S is channel slope. Among these parameters, A, P and R are shape-dependent functions of channel geometries. Additionally, S may be replaced with the bottom channel slope, while this substitution gives the normal water depth in Manning's equation. Moreover, n may be estimated using one of the bed roughness predictors available in the literature (Niazkar *et al.* 2019b), while a flow-independent n is utilized for simplification purposes in the literature (Swamee *et al.* 2000; Aksoy & Altan-Sakarya 2006; Tabari & Mari 2016). In order to enhance the generality of the final solutions, the parameters involved in the objective function and constraint are all turned into unitless parameters using a length-scale parameter:

$$\lambda = \left(\frac{Qn}{\sqrt{S}} \right)^{3/8} \tag{2}$$

The dimensionless Manning's equation and objective functions for optimum design of trapezoidal, rectangular and triangular channels are presented in Equations (3)–(6), respectively:

$$A_* R_*^{2/3} - 1 = 0 \tag{3}$$

$$C_* = \beta_{L*}(2y_* \sqrt{1+m^2} + b_*) + \beta_{A*}\left(0.5 b_* y_*^2 + \frac{m y_*^3}{3}\right) + b_* y_* + m y_*^2 \tag{4}$$

$$C_* = \beta_{L*}(2y_* + b_*) + 0.5 b_* y_*^2 \beta_{A*} + b_* y_* \tag{5}$$

$$C_* = 2\beta_{L*} y_* \sqrt{1+m^2} + \beta_{A*} \frac{m y_*^3}{3} + m y_*^2 \tag{6}$$

where subscript * denotes the dimensionless form of a variable, $A_* = \frac{A}{\lambda^2}$, $R_* = \frac{R}{\lambda}$, $\beta_{L*} = \frac{\beta_L}{\beta_E \lambda}$, $\beta_{A*} = \frac{\beta_A}{\beta_E \lambda}$, $C_* = \frac{C}{\beta_E \lambda^2}$ is the total dimensionless cost of constructing a lined channel per unit length, y_* is dimensionless water depth, m is side slope and b_* is dimensionless bottom width of the channel under construction.

Equations (5) and (6) can be determined by substituting $m = 1$ and $b = 0$ into Equation (4), respectively. Moreover, Equations (3) and (4) are the governing equations for optimum design of trapezoidal channels, while the problem statement of design of rectangular channels consists of Equations (3) and (5). Finally, Equations (3) and (6) can be used for the design of lined triangular channels.

Explicit relations for optimum design of trapezoidal-family channels

The problem statement described in the previous section has been solved in the literature. In this regard, several explicit equations have been recommended for the design of optimum lined channels. Tables 1–3 list the explicit equations previously suggested for trapezoidal, rectangular and triangular channels, respectively. As shown, four different models have already been proposed for the optimum design of lined channels (Swamee *et al.* 2000; Aksoy &

Table 1 | Chronological review of explicit relations available for optimum design of trapezoidal channels

Models	Equation no.	Relations
Swamee *et al.* (2000)	(7)	$m = 0.57735 + \dfrac{0.12485\,\beta_A L^2}{\beta_E L + 14.2772\,\beta_L}$
	(8)	$b = 0.43407L + \dfrac{0.15121\,\beta_A L^3}{\beta_E L + 14.2425\,\beta_L}$
	(9)	$y = 0.37592L\left(1 + \dfrac{0.22332\,\beta_A L^2}{\beta_E L + 14.2274\,\beta_L}\right)^{-1}$
Aksoy & Altan-Sakarya (2006) – First model	(10)	$m = 0.577 + \dfrac{0.065\,\beta_{A*}}{\beta_{L*}}$
	(11)	$b_* = 1.118 + \dfrac{0.177\,\beta_{A*}}{\beta_{L*}}$
	(12)	$y_* = 0.968\left(1 + \dfrac{0.104\,\beta_{A*}}{\beta_{L*}}\right)^{-1}$
Aksoy & Altan-Sakarya (2006) – Second model	(13)	$m = 0.577 + \dfrac{0.331\,\beta_{A*}}{1 + 4.994\,\beta_{L*}}$
	(14)	$b_* = 1.118 + \dfrac{0.938\,\beta_{A*}}{1 + 5.200\,\beta_{L*}}$
	(15)	$y_* = 0.968\left(1 + \dfrac{0.541\,\beta_{A*}}{1 + 5.101\,\beta_{L*}}\right)^{-1}$
Niazkar & Afzali (2015)	(16)	$m = 0.5774 + 0.062\,\beta_{A*}^{0.9219}\,\beta_{L*}^{-0.7096}$
	(17)	$b_* = 1.1175 + 0.1502\,\beta_{A*}^{1.0611}\,\beta_{L*}^{-0.8094}$
	(18)	$y_* = 0.9678\,(1.0018 + 0.7401\,\beta_{A*}^{1.1482}\beta_{L*}^{-0.9071})^{-0.1589}$

Table 2 | Chronological review of explicit relations available for optimum design of rectangular channels

Models	Equation no.	Relations
Swamee *et al.* (2000)	(19)	$b = 0.71136\,L + \dfrac{0.22772\,\beta_A L^3}{\beta_E L + 15.0284\,\beta_L}$
	(20)	$y = 0.35568\,L\left(1 + \dfrac{0.30657\,\beta_A L^2}{\beta_E L + 15.0234\,\beta_L}\right)^{-1}$
Aksoy & Altan-Sakarya (2006) – First model	(21)	$b_* = 1.834 + \dfrac{0.246\,\beta_{A*}}{\beta_{L*}}$
	(22)	$y_* = 0.917\left(1 + \dfrac{0.129\,\beta_{A*}}{\beta_{L*}}\right)^{-1}$
Aksoy & Altan-Sakarya (2006) – Second model	(23)	$b_* = 1.834 + \dfrac{1.347\,\beta_{A*}}{1 + 5.359\,\beta_{L*}}$
	(24)	$y_* = 0.917\left(1 + \dfrac{0.695\,\beta_{A*}}{1 + 5.272\,\beta_{L*}}\right)^{-1}$
Niazkar & Afzali (2015)	(25)	$b_* = 1.834 + 0.2227\,\beta_{A*}^{0.9861}\beta_{L*}^{-0.7636}$
	(26)	$y_* = 0.917(1.0091 + 0.9838\beta_{A*}^{1.1369}\beta_{L*}^{-0.9151})^{-0.163}$

Altan-Sakarya 2006; Niazkar & Afzali 2015). In Tables 1–3,

$$L = \frac{\lambda^{32/30}}{n^{0.4} g^{0.2}}$$

where g is the gravitational acceleration. Furthermore, geometries for trapezoidal-family channels can be computed by knowing three parameters: (1) β_{L*}, (2) β_{A*} and (3) λ. Hence, the relations presented in Tables 1–3 can be used

Table 3 | Chronological review of explicit relations available for optimum design of triangular channels

Models	Equation no.	Relations
Swamee *et al.* (2000)	(27)	$m = 1.0 + \frac{0.30389\beta_A L^2}{\beta_E L + 15.0491\beta_L}$
	(28)	$y = 0.50301L\left(1 + \frac{0.13973\beta_A L^2}{\beta_E L + 15.0389\beta_L}\right)^{-1}$
Aksoy & Altan-Sakarya (2006) – First model	(29)	$m = 1.0 + \frac{0.135\beta_{A*}}{\beta_{L*}}$
	(30)	$y_* = 1.297\left(1 + \frac{0.063\beta_{A*}}{\beta_{L*}}\right)^{-1}$
Aksoy & Altan-Sakarya (2006) – Second model	(31)	$m = 1.0 + \frac{0.741\beta_{A*}}{1 + 5.375\beta_{L*}}$
	(32)	$y_* = 1.297\left(1 + \frac{0.342\beta_{A*}}{1 + 5.298\beta_{L*}}\right)^{-1}$
Niazkar & Afzali (2015)	(33)	$m = 1.0 + 0.2885\beta_{A*}^{0.9034}\beta_{L*}^{-0.7034}$
	(34)	$y_* = 1.2968(1.0018 + 1.8568\beta_{A*}^{1.0955}\beta_{L*}^{-0.9007})^{-0.1168}$

for any hydraulically possible conditions while the exclusive limitation of these empirical formulas is $0 < \frac{\beta_{A*}}{\beta_{L*}} < 2$ (Aksoy & Altan-Sakarya 2006; Niazkar & Afzali 2015).

Artificial intelligence models

Two AI models (ANN and GP) were used to design lined channels with optimum trapezoidal, rectangular and triangular shapes. To the author's knowledge, this is the first time that these AI models has been used for the optimum design of open channels while they have been utilized for various applications in water resources (Babovic & Keijzer 2000; Niazkar 2019; Niazkar *et al.* 2019b, 2020). A brief summary of these models are presented below.

Generally, ANNs consist of several layers while each layer has some neurons. The interconnection between neurons in different layers builds a flexible architecture. This characteristic basically enables the prediction of a relation between a vector of input and a vector of output data. In this study, a three-layered (input, hidden and output) feed-forward network was used to predict optimum channel geometries. Each row of the input layer has two normalized values of β_{L*} and β_{A*}, while the output vector includes a normalized channel geometry.

GP is an AI model which not only adopts genetic algorithm characteristics but also extends them to improve the prediction capability of this well-known optimization algorithm. To be more specific, GP uses initialization, mutation, reproduction, and survival principles not only to find an optimum relation between known input and output vectors, but also to estimate an unknown output vector for a vector of input values. This AI model consists of functions and terminals. The former are mathematical and logical operators and logical conditions while the latter includes variables and coefficients. The tree-structure between the functions and terminals typically creates not only powerful but also flexible estimators. In this study, Discipulus (Francone 1998) software, which has been successfully utilized for solving other hydraulic engineering problems (Niazkar *et al.* 2018), was used to employ GP for the optimal design of trapezoidal-family channels.

New regression-based explicit relations for optimum design of trapezoidal-family channels

The solutions of the optimum design of trapezoidal-family channels were used to develop three types of regression-based explicit equations using MATLAB, which has been successfully used for numerical modeling and solving engineering problems (Niazkar & Afzali 2017c, 2017d; Motaman *et al.* 2018). The proposed relations include (1) linear, (2) the second-order polynomial and (3) the third-order polynomial equations. Because of the nonlinear relations

Table 4 | New second-order nonlinear explicit relations developed by regression for optimum design of lined channels

Channel type	Equation no.	Relations
Trapezoidal channel	(35)	$m = 0.5954 - 0.0165\beta_{L*} + 0.08641\beta_{A*} + 0.002831\beta_{L*}^2 - 0.01313\beta_{L*}\beta_{A*} - 0.01345\beta_{A*}^2$
	(36)	$b_* = 1.161 - 0.04151\beta_{L*} + 0.1975\beta_{A*} + 0.007338\beta_{L*}^2 - 0.03368\beta_{L*}\beta_{A*} - 0.02153\beta_{A*}^2$
	(37)	$y_* = 0.9444 + 0.02165\beta_{L*} - 0.1114\beta_{A*} - 0.003735\beta_{L*}^2 + 0.01729\beta_{L*}\beta_{A*} + 0.01646\beta_{A*}^2$
Rectangular channel	(38)	$b_* = 1.899 - 0.06121\beta_{L*} + 0.3019\beta_{A*} + 0.01065\beta_{L*}^2 - 0.04859\beta_{L*}\beta_{A*} - 0.04081\beta_{A*}^2$
	(39)	$y_* = 0.8888 + 0.0256\beta_{L*} - 0.135\beta_{A*} - 0.004363\beta_{L*}^2 + 0.02004\beta_{L*}\beta_{A*} + 0.02252\beta_{A*}^2$
Triangular channel	(40)	$m = 1.085 - 0.07673\beta_{L*} + 0.4048\beta_{A*} + 0.01309\beta_{L*}^2 - 0.05977\beta_{L*}\beta_{A*} - 0.06905\beta_{A*}^2$
	(41)	$y_* = 1.253 + 0.03746\beta_{L*} - 0.2207\beta_{A*} - 0.006158\beta_{L*}^2 + 0.0283\beta_{L*}\beta_{A*} + 0.04899\beta_{A*}^2$

between channel geometries, β_{L*} and β_{A*}, the linear equations may have relatively large errors in the direct calculation of channel properties, while the third-order polynomial equations have relatively more terms than the explicit equations available in the literature. Thus, these two regression-based equations and their performances are presented in the Appendix, while the second-order regression-based equations, as a suitable choice between a trade-off between accuracy and formula complexity, are shown in Table 4 for trapezoidal-family channels. Although the regression-based explicit equations enable direct calculation of channel geometries, it requires specifying the type of relation before curve fitting. However, the AI models, like ANN and GP, do not require not only coefficient values, but also the type of formulation in advance, which is considered as one of the advantages of AI models over regression analysis (Niazkar & Niazkar 2020). The advantage of explicit equations, which are available in the literature and shown in Tables 1–3, to the proposed regression-based models is that the former, unlike the latter, can give the exact optimum solutions for $\beta_{A*} = 0$, which is a simplified channel-construction condition (Niazkar & Afzali 2015).

Performance evaluation metrics

For comparison purposes, five criteria were adopted from the literature (Niazkar & Afzali 2018; Niazkar *et al.* 2019c). These performance evaluation metrics are: (1) Root Mean Square Error (RMSE), (2) mean absolute error (MAE), (3) mean Absolute Relative Error (MARE), (4) Relative Error (RE), and (5) coefficient of determination (R^2). Lower values of the first four of these metrics indicate better results, whereas the higher the value of R^2, the closer the results to the benchmark solutions. Furthermore, RMSE, MAE, MARE and R^2 compare the results for the whole test data while RE is a local criterion that needs to be computed for each data point. The five criteria can be calculated for each channel geometry, and they are written for y_* in the following equations, respectively:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_{*,\text{database}} - y_{*,\text{estimated}})^2} \quad (42)$$

$$\text{MAE} = \frac{1}{N}\sum_{i=1}^{N}|y_{*,\text{database}} - y_{*,\text{estimated}}| \quad (43)$$

$$\text{MARE} = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{y_{*,\text{database}} - y_{*,\text{estimated}}}{y_{*,\text{database}}}\right| \times 100 \tag{44}$$

$$\text{RE} = \frac{y_{*,\text{estimated}} - y_{*,\text{database}}}{y_{*,\text{database}}} \tag{45}$$

$$R^2 = \left(\frac{\sum_{i=1}^{N}\left[\left(y_{*,\text{database}} - \frac{\sum_{i=1}^{N} y_{*,\text{database}}}{N}\right)\left(y_{*,\text{estimated}} - \frac{\sum_{i=1}^{N} y_{*,\text{estimated}}}{N}\right)\right]}{\sqrt{\sum_{i=1}^{N}\left[\left(y_{*,\text{database}} - \frac{\sum_{i=1}^{N} y_{*,\text{database}}}{N}\right)^2\left(y_{*,\text{estimated}} - \frac{\sum_{i=1}^{N} y_{*,\text{estimated}}}{N}\right)^2\right]}}\right)^2 \tag{46}$$

where $y_{*,\text{database}}$ and $y_{*,\text{estimated}}$ are dimensionless water depths which are the final solution and predicted results, respectively.

RESULTS AND DISCUSSION

The problems of optimum design of lined trapezoidal-family channels were separately solved by the MHBMO algorithm. This algorithm has been successfully used for the optimum design of lined channels (Niazkar & Afzali 2015; Niazkar *et al.* 2018) and other optimization problems in hydraulic and water resources engineering (Niazkar & Afzali 2014, 2016, 2017a, 2017b). The design problem for each channel shape (trapezoidal, rectangular and triangular) was solved for 146 pairs of different values β_{A*} and β_{L*}. Hence, one set of data (146 data points) for the optimum design of trapezoidal channels, one set of data (146 data points) for the optimum design of rectangular channels and one set of data (146 data points) for the optimum design of triangular channels were developed.

The bound considered for β_{A*} and β_{L*}

$$\left(0 < \frac{\beta_{A*}}{\beta_{L*}} < 2\right)$$

was adopted from previous studies in the literature (Aksoy & Altan-Sakarya 2006; Niazkar & Afzali 2015). The upper bound of this range is applicable for $Q = 100\ \text{m}^3/\text{s}$, $n = 0.033$ and $S = 0.0002$ ($\lambda = 7.727$). These data were used to either develop equations or were used by AI models for calculating each channel geometry, like b_*, while each row of data consists of three values: (1) β_{A*}, (2) β_{L*}, and (3) b_*. Thus, each row of data is not designated to a set of specific values of Q, b, S_0, n, and y, whereas it corresponds to a set dimensionless parameters (β_{A*}, β_{L*}, and b_*). This dimensionless set of parameters can cover many combinations of Q, b, S_0, n, and y, while each combination can have a different set of values for Q, b, S_0, n, and y. Therefore, the 146 data used for each channel property can cover a wide range of values.

Each one of the data sets was normalized using the maximum and minimum values of each dataset. Afterwards, each one of these three datasets was randomly divided into two parts. The first part, which includes 110 data points, was applied to train the AI models, while the rest (36 data points in each one of the three data sets) was utilized as the test data. The latter provides an opportunity to compare the performance of AI models with those of explicit equations available in the literature.

The solutions proposed by the explicit equations shown in Tables 1–4, the ones in the Appendix and those by ANN and GP are applicable to the problem statement defined in Equations (3)–(6). They are valid when the parameters involved in this problem satisfy $0 < \frac{\beta_{A*}}{\beta_{L*}} < 2$. Obviously, any change in either problem constraints or the factors that play the key role in the cost function yield to a new problem with a different governing equation. In that case, the aforementioned solutions need to be revised.

Results for optimum design of trapezoidal channels

The performances of the two AI models for the optimum design of trapezoidal channels are compared with those of nonlinear regression-based models and four models available in the literature. Table 5 indicates that two AI models employed in this study outperformed other explicit models for calculating optimum m of trapezoidal channels in terms of all four criteria considered. Among the explicit equations listed in Table 5, Equation (16) reached the closest values to the final solutions for computing m for trapezoidal channels. The comparison carried out in Table 5 demonstrates that ANN is the best model whereas GP did not achieve better results than available explicit equations for predicting optimum b_* of trapezoidal channels. Furthermore, the second best model in Table 5 is Equation (14) in

Table 5 | Comparison of different models for optimum design of lined trapezoidal channels

Model	Equation no.	RMSE	MAE	R^2	MARE
(a) For calculating optimum m					
Swamee *et al.* (2000)	(7)	0.0079	0.0064	0.9881	0.0104
Aksoy & Altan-Sakarya (2006) – First model	(10)	0.0091	0.0055	0.9581	0.0087
Aksoy & Altan-Sakarya (2006) – Second model	(13)	0.0091	0.0055	0.9581	0.0087
Niazkar & Afzali (2015)	(16)	0.0015	0.0011	0.9960	0.0018
Nonlinear regression (this study)	(35)	0.0065	0.0041	0.9390	0.0067
ANN (this study)	–	0.0001	0.0001	1.0000	0.0001
GP (this study)	–	0.0000	0.0000	1.0000	0.0000
(b) For calculating optimum b_*					
Swamee *et al.* (2000)	(8)	0.0030	0.0029	0.9998	0.0025
Aksoy & Altan-Sakarya (2006) – First model	(11)	0.0271	0.0109	0.9837	0.0084
Aksoy & Altan-Sakarya (2006) – Second model	(14)	0.0009	0.0007	0.9999	0.0006
Niazkar & Afzali (2015)	(17)	0.0026	0.0019	0.9982	0.0016
Nonlinear regression (this study)	(36)	0.0208	0.0124	0.9105	0.0101
ANN (this study)	–	0.0006	0.0002	0.9999	0.0001
GP (this study)	–	0.0043	0.0011	0.9975	0.0008
(c) For calculating optimum y_*					
Swamee *et al.* (2000)	(9)	0.0019	0.0016	0.9975	0.0017
Aksoy & Altan-Sakarya (2006) – First model	(12)	0.0110	0.0048	0.9789	0.0054
Aksoy & Altan-Sakarya (2006) – Second model	(15)	0.0030	0.0026	0.9981	0.0028
Niazkar & Afzali (2015)	(18)	0.0012	0.0009	0.9987	0.0009
Nonlinear regression (this study)	(37)	0.0089	0.0056	0.9340	0.0062
ANN (this study)	–	0.0028	0.0010	0.9948	0.0012
GP (this study)	–	0.0005	0.0002	0.9998	0.0003

terms of RMSE, MAE, R^2 and MARE. For calculating optimum y_* of trapezoidal channels, Table 5 obviously shows that GP yielded to the best results comparing to other models, while Equation (13) obtains the best second results in this table. Also, ANN may be recognized as the third best model in Table 5 based on RMSE, MAE and MARE.

The relative errors of trapezoidal geometries predicted by the AI models are depicted in Figure 1 for the test data. As shown, RE values of $y*$ predicted by ANN and GP are placed within [–0.0035, 0.0179] and [–0.0022, 0.0010], respectively. Additionally, the averages of absolute RE for y_* calculated by ANN and GP are 0.0012 and 0.0003, respectively. These results confirm the ones reported in Table 5 which indicate that GP estimated $y*$ values closer to the final solutions than ANN. According to Figure 1, the bounds of RE values for b_* obtained by ANN and GP are [–0.0024, 0.0003] and [–0.0150, 0.0003], respectively. Furthermore, the averages of absolute RE for $y*$ computed by ANN and GP are 0.0001 and 0.0008, respectively. These results are in line with those shown in Table 5 indicating that ANN performs better than GP in predicting optimum b_* for trapezoidal channels. Also, the RE values of optimum m achieved by ANN and GP vary within [–0.0005, 0. 0005] and [–0.0001, 0.0001], respectively, while the averages of absolute RE obtained by ANN and GP are 0.0001 and zero. Based on these results, GP performs better than ANN in the computation of optimum m of trapezoidal channels while they both reach very close results to the final solution. Comparing different bounds of RE

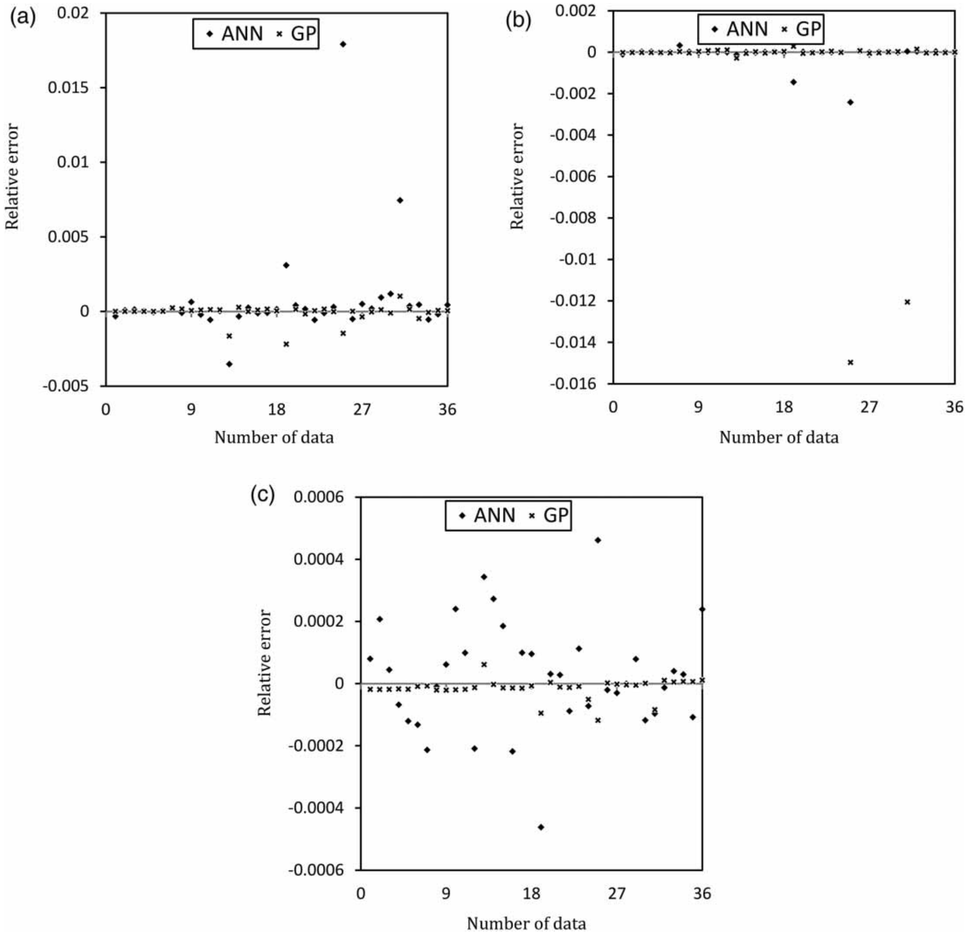

Figure 1 | Relative errors computed for optimum trapezoidal channels predicted by ANN and GP: (a) y_*, (b) b_* and (c) m.

values shown in Figure 1 reveals that the AI models achieved much closer results for optimum m than y_* and b_*.

Results for optimum design of rectangular channels

The performances of five explicit equations, ANN and GP are compared for predicting optimum b_* and y_* of rectangular channels in Table 6, respectively. In Table 6 both AI models performed not only quite the same but also much better than other explicit models for calculating optimum b_* in terms of all four criteria considered. Likewise, Table 6 indicates that both AI models applied in this study outperformed explicit equations available for predicting optimum y_* of rectangular channels. Among the explicit equations compared in Table 6, Equations (25) and (26) achieved the highest R^2 and the lowest RMSE, MAE and MARE for optimum b_* and y_*, respectively. Finally, Table 6 shows that both ANN and GP significantly improved the accuracy of predicting b_* and y_* in the optimum design of rectangular channels.

Figure 2 shows the RE values of estimated y_* and b_* of rectangular channels for 36 test data points. Based on Figure 2, the optimum values of y_* predicted by ANN and GP result to RE values placed within [–0.0009, 0.0] and [–0.0005, 0.0021], respectively. Furthermore, the averages of absolute RE for y_* computed by ANN and GP are 0.0001 and 0.0003, respectively. These results are consistent with the ones mentioned in Table 6 which shows that ANN obtained optimum y_* values much closer to the final solutions than GP, while they both have increased the accuracy of available explicit equations. According to Figure 2, the boundaries of RE values for calculating

Table 6 | Comparison of different models for optimum design of lined rectangular channels

Model	Equation no.	RMSE	MAE	R^2	MARE
(a) For calculating optimum b_*					
Swamee *et al.* (2000)	(19)	0.0129	0.0121	0.9966	0.0063
Aksoy & Altan-Sakarya (2006) – First model	(21)	0.0335	0.0144	0.9754	0.0070
Aksoy & Altan-Sakarya (2006) – Second model	(23)	0.0099	0.0082	0.9972	0.0042
Niazkar & Afzali (2015)	(25)	0.0041	0.0030	0.9977	0.0015
Nonlinear regression (this study)	(38)	0.0276	0.0169	0.9234	0.0085
ANN (this study)	–	0.0002	0.0001	1.0000	0.0000
GP (this study)	–	0.0002	0.0001	1.0000	0.0001
(b) For calculating optimum y_*					
Swamee *et al.* (2000)	(20)	0.0026	0.0022	0.9948	0.0025
Aksoy & Altan-Sakarya (2006) – First model	(22)	0.0116	0.0058	0.9749	0.0071
Aksoy & Altan-Sakarya (2006) – Second model	(24)	0.0050	0.0042	0.9955	0.0048
Niazkar & Afzali (2015)	(26)	0.0013	0.0010	0.9988	0.0011
Nonlinear regression (this study)	(39)	0.0089	0.0063	0.9412	0.0074
ANN (this study)	–	0.0002	0.0001	1.0000	0.0001
GP (this study)	–	0.0004	0.0003	0.9999	0.0003

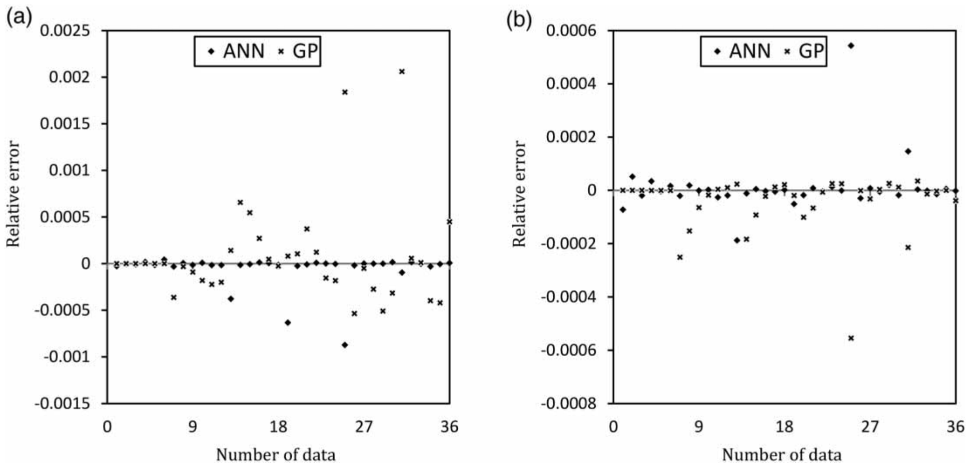

Figure 2 | Relative errors computed for optimum rectangular channels predicted by ANN and GP: (a) y_* and (b) b_*.

optimum b_* achieved by ANN and GP vary within [–0.0002, 0.0005] and [–0.0006, 0.0], respectively. Furthermore, the averages of absolute RE for y_* computed by ANN and GP are zero and 0.0001, respectively. These results align with those shown in Table 6 obviously indicate that both AI models improve the results of optimum $b*$ predicted for rectangular channels. Finally, investigating the variation of RE values achieved by the AI models indicate that they can improve the prediction of optimum geometries of lined rectangular channels in comparison with the available explicit models.

Results for optimum design of triangular channels

Table 7 compares the performances of the AI models with those of available explicit equations for predicting optimum m of lined triangular channels. As shown, ANN outperformed other models for calculating optimum m, while GP

Table 7 | Comparison of different models for optimum design of lined triangular channels

Model	Equation no.	RMSE	MAE	R^2	MARE
(a) For calculating optimum m					
Swamee *et al.* (2000)	(27)	0.0947	0.0699	0.9878	0.0585
Aksoy & Altan-Sakarya (2006) – First model	(29)	0.0777	0.0608	0.9594	0.0515
Aksoy & Altan-Sakarya (2006) – Second model	(31)	0.0931	0.0687	0.9893	0.0575
Niazkar & Afzali (2015)	(33)	0.0059	0.0043	0.9972	0.0038
Nonlinear regression (this study)	(40)	0.0308	0.0194	0.9365	0.0163
ANN (this study)	–	0.0005	0.0002	1.0000	0.0001
GP (this study)	–	0.0027	0.0011	0.9997	0.0008
(b) For calculating optimum y_*					
Swamee *et al.* (2000)	(28)	0.0442	0.0347	0.9664	0.0291
Aksoy & Altan-Sakarya (2006) – First model	(30)	0.0394	0.0327	0.9315	0.0272
Aksoy & Altan-Sakarya (2006) – Second model	(32)	0.1167	0.0842	0.9600	0.0712
Niazkar & Afzali (2015)	(34)	0.0017	0.0013	0.9990	0.0011
Nonlinear regression (this study)	(41)	0.0120	0.0080	0.9569	0.0067
ANN (this study)	–	0.0030	0.0010	0.9987	0.0009
GP (this study)	–	0.0018	0.0007	0.9990	0.0006

is the second best model in terms of all four criteria considered. Among the explicit equations in Table 7, Equation (33) achieved the best results. Additionally, Table 7 indicates that GP yielded to the best optimum y_* values for lined triangular channels, while ANN obtained the second best values of RMSE, MAE and MARE for optimum y_*. Among the explicit equations in Table 7, Equation (34) reached the closest optimum y_* to the final solutions compared with other models. Finally, Table 7 demonstrates the superiority of ANN and GP in the estimation of optimum m and $y*$ in the design of lined triangular channels.

The variations of RE values of $y_*$and m of lined triangular channels predicted by ANN and GP are shown in Figure 3 for the test data. According to Figure 2, RE values of the optimum y_* predicted by ANN and GP are within [–0.0105, 0.0014] and [–0.0093, 0.0026], respectively. Moreover, the averages of absolute RE for y_* obtained by ANN and GP are 0.0009 and 0.0006, respectively. These results are in agreement with those mentioned in Table 7 that implies the superiority of the AI models in the prediction of optimum y_* in the design of lined channels with triangular shapes. Additionally, Figure 3 shows that the variations of RE values for optimum m estimated by ANN and GP are placed within [–0.0002, 0.0018] and [–0.0017, 0.0085], respectively. In addition, the averages of absolute RE for m calculated by ANN and GP are 0.0001 and 0.0008, respectively. These results and those shown in Table 7 clearly indicate that ANN and GP are the first and second best models in the prediction of optimum m for the design of lined triangular channels. Finally, the comparison made between the performances of different models for the optimum design of trapezoidal-family channels clearly indicate that the AI models considerably improved such designs.

Comparison of dimensionless cost for optimum design of lined channels

Based on the channel geometries obtained by ANN, GP and five explicit equations, the dimensionless costs were calculated for trapezoidal, rectangular and triangular canals using Equations (4)–(6), respectively. The dimensionless costs computed for the test data are compared in Table 8 using the four metrics considered. For trapezoidal canals, the second-order regression-based model resulted in the lowest RMSE, while GP yielded to the best MAE value and the second lowest MARE. Among the different models shown in Table 8, GP achieved the lowest dimensionless costs for triangular channels based on three metrics, while ANN

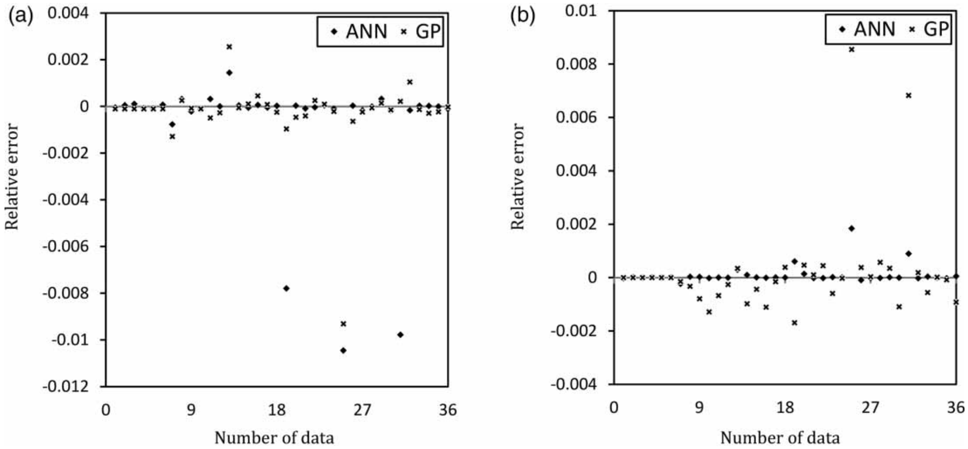

Figure 3 | Relative errors computed for optimum rectangular channels predicted by ANN and GP: (a) y_* and (b) m.

Table 8 | Comparison of different models for dimensionless cost of lined channels

Model	RMSE	MAE	R^2	MARE
(a) Trapezoidal channels				
Swamee *et al.* (2000)	0.0072	0.0059	1.0000	0.0008
Aksoy & Altan-Sakarya (2006) – First model	0.0262	0.0247	1.0000	0.0025
Aksoy & Altan-Sakarya (2006) – Second model	0.0066	0.0059	1.0000	0.0006
Niazkar & Afzali (2015)	0.0066	0.0058	1.0000	0.0006
Nonlinear regression – the 2nd-order polynomial (this study)	0.0064	0.0047	1.0000	0.0006
ANN (this study)	0.0136	0.0061	1.0000	0.0012
GP (this study)	0.0077	0.0030	1.0000	0.0007
(b) Rectangular channels				
Swamee *et al.* (2000)	0.0459	0.0428	1.0000	0.0040
Aksoy & Altan-Sakarya (2006) – First model	0.0060	0.0050	1.0000	0.0006
Aksoy & Altan-Sakarya (2006) – Second model	0.0059	0.0048	1.0000	0.0005
Niazkar & Afzali (2015)	0.0085	0.0070	1.0000	0.0008
Nonlinear regression – the 2nd-order polynomial (this study)	0.0082	0.0065	1.0000	0.0007
ANN (this study)	0.0005	0.0003	1.0000	0.0001
GP (this study)	0.0024	0.0017	1.0000	0.0002
(c) Triangular channels				
Swamee *et al.* (2000)	0.0239	0.0192	1.0000	0.0022
Aksoy & Altan-Sakarya (2006) – First model	0.0495	0.0471	1.0000	0.0046
Aksoy & Altan-Sakarya (2006) – Second model	0.0107	0.0063	1.0000	0.0010
Niazkar & Afzali (2015)	0.5368	0.4135	0.9958	0.0577
Nonlinear regression – the 2nd-order polynomial (this study)	0.0289	0.0225	1.0000	0.0026
ANN (this study)	0.0174	0.0066	1.0000	0.0014
GP (this study)	0.0099	0.0068	1.0000	0.0010

outperformed others in the calculation of dimensionless costs for the rectangular channels. Therefore, Table 8 demonstrates that AI models achieved the lowest dimensionless costs of trapezoidal-family lined channels in ten out of twelve scenarios.

CONCLUSIONS

One of the important challenges facing engineers in water resources management is the design of manmade canals for cost-effective water conveyance. Since the water resources are not necessarily close to where water is in need, water conveyance through artificial and mostly lined canals are practically inevitable. In the hydraulic viewpoint, optimum channel design has been treated as an optimization problem, which comprises a channel cost and a resistance equation as the objective function and constraint, respectively. Evidently, each and every variable that plays a role in real-life projects cannot be considered in the channel cost. However, the most prominent factors such as earthwork and lining costs have been taken into account in previous studies. In this regard, a well-established design optimization problem adopted from the literature was solved to determine optimal dimensions of three widely-common channels shapes: trapezoidal, rectangular and triangular sections. The main contribution of this study is the application of two AI models (ANN and GP) to design optimum trapezoidal-family lined channels. For each dimensionless property of these channels, the design problem was solved for 146 different conditions, while these data were randomly divided into two parts. The first 110 data were used to train the two AI models, while the rest was utilized for comparison purposes. Additionally, three regression-based explicit models were developed for each of the channel's properties. The performances of ANN and GP were compared with those of new regression-based relations and explicit equations available in the literature. The comparison, which was carried out for five performance evaluation criteria, indicates that the AI models outperformed both the previously recommended explicit formulas and the new regression-based models. Moreover, the range of relative errors achieved by the AI models for dimensionless channel geometries placed within [–0.0150, 0.0179] for the trapezoidal section, [–0.0009, 0.0021] for the rectangular section, and [–0.0105, 0.0085] for the triangular section for the test data, respectively. These error bounds obviously demonstrate the high precision of AI models in the optimum design of trapezoidal-family channels. Finally, comparison of dimensionless costs of different models demonstrates that the AI models achieved the lowest dimensionless costs of the trapezoidal-family lined channels in ten out of twelve scenarios.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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