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Assessment of Smart Grid Systems for Electricity Using Power Maclaurin Symmetric Mean Operators Based on T-Spherical Fuzzy Information

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Abstract: Traditional electricity networks are replaced by smart grids to increase efficiency at a low cost. Several energy projects in Pakistan have been developed, while others are currently in the planning stages. To assess the performance of the smart grids in Pakistan, this article employs a multi-attribute group decision-making (MAGDM) strategy based on power Maclaurin symmetric mean (PMSM) operators. We proposed a T-spherical fuzzy (TSF) power MSM (TSFPMSM), and a weighted TSFPMSM (WTSFPMSM) operator. The proposed work aims to analyze the problem involving smart grids in an uncertain environment by covering four aspects of uncertain information. The idempotency, boundedness, and monotonicity features of the proposed TSFPMSM are investigated. In order to assess Pakistan's smart grid networks based on the suggested TSFPMSM operators, a MAGDM algorithm has been developed. The sensitivity analysis of the proposed numerical example is analyzed based on observing the reaction of the variation of the sensitive parameters, followed by a comprehensive comparative study. The comparison results show the superiority of the proposed approach.

Keywords: T-Spherical fuzzy Maclaurin symmetric means; T-Spherical fuzzy power Maclaurin symmetric means; smart grid technology; MULTI-attribute group decision making

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1. Introduction

Zadeh [1] presented the idea of a fuzzy set (FS) in 1965, where a membership function expresses the human opinion to express the vagueness and uncertainties in real-life problems. A FS defined the membership grade (MG) of elements on the interval $[0,1]$ to mathematically represent the uncertainty in the information. FS became the most powerful tool to deal with ambiguity rather than the crisp or classical sets. In addition to FSs, Atanassov introduced the idea of an intuitionistic fuzzy set (IFS), which expands the FSs by incorporating the non-membership grade (NMG) together with the (MG) on the interval $[0,1]$, which is the ideal approach to define the human's point of view. According to IFS theory, only those duplets of information whose sum of MG and NMG lies between [0, 1] are allowed. Atanassov [2] limited the allocation of MG and NMG of (IF) pairs with the condition that the sum of MG and NMG lies in the interval $[0,\ 1]$, which provides much less flexibility in choosing the MGs and NMGs. In real-world problems such as pattern identification and decision-making, the theory of IFS becomes more robust and pervasive. However, if the sum of the duplets becomes greater than 1, IFS fails. Contrarily, to address such issues, Yager [3] suggested the concept of Pythagorean fuzzy sets (PyFS), which only permits the sum of the squares of the MG and NMG within $[0,1]$. PyFS is the generalization of the IFS, which can define ambiguity more accurately and with more considerable flexibility. However, when the sum of the square of the duplets exceeds 1, PyFS fails to be applicable. To cope with such situations, Yager [4] presented the concept of a q-rung orthopair fuzzy set (q-ROFS) that permits MG and NMG between $[0, 1]$ and is more capable and extensive in tackling the ambiguities compared to IFS and PyFS.

Cuong [5] presented the term picture fuzzy set (PFS) with restrictions on MG, NMG, and AG, represented by $\hat{\alpha}$. The theory of PFS is not beneficial for many experimental situations. Due to this fact, Mahmood et al. [6] introduced a spherical fuzzy set (SFS) which enlarges by the sum of squares of MG, NMG, and AG between $[0, 1]$. This is because the term SFS impressively increases the range of PFS for MG, NMG, and AG, but is still not beneficial for some triplets. This gave direction to Mahmood et al. [6] to develop the idea of the TSF set (TSFS) with the parameter q that classifies every triplet as a TSF value (TSFV). The remarkable literature can be found in [7–11].

The aggregation operator is the most effective process for information alliance. During the last tenner, many authors present many aggregation operators. The average mean (AM) operator is the most frequently utilized AO because it easily combines all the various data in a complete form. In addition, several convenient AOs have been created that are beneficial for gathering information in uncertain and complex fuzzy decision-making environments, including arithmetic mean (AM) operator, geometric mean (GM) operator, Bonferroni mean (BM) operator, Heronian mean (HM) operator, etc. Yager [12] suggested utilizing the power average (PA) mean operator to include fuzzy information where the element values support one another throughout the aggregation. IF geometric Heronian mean (IFGHM) AOs were proposed by Yu [13], which were further utilized in MADM. In previous years, other AM AOs have been developed for MADM. None of the above-described operators consider the relationship between the values being used. Yager developed the idea of a power AO to solve this problem [12]. Power AOs strongly influence the relationship of the data being aggregated. Several researchers have used power AOs to handle the numerous MADM issues. For example, Heronian mean HM [14,15], the BM [16], the Hammy mean [17], the power AOs [18], and the MSM operators [19].

MSM operators are one of the subjects in the theory of aggregation that has received the most attention. Maclaurin [20] first proposed the Maclaurin symmetric mean (MSM), which was later popularized by Detemple and Robertson [21]. The fundamental property of MSM is that it conquers the relation of various input arguments. The main distinction between MSM and BM is that, in contrast to BM, MSM can express relationships between more than two input arguments. With regards to the parameter value, MSM monotonically declines for the arguments that are provided. MSM is the one that takes the case in which arguments are converted to crisp numbers. Qin and Liu presented IF MSM (IFMSM) operators [22]; however, Liu et al. [23] examined partitioned MSM operators for MADM applications in the context of IFSs. Wei and Lu [24] studied the Pythagorean fuzzy MSM (PyFMSM) operators in the application of the technology. Yang and Pang [25] also modified the PyFMSM operators by developing transactional PyFMSM operators for MADM applications. Wei et al. [26] invented q-Rung orthopair fuzzy MSM (QROFMSM) operators for the MADM. Power QROFMSM operators were presented by Liu et al. [27] by merging power AO with QROFMSM operators for the decision-making process, and the concept of QROFMSM operators was expanded by Wang et al. [28]. Using an IF layout, Liu and Qin [29] investigated MSM operations for linguistic variables, and Qin et al. presented the MSM operators in hesitant fuzzy settings [30]. Refer to [31–35] for additional helpful research on the theory and uses of MSM operators.

Energy resources and smart grids play an essential role in the energy sector of any country and the investigation of such areas using fuzzy mathematical tools is a hot research topic. The use of fuzzy MAGDM in the energy sector is becoming more popular, and numerous studies have been conducted in this field. As an illustration, Ibrahim Mashal [36] defines the evaluation and assessment of smart grid reliability with the help of fuzzy MADM. The applications of MADM methods in vertical hydroponic farming are investigated by Tolga and Basar [37]. The fuzzy Gaussian number-based TODIM method is utilized in healthcare systems by Tolga et al. [38]. Tolga et al. [39] also investigated the performance of energy power plants for renewable energies. Some other recent approaches for assessing and evaluating energy systems can be seen in [40–43]. The significance of the proposed work is twofold; first, the notion that TSFS can handle uncertain situations based on MADM problems efficiently, as Akram et al. [8]. Other fuzzy frameworks handle uncertainty with less independency, showing our proposed work's superiority. Secondly, the applications of smart grids in electric systems for energy projects play an essential role. Based on these facts, in this paper, we give an overview of the current situation of Pakistan's energy sector. The proposed T-SFPMSM operators are intended to be used in MAGDM issues in the field of electric energy, where the most dependable smart grids are chosen based on a complete numerical example.

This article investigates the novelty of TSF power MSM (T-SFPMSM) and defines their basic operational laws. The central aspect of this paper is to develop the idea of T-SFMSM in the pattern of T-SFPMSM because it is more flexible.

The following are the objective of this paper:

(1) In this work, we present the idea of T-SFPMSM and a few operational rules, then discuss and compare their aspects.
(2) We develop some more AOs, such as WT-SFPMSM (Weighted T-spherical fuzzy power Maclaurin symmetric mean operator).
(3) Using the suggested operators, we propose a new MADM method.
(4) The dominancy of the developed method is illustrated by some examples.

This paper is structured as follows: the purpose of Section 2 is to introduce some basic concepts of T-SFSs and MSM as well as their properties. Section 3 defines the (T-SFPMSM) operator and its basic operating laws. In Section 4, we developed the (WTSFPMSM) operator and investigated its properties. The steps of MAGDM problems are explained in Section 5 using an example. Moreover, this defines the benefits of our new work and presents the comparison study. In the last section, you will find the conclusion of this paper.

2. Preliminaries

In this study, the concept of TSFS will be defined coupled with examining some fundamental attributes of TSFS. Moreover, the idea of MSM operators is also described. The symbol S denotes the universal set, and the triplet $(m,\ d,\ n)$ represents the MG, AG, and NG.

Definition 1. [44] *Let a TSFS $\overline{\alpha}$ in a fixed set X is defined as follows:*

$$\overline{\alpha} = \{\langle s, m(s), d(s), n(s)\rangle | s \in S\} \tag{1}$$

where

$$0 \leq m^z(s) + d^z(s) + n^z(s) \leq 1,\ z \in \mathbb{Z}^+.$$

The term $R(s)$ is expressed as RD, and

$$R(s) = \sqrt[z]{1 - (m^z(s) + d^z(s) + n^z(s))} \tag{2}$$

The membership, abstinence, and non-membership grade represent the numbers m(s), d(s), and n(s), respectively. We say $\overline{\alpha} = (m, d, n)$ be a T-spherical fuzzy value (TSFV).

Definition 2. [45] *Let $\overline{\alpha}_\iota = (m_i, d_i, n_i)$ $(\iota = 1, 2)$ be two TSFVs, eventually various operations of TSFVs are defined as:*

1. $\overline{\alpha}_1 \oplus \overline{\alpha}_2 = \left(\sqrt[z]{(m_1)^z + (m_2)^z - (m_1)^z(m_2)^z}, d_1 d_2, n_1 n_2\right)$
2. $\overline{\alpha}_1 \otimes \overline{\alpha}_2 = \left(m_1 m_2, \sqrt[z]{(d_1)^z + (d_2)^z - (d_1)^z(d_2)^z}, \sqrt[z]{(n_1)^z + (n_2)^z - (n_1)^z(n_2)^z}\right)$
3. $\rho\overline{\alpha} = \left(\sqrt[z]{1 - (1 - m^z)^\rho}, d^\rho, n^\rho\right)$

4. $(\overline{\alpha})^{\rho} = \left(m^{\rho}, \sqrt[z]{1-(1-d^{z})^{\rho}}, \sqrt[z]{1-(1-n^{z})^{\rho}}\right)$
5. $(\overline{\alpha}') = (n, d, m)$

Definition 3. *We define the notion of score function $\mathfrak{P}(\overline{\alpha})$ for TSFVs $\overline{\alpha}$ by:*

$$\mathfrak{P}(\overline{\alpha}) = m^{z} - d^{z}.R^{z} \tag{3}$$

where $\mathfrak{P}(\overline{\alpha}) \in [-1, 1]$.

Definition 4. [22] *The comparison of two TSFVs $\overline{\alpha}_1$ and $\overline{\alpha}_2$ are defined as:*

1. *If $\mathfrak{P}(\overline{\alpha}_1) > \mathfrak{P}(\overline{\alpha}_2)$, then $\overline{\alpha}_1$ shall be given preference over $\overline{\alpha}_2$.*
2. *If $\mathfrak{P}(\overline{\alpha}_1) = \mathfrak{P}(\overline{\alpha}_2)$, then $\overline{\alpha}_1$ is indifferent to $\overline{\alpha}_2$*

Definition 5. [44] *The Hamming distance of two TSFVs $\overline{\alpha}_1$ and $\overline{\alpha}_2$ is defined as:*

$$d(\overline{\alpha}_1, \overline{\alpha}_2) = \frac{1}{3}(|m_1 - m_2| + |d_1 - d_2| + |n_1 - n_2|) \tag{4}$$

Note: *Some cases of TSFS are defined as:*

1. *If $z = 2$ in TSFS, then it becomes SFS.*
2. *If $z = 1$ in TSFS, then it becomes PFS.*
3. *If $d(s) = 0$ in TSFS, then it becomes QROFS.*
4. *If $d(s) = 0$ and $z = 2$ in TSFS, then it becomes PyFS.*
5. *If $d(s) = 0$ and $z = 1$ in TSFS, then it becomes IFS.*

Definition 6 *The MSM is defined as the collection of positive real values $(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n)$ (r = 1, 2, . . ., n) as follows:*

$$\mathrm{MSM}^{(r)}(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) = \left(\frac{\sum_{1 \le \iota_1 < \ldots < \iota_r \le n} \prod_{\mathfrak{j}=1}^{r} \overline{\alpha}_{\iota_{\mathfrak{j}}}}{C_n^r}\right)^{1/r} \tag{5}$$

3. Power Maclaurin Symmetric Mean Operators Based on TSFVs

The complicated decision-making issues stated in Section 1 are addressed in this section, and the standard PA operators will be integrated into the canonic MSM coupled with some new TSF AOs that will be intended and disclosed.

Definition 7. *Let $\overline{\alpha}_n = (m_n, d_n, n_n)$, $(\iota = 1, 2)$ be three TSFVs. Then mapping of the TSFPMSM aggregation operator is $TSFPMSM : \Omega^n \to \Omega$ defined as;*

$$\mathrm{TSFPMSM}^{(r)}(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) = \left\{ \frac{\left(\sum_{1 \le \iota_1 < \ldots < \iota_r \le n} \prod_{\mathfrak{j}=1}^{r} \frac{n\left(1+\mathcal{T}\left(\overline{\alpha}_{\iota_{\mathfrak{j}}}\right)\right)}{\sum_{t=1}^{n}(1+\mathcal{T}(\overline{\alpha}_t))} \right)}{C_n^r} \right\}^{\frac{1}{r}} \tag{6}$$

where

$$\mathcal{T}\left(\overline{\alpha}_{\mathfrak{j}}\right) = \sum_{t=1, t \ne \mathfrak{j}}^{n} \mathrm{Sup}\left(\overline{\alpha}_{\iota}, \overline{\alpha}_{\mathfrak{j}}\right) \tag{7}$$

and $(\iota_1, \iota_2, \ldots, \iota_r)$ Transverses every k-tuple combination of the elements $(1,\ 2,\ \ldots,\ n)$. The above denominator of fractions is composed of the binomial coefficient C_n^r, n is the balancing coefficient, *and r is the number of choices.* $Sup\left(\overline{\alpha}_{\iota},\overline{\alpha}_{\hat{\jmath}}\right)$ *is the measure for TSFN* $\tilde{\overline{\alpha}}_{\iota}$ *from* $\overline{\alpha}_{\hat{\jmath}}$*, satisfying the three criteria listed below:*

(1) $Sup\left(\overline{\alpha}_{\iota},\overline{\alpha}_{\hat{\jmath}}\right) \in [0,1]$

(2) $Sup\left(\overline{\alpha}_{\iota},\overline{\alpha}_{\hat{\jmath}}\right) = Sup\left(\overline{\alpha}_{\hat{\jmath}},\overline{\alpha}_{\iota}\right)$

(3) *If* $d\left(\overline{\alpha}_{\iota},\overline{\alpha}_{\hat{\jmath}}\right) \leq d(\overline{\alpha}_m,\overline{\alpha}_n)$ *, then* $Sup\left(\overline{\alpha}_{\iota},\overline{\alpha}_{\hat{\jmath}}\right) \geq Sup(\overline{\alpha}_m,\overline{\alpha}_n)$ *, where* $d\left(\overline{\alpha}_{\iota},\overline{\alpha}_{\hat{\jmath}}\right)$ *represents the distance measure between two TSFVs.*

To clarify Equation(5)*,we represent*

$$\rho_{\iota} = \frac{1+\mathcal{T}(\overline{\alpha}_{\iota})}{\sum_{t=1}^{n}(1+\mathcal{T}(\overline{\alpha}_{t}))} \tag{8}$$

and call $(\rho_1, \rho_2, \ldots, \rho_n)$ *the power-weighting vector. Apparently,* $\rho_{\iota} \geq 0$ and $\sum_{\iota=1}^{n}\rho_{\iota} = 1$*. So Equation (5) can be more clearly stated as follows:*

$$\mathrm{TSFPMSM}^{(r)}(\overline{\alpha}_1,\overline{\alpha}_2,\ldots,\overline{\alpha}_n) = \left(\frac{\sum_{1\leq\iota_1\leq\ldots\leq\iota_r\leq n}\prod_{\hat{\jmath}=1}^{r} n\rho_{\iota_{\hat{\jmath}}}\overline{\alpha}_{\iota_{\hat{\jmath}}}}{C_n^r}\right)^{1/r} \tag{9}$$

Theorem 1. *Let* $\overline{\alpha}_{\iota} = (a_{\iota}, b_{\iota}, c_{\iota})$ $(\iota = 1,2,\ldots,n)$ *be set of TSFVs and* $r = 1,\ 2,\ \ldots,\ n$*. Then by using* $TSFPMSM^{(r)}$ *we obtain the aggregated value, which is also TSFV.*

$$\begin{aligned} &TSFPMSM^{(r)}(\overline{\alpha}_1,\overline{\alpha}_2,\ldots,\overline{\alpha}_n) \\ &= \left[\begin{array}{c} \left[\sqrt[z]{1-\left(1-\left(1-\prod_{1\leq\iota_1<\ldots<\iota_r\leq n}\left(\left(1-\left(\prod_{\hat{\jmath}=1}^{r}\left(1-\left(1-\left(m_{\iota_{\hat{\jmath}}}\right)^{z}\right)^{n\rho_{\iota_{\hat{\jmath}}}}\right)\right)\right)\right)\right)\right)^{1/C_n^r}}\right]^{1/r}, \\ \sqrt[z]{1-\left\{1-\left(\prod_{1\leq\iota<\ldots<\iota_r\leq n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-d_{\iota_{\hat{\jmath}}}^{\,zn\rho_{\iota_{\hat{\jmath}}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}}, \\ \sqrt[z]{1-\left\{1-\left(\prod_{1\leq\iota_1<\ldots<\iota_r\leq n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-n_{\iota_{\hat{\jmath}}}^{\,zn\rho_{\iota_{\hat{\jmath}}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}} \end{array}\right] \end{aligned} \tag{10}$$

Proof. Following the TSFV's operational law, we have

$$n\rho_{\iota_{\hat{\jmath}}}\overline{\alpha}_{\iota_{\hat{\jmath}}} = \left[\sqrt[z]{1-\left(1-(m_{\iota})^{z}\right)^{n\rho_{\iota_{\hat{\jmath}}}}}, d_{\iota_{\hat{\jmath}}}^{\,n\rho_{\iota_{\hat{\jmath}}}}, n_{\iota_{\hat{\jmath}}}^{\,n\rho_{\iota_{\hat{\jmath}}}}\right]$$

and

$$\prod_{\hat{\jmath}=1}^{r} n\rho_{\iota_{\hat{\jmath}}}\overline{\alpha}_{\iota_{\hat{\jmath}}} = \left[\begin{array}{c} \prod_{\hat{\jmath}=1}^{r}\left(\sqrt[z]{1-\left(1-\left(m_{\iota_{\hat{\jmath}}}\right)^{z}\right)^{n\rho_{\iota_{\hat{\jmath}}}}}\right), \sqrt[z]{1-\prod_{\hat{\jmath}=1}^{r}\left(1-d_{\iota_{\hat{\jmath}}}^{\,zn\rho_{\iota_{\hat{\jmath}}}}\right)}, \\ \sqrt[z]{1-\prod_{\hat{\jmath}=1}^{r}\left(1-n_{\iota_{\hat{\jmath}}}^{\,zn\rho_{\iota_{\hat{\jmath}}}}\right)} \end{array}\right]$$

then

$$\sum_{1\le\iota_1<\ldots<\iota_r\le n}\prod_{\hat{\jmath}=1}^{r} n\rho_{\iota_{\hat{\jmath}}}\overline{\alpha}_{\iota\hat{\jmath}}=\left(\begin{array}{c}\sqrt[z]{1-\prod\limits_{1\le\iota_1<\ldots<\iota_r\le n}\left(\left(1-\left(\prod\limits_{\hat{\jmath}=1}^{r}\left(1-\left(1-\left(m_{\iota_{\hat{\jmath}}}\right)^{z}\right)^{n\rho_{\iota_{\hat{\jmath}}}}\right)\right)\right)\right)},\\ \prod\limits_{1\le\iota_1<\ldots<\iota_r\le n}\sqrt[z]{1-\prod\limits_{\hat{\jmath}=1}^{r}\left(1-d_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)},\\ \prod\limits_{1\le\iota_1<\ldots<\iota_r\le n}\sqrt[z]{1-\prod\limits_{\hat{\jmath}=1}^{r}\left(1-n_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)}\end{array}\right)$$

and

$$\frac{1}{C_n^r}\sum_{1\le\iota_1<\ldots<\iota_r\le n}\prod_{\hat{\jmath}=1}^{r} n\rho_{\iota_{\hat{\jmath}}}\overline{\alpha}_{\iota\hat{\jmath}}=\left(\begin{array}{c}\sqrt[z]{1-\left(1-\left(1-\prod\limits_{1\le\iota_1<\ldots<\iota_r\le n}\left(\left(1-\left(\prod\limits_{\hat{\jmath}=1}^{r}\left(1-\left(1-\left(m_{\iota_{\hat{\jmath}}}\right)^{z}\right)^{n\rho_{\iota_{\hat{\jmath}}}}\right)\right)\right)\right)\right)\right)^{1/C_n^r}},\\ \left(\prod\limits_{1\le\iota_1<\ldots<\iota_r\le n}\sqrt[z]{1-\prod\limits_{\hat{\jmath}=1}^{r}\left(1-d_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)}\right)^{1/C_n^r},\\ \left(\prod\limits_{1\le\iota_1<\ldots<\iota_r\le n}\sqrt[z]{1-\prod\limits_{\hat{\jmath}=1}^{r}\left(1-n_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)}\right)^{1/C_n^r}\end{array}\right)$$

Therefore,

$$\left(\frac{\sum_{1\le\iota_1<\ldots<\iota_r\le n}\prod_{\hat{\jmath}=1}^{r} n\rho_{\iota_{\hat{\jmath}}}\overline{\alpha}_{\iota_{\hat{\jmath}}}}{C_n^r}\right)^{1/r}$$
$$=\left[\begin{array}{c}\left[\sqrt[z]{1-\left(1-\left(1-\prod\limits_{1\le\iota_1<\ldots<\iota_r\le n}\left(\left(1-\left(\prod\limits_{\hat{\jmath}=1}^{r}\left(1-\left(1-\left(m_{\iota_{\hat{\jmath}}}\right)^{z}\right)^{n\rho_{\iota_{\hat{\jmath}}}}\right)\right)\right)\right)\right)\right)^{1/C_n^r}}\right]^{1/r},\\ \sqrt[z]{1-\left(1-\left(\prod\limits_{1\le\iota_1<\ldots<\iota_r\le n}\left(1-\prod\limits_{\hat{\jmath}=1}^{r}\left(1-d_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right)^{1/z}\right)^{1/C_n^r}\right)^{1/r}},\\ \sqrt[z]{1-\left(1-\left(\prod\limits_{1\le\iota_1<\ldots<\iota_r\le n}\left(1-\prod\limits_{\hat{\jmath}=1}^{r}\left(1-n_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right)^{1/z}\right)^{1/C_n^r}\right)^{1/r}}\end{array}\right]$$

Hence proved.□

Example 1. *Let* $\overline{\alpha}_1=(0.3,\ 0.2,\ 0.4)$, $\overline{\alpha}_2=(0.2,\ 0.5,\ 0.6)$, $\overline{\alpha}_3=(0.7,\ 0.2,\ 0.3)$ *and* $\overline{\alpha}=(0.2,\ 0.6,\ 0.7)$ *be four TSFVs, where* $z=2$, $r=2$ *and* $n=3$; *then these four TSFVs are combined to produce a TSFV using the TSFPMSM operator.*

The following are the steps:

Step 1. *Evaluate support* $Sup\left(\overline{\alpha}_i,\overline{\alpha}_{\hat{\jmath}}\right)=1-d\left(\overline{\alpha}_\iota,\overline{\alpha}_{\hat{\jmath}}\right)\ (\iota,\hat{\jmath}=1,2,3,4)$. *Then we have*

$Sup(\overline{\alpha}_1,\overline{\alpha}_2)=Sup(\overline{\alpha}_2,\overline{\alpha}_1)=0.8467,\ Sup(\overline{\alpha}_1,\overline{\alpha}_3)=Sup(\overline{\alpha}_3,\overline{\alpha}_1)=0.8433$
$Sup(\overline{\alpha}_1,\overline{\alpha}_4)=Sup(\overline{\alpha}_4,\overline{\alpha}_1)=0.7667,\ Sup(\overline{\alpha}_2,\overline{\alpha}_3)=Sup(\overline{\alpha}_3,\overline{\alpha}_2)=0.69$
$Sup(\overline{\alpha}_2,\overline{\alpha}_4)=Sup(\overline{\alpha}_4,\overline{\alpha}_2)=0.92,\ Sup(\overline{\alpha}_3,\overline{\alpha}_4)=Sup(\widetilde{\alpha}_4,\widetilde{\alpha}_3)=0.61$

Step 2. Evaluate the vector with power weight by Equations (5) and (7), we have

$\mathcal{T}(\overline{\alpha}_1)=Sup(\overline{\alpha}_1,\overline{\alpha}_2)+Sup(\overline{\alpha}_1,\overline{\alpha}_3)+Sup(\overline{\alpha}_1,\overline{\alpha}_4)=0.8467+0.8433+0.7667=2.4567$
$\mathcal{T}(\overline{\alpha}_2)=Sup(\overline{\alpha}_2,\overline{\alpha}_1)+Sup(\overline{\alpha}_2,\overline{\alpha}_3)+Sup(\overline{\alpha}_2,\overline{\alpha}_4)=0.8467+0.69+0.0.92=2.4567$
$\mathcal{T}(\overline{\alpha}_3)=Sup(\overline{\alpha}_3,\overline{\alpha}_1)+Sup(\overline{\alpha}_3,\overline{\alpha}_2)+Sup(\overline{\alpha}_3,\overline{\alpha}_4)=0.8433+0.69+0.61=2.1433$
$\mathcal{T}(\overline{\alpha}_4)=Sup(\overline{\alpha}_4,\overline{\alpha}_1)+Sup(\overline{\alpha}_4,\overline{\alpha}_2)+Sup(\overline{\alpha}_4,\overline{\alpha}_3)=0.7667+0.92+0.61=2.2967$

and

$$\rho_1 = \frac{1+T(\overline{\alpha}_1)}{\sum_{t=1}^{n}(1+T(\overline{\alpha}_t))} = 0.2589,\ \rho_2 = \frac{1+T(\overline{\alpha}_2)}{\sum_{t=1}^{n}(1+T(\overline{\alpha}_t))} = 0.2589$$

$$\rho_3 = \frac{1+T(\overline{\alpha}_3)}{\sum_{t=1}^{n}(1+T(\overline{\alpha}_t))} = 0.2354,\ \rho_4 = \frac{1+T(\overline{\alpha}_4)}{\sum_{t=1}^{n}(1+T(\overline{\alpha}_t))} = 0.246$$

Step 3. *Evaluate the TSF value $\overline{\alpha} = (m, d, n)$ by Equation (9) where $r = 2$, then we have*

$$m = \left[\sqrt[z]{1-\left(1-\left(1-\prod_{1\le\iota_1<\ldots<\iota_r\le n}\left(\left(1-\left(\prod_{\hat{\jmath}=1}^{r}\left(1-\left(1-\left(m_{\iota_{\hat{\jmath}}}\right)^{z}\right)^{n\rho_{\iota_{\hat{\jmath}}}}\right)\right)\right)\right)\right)\right)^{1/C_3^2}}\,\right]^{1/2}$$

$$= \left[\left(1-\left(1-\left(1-\left(\begin{array}{c}\left\{1-\left(1-\left(1-(m_1)^2\right)^{3\rho_1}\right)*\left(1-\left(1-(m_2)^2\right)^{3\rho_2}\right)\right\}\\ *\left\{1-\left(1-\left(1-(m_1)^2\right)^{3\rho_1}\right)*\left(1-\left(1-(m_3)^2\right)^{3\rho_3}\right)\right\}\\ *\left\{1-\left(1-\left(1-(m_1)^2\right)^{3\rho_1}\right)*\left(1-\left(1-(m_4)^2\right)^{3\rho_4}\right)\right\}\\ *\left\{1-\left(1-\left(1-(m_2)^2\right)^{3\rho_2}\right)*\left(1-\left(1-(m_3)^2\right)^{3\rho_3}\right)\right\}\\ *\left\{1-\left(1-\left(1-(m_2)^2\right)^{3\rho_2}\right)*\left(1-\left(1-(m_4)^2\right)^{3\rho_4}\right)\right\}\\ *\left\{1-\left(1-\left(1-(m_3)^2\right)^{3\rho_3}\right)*\left(1-\left(1-(m_4)^2\right)^{3\rho_4}\right)\right\}\end{array}\right)\right)\right)^{1/3}\right)^{1/2}\right]^{1/2}$$

$$= 0.4$$

$$d = \sqrt[z]{1-\left(1-\left(\prod_{1\le\iota_1<\ldots<\iota_r\le n}\left(1-\prod_{\hat{\jmath}=1}^{r}\left(1-d_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right)^{1/z}\right)^{1/C_3^2}\right)^{1/2}}$$

$$d = \left[1-\left(1-\left(\begin{array}{c}\left(1-\left(1-d_1^{2n\rho_1}\right)*\left(1-d_2^{2n\rho_2}\right)\right)^{\frac{1}{z}}\\ *\left(1-\left(1-d_1^{2n\rho_1}\right)*\left(1-d_3^{2n\rho_3}\right)\right)^{\frac{1}{z}}\\ *\left(1-\left(1-d_1^{2n\rho_1}\right)*\left(1-d_4^{2n\rho_4}\right)\right)^{\frac{1}{z}}\\ *\left(1-\left(1-d_2^{2n\rho_2}\right)*\left(1-d_3^{2n\rho_3}\right)\right)^{\frac{1}{z}}\\ *\left(1-\left(1-d_2^{2n\rho_2}\right)*\left(1-d_3^{2n\rho_3}\right)\right)^{\frac{1}{z}}\\ *\left(1-\left(1-d_3^{2n\rho_3}\right)*\left(1-d_4^{2n\rho_4}\right)\right)^{1/z}\end{array}\right)^{1/3}\right)^{1/2}\right]^{1/2}$$

$$= 0.5$$

similarly,

$$n = \sqrt[z]{1-\left(1-\left(\prod_{1\le\iota_1<\ldots<\iota_r\le n}\left(1-\prod_{\hat{\jmath}=1}^{r}\left(1-n_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right)^{1/z}\right)^{1/C_3^2}\right)^{1/2}} = 0.6$$

$$TSFPMSM^{(2)}(\overline{\alpha}_1, \overline{\alpha}_2, \overline{\alpha}_3, \overline{\alpha}_4) = (0.4,\ 0.5,\ 0.6)$$

We easily understand the following list of desirable characteristics of TSFPMSM.

Property 1. *(Idempotency). Let $\overline{\alpha}_{\iota} = \alpha = (m, d, n)$ $\forall$ ι be some TSFVs and $\overline{\alpha}_{\iota} = \alpha$ and for all α. Then*

$$TSFPMSM^{(r)}(\alpha, \alpha, \ldots, \alpha) = \alpha \tag{11}$$

Proof. If $\overline{\alpha}_{\iota_{\hat{j}}} = \alpha = (m, d, n)$ then

$$n\rho_{\iota_{\hat{j}}} = \frac{n\left[1 + \mathcal{T}\left(a_{\iota_{\hat{j}}}\right)\right]}{\sum_{t=1}^{n}[1 + \mathcal{T}(a_t)]} = 1.$$

$$= \sqrt[z]{\prod_{1 \le \iota_1 < \ldots < \iota_r \le n}\left(1 - \prod_{\hat{j}=1}^{r}\left(1 - \left(1 - \left(m_{\iota_{\hat{j}}}\right)^{z}\right)^{n\rho_{\iota_{\hat{j}}}}\right)\right)^{1/C_n^r}} = \sqrt[z]{\prod_{1 \le \iota_1 < \ldots < \iota_r \le n}\left(1 - \left(m_{\iota_{\hat{j}}}\right)^{zr}\right)^{1/C_n^r}}$$
$$= \sqrt[z]{\left(\left(1 - \left(m_{\iota_{\hat{j}}}\right)^{zr}\right)^{1/C_n^r}\right)^{C_n^r}} = \sqrt[z]{1 - \left(m_{\iota_{\hat{j}}}\right)^{zr}}$$

then

$$\left[\sqrt[z]{1 - \left(\prod_{1 \le \iota_1 < \ldots < \iota_r \le n}\left(1 - \prod_{\hat{j}=1}^{r}\left(1 - \left(1 - \left(m_{\iota_{\hat{j}}}\right)^{z}\right)^{n\rho_{\iota_{\hat{j}}}}\right)\right)\right)^{1/C_n^r}}\right]^{1/r} =$$

$$\left[\sqrt[z]{1 - (1 - m^{zr})}\right]^{1/r} = \left[\sqrt[z]{(m^{zr})}\right]^{1/r} = m$$

$$\sqrt[z]{\prod_{1 \le \iota_1 < \ldots < \iota_r \le n}\left(1 - \prod_{\hat{j}=1}^{r}\left(1 - \left(d_{\iota_{\hat{j}}}\right)^{zn\rho_{\iota_{\hat{j}}}}\right)\right)^{1/C_n^r}} = \sqrt[z]{\prod_{1 \le \iota_1 < \ldots < \iota_r \le n}\left(1 - \left(1 - d_{\iota_{\hat{j}}}\right)^{zr}\right)^{1/C_n^r}}$$
$$= \sqrt[z]{1 - \left(1 - d_{\iota_{\hat{j}}}\right)^{zr}}$$

Then

$$\sqrt[z]{1 - \left(1 - \prod_{1 \le \iota_1 < \ldots < \iota_r \le n}\left(1 - \prod_{\hat{j}=1}^{r}\left(1 - \left(d_{\iota_{\hat{j}}}\right)^{zn\rho_{\iota_{\hat{j}}}}\right)\right)^{1/C_n^r}\right)^{1/r}}$$

$$= \sqrt[z]{1 - \left(1 - \left(1 - \left(1 - d_{\iota_{\hat{j}}}\right)^{zr}\right)\right)^{1/r}} = d$$

similarly,

$$\sqrt[z]{\prod_{1 \le \iota_1 < \ldots < \iota_r \le n}\left(1 - \prod_{\hat{j}=1}^{r}\left(1 - n_{\iota_{\hat{j}}}^{zn\rho_{\iota_{\hat{j}}}}\right)\right)^{1/C_n^r}} = \sqrt[z]{\prod_{1 \le \iota_1 < \ldots < \iota_r \le n}\left(1 - \left(1 - n_{\iota_{\hat{j}}}\right)^{zr}\right)^{1/C_n^r}}$$

$$= \sqrt[z]{\left(\left(1 - \left(1 - n_{\iota_{\hat{j}}}\right)^{zr}\right)^{1/C_n^r}\right)^{C_n^r}} = \sqrt[q]{1 - \left(1 - n_{\iota_{\hat{j}}}\right)^{zr}}$$

Then,

$$\sqrt[z]{1 - \left(1 - \prod_{1 \le \iota_1 < \ldots < \iota_r \le n}\left(1 - \prod_{\hat{j}=1}^{r}\left(1 - n_{\iota_{\hat{j}}}^{zn\rho_{\iota_{\hat{j}}}}\right)\right)^{1/C_n^r}\right)^{1/r}}$$

$$= \sqrt[z]{1-\left(1-\left(1-\left(1-n_{\iota_{\jmath}}\right)^{zr}\right)\right)^{\frac{1}{r}}} = n$$

therefore, if $\overline{\alpha}_{\iota} = \alpha = (m, d, n)$, then

$$TSFPMSM^{(r)}(\alpha, \alpha, \ldots, \alpha) = \alpha = (m, d, n)$$

hence proof. □

Property 2. (Boundedness): *Let $\overline{\alpha}_{\iota} = (m_{\iota}, d_{\iota}, n_{\iota})$ $(\iota = 1, 2, \ldots, n)$ be a collection of TSFVs, and $\overline{\alpha}^{-} = min\left(\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots, \widetilde{\alpha}_n\right) = \left(\overline{m}, \overline{d}, \overline{n}\right)$, $\overline{\alpha}^{+} = max(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) = \left(\hat{m}, \hat{d}, \hat{n}\right)$. Then, the TSFPMSM operator gives:*

$$\overline{\alpha}^{-} \leq TSFPMSM(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) \leq \overline{\alpha}^{+}$$

Proof. Since $\rho_{\iota} = \frac{n[1+\mathcal{T}(\overline{\alpha}_{\iota})]}{\sum_{t=1}^{n}[1+\mathcal{T}(\overline{\alpha}_{t})]}$ and $\sum_{t=1}^{n} \rho_{\iota} = 1$, by using Equation (5) and TSFMSM's boundedness, we have

$$TSFPMSM(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) = \left(\frac{\sum_{1\leq\iota_1<\ldots<\iota_k\leq n} \prod_{j=1}^{r} n\rho_{\iota_{\jmath}}\overline{\alpha}_{\iota_{\jmath}}}{C_n^r}\right)^{1/r}$$

$$\leq \left(\frac{\sum_{1\leq\iota_1<\ldots<\iota_k\leq n} \prod_{\hat{\jmath}=1}^{r} n\rho_{\iota_{\jmath}}\overline{\alpha}^{+}}{C_n^r}\right)^{1/r} = \overline{\alpha}^{+}$$

Similarly, we have $TSFPMSM(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) \leq \overline{\alpha}^{-}$. Thus,

$$\overline{\alpha}^{+} \leq TSFPMSM(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) \leq \overline{\alpha}^{-}$$

□

Property 3. (Monotonicity): *Suppose there are two collections of TSFVs $\overline{\alpha}_{\iota}$ and $\hat{\alpha}_{\iota}$ $(\iota = 1, 2, \ldots, n)$, if $\overline{\alpha}_{\iota} \leq \hat{\alpha}_{\iota}\ \forall\ \iota$, then*

$$TSFPMSM^{(r)}(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) \leq TSFPMSM^{(r)}(\hat{\alpha}_1, \hat{\alpha}_2, \ldots, \hat{\alpha}_n)$$

Proof. As we have

$$TSFPMSM^{(r)}(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n)$$
$$= \left[\begin{array}{c} \left[\sqrt[z]{1-\left(1-\left(1-\prod_{1\leq\iota_1<\ldots<\iota_r\leq n}\left(\left(1-\left(\prod_{\hat{\jmath}=1}^{r}\left(1-\left(1-\left(m_{\iota_{\jmath}}\right)^{z}\right)^{n\rho_{\iota_{\jmath}}}\right)\right)\right)\right)\right)\right)^{1/C_n^r}}\right]^{1/r}, \\ \sqrt[z]{1-\left\{1-\left(\prod_{1\leq\iota<\ldots<\iota_r\leq n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-d_{\iota_{\jmath}}^{zn\rho_{\iota_{\jmath}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}}, \\ \sqrt[z]{1-\left\{1-\left(\prod_{1\leq\iota_1<\ldots<\iota_r\leq n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-n_{\iota_{\jmath}}^{zn\rho_{\iota_{\jmath}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}} \end{array}\right]$$

$$TSFPMSM^{(r)}(\hat{\alpha}_1,\hat{\alpha}_2,\ldots,\hat{\alpha}_n)$$
$$= \left[\begin{array}{c} \left[\sqrt[z]{1-\left(1-\left(1-\prod_{1\le\iota_1<\ldots<\iota_r\le n}\left(\left(1-\left(\prod_{\hat{\jmath}=1}^{r}\left(1-\left(1-\left(\hat{m}_{\iota_{\hat{\jmath}}}\right)^z\right)^{n\rho_{\iota_{\hat{\jmath}}}}\right)\right)\right)\right)\right)\right)^{1/C_n^r}}\right]^{1/r} \\ \sqrt[z]{1-\left\{1-\left(\prod_{1\le\iota<\ldots<\iota_r\le n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-\hat{d}_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}}, \\ \sqrt[z]{1-\left\{1-\left(\prod_{1\le\iota_1<\ldots<\iota_r\le n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-\hat{n}_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}} \end{array}\right]$$

as we know that for monotonicity, we have

$$m_\iota \le \hat{m}_\iota, d_\iota \le \hat{d}_\iota,\ n_\iota \le \hat{n}_\iota.$$

then

$$= \sqrt[z]{1-\left(1-\left(1-\prod_{1\le\iota_1<\ldots<\iota_r\le n}\left(\left(1-\left(\prod_{\hat{\jmath}=1}^{r}\left(1-\left(1-\left(m_{\iota_{\hat{\jmath}}}\right)^z\right)^{n\rho_{\iota_{\hat{\jmath}}}}\right)\right)\right)\right)\right)\right)^{1/C_n^r}}$$
$$\le \sqrt[z]{1-\left(1-\left(1-\prod_{1\le\iota_1<\ldots<\iota_r\le n}\left(\left(1-\left(\prod_{\hat{\jmath}=1}^{r}\left(1-\left(1-\left(\hat{m}_{\iota_{\hat{\jmath}}}\right)^z\right)^{n\rho_{\iota_{\hat{\jmath}}}}\right)\right)\right)\right)\right)\right)^{1/C_n^r}}$$

and

$$= \sqrt[z]{1-\left\{1-\left(\prod_{1\le\iota<\ldots<\iota_r\le n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-d_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}} \ge$$
$$\sqrt[z]{1-\left\{1-\left(\prod_{1\le\iota<\ldots<\iota_r\le n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-\hat{d}_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}}$$

similarly,

$$\sqrt[z]{1-\left\{1-\left(\prod_{1\le\iota_1<\ldots<\iota_r\le n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-n_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}} \ge$$
$$\sqrt[z]{1-\left\{1-\left(\prod_{1\le\iota_1<\ldots<\iota_r\le n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-\hat{n}_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}}$$

by utilizing this, we can write

$$= \begin{bmatrix} \left[\sqrt[z]{1-\left(1-\left(1-\prod_{1\le\iota_1<\ldots<\iota_r\le n}\left(\left(1-\left(\prod_{\hat{\jmath}=1}^{r}\left(1-\left(1-\left(m_{\iota_{\hat{\jmath}}}\right)^z\right)^{n\rho_{\iota_{\hat{\jmath}}}}\right)\right)\right)\right)\right)\right)^{1/C_n^r}}\right]^{1/r}, \\ \sqrt[z]{1-\left\{1-\left(\prod_{1\le\iota<\ldots<\iota_r\le n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-d_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}}, \\ \sqrt[z]{1-\left\{1-\left(\prod_{1\le\iota_1<\ldots<\iota_r\le n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-n_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}} \end{bmatrix} \le$$

$$\begin{bmatrix} \left[\sqrt[z]{1-\left(1-\left(1-\prod_{1\le\iota_1<\ldots<\iota_r\le n}\left(\left(1-\left(\prod_{\hat{\jmath}=1}^{r}\left(1-\left(1-\left(\hat{m}_{\iota_{\hat{\jmath}}}\right)^z\right)^{n\rho_{\iota_{\hat{\jmath}}}}\right)\right)\right)\right)\right)\right)^{1/C_n^r}}\right]^{1/r}, \\ \sqrt[z]{1-\left\{1-\left(\prod_{1\le\iota<\ldots<\iota_r\le n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-\hat{d}_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}}, \\ \sqrt[z]{1-\left\{1-\left(\prod_{1\le\iota_1<\ldots<\iota_r\le n}\left[1-\prod_{\hat{\jmath}=1}^{r}\left(1-\hat{n}_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right]^{1/z}\right)^{1/C_n^r}\right\}^{1/r}} \end{bmatrix}$$

thus,

$$TSFPMSM^{(r)}(\overline{\alpha}_1,\overline{\alpha}_2,\ldots,\overline{\alpha}_n) \le TSFPMSM^{(r)}(\hat{\alpha}_1,\hat{\alpha}_2,\ldots,\hat{\alpha}_n)$$

the parameter r of the following, $TSFPMSM^{(r)}$ operator can be changed to attain three particular cases. □

Case 1. *When $r = 1$, since we get:*

$$TSFPMSM^{(1)}(\overline{\alpha}_1,\overline{\alpha}_2,\ldots,\overline{\alpha}_{\mathrm{n}}) = \sqrt[z]{\left(\frac{\sum_{1\le\iota_1\le\iota_2\le\ldots\le\iota_k\le n}\prod_{\hat{\jmath}=1}^{k} n\rho_{\iota_{\hat{\jmath}}}\overline{\alpha}_{\iota_{\hat{\jmath}}}}{C_n^1}\right)^{1/1}}$$

$$= \begin{bmatrix} \sqrt[z]{1-\prod_{1\le\iota_1\le n}\left(1-\prod_{\hat{\jmath}=1}^{1}\left(1-\left(1-\left(m_{\iota_{\hat{\jmath}}}\right)^z\right)^{n\rho_{\iota_{\hat{\jmath}}}}\right)\right)^{1/C_n^1}}, \\ \sqrt[z]{1-\left(1-\prod_{1\le\iota_1\le n}\left(1-\prod_{\hat{\jmath}=1}^{1}\left(1-d_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right)^{1/C_n^1}\right)}, \\ \sqrt[z]{1-\left(1-\prod_{1\le\iota_1\le n}\left(1-\prod_{\hat{\jmath}=1}^{1}\left(1-n_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}}\right)\right)^{1/C_n^1}\right)} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[z]{1-\prod_{1\le\iota_1\le \mathrm{n}}\left((1-(\mathrm{m}_{\iota_1})^z)^{\mathrm{n}\rho_{\iota_1}}\right)^{1/\mathrm{n}}}, \\ \sqrt[z]{1-\left(1-\prod_{1\le\iota_1\le\mathrm{n}}\left(\mathrm{d}_{\iota_1}^{\mathrm{zn}\rho_{\iota}}\right)^{1/\mathrm{n}}\right)}, \\ \sqrt[z]{1-\left(1-\prod_{1\le\iota_1\le\mathrm{n}}\left(\mathrm{n}_{\iota_1}^{\mathrm{zn}\rho_{\iota_1}}\right)^{1/\mathrm{n}}\right)} \end{bmatrix}$$

let $\iota_1 = \iota$

$$= \left[\sqrt[z]{1 - \prod_{1 \le \iota \le n} \left(1 - (m_\iota)^z\right)^{\rho_\iota}},\ \sqrt[z]{\prod_{1 \le \iota \le n} d_\iota^{z\rho_\iota}},\ \sqrt[z]{\prod_{1 \le \iota \le n} n_\iota^{z\rho_i}} \right]$$

under certain conditions, the proposal transforms the TSFPMSM operator into T-Spherical fuzzy power average operator (TSFPA).

Case 2. *By taking $r = 2$, we obtain:*

$$TSFPMSM^{(2)}(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) = \sqrt[z]{\left(\frac{\sum_{1 \le \iota_1 \le \iota_2 \le \ldots \le \iota_r \le n} \prod_{\hat{\jmath}=1}^{2} n\rho_{\iota_{\hat{\jmath}}} \overline{\alpha}_{\iota_{\hat{\jmath}}}}{C_n^2} \right)^{1/2}}$$

$$= \left[\begin{array}{c} \left\{ \sqrt[z]{\left(1 - \prod_{1 \le \iota_1 \le \ldots \le \iota_r \le n} \left(1 - \prod_{\hat{\jmath}=1}^{2} \left(1 - \left(1 - \left(m_{\iota_{\hat{\jmath}}} \right)^z \right)^{n\rho_{\iota_{\hat{\jmath}}}} \right) \right)^{1/C_n^2} \right)} \right\}^{1/2}, \\ \sqrt[z]{1 - \left(1 - \prod_{1 \le \iota_1 \le \ldots \le \iota_r \le n} \left(1 - \prod_{\hat{\jmath}=1}^{2} \left(1 - d_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}} \right) \right)^{1/C_n^2} \right)^{1/2}}, \\ \sqrt[z]{1 - \left(1 - \prod_{1 \le \iota_1 \le \ldots \le \iota_r \le n} \left(1 - \prod_{\hat{\jmath}=1}^{2} \left(1 - n_{\iota_{\hat{\jmath}}}^{zn\rho_{\iota_{\hat{\jmath}}}} \right) \right)^{1/C_n^2} \right)^{1/2}} \end{array} \right]$$

$$= \left[\begin{array}{c} \left(\sqrt[z]{\left(1 - \prod_{\substack{\iota_1, \iota_2 = 1 \\ \iota_1 \ne \iota_2}} \left(\left(1 - \left[1 - (1 - m_\iota)^{n\rho_{\iota_1}} \right] \left[1 - (1 - m_{\iota_2})^{n\rho_{\iota_2}} \right] \right)^{1/2} \right)^{2/n(n-1)} \right)} \right)^{1/2}, \\ \sqrt[z]{1 - \left(1 - \prod_{\substack{\iota_1, \iota_2 = 1 \\ \iota_1 \ne \iota_2}} \left(1 - \left(1 - d_{\iota_1}^{n\rho_{\iota_1}} \right) \left(1 - d_{\iota_2}^{n\rho_{\iota_2}} \right) \right)^{1/n(n-1)} \right)^{1/2}}, \\ \sqrt[z]{1 - \left(1 - \prod_{\substack{\iota_1, \iota_2 = 1 \\ \iota_1 \ne \iota_2}} \left(1 - \left(1 - n_{\iota_1}^{n\rho_{\iota_1}} \right) \left(1 - n_{\iota_2}^{n\rho_{\iota_2}} \right) \right)^{1/n(n-1)} \right)^{1/2}} \end{array} \right]$$

$$= \left[\begin{array}{c} \left\{ \sqrt[z]{\left(1 - \prod_{\substack{\iota_1, \iota_2 = 1 \\ \iota_1 \ne \iota_2}} \left(1 - \left[1 - \left(1 - (m_{\iota_1})^z\right)^{n\rho_{\iota_1}} \right] \left[1 - \left(1 - (m_{\iota_2})^z\right)^{n\rho_{\iota_2}} \right] \right)^{1/n(n-1)} \right)} \right\}^{1/2}, \\ \sqrt[z]{1 - \left(1 - \prod_{\substack{\iota_1, \iota_2 = 1 \\ \iota_1 \ne \iota_2}} \left(1 - \left(1 - d_{\iota_1}^{zn\rho_{\iota_1}} \right) \left(1 - d_{\iota_2}^{zn\rho_{\iota_2}} \right) \right)^{1/n(n-1)} \right)^{1/2}}, \\ \sqrt[z]{1 - \left(1 - \prod_{\substack{\iota_1, \iota_2 = 1 \\ \iota_1 \ne \iota_2}} \left(1 - \left(1 - n_{\iota_1}^{zn\rho_{\iota_1}} \right) \left(1 - n_{\iota_2}^{zn\rho_{\iota_2}} \right) \right)^{1/n(n-1)} \right)^{1/2}} \end{array} \right]$$

therefore, the TSFPMSM lessens the T-spherical fuzzy power Bonferroni mean (TSFPB) $(p = z = 1)$ operator. Remember that TSFPB is straightforward to obtain; see [46]. *Moreover, Equation (9) can be transformed as:*

$$TSFPMSM^{(2)}(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) = \sqrt[z]{\left(\frac{\sum_{1 \le \iota_1 \le \iota_2 \le \ldots \le \iota_r \le n} \prod_{\hat{\jmath}=1}^{r} n\rho_{\iota_{\hat{\jmath}}} \overline{\alpha}_{\iota_{\hat{\jmath}}}}{C_n^2} \right)^{1/2}}$$

$$= \sqrt[z]{\left(\left(\sum_{1 \le \iota_1 \le \iota_2 \le \ldots \le \iota_r \le n} \prod_{\jmath=1}^{2} \overline{\alpha}_{\iota_\jmath}\right)^{1/C_n^2}\right)^{1/2}}$$

$$= \left[\begin{array}{c} \left(\sqrt[z]{\left(1 - \prod_{\substack{\iota_1,\iota_2=1 \\ \iota_1 \ne \iota_2}} (1 - m_{\iota_1} m_{\iota_2})^{z^{1/n(n-1)}}\right)}\right)^{1/2}, \\ \sqrt[z]{1 - \left(1 - \prod_{\substack{\iota_1,\iota_2=1 \\ \iota_1 \ne \iota_2}} (1 - (1 - d_{\iota_1})(1 - d_{\iota_2}))^{z^{1/n(n-1)}}\right)^{1/2}}, \\ \sqrt[z]{1 - \left(1 - \prod_{\substack{\iota_1,\iota_2=1 \\ \iota_1 \ne \iota_2}} (1 - (1 - n_{\iota_1})(1 - n_{\iota_2}))^{z^{1/n(n-1)}}\right)^{1/2}} \end{array}\right]$$

$$= TSFBM^{(1,1)}(\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots, \widetilde{\alpha}_n)$$

Xu and Chen [47] proposed TSFBM referred to as $(p = z = 1)$

Case 3. *If* $r = n$ *, Equation (9) will become as follows:*

$$TSFPMSM^{(n)}(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) = \sqrt[z]{\left(\frac{\sum_{1 \le \iota_1 \le \ldots \le \iota_r \le n} \prod_{\jmath=1}^{n} n\rho_{\iota_\jmath} \overline{\alpha}_{\iota_\jmath}}{C_n^n}\right)^{1/n}}$$

$$= \left[\begin{array}{c} \left(\sqrt[z]{\left(1 - \prod_{1 \le \iota_1 \le \ldots \le \iota_r \le n} \left(1 - \prod_{\jmath=1}^{n} \left(1 - \left(1 - \left(m_{\iota_\jmath}\right)^z\right)^{n\rho_{\iota_\jmath}}\right)\right)^{1/C_n^n}\right)}\right)^{1/n}, \\ \sqrt[z]{1 - \left(1 - \prod_{1 \le \iota_1 \le \ldots \le \iota_r \le n} \left(1 - \prod_{\jmath=1}^{n} \left(1 - d_{\iota_\jmath}^{zn\rho_{\iota_\jmath}}\right)\right)^{1/C_n^n}\right)^{1/n}}, \\ \sqrt[z]{1 - \left(1 - \prod_{1 \le \iota_1 \le \ldots \le \iota_r \le n} \left(1 - \prod_{\jmath=1}^{n} \left(1 - n_{\iota_\jmath}^{zn\rho_{\iota_\jmath}}\right)\right)^{1/C_n^n}\right)^{1/n}} \end{array}\right]$$

$$= \left[\sqrt[z]{\prod_{\jmath=1}^{n} \left(1 - \left(1 - \left(m_{\iota_\jmath}\right)^z\right)^{n\rho_{\iota_\jmath}}\right)^{1/n}}, \sqrt[z]{1 - \left(\prod_{\jmath=1}^{n} \left(1 - d_{\iota}^{zn\rho_\iota}\right)\right)^{1/n}}, \sqrt[z]{1 - \left(\prod_{\jmath=1}^{n} \left(1 - n_{\iota}^{zn\rho_\iota}\right)\right)^{1/n}}\right]$$

moreover, if we assume $Sup\left(\overline{\alpha}_\iota, \overline{\alpha}_\jmath\right) = \zeta \; \forall \; \iota \ne \jmath$ *, then* $n\rho_{\iota_\jmath} = \frac{n\left[1 + \mathcal{T}\left(a_{\iota_\jmath}\right)\right]}{\sum_{t=1}^{n} [1 + \mathcal{T}(a_t)]} = 1$*, as well as the Equation (9) can be transformed as follows:*

$$TSFPMSM^{(n)}(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) = \sqrt[z]{\left(\frac{\sum_{1 \le \iota_1 \le \iota_2 \le \ldots \le \iota_r \le n} \prod_{\jmath=1}^{n} n\rho_{\iota_\jmath} \overline{\alpha}_{\iota_\jmath}}{C_n^n}\right)^{1/n}}$$

$$= \left[\sqrt[z]{\left(\prod_{\iota=1}^{n} m_\iota\right)^{1/n}}, \sqrt[z]{1 - \left(\prod_{\iota=1}^{n} (1 - d_\iota)\right)^{1/n}}, \sqrt[z]{1 - \left(\prod_{\iota=1}^{n} (1 - n_\iota)\right)^{1/n}}\right]$$

This indicates that the TSFPMSM operator becomes the TSF geometric mean (TSFGM) operator if all the supports are the same. It is clear that the TSFPMSM operator only takes the interaction between the power weighting vector and the input arguments, not with the aggregated arguments. However, there are cases, particularly in MAGDM, where the attribute weight vectors play a crucial role in the aggression process. When different weights are assigned to various attributes in the following, the weighted form of the TSFPMSM operators can be defined as follows:

Definition 8. *Let a set of TSFVs be* $\overline{\alpha}_{\iota}$ $(\iota = 1, 2, \ldots, n)$ *and* $r = 1, 2, \ldots, n$. *The weighted TSFPMSM operator is represented by the mapping* $WTSFPMSM : \varphi^n \to \varphi$, *which is defined as follows:*

$$WTSFPMSM^{(r)}(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) = \sqrt[z]{\left(\frac{1}{C_n^r} \sum_{1 \le \iota_1 \le \ldots \le \iota_r \le n} \prod_{\hat{\jmath}=1}^{r} n\omega_{\iota_{\hat{\jmath}}} \overline{\alpha}_{\iota_{\hat{\jmath}}}\right)^{1/r}} \tag{12}$$

where $(\iota_1, \iota_2, \ldots, \iota_r)$ *traverse every k-tuple combination of* $(1, 2, \ldots, n)$,

$$\widetilde{\omega}_{\iota} = \frac{w_{\iota}[1 + \mathcal{T}(\overline{\alpha}_{\iota})]}{\sum_{t=1}^{n} w_{\iota}[1 + \mathcal{T}(\overline{\alpha}_t)]}$$

and

$$\mathcal{T}\left(\overline{\alpha}_{\hat{\jmath}}\right) = \sum_{\substack{t=1 \\ t \ne \hat{\jmath}}}^{n} Sup\left(\overline{\alpha}_t, \overline{\alpha}_{\hat{\jmath}}\right)$$

$Sup\left(\overline{\alpha}_t, \overline{\alpha}_{\hat{\jmath}}\right)$ *and satisfies the characteristics listed in Definition 5. The weight vector* $w = (w_1, w_2, \ldots, w_n)^T$ *of* $(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n)$, *with* $w_{\iota} \in [0, 1]$ *and* $\sum_{\iota=1}^{n} w_{\iota} = 1$. *The balancing coefficient is n, while the binomial coefficient is* C_n^r. *Equation (13) can be further transformed based on TSFVs operations as follows:*

$$\begin{aligned} &WTSFPMSM^{(r)}(\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n) \\ &= \left[\begin{array}{c} \left[\sqrt[z]{1 - \left(1 - \left(1 - \prod_{1 \le \iota_1 < \ldots < \iota_r \le n} \left(\left(1 - \left(\prod_{\hat{\jmath}=1}^{r} \left(1 - \left(1 - \left(m_{\iota_{\hat{\jmath}}}\right)^z\right)^{n\widetilde{\omega}_{\iota_{\hat{\jmath}}}}\right)\right)\right)\right)\right)\right)^{1/C_n^r}}\right]^{1/r}, \\ \sqrt[z]{1 - \left(1 - \left(\prod_{1 \le \iota_1 < \ldots < \iota_r \le n} \left(1 - \prod_{\hat{\jmath}=1}^{r} \left(1 - d_{\iota_{\hat{\jmath}}}^{zn\widetilde{\omega}_{\iota_{\hat{\jmath}}}}\right)\right)^{1/z}\right)^{1/C_n^r}\right)^{1/r}}, \\ \sqrt[z]{1 - \left(1 - \left(\prod_{1 \le \iota_1 < \ldots < \iota_{rr} \le n} \left(1 - \prod_{\hat{\jmath}=1}^{r} \left(1 - n_{\iota_{\hat{\jmath}}}^{zn\widetilde{\omega}_{\iota_{\hat{\jmath}}}}\right)\right)^{1/z}\right)^{1/C_n^r}\right)^{1/r}} \end{array}\right] \end{aligned} \tag{13}$$

4. MAGDM Methods by Using Investigated Operators Based on SFSs

For this problem, assuming that $\{r_1, r_2, \ldots, r_n\}$ be a set of Alternatives for TSF-MAGDM issues and a group of experts $\{e_1, e_2, \ldots, e_t\}$ that have the weight vector $\{w_1, w_2, \ldots, w_n\}^T$, with $w_{\iota} \epsilon [0, 1]$ and $\sum_{\iota=1}^{n} w_{\iota} = 1$. Let $C = \{c_1, c_2, \ldots, c_n\}$ be a set of attributes, $w = \{w_1, w_2, \ldots, w_n\}^T$ is the weighting vector of attributes, satisfying $w_{\iota} \ge 0$ $(\iota = 1, 2, \ldots, n)$ and $\sum_{\iota=1}^{n} w_{\iota} = 1$. Let $A^{(h)} = \left(\alpha_{\iota_{\hat{\jmath}}}^{(h)}\right)_{m \times n}$ be a T-Spherical fuzzy decision matrix given by the expert $e_l (1 \le h \le t)$, where the experts e_l $(l = 1, 2, \ldots, t)$ gives a TSFV $\alpha_{\iota_{\hat{\jmath}}}^{(h)} = \left(m_{\iota_{\hat{\jmath}}}^{h}, d_{\iota_{\hat{\jmath}}}^{h}, n_{\iota_{\hat{\jmath}}}^{h}\right)$ for the alternative s_{ι} $(\iota = 1, 2, \ldots, n)$ under the attribute

$c_{\hat{\jmath}} = (\hat{J} = 1, 2, \ldots, m)$, then the following steps are used for the procedure of MAGDM problems.

Step 1: Convert the given TSF matrix $A^{(h)} = \left(\alpha_{\iota\hat{\jmath}}^{(h)}\right)_{n \times m}$ in the form of a normal decision matrix $\overline{A}^{(h)} = \left(\overline{\alpha}_{\iota\hat{\jmath}}^{(h)}\right)_{n \times m}$. By using the following method, cost-type attribute values can be transformed into benefit-type attribute values:

$$\overline{\alpha}_{\iota\hat{\jmath}}^{(h)} = \begin{Bmatrix} \alpha_{\iota\hat{\jmath}}^{(h)} & for\ benifit\ attribute\ c_{\hat{\jmath}} \\ \left(\alpha_{\iota\hat{\jmath}}^{(h)}\right)' & for\ cost\ attribute\ c_{\hat{\jmath}} \end{Bmatrix} \tag{14}$$

where $\iota = 1, 2, \ldots, m$; $\hat{J} = 1, 2, \ldots, n$ and $\left(\alpha_{\iota\hat{\jmath}}^{(h)}\right)'$ is the complement of $\alpha_{\iota\hat{\jmath}}^{(h)}$ such that $\left(\alpha_{\iota\hat{\jmath}}^{(h)}\right)' = \left(m_{\iota\hat{\jmath}}^{h}, d_{\iota\hat{\jmath}}^{h}, n_{\iota\hat{\jmath}}^{h}\right)$.

Step 2: Evaluate the support degrees.

$$Sup\left(\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}, \overline{\alpha}_{\iota\hat{\jmath}}^{(l)}\right) = 1 - d\left(\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}, \overline{\alpha}_{\iota\hat{\jmath}}^{(l)}\right) \tag{15}$$

where $(k, l = 1, 2, \ldots, t)$ And satisfies the conditions defined in Equation (13). Here, $d\left(\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}, \overline{\alpha}_{\iota\hat{\jmath}}^{(l)}\right)$ denotes the distance between $\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}$ and $\overline{\alpha}_{\iota\hat{\jmath}}^{(l)}$ determined by Equation (1).

Step 3: Evaluate the support $\mathcal{T}\left(\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}\right)$ of the TSFVs $\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}$ by other TSFVs$\overline{\alpha}_{\iota\hat{\jmath}}^{(l)}$ $(l = 1, 2, \ldots, t \text{ and } l \neq k)$

$$\mathcal{T}\left(\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}\right) = \sum_{l=1; l \neq d}^{t} w_l Sup\left(\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}, \overline{\alpha}_{\iota\hat{\jmath}}^{(l)}\right) \tag{16}$$

where $(k, l = 1, 2, \ldots, t)$; $(\iota = 1, 2, \ldots, m)$; $(\hat{J} = 1, 2, \ldots, r)$.

Next, the weights $w_k (k = 1, 2, \ldots, t)$ of the experts $e_h (h = 1, 2, \ldots, t)$ are used to compute the weights

$$\widetilde{\omega}_{\iota\hat{\jmath}}^{(k)} = \frac{w_k\left[1 + \mathcal{T}\left(\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}\right)\right]}{\sum_{l=1}^{t} w_l\left[1 + \mathcal{T}\left(\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}\right)\right]} \tag{17}$$

where $(k = 1, 2, \ldots, t)$; $(\iota = 1, 2, \ldots, m)$; $(\hat{J} = 1, 2, \ldots, r)$, $\widetilde{\omega} \geq 0$ and $\sum_{d=1}^{t} \widetilde{\omega}_{\iota\hat{\jmath}}^{(k)} = 1$.

Step 4: Utilize the WTSFPMSM operator Equation (14).

$$\overline{\alpha}_{\iota\hat{\jmath}} = WTSFPMSM^{(r)}\left(\overline{\alpha}_{\iota\hat{\jmath}}^{(1)}, \overline{\alpha}_{\iota\hat{\jmath}}^{(2)}, \ldots, \overline{\alpha}_{\iota\hat{\jmath}}^{(t)}\right)$$

$$\begin{gathered} \left[\sqrt[z]{1 - \left(1 - \left(1 - \prod_{1 \leq \iota_1 < \ldots < \iota_r \leq n}\left(\left(1 - \left(\prod_{\hat{\jmath}=1}^{r}\left(1 - \left(1 - \left(m_{\iota\hat{\jmath}}\right)^{z}\right)^{n\widetilde{\omega}_{\iota\hat{\jmath}}}\right)\right)\right)\right)\right)\right)^{1/C_n^r}}\right]^{1/r}, \\ \sqrt[z]{1 - \left(1 - \left(\prod_{1 \leq \iota_1 < \ldots < \iota_r \leq n}\left(1 - \prod_{\hat{\jmath}=1}^{r}\left(1 - d_{\iota\hat{\jmath}}^{zn\widetilde{\omega}_{\iota\hat{\jmath}}}\right)\right)^{1/z}\right)^{1/C_n^r}\right)^{1/r}}, \\ \sqrt[z]{1 - \left(1 - \left(\prod_{1 \leq \iota_1 < \ldots < \iota_r \leq n}\left(1 - \prod_{\hat{\jmath}=1}^{r}\left(1 - n_{\iota\hat{\jmath}}^{zn\widetilde{\omega}_{\iota\hat{\jmath}}}\right)\right)^{1/z}\right)^{1/C_n^r}\right)^{1/r}} \end{gathered} \tag{18}$$

In order to combine all the decision matrices $\overline{\mathrm{A}}^{(\mathrm{h})} = \left(\overline{\alpha}_{\iota_{\hat{\jmath}}}^{(\mathrm{h})}\right)_{\mathrm{m}\times\mathrm{n}} (\mathrm{h} = 1, 2, \ldots, \mathrm{h})$ given by experts into the comprehensive decision matrices $\overline{\mathrm{A}} = \left(\overline{\alpha}_{\iota_{\hat{\jmath}}}\right)_{\mathrm{m}\times\mathrm{n}}$.

Step 5: Calculate the support degrees.

$$\mathrm{Sup}\left(\overline{\alpha}_{\iota_{\hat{\jmath}}}, \overline{\alpha}_{\iota_{\mathrm{h}}}\right) = 1 - \mathrm{d}\left(\overline{\alpha}_{\iota_{\hat{\jmath}}}, \overline{\alpha}_{\iota_{\mathrm{h}}}\right) \tag{19}$$

where $\mathrm{d}\left(\overline{\alpha}_{\iota_{\hat{\jmath}}}, \overline{\alpha}_{\iota_{\mathrm{h}}}\right)$ is the distance between the TSFVs $\overline{\alpha}_{\iota_{\hat{\jmath}}}$ and $\overline{\alpha}_{\iota_{\mathrm{h}}}$ determined by Equation (1) and satisfies the conditions defined in Equation (13).

Step 6: Determine weighted supports $\mathcal{T}\left(\overline{\alpha}_{\iota_{\hat{\jmath}}}\right)$ of $\overline{\alpha}_{\iota_{\hat{\jmath}}}$ $\left(\iota = 1, 2, \ldots, m; \hat{\jmath} = 1, 2, \ldots, \mathrm{r}\right)$ using the weights $\mathrm{w}_{\hat{\jmath}}$ of the attributes $\mathrm{c}_{\hat{\jmath}}$ and the weights $\chi_{\iota\hat{\jmath}}$ associated with $\overline{\alpha}_{\iota_{\hat{\jmath}}}$ by the attributes $\mathrm{c}_{\hat{\jmath}}$ weights $\mathrm{w}_{\hat{\jmath}}$.

$$\mathcal{T}\left(\overline{\alpha}_{\iota_{\hat{\jmath}}}\right) = \sum_{\substack{h=1; \\ h\neq\hat{\jmath}}}^{n} w_{\hat{\jmath}}\, Sup\left(\overline{\alpha}_{\iota_{\hat{\jmath}}}, \overline{\alpha}_{\iota_h}\right) \tag{20}$$

$$\chi_{\iota\hat{\jmath}} = \frac{w_{\hat{\jmath}}\left[1 + \mathcal{T}\left(\overline{\alpha}_{\iota_{\hat{\jmath}}}\right)\right]}{\sum_{\hat{\jmath}=1}^{n} w_{\hat{\jmath}}\left[1 + \mathcal{T}\left(\overline{\alpha}_{\iota_{\hat{\jmath}}}\right)\right]} \tag{21}$$

Step 7: By using WTSFPMSM operator, we calculate the TSF evaluation value α_ι of the alternative s_ι $(\iota = 1, 2, \ldots, \mathrm{m})$.

$$\alpha_\iota = WTSFPMSM^{(r)}\left(\overline{\alpha}_{\iota_1}, \overline{\alpha}_{\iota_2}, \ldots, \overline{\alpha}_{\iota_n}\right)$$
$$= \left[\begin{array}{c} \left[\sqrt[z]{1-\left(1-\left(1-\prod\limits_{1\leq\iota_1<\ldots<\iota_r\leq n}\left(\left(1-\left(\prod\limits_{\hat{\jmath}=1}^{r}\left(1-\left(1-\left(m_{\iota_{\hat{\jmath}}}\right)^{z}\right)^{n\chi_{\iota_{\hat{\jmath}}}}\right)\right)\right)\right)\right)\right)^{1/C_n^r}}\right]^{1/r}, \\ \sqrt[z]{1-\left(1-\left(\prod\limits_{1\leq\iota_1<\ldots<\iota_r\leq n}\left(1-\prod\limits_{\hat{\jmath}=1}^{r}\left(1-d_{\iota_{\hat{\jmath}}}^{zn\chi_{\iota_{\hat{\jmath}}}}\right)\right)^{1/z}\right)^{1/C_n^r}\right)^{1/r}}, \\ \sqrt[z]{1-\left(1-\left(\prod\limits_{1\leq\iota_1<\ldots<\iota_r\leq n}\left(1-\prod\limits_{\hat{\jmath}=1}^{r}\left(1-n_{\iota_{\hat{\jmath}}}^{zn\chi_{\iota_{\hat{\jmath}}}}\right)\right)^{1/z}\right)^{1/C_n^r}\right)^{1/r}} \end{array}\right] \tag{22}$$

Step 8: Rank α_ι $(\iota = 1, 2, \ldots, m)$ in decreasing order according to the proposed method defined in Definition (2.3).

Step 9: The best option is chosen based on the ranking of all the alternatives s_ι $(\iota = 1, 2, \ldots, m)$, which are all ranked.

Example 1. *A smart grid is a digitally based power network that uses two-way digital communication to deliver electricity to customers. This system enables supply chain monitoring, analysis, control, and communication to improve efficiency, lower energy consumption, and costs, and increase the energy supply chain's transparency and reliability. The smart grids were developed to use smart net meters to overcome the flaws of traditional electrical grids. Several governments worldwide are promoting the adoption of smart grids because of their ability to regulate and reduce global warming, disaster resistance, and energy independence situations. The United States wants to collaborate with Water and Power Development Authority (WAPDA) on a study to see if a Pakistani Smart Grid system is feasible. This project opens the way for a large-scale Smart Grid development program. WAPDA produced 37,402MW of electricity in 2020 and supplied electric power to all over Pakistan; it decided to construct a Smart Grid that combines the electricity distribution grid with an information and net metering system and provides electricity to companies using technological tools and multiple communications to save electricity, lower costs, and enhance reliability. There are four electrical companies: the Faisalabad Electric Supply Company (FESCO), Islamabad Electric Supply Company (IESCO), Lahore Electric Supply Company (LESCO), and Peshawar Electric Supply Company (PESCO) are called alternative, with the four attributes of*

$\check{C}_1$: *Dynamic control of voltage,*
$\check{C}_2$: *Weather data integration,*
$\check{C}_3$: *Fault protection, and*
$\check{C}_4$: *Outage management.*

The steps of the algorithm under consideration are discussed in the following manners, where $w = (0.2,\ 0.3,\ 0.35,\ 0.15)^T$ *is the weight vectors. The experts express their opinion using TSFVs and construct T-spherical fuzzy decision matrices.*

Step 1: We create decision matrices, presented in Tables 1–3.

Table 1. T-spherical fuzzy decision matrix A^1.

Alternative/Attributes	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	$(0.5, 0.3, 0.4)$	$(0.4, 0.2, 0.1)$	$(0.6, 0.1, 0.2)$	$(0.6, 0.1, 0.3)$
$\overline{A}_2$	$(0.7, 0.1, 0.2)$	$(0.5, 0.10.2)$	$(0.3, 0.3, 0.4)$	$(0.7, 0.1, 0.2)$
$\overline{A}_3$	$(0.4, 0.2, 0.3)$	$(0.5, 0.1, 0.2)$	$(0.4, 0.3, 0.4)$	$(0.5, 0.2, 0.3)$
$\overline{A}_4$	$(0.6, 0.1, 0.3)$	$(0.5, 0.2, 0.3)$	$(0.4, 0.2, 0.4)$	$(0.4, 0.2, 0.3)$

Table 2. T-spherical fuzzy decision matrix A^2.

Alternative/Attributes	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	$(0.7, 0.8, 0.1)$	$(0.5, 0.6, 0.1)$	$(0.4, 0.5, 0.2)$	$(0.5, 0.8, 0.1)$
$\overline{A}_2$	$(0.5, 0.6, 0.2)$	$(0.6, 0.7, 0.2)$	$(0.5, 0.5, 0.2)$	$(0.6, 0.7, 0.1)$
$\overline{A}_3$	$(0.4, 0.5, 0.1)$	$(0.6, 0.8, 0.1)$	$(0.5, 0.7, 0.2)$	$(0.6, 0.7, 0.3)$
$\overline{A}_4$	$(0.5, 0.6, 0.2)$	$(0.4, 0.5, 0.3)$	$(0.6, 0.8, 0.1)$	$(0.5, 0.8, 0.1)$

Table 3. T-spherical fuzzy decision matrix A^3.

Alternative/Attributes	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	$(0.6, 0.6, 0.2)$	$(0.5, 0.8, 0.1)$	$(0.5, 0.7, 0.1)$	$(0.6, 0.7, 0.2)$
$\overline{A}_2$	$(0.7, 0.8, 0.1)$	$(0.4, 0.5, 0.3)$	$(0.6, 0.7, 0.1)$	$(0.5, 0.6, 0.2)$
$\overline{A}_3$	$(0.6, 0.6, 0.2)$	$(0.5, 0.7, 0.2)$	$(0.6, 0.8, 0.1)$	$(0.5, 0.6, 0.3)$
$\overline{A}_4$	$(0.4, 0.5, 0.3)$	$(0.6, 0.8, 0.1)$	$(0.5, 0.6, 0.2)$	$(0.7, 0.8, 0.1)$

Step 2: Evaluate the support function using Tables 1–3. The resulting information given in Tables 4–6.

Table 4. Decision matrix of calculated support values established from Tables 1 and 2.

Sup(1,2)/(2,1)	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	(0.89)	(0.942)	(0.91)	(0.79)
$\overline{A}_2$	(0.86)	(0.916)	(0.92)	(0.84)
$\overline{A}_3$	(0.95)	(0.858)	(0.9)	(0.86)
$\overline{A}_4$	(0.89)	(0.941)	(0.86)	(0.8)

Table 5. Decision matrix of calculated support values established from Tables 1 and 3.

Sup(1,3)/(3,1)	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	(0.89)	(0.844)	(0.85)	(0.88)
$\overline{A}_2$	(0.83)	(0.932)	(0.94)	(0.86)
$\overline{A}_3$	(0.87)	(0.886)	(0.87)	(0.93)
$\overline{A}_4$	(0.91)	(0.854)	(0.93)	(0.92)

Table 6. Decision matrix of calculated support values established from Tables 2 and 3.

Sup(2,3)/(3,2)	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	(0.86)	(0.901)	(0.95)	(0.91)
$\overline{A}_2$	(0.97)	(0.87)	(0.96)	(0.93)
$\overline{A}_3$	(0.98)	(0.911)	(0.91)	(0.93)
$\overline{A}_4$	(0.94)	(0.913)	(0.87)	(0.93)

Step 3: Evaluate the weighted supported $\mathcal{T}\left(\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}\right)$ of TSFV$\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}$ by using other TSFV$\overline{\alpha}_{\iota\hat{\jmath}}^{(l)}$ $(l = 1,2,3)$ and $l \neq k$ by Equation (17) and calculate the weight $\widetilde{\omega}_{\iota_{\hat{\jmath}}}^{(k)}$ $(\iota, \hat{\jmath} = 1,2,3,4;\ d = 1,2,3)$ of TSFV$\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}$ $(\iota, \hat{\jmath} = 1,2,3,4;\ k = 1,2,3)$ by Equation (18). In the following, we express $\left[\mathcal{T}\left(\overline{\alpha}_{\iota\hat{\jmath}}^{(k)}\right)\right]_{4\times 4}$ as T_k $(k = 1,2,3)$ and $\left(\widetilde{\omega}_{\iota_{\hat{\jmath}}}^{(k)}\right)_{4\times 4}$ as W_k $(k = 1,2,3)$, which shows as follows in Tables 7–12:

Table 7. Decision matrix, which calculates the value of $\mathcal{T}_1$.

$\mathcal{T}_1$	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	(0.36)	(0.5)	(0.62)	(0.3)
$\overline{A}_2$	(0.34)	(0.6)	(0.65)	(0.3)
$\overline{A}_3$	(0.37)	(0.5)	(0.62)	(0.3)
$\overline{A}_4$	(0.36)	(0.5)	(0.63)	(0.3)

Table 8. Decision matrix, which calculates the value of $\mathcal{T}_2$.

$\mathcal{T}_2$	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	(0.35)	(0.6)	(0.65)	(0.3)
$\overline{A}_2$	(0.37)	(0.5)	(0.65)	(0.3)
$\overline{A}_3$	(0.39)	(0.5)	(0.63)	(0.3)
$\overline{A}_4$	(0.37)	(0.6)	(0.61)	(0.3)

Table 9. Decision matrix, which calculates the value of $\mathcal{T}_3$.

$\mathcal{T}_3$	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	(0.35)	(0.5)	(0.63)	(0.3)
$\overline{A}_2$	(0.36)	(0.5)	(0.66)	(0.3)
$\overline{A}_3$	(0.37)	(0.5)	(0.62)	(0.3)
$\overline{A}_4$	(0.37)	(0.5)	(0.63)	(0.3)

Table 10. Decision matrix in which find the value of W_1.

W_1	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	(0.32)	(0.32)	(0.31)	(0.32)
$\overline{A}_2$	(0.31)	(0.32)	(0.32)	(0.32)
$\overline{A}_3$	(0.32)	(0.32)	(0.32)	(0.32)
$\overline{A}_4$	(0.32)	(0.32)	(0.32)	(0.32)

Table 11. Decision matrix in which find the value of W_2.

W_2	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	(0.45)	(0.45)	(0.45)	(0.45)
$\overline{A}_2$	(0.45)	(0.45)	(0.45)	(0.45)
$\overline{A}_3$	(0.45)	(0.45)	(0.45)	(0.45)
$\overline{A}_4$	(0.45)	(0.45)	(0.45)	(0.45)

Table 12. Decision matrix in which find the value of W_3.

W_3	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	(0.23)	(0.23)	(0.23)	(0.23)
$\overline{A}_2$	(0.23)	(0.23)	(0.23)	(0.23)
$\overline{A}_3$	(0.23)	(0.23)	(0.23)	(0.23)
$\overline{A}_4$	(0.23)	(0.23)	(0.23)	(0.23)

Step 4: Utilizing the WTSFPMSM Equation (19), we aggregate three decision matrices $\overline{A}^{(h)} = \left(\overline{\alpha}^{(h)}_{\iota\hat{\jmath}}\right)_{m\times n} (h = 1,2,3)$. Acquire an aggregated decision matrix $\overline{A} = \left(\overline{\alpha}_{\iota\hat{\jmath}}\right)_{m\times n}$ given by experts. Let $r = 2$ and $z = 3$ in Table 13.

Table 13. Aggregated matrix by using data in Tables 1–3.

Alternative/Attributes	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	$(0.598, 0.804, 0.609$	$(0.5, 0.8, 0.5)$	$(0.5, 0.7, 0.5)$	$(0.6, 0.82, 0.577)$
$\overline{A}_2$	$(0.625, 0.779, 0.5)$	$(0.5, 0.70.6)$	$(0.5, 0.7, 0.6)$	$(0.6, 0.77, 0.542)$
$\overline{A}_3$	$(0.458, 0.714, 0.578$	$(0.5, 0.8, 0.5)$	$(0.5, 0.8, 0.6)$	$(0.5, 0.77, 0.618)$
$\overline{A}_4$	$(0.507, 0.716, 0.612)$	$(0.5, 0.8, 0.6)$	$(0.5, 0.8, 0.6)$	$(0.5, 0.84, 0.532)$

Step 5: Evaluate the supports $\mathrm{Sup}\left(\overline{\alpha}_{\iota_{\hat{\jmath}}}, \overline{\alpha}_{\iota_h}\right)$ $(\iota, \hat{\jmath}, h = 1,2,3,4 \text{ and } \hat{\jmath} \neq h)$ by Equation (16). For simplicity, $\left[\mathrm{Sup}\left(\overline{\alpha}_{\iota_{\hat{\jmath}}}, \overline{\alpha}_{\iota_h}\right)\right]_{4\times 1}$ indicate the support between the jth and hth columns of $\overline{A}$. The information is given in Table 14.

Table 14. Matrix of support values.

Sup(1,2)	**Sup(1,3)**	**Sup(1,4)**	**Sup(2,3)**	**Sup(2,4)**	**Sup(3,4)**	**Sup(2,4)**
0.92	0.891	0.97	0.97	0.99	0.94	0.99
0.92	0.92	0.97	0.99	0.99	0.98	0.99
0.94	0.948	0.97	0.98	0.94	0.92	0.94
0.96	0.948	0.9	0.96	0.95	0.96	0.95

Step 6: Evaluate the weighted support $\mathcal{T}\left(\overline{\alpha}_{\iota_{\hat{\jmath}}}\right)$ of TSFV$\overline{\alpha}_{\iota_{\hat{\jmath}}}$ by Equation (21) and the weights $\omega_{\iota\hat{\jmath}} \left(\hat{\jmath} = 1,2,3,4\right)$ of TSFV $\overline{\alpha}_{\iota\hat{\jmath}}$ $\left(\hat{\jmath} = 1,2,3\right)$ are calculated using Equation (22) and given in Tables 15 and 16.

Table 15. Matrix in which we find the value of T.

$\mathcal{T}$	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_B$	0.56	0.86	0.98	0.43
$\overline{A}_2$	0.56	0.87	1.01	0.44
$\overline{A}_3$	0.57	0.86	1	0.43
$\overline{A}_4$	0.56	0.86	1	0.42

Table 16. Matrix in which we find the value of W.

W	$\breve{C}_1$	$\breve{C}_2$	$\breve{C}_3$	$\breve{C}_4$
$\overline{A}_1$	0.17	0.31	0.39	0.12
$\overline{A}_2$	0.17	0.31	0.392	0.12
$\overline{A}_3$	0.18	0.31	0.391	0.12
$\overline{A}_4$	0.18	0.31	0.393	0.12

Step 7: Using WTSFPMSM Equation (18), all the T-spherical values $\overline{\alpha}_{\iota\hat{\jmath}}$ $\left(\hat{\jmath} = 1,2,3,4\right)$ in the ith row of $\overline{A}$ are aggregated to determine the comprehensive values $\overline{\alpha}_{\iota}$ $(\iota = 1,2,3,4)$ as follows in Table 17.

Table 17. T-spherical values by using data in Table 13 and the WTSFPMSM operator.

	WTSFPMSM Operator		
	MD	**AD**	**NMD**
$\overline{A}_1$	0.52744	0.8723	0.73845
$\overline{A}_2$	0.54891	0.8593	0.733
$\overline{A}_3$	0.51278	0.8753	0.74951
$\overline{A}_4$	0.51037	0.8696	0.75134

Step 8: Calculate the RD by using Definition 3. All the RDs are given in Table 18.

Table 18. Calculated values of Refusal degree.

Refusal Degree	
$\pi(s_1)$	−0.5973
$\pi(s_2)$	−0.5788
$\pi(s_3)$	−0.6099
$\pi(s_4)$	−0.5988

Step 9: Determine the score values $\mathfrak{P}(\overline{\alpha}_i)$ of $\overline{\alpha}_i$ $(i = 1, 2, 3, 4)$ by Equation (3). The results are given in Table 19.

Table 19. Score Values of the aggregated values in Table 17.

Alternatives	Scores
$\overline{A}_1$	0.28826
$\overline{A}_2$	0.28824
$\overline{A}_3$	0.28697
$\overline{A}_4$	0.27408

Then, the alternatives $\overline{\alpha}_\iota$ $(\iota = 1, 2, 3, 4)$ are written in decreasing order according to the values of $\mathfrak{P}(\overline{\alpha}_\iota)$

$$\overline{\alpha}_1 > \overline{\alpha}_2 > \overline{\alpha}_3 > \overline{\alpha}_4$$

Step 10: Based on the score function's value, all the alternatives r_ι $(\iota = 1, 2, 3, 4)$ are ranked as follows:

$$\overline{A}_1 > \overline{A}_2 > \overline{A}_3 > \overline{A}_4$$

By using score function values, the ranking result is $\overline{A}_1 > \overline{A}_2 > \overline{A}_3 > \overline{A}_4$. Hence, the best alternative we obtained is $\overline{A}_1$ by using the TSFPMSM operator among the four alternatives of companies. Thus, we can conclude that the most appropriate company for this project is FESCO. For better understanding, we present this graphically, as shown in Figure 1. The results obtained here are based on TSF information and power MSM operators. First of all, TSFS provides a larger ground for the decision-makers to establish their opinion with no limitations, as seen in the tables above. Secondly, the relationship of the information in the aggregation process is important, and the proposed power MSM operators interrelate the information aggregation. In our next section, we show the superiority of using the proposed AOs after comparing the results of proposed and existing methods.

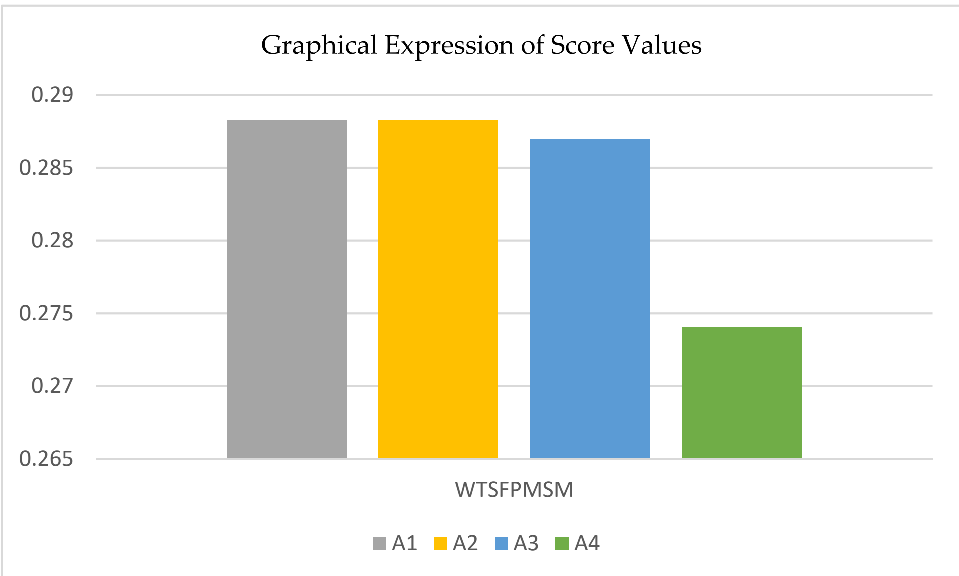

Figure 1. Illustration of the information in Table 6.

5. Comparative Study

To demonstrate the effectiveness and validity of the proposed approaches, many mathematicians use different types of MAGDM [48–58]. We utilize T-spherical fuzzy information through the existing concept of MAGDM, which can be successively used in the weighted and Power Maclaurin Symmetric Mean operator field. Moreover, through the data in Table 4, we can analyze the advantage of a proposed operator with the concept of TSFSs. We compared the proposed method with TSFAAWA and TSFAAWG by Hussain et al. [7], TSFPWA and TSFPWG by Garg et al. [48], TSFWNA by Javed.et.al [59], TSFFWA and TSFFWG by Mahnaz.et.al [60], and TSFDBM by Yang.et.al [61] . The common feature of these methods is their ability to characterize the interrelationship among the input arguments. The comparative anatomization of the proposed work and prevailing operators are discussed in Table 20.

Table 20. Comparative analysis of the proposed and existing operators by using the information in Table 4.

Methods	**Operator**	**Ranking Values**
Proposed	WTSFPMSM	$\overline{A}_1 > \overline{A}_2 > \overline{A}_3 > \overline{A}_4$
Hussain et.al. [7]	TSFAAWA	$\overline{A}_3 > \overline{A}_1 > \overline{A}_4 > \overline{A}_2$
Hussain et.al. [7]	TSFAAWG	$\overline{A}_4 > \overline{A}_2 > \overline{A}_1 > \overline{A}_3$
Garg et.al. [48]	TSFPWA	$\overline{A}_2 > \overline{A}_1 > \overline{A}_4 > \overline{A}_3$
Garg et.al. [48]	TSFPWG	$\overline{A}_2 > \overline{A}_1 > \overline{A}_4 > \overline{A}_3$
Javed et.al. [59]	TSFWNA	$\overline{A}_3 > \overline{A}_2 > \overline{A}_1 > \overline{A}_4$
Mahnaz et.al. [60]	TSFFWA	$\overline{A}_2 > \overline{A}_1 > \overline{A}_4 > \overline{A}_3$
Mahnaz.et.al. [60]	TSFFWG	$\overline{A}_2 > \overline{A}_1 > \overline{A}_3 > \overline{A}_4$
Yang et.al. [61]	TSFDBM	$\overline{A}_2 > \overline{A}_1 > \overline{A}_4 > \overline{A}_3$

The pictorial view of the information in Table 7 is expressed in Figure 2.

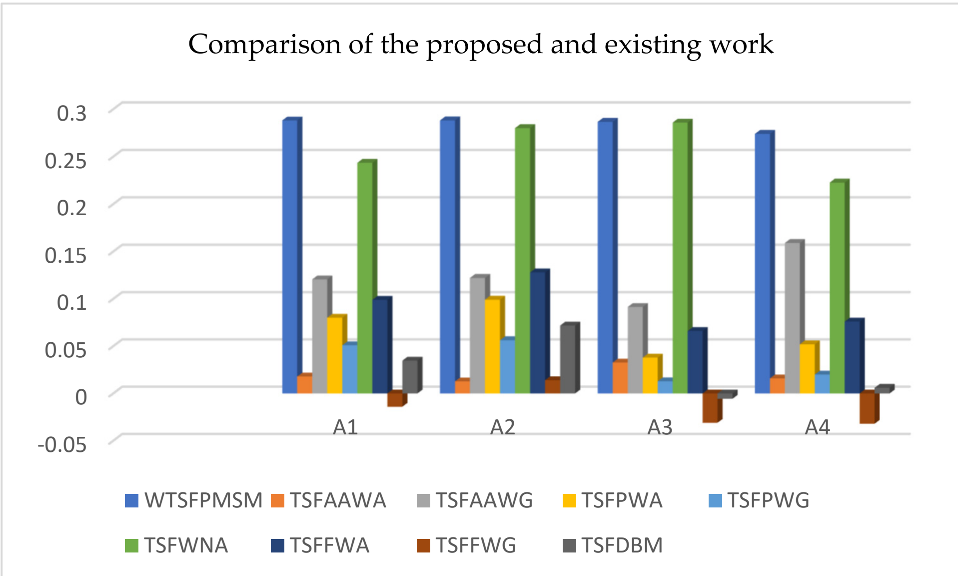

Figure 2. Demonstration of the MSM operating area with different TSFS.

To demonstrate the efficiency of the proposed method, we can use some existing MAGDM methods to resolve the application example mentioned above, given that the proposed technique combines the PA and MSM operators. To assess its benefits, we may compare the proposed method with eight different MAGDM methods based on various TSF AOs, which are:

(1) The simple and conventional approach suggested by Garg et al. [48], based on the TSF power-weighted average (TSFPWA) operator and TSF power-weighted geometric (TSFPWG) operator.
(2) The method proposed by Hussain.et.al [7], based on TSF Aczel-Alsina weighted average (TSFAAWA) operator and TSF Aczel-Alsina weighted geometric (TSFAAWG) operator.
(3) The existing method proposed by Javed.et.al [59], based on TSF weighted neutral aggregation (TSFWNA) operator.

(4) The method proposed by Mahnaz.et.al [60], based on TSF Frank weighted average (TSFFWA) operator and TSF Frank weighted geometric (TSFFWG) operator.
(5) The method proposed by Yang.et.al [61], based on TSF Dombi Bonferroni mean (TSFDBM) operator.

The ranking results acquired using the eight approaches mentioned above and the method that is being suggested in this paper are shown in Table 19.

Ranking of these existing methods differs from the ranking of the suggested method. The best alternative determined by the proposed method is $\overline{A}_1$ but the optimal choice for TSFPWA, TSFPWG, TSFFWA, TSFFWG, and TSFDBM is $\overline{A}_2$, the best choice for TSFAAWG is $\overline{A}_4$, the best choice for TSFAAWA is $\overline{A}_3$,similarly, the best choice for TSFWNA is $\overline{A}_3$.We represent this graphically in Figure 2 for better understanding.

Sensitivity Analysis of Different z in WTSFPMSM Operator

This section analyzes the sensitivity of the parameter involved and its impact on the aggregation results. We vary the variable parameter and display the ranking results in Table 21 to see the impact.

Table 21. Influence of variable parameter.

	$\overline{\mathfrak{S}}(\alpha_1)$	$\overline{\mathfrak{S}}(\alpha_2)$	$\overline{\mathfrak{S}}(\alpha_3)$	$\overline{\mathfrak{S}}(\alpha_4)$	**Ranking of Alternatives**	**Outcome**
$z=1$	0.65648	**0.65306**	**0.65567**	**0.63718**	$\overline{A}_1 > \overline{A}_3 > \overline{A}_2 > \overline{A}_4$	$\overline{A}_1$
$z=3$	**0.28826**	**0.28824**	**0.28697**	**0.27408**	$\overline{A}_1 > \overline{A}_2 > \overline{A}_3 > \overline{A}_4$	$\overline{A}_1$
$z=5$	**0.2511**	**0.241**	**0.25674**	**0.24741**	$\overline{A}_3 > \overline{A}_1 > \overline{A}_4 > \overline{A}_2$	$\overline{A}_3$
$z=7$	**0.29053**	**0.27201**	**0.30017**	**0.29187**	$\overline{A}_3 > \overline{A}_4 > \overline{A}_1 > \overline{A}_2$	$\overline{A}_3$
$z=25$	**−0.10932**	**−0.1184**	**−0.10838**	**−0.11175**	$\overline{A}_3 > \overline{A}_1 > \overline{A}_4 > \overline{A}_2$	$\overline{A}_3$
$Z=35$	**−0.10042**	**−0.1092**	**−0.11252**	**−0.10943**	$\overline{A}_1 > \overline{A}_2 > \overline{A}_4 > \overline{A}_3$	$\overline{A}_1$

Other z can also be considered in the WTSFPMSM operator. If z is considered, the results are shown in Table 21. The ranking values of $z = 1, 3$ and 35 are the same and $\overline{A}_1$ is the optimal alternative; $\overline{A}_3$ becomes the optimal alternative when $z = 5, 7, \ldots, 25$. Hence, different ranking values are reasonable.

6. Conclusions

Combining the standard PA operator and the Maclaurin symmetric mean can resolve complicated decision-making problems. This article uses a multi-attribute group decision-making (MAGDM) approach based on power Maclaurin symmetric mean (PMSM) operators. We proposed T-spherical fuzzy (TSF) power MSM (TSFPMSM) operators because TSF information covers four aspects of uncertain information. The main advantage of the suggested operators is that they can consider the relations between the various arguments; additionally, they can also consider the relationships between the combined values simultaneously. To check the effectiveness of weights, we introduced a weighted TSFPMSM (WTSFPMSM) operator. The idempotency, boundedness, and monotonicity features of the proposed TSFPMSM are investigated. A MAGDM algorithm based on the proposed TSFPMSM operators is established to be applied to assess smart grid systems in Pakistan. The sensitivity analysis of the proposed numerical example is analyzed based on observing the reaction of the variation of the sensitive parameters, followed by a comprehensive comparative study.

The benefits of using power MSM operators for TSFSs cover two aspects. First, it provides a flexible ground for the expression of expert opinion. Secondly, it interrelates the information of aggregation, which other operators cannot. The power Maclaurin symmetric mean operator can also be extended to handle more decision-making contexts, for example, Prioritized Power Maclaurin symmetric mean, interactive power Maclaurin symmetric mean, etc. Some possible directions of the present work can be found in [62,63].

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