Zhou, Yifan and Ma, Lin and Wolff, Rodney C. and Kim, Hack-Eun (2009) ***Asset life prediction using multiple degradation indicators and lifetime data : a Gamma-based state space model approach.*** In: The 8th International Conference on Reliability, Maintainability and Safety, 20-24 July 2009, Chengdu, China.

Asset life prediction using multiple degradation indicators and lifetime data: a Gamma-based state space model approach

Yifan Zhou, Lin Ma, Joseph Mathew, Hack-Eun Kim
CRC of Integrated Engineering Asset Management (CIEAM), School of Engineering Systems
Queensland University of Technology
Brisbane, Australia
E-mail: yifan.zhou@qut.edu.au

Rodney Wolff
School of Mathematical Sciences
Queensland University of Technology
Brisbane, Australia

***Abstract*— This paper proposes a Gamma-based state space model to predict engineering asset life when multiple degradation indicators are involved and the failure threshold on these indicators are uncertain. Monte Carlo-based parameter estimation and model inference algorithms are developed to deal with the proposed Gamma-based state space model. A case study using real data from industry is conducted to compare the performance of the proposed model with the commonly used proportional hazard model (PHM). The result shows that the Gamma-based state space model is more appropriate to deal with the situation when the failure data is insufficient.**

Keywords- Expectation-Maximization algorithm; Gamma process; Proportional hazard model; State space model

I. INTRODUCTION

Predicting the failures of engineering assets is important in modern industry. An unexpected failure of a critical engineering asset can cause the breakdown of a whole plant. Therefore, a lot of research has been conducted to predict asset failures through failure history and degradation indicators. However, three issues make asset failures difficult to predict in practice. The first issue is the stochastic property of degradation indicator development processes. To deal with this issue, a flexible mathematic model should be developed to fit to these stochastic curves. The second issue is the uncertain failure thresholds on degradation indicators. When a failure happens, the same degradation indicator of different subjects may have different values. This issue is also called “gray boundary” in some literature [1]. The last issue is the fusion of multiple indicators collected during asset degradation processes.

The three issues have been partially addressed by existing research. The proportional hazard model (PHM), which was originally proposed in biomedical research, can model the uncertain relationships between multiple indicators and time dependent failure rate. However, the PHM has the assumption of time independent covariates effects [2]. Furthermore, when using the PHM to deal with time dependent covariates, an integral is involved. To make the integral of the covariates tractable, the time dependent covariates are often assumed to follow some deterministic functions of time [1], or some special stochastic process(es), e.g., discrete state assumption [3]. Another model to deal with multiple degradation indicators and uncertain failure thresholds is the logistic regression model. The logistic regression model has the ability to model the uncertain relationship between the failure probability and the values of multiple degradation indicators. However, the logistic regression model only considers the covariate values at the failure time or at the last inspection. Therefore, the logistic regression model cannot obtain as good a result as the PHM [1].

Compared with the approaches above, the state space model is a more flexible and mathematical tractable method to handle multiple stochastic degradation indicators and uncertain failure thresholds. In the state space model, the dynamic characteristics of a system are modelled by a time dependent system process, and the system states are supposed to be partially revealed by observations. In degradation modelling, the system development process denotes the underlying degradation process of an asset, while the observations represent degradation indicators. Wang develops a state space model to predict the remaining useful life (RUL) of bearings using the RMS of vibration signals [4]. However, the state space model developed by Wang assumes that the vibration RMS values of all the bearings at the same residual life follow an identical distribution. Therefore, Wang’s model does not consider the heterogeneous longitude health degradation processes of different subjects. Makis and Jiang develop a state space model based on a discrete state continuous time Markov process [5]. The discrete state requires discretising the continuous degradation process which needs expert knowledge and may introduce additional errors. Whitmore, Crowder and Lawless propose a bivariate Wiener process [6]. However, in the bivariate Wiener process, only the covariates obtained at failure times or censoring times are considered.

This research extends the existing research on the state space model for asset life prediction by proposing a Gamma-based state space model. Monte Carlo-based parameter estimation and model inference algorithms are developed to handle the non-Gaussian property. Finally, a case study using vibration data from bearings on high pressure liquefied nature gas pumps is conducted to compare the performance of the proposed Gamma-based state space model and the PHM.

II. Model development

The proposed Gamma-based state space degradation model consists of two components. One is the system equation modelling the underlying degradation process; the other is the observation equation describing the relationship between the underlying degradation process and the degradation indicators. The system equation is given by (1), where the scalar variable $\Lambda(t)$ denotes the underlying health state of an asset at time t. The increments of $\Lambda(t)$ follow a Gamma distribution. Further, ξ is the scale parameter of the Gamma distribution. The shape parameter (i.e., $a \cdot \Delta t$) of the Gamma distribution is assumed to be proportional to the length of the time interval. The initial health state of the asset is assumed to be brand new, i.e., $\Lambda(t_0)=0$. A larger value of $\Lambda(t)$ indicates a worse health state. A failure is supposed to happen when the underlying degradation process, $\Lambda(t)$, hits a failure threshold Λ_f. The observation equation is given by (2), where the degradation indicator vector, $\vec{X}(t)$, is assumed to follow a multivariate normal distribution, where $\vec{\Sigma}$ is the covariance matrix.

$$\Lambda(t+\Delta t)-\Lambda(t) \sim Ga(a\cdot\Delta t,\xi) \tag{1}$$

$$\vec{X}(t) \sim N\left(\vec{c}\cdot\Lambda(t),\vec{\Sigma}\right) \tag{2}$$

In this research, the Gamma process is adopted as the underlying degradation process. The reasons are as follows: Firstly, the Gamma process is continuous in time and state. Therefore, irregular inspection intervals can be handled naturally, and discretisation of the health state can be avoided. Secondly, compared with another commonly used continuous stochastic process, the Wiener process, the increments of the Gamma process are positive (they increase monotonically). This monotonic property can avoid constructing a conditional probability function to ensure the underlying degradation process dose not drift to a failure threshold between two normal states. The monotonic increasing property is also consistent with irreversible degradation processes (e.g., wear, corrosion, crack growth) of most engineering assets.

To make the formulations involved in parameter estimation more concise, only one sequence of indicators is considered in the parameter estimation algorithm in the present paper. The formulations can be extended to multiple data sequences without any theoretical difficulties. Denote the inspection times as $t_i; \quad i=1,2,\cdots,n$, where n is the number of inspections. The values of the underlying degradation health state and the degradation indicator vector at the i th inspection are denoted as λ_i and $\vec{x}_i$ respectively. The failure time and the failure threshold on the underlying degradation process are denoted as T_f and Λ_f. The degradation indicators at T_f are not required by the Gamma-based state space model. Note that Λ_f is assumed equal to 1 because, for different values of Λ_f, the same life time distribution can be obtained by changing the scale parameter ξ.

III. Parameter estimation

This research uses the Expectation-Maximisation (EM) algorithm to estimate the parameters of the Gamma-based state space model. The Monte Carlo-based particle smoother is used in the E step of the EM algorithm to deal with the non-Gaussian underlying degradation process.

The application of the EM algorithm can be divided into four steps. The first step is the estimation of initial parameters. The second step, namely the E step, is to estimate the expectation of the complete likelihood function. After that, a new set of parameters is obtained by maximising the expected complete likelihood function. This step is called the M step. The final step is to check the convergence of EM loops. If the convergence condition is satisfied, the parameters acquired during the M step are regarded as the final result. Otherwise, another iteration of EM loop begins using the parameters acquired in the M step as the initial parameters. The four steps are detailed as follows.

In this research, the initial parameters used by the first EM iteration are largely obtained based on human experience according to failure times and degradation indicators. A more objective initial parameter selection method is to be investigated in the future research.

The E step is to obtain the expectation of the complete likelihood function given by (3), where $\theta_1=\{a \quad \xi\}$, $\theta_2=\{\vec{c} \quad \vec{\Sigma}\}$. According to the Gamma Bridge theory [7], the underlying degradation process changes from the original Gamma process to a new stochastic process given by (5) after the failure time is considered. The Monte Carlo-based particle smoothing algorithm is adopted to deal with the state space model based on the new stochastic process. This paper adopts the particle smoother using the backwards simulation method proposed by Simon, Arnaud et al [8]. The detail of implementing the particle smoothing is not discussed in this research. After the particle smoothing, N_f sequences of samples are generated, i.e., $s_{1:n}^{1:N_f}$, where N_f is the number of particles used in the particle smoothing. The expectation of functions of the underlying degradation state $\lambda_k; k=1,2,\ldots n$, can be approximated using the smoothing particles $s_{1:n}^{1:N_f}$, as in (4).

$$E\left(\log f\left(\vec{x}_{1:n},\lambda_{1:n}\middle|\theta\right)\right)=E\left(\log f\left(\lambda_{1:n}\middle|\theta_1\right)\right)+E\left(\log f\left(\vec{x}_{1:n}\middle|\theta_2,\lambda_{1:n}\right)\right) \tag{3}$$

$$E\left(g(\lambda_k)\middle|\vec{x}_{1:n},T_f\right)\approx\sum_{i=1}^{N_f} g\left(s_k^i\right) \quad k=1,2,\ldots,n \tag{4}$$

The two components of (3) can be written as (6) and (7) respectively, where $v_i = \lambda_i - \lambda_{i-1}$, $u_i = at_i - at_{i-1}$; $i = 2,3,\ldots,n+1$, $\lambda_{n+1} = \Lambda_f$ and $t_{n+1} = T_f$, where m is the size of the degradation indicator vector. To calculate (6) and (7), the three components (i.e., $E(\lambda_i)$, $E(\lambda_i^2)$, and $E(\log(v_i))$) should be approximated using the smoothing particles $s_{1:n}^{1:N_f}$ according to (4).

After the expected complete likelihood function is obtained, the M step is carried out. During the M step, (6) and (7) are optimized separately. The optimization algorithms are straightforward, and are not discussed here.

Compared with the EM algorithm based on an analytic method, the EM algorithm using the particle smoother does not converge smoothly. The parameter estimates fluctuate in a certain range after some EM iterations. After increasing the number of particles, the estimates fluctuate in a smaller range. In this research, the EM iteration is regarded as having converged when the maximum and minimum values of $\hat{a}$, $\hat{\xi}$, and $\hat{\vec{c}}$ are not in the results of the recent five EM loops. The EM algorithm is carried out in two stages. Firstly, 1000 particles are used. After the EM iteration converges, 2000 particles are adopted to obtain a better estimation result.

IV. LIFETIME PREDICTION

The survival function in (8), inferred from the Gamma-based state space model, consists of two components. The first component is the probability density function (PDF) of the current underlying degradation state λ_c given the degradation indicators up to current inspections, and the fact that the failure has not happened yet, i.e., $f(\lambda_c | \vec{x}_{1:c}, \lambda_c < \Lambda_f)$, where c denotes the current inspection index. Then $f(\lambda_c | \vec{x}_{1:c}, \lambda_c < \Lambda_f)$ can be approximated by the particle filtering results $f_c^{1:N_f}$. For details of the implementation process of the particle filter, readers can refer to [9]. The second component is the survival function given the current degradation state, i.e., $\Pr(t \geq T_f | \lambda_c)$. In this paper, the development of the underlying degradation process is the Gamma process. Therefore, $\Pr(t \geq T_f | \lambda_c)$ can be acquired as (9). After substituting (9) into (8), and using the result of the particle filter, the survival function is obtained as (10). After differentiating (10), the PDF of the lifetime conditional on the degradation observations up to the current time can be obtained as (11).

$$f\left(\Lambda(t_i) \middle| \Lambda(t_{i-1}), \Lambda(T_f) = \Lambda_f\right) = Be\left(\left(\Lambda(t_i) - \Lambda(t_{i-1})\right) / \left(\Lambda_f - \Lambda(t_{i-1})\right); a(t_i - t_{i-1}), a(T_f - t_i)\right) \quad i = 2,\ldots,n \tag{5}$$

$$E\left(\log f(\lambda_{1:n+1} | \theta_1)\right) = E\left(\log\left(\prod_{i=2}^{n+1} gam(v_i; u_i, \xi)\right)\right) = \sum_{i=2}^{n+1}\left(-u_i \log\xi - \log\Gamma(u_i) + (u_i - 1)E(\log(v_i)) - \frac{1}{\xi}\left(E(\lambda_i) - E(\lambda_{i-1})\right)\right) \tag{6}$$

$$E\left(\log f(\vec{x}_{1:n} | \theta_2, \lambda_{1:n})\right) = E\left(\log\left(\prod_{i=1}^{n} N(\vec{x}_i - \vec{c}\cdot\lambda_i; 0, \mathbf{\Sigma})\right)\right) = \frac{-n}{(2\pi)^{m/2}} - \frac{n}{2}\ln|\mathbf{\Sigma}| - \frac{1}{2} tr\left(\mathbf{\Sigma}^{-1}\sum_{i=1}^{n} E\left((\vec{x}_i - \vec{c}\cdot\lambda_i)\cdot(\vec{x}_i - \vec{c}\cdot\lambda_i)'\right)\right) \tag{7}$$

$$\Pr(t \geq T_f | x_{1:c}, \lambda_c < \Lambda_f) = \int_0^{\Lambda_f} \Pr(t \geq T_f | \lambda_c) f(\lambda_c | x_{1:c}, \lambda_c < \Lambda_f) d\lambda_c \tag{8}$$

$$\Pr(t \geq T_f | \lambda_c) = \Pr(\Lambda(t) \geq \Lambda_f | \lambda_c) = \Gamma\left(a\cdot(t - t_c), (\Lambda_f - \lambda_c)/\xi\right) / \Gamma\left(a\cdot(t - t_c)\right) I_{(t_c \quad \infty)}(t) \tag{9}$$

$$\Pr(t \geq T_f | x_{1:c}, \lambda_c < \Lambda_f) = \int \Pr(\Lambda(t) \geq \Lambda_f | \lambda_c) f(\lambda_c | x_{1:c}) d\lambda_c \approx \frac{1}{N_f}\left(\sum_{i=1}^{N_f} \Pr\left(\Lambda(t) \geq \Lambda_f \middle| \lambda_c = f_c^i\right)\right) I_{(t_c \quad \infty)}(t) \tag{10}$$

$$f(t = T_f | x_{1:c}, \lambda_c < \Lambda_f) \approx \frac{1}{N_f}\sum_{i=1}^{N_f}\left(\frac{a}{\Gamma(a(t - t_c))}\int_{(\Lambda_f - f_c^i)/\xi}^{\infty} \{\ln(z) - \psi(a(t - t_c))\} z^{a(t - t_c) - 1} e^{-z} dz\right) I_{(t_c \quad \infty)}(t) \tag{11}$$

V. CASE STUDY

Liquefied natural gas (LNG) pumps are critical equipment in the LNG industry. An unexpected breakdown of an LNG pump can reduce the amount of LNG at the receiving terminal and cause performance dropdown of the whole plant. The structure of an LNG pump is shown in Fig 1. The LNG pump is enclosed within a suction vessel and mounted with a vessel top plate. Three ball bearings are installed to support the entire dynamic load of the integrated shaft of a pump and a motor. For each bearing, three accelerometers are installed on housing near the bearing assembly in horizontal, vertical and axial directions respectively.

The degradation processes of two bearings installed on different LNG pumps were recorded. During the degradation processes vibration signals were measured at inspections with irregular intervals. At the beginning and the last stages of lifetime, vibration signals were measured more frequently; while at the middle stage of lifetime, the intervals between inspections were relatively longer. The vibration signals were all measured in the horizontal direction. The characteristics of the vibration signals are listed in TABLE I. .

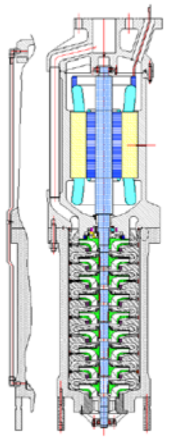

Figure 1. Pump schematic

Bearing failures often generate shock pulses whose energy locates at a relatively high frequency band. Therefore, vibration signals, after high pass filtering (HPF), are often more sensitive to defects at an early stage. For raw vibration signals, the kurtosis and the crest factor which reveals the number of extreme deviations can also indicate early defects. After investigating the different features of the vibration data, three features were used as degradation indicators: the crest factor of the raw vibration signals, the crest factor of the vibration signals after HPF at 2500 Hz, and the entropy of the vibration signals after HPF at 3000 Hz.

TABLE I. THE VIBRATION DATA FEATURES

Machine No	Life Time (Hrs)	Failure Mode	Sample Number	Sampling Frequency
P301C	4,698	Outer raceway spalling	120	12,800 Hz
P301D	3,511	Inner raceway flaking	136	12,800 Hz

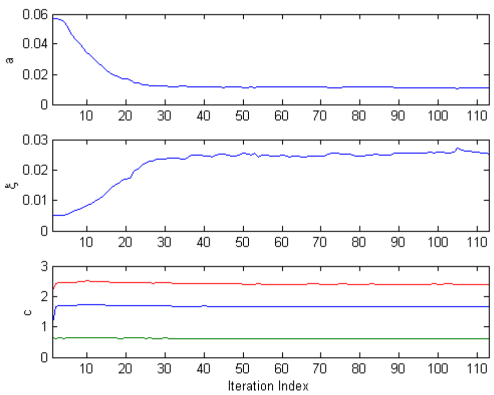

Figure 2. The convergence process of the parameters

Using the selected degradation indicators of the bearing installed on P301D, the Gamma-based state space model was trained by the Monte Carlo-based EM algorithm developed in Section III. The convergence processes of the parameter estimates are shown in Fig 2. The final training results are $\hat{a} = 0.0113$, $\hat{\xi} = 0.0253$, $\hat{\vec{c}} = (1.6663 \quad 0.6169 \quad 2.4058)'$, $\hat{\vec{\Sigma}} = \left((5.38 \quad -1.43 \quad -1.71)' \quad (-1.43 \quad 5.35 \quad -1.11)' \quad (-1.71 \quad -1.11 \quad 5.72)' \right) \times 10^{-2}$. According to the training results, the RUL of the bearing from P301C was estimated using the algorithms proposed in Section IV. The prediction results at different inspections and the corresponding 95% confidence intervals are illustrated in Fig 3.

Fig 3 shows that, at the beginning, the prediction error was very large. This was caused by the differences between the lifetimes of the training dataset and the test dataset. At the beginning of the lifetime, only few condition monitoring observations were collected. The RULs were largely predicted based on the lifetime of the training dataset which was much shorter than that of the test dataset. Consequently, the predicted RULs were shorter than the actual values. When a longer indicator history was considered, the quicker degradation progress of the bearing from P301C was detected. As a result, the prediction error decreased. Especially at the last stage of the life, the prediction results were very close to the real values. Fig 3 also illustrates that most of the actual RULs fell in the 95% confidence intervals, even at the beginning of the life. The estimated RULs of the bearing from P301 at different operation hours are listed in TABLE II. .

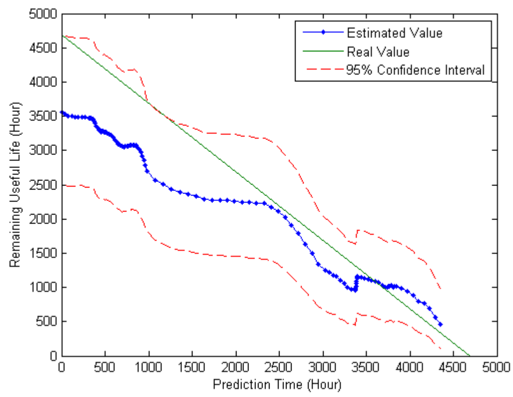

Figure 3. The RUL prediction results of P301C

TABLE II. THE ESTIMATED RUL OF THE BEARING FROM P301 AT DIFFERENT OPERATION HOURS

Operation Hours (Hrs)	1076	1546	2072	2565	3032	3522	4035	4356
Estimated RUL (Hrs)	2572	2326	2251	2019	1246	1113	885	453
Actual RUL (Hrs)	3622	3152	2626	2133	1666	1176	663	342

The same data were also treated using the PHM algorithm developed in [1]. The vibration collected from the bearing installed in P301D was again used as the training dataset. The parameters of the PHM model given by (12) were estimated as:

$\hat{\gamma} = 1265.9$, $\hat{\eta} = 5430.4$, $\hat{\beta}' = (19.8 \quad 102.5 \quad 28.7)$. Using these parameters, the RULs of the bearing from P301D and P301C were estimated as Fig 4 and Fig 5. Fig 4 shows that, the PHM even cannot give reasonable prediction results for the original training dataset until the last stage of the life. For the test dataset, even at the late stage, the predicted RULs still have significant errors.

$$\lambda(t|Z(t)) = \gamma/\eta(t/\eta)^{\gamma-1}\exp(\beta' Z(t)) \tag{12}$$

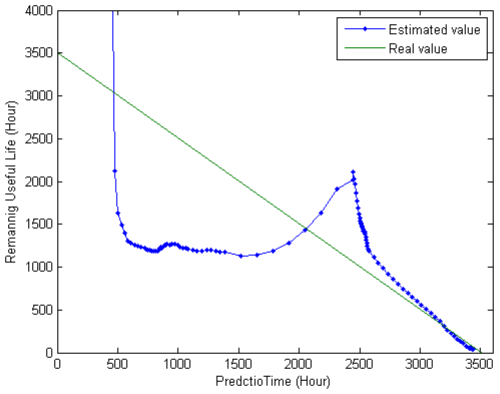

Figure 4. RUL prediction results of P301D using the PHM

The RUL prediction results verify that the proposed Gamma-based state space model can achieve a more accurate prediction result than the commonly used PHM in this case study. The reasons are as follows. Firstly, the PHM model requires substantial failure history. In this case study, however, only one failure event is available. In this situation, the PHM will have a large prediction error when the lifetime of the test data is considerably different from that of the training data. In Fig 5, the RUL sharply decreases to zero after 3511 hour, which is the failure time of the training data. In contrast, the Gamma-based state space model uses the increments of the underlying degradation process instead of only the failure time. Therefore, the degradation processes of indicators and the failure times are combined more efficiently. Secondly, the RUL estimation using the PHM is based on the prediction of degradation indicators. This paper uses the nonlinear model fitting method as [1] which may suffer significant error when a degradation process is highly stochastic. Using more sophisticated prediction methods such as stochastic process fitting may get better results. However, the integral of the predicted hazard process may become intractable. In addition, the problem which is caused by insufficient failure history cannot be solved even the degradation indicators are predicted appropriately.

VI. Conclusions

This paper has developed a Gamma-based state space degradation model to predict the RUL of engineering assets. The information from the stochastic degradation processes of multiple degradation indicators and uncertain failure thresholds has been combined by the Gamma-based state space model. Furthermore, Monte Carlo-based parameter estimation and RUL prediction algorithms have also been developed to deal with the Gamma-based state space model. In addition, a case study using the vibration data of bearings from LNG pumps has been conducted. The results of the case study demonstrate that the proposed Gamma-based state space model has a better performance than the commonly used PHM when the failure history is insufficient. Therefore, the proposed Gamma-based state space model is expected to be more appropriate to real industry where event data can often be sparse.

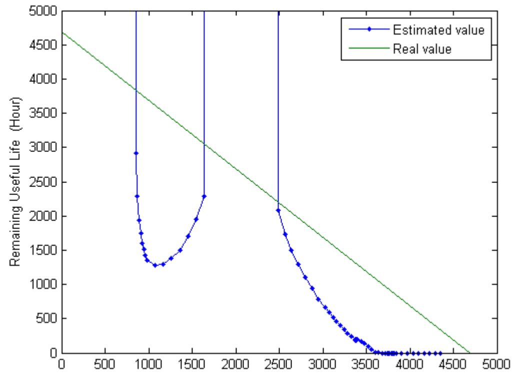

Figure 5. RUL prediction results of P301C using the PHM

Acknowledgment

This research was concluded within the CRC for Integrated Engineering Asset Management, established and supported under the Australian Government's Cooperative Research Centres Programme.

References

[1] H. Liao, W. Zhao, and H. Guo, "Predicting remaining useful life of an individual unit using proportional hazards model and logistic regression model," in Reliability and Maintainability Symposium, 2006. RAMS '06. Annual, 2006, pp. 127-132.

[2] D. Kumar and U. Westberg, "Proportional hazards modeling of time-dependent covariates using linear regression: a case study [mine power cable reliability]," Reliability, IEEE Transactions on, vol. 45, pp. 386-392, 1996.

[3] D. Banjevic and A. K. S. Jardine, "Calculation of reliability function and remaining useful life for a Markov failure time process," IMA J Management Math, vol. 17, pp. 115-130, April 1, 2006 2006.

[4] W. Wang, "A model to predict the residual life of rolling element bearings given monitored condition information to date," IMA Journal of Management Mathematics, vol. 13, p. 3, 2002.

[5] V. Makis and X. Jiang, "Optimal replacement under partial observations," Mathematics of Operations Research, vol. 28, p. 382, 2003.

[6] G. A. Whitmore, M. J. Crowder, and J. F. Lawless, "Failure Inference From a Marker Process Based on a Bivariate Wiener Model," Lifetime Data Analysis, vol. 4, pp. 229-251, 1998.

[7] J. M. van Noortwijk, "A survey of the application of gamma processes in maintenance," Reliability Engineering & System Safety, vol. In Press, Corrected Proof.

[8] J. G. Simon, D. Arnaud, and W. Mike, "Monte Carlo Smoothing for Nonlinear Time Series," Journal of the American Statistical Association, vol. 99, p. 156, 2004.

[9] M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking," Signal Processing, IEEE Transactions on, vol. 50, pp. 174-188, 2002.