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Asset Prices in General Equilibrium with Recursive Utility and Illiquidity Induced by Transactions Costs

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Non-Technical Summary

In recent years, which have been characterized by extremely turbulent financial markets, a number of economists and politicians have suggested to introduce tax on financial transactions, so that the speculative trading in financial markets is eliminated and the volatility of asset returns is reduced. John Maynard Keynes was among the first proponents of a securities transaction tax. In 1936, he proposed that a small tax should be levied on dealings on Wall Street, in the United States, where he argued that excessive speculation by uninformed financial traders increased volatility. For Keynes, the key issue was the proportion of 'speculators' in the market, and his concern that, if left unchecked, these types of players would become too dominant. Keynes writes: "The introduction of a substantial Government transfer tax on all transactions might prove the most serviceable reform available, with a view to mitigating the predominance of speculation over enterprise in the United States." (1936:159-60)

In September 2011, the European Commission proposed a harmonized Financial Transaction Tax for the EU (see IP/11/1085). The objectives of the proposed FTT were:

- to prevent the fragmentation of the Single Market that could result from numerous uncoordinated national approaches to taxing financial transactions,
- to ensure that the financial sector made a fair and substantial contribution to public finances, and
- to discourage financial transactions which do not contribute to the efficiency of financial markets or of the real economy. [source: http://ec.europa.eu/taxation_customs/taxation/other_taxes/financial_sector/index_en.htm]

The idea is also widely discussed in the press:

> "It's a simple tweak that would reign in an out-of-control financial sector, stimulate jobs, generate billions of revenue, and possibly prevent another heart-wrenching crisis. Nobel Prize-winning economists like Joseph Stiglitz and Paul Krugman want it. Billionaires like Warren Buffett and Bill Gates want it. Polls show the majority of Americans want it. Even the Pope wants it. We're talking about a financial transaction tax (FTT) – a tiny tax of, say, less than half a percent: maybe 3 cents per $100 – on Wall Street trading. It's simple, more than fair, widely supported by the public, and long overdue."
> [source:http://www.salon.com/2013/10/18/the_tax_that_could_save_america_from_wall_street_partner/]

While the idea of a financial transaction tax is evidently not new, and while it is widely discussed in the media, its economic effects are not well understood, and there is very limited empirical and theoretical support for its effectiveness. The reason for scarce academic research on the area is the complexity of the general equilibrium analysis in incomplete markets, especially in the presence of transaction costs. We show how one can solve for a general equilibrium in an economy with heterogeneous investors, where the transactions in one of the risky assets are subject to the financial transaction tax. We analyze the solution in a number of settings, paying a special attention to the effects of the introduction of the financial transaction tax on the economy in general, on financial markets, and on the welfare of the agents.

In particular, we address the following questions: How do investors change their consumption and portfolio decisions in the presence of proportional transaction costs in general equilibrium? What are the effects of these optimal trading and consumption decisions of individuals on asset prices and risk premia? How do transaction costs affect the liquidity of the assets, and what is the magnitude of the resulting liquidity premium, which is defined to be the difference in the expected returns of the low-liquidity and high-liquidity assets? How do stochastic labor income and heterogeneous beliefs influence the equilibrium effects of transaction costs? How sensitive are the answers to these questions to whether the analysis is undertaken in general or partial equilibrium, and to the particular functional form assumed for individual utility functions?

Our key findings are summarized below. As one would expect, the main effect of transaction costs is on portfolio turnover. In the presence of transaction costs, investors reduce substantially the frequency with which they trade Stock 1, which incurs a transaction cost. Typically, for a transaction cost of 1%, the average trading volume in this stock drops by more than 50%. Trading in the bond also drops, but by a smaller proportion, whereas trading in Stock 2 changes negligibly. Moreover, in about 85% of all states, investors decide not to trade Stock 1, rendering the stock illiquid. Consequently, investors alter their consumption and portfolio choices: the consumption of each investor is more highly correlated to her endowment, because the more risk tolerant investor reduces the riskiness of her portfolio, while the more risk averse investor increases the riskiness of her portfolio. Asset prices respond to these changes in asset demands, but because investors can optimize their trading decisions in the presence of transaction costs, and because the changes in demands of the two investors offset each other, the net impact on expected returns is small, and the impact on volatility of stock returns is essentially zero.

The major practical implication of the financial transaction tax is that it basically does not improve the situation with volatility on financial markets, and in fact it makes the market participants worse off by preventing the optimal risk sharing on the financial markets.

Asset Prices in General Equilibrium with Recursive Utility and Illiquidity Induced by Transactions Costs*

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Abstract

In this paper, we study the effect of proportional transaction costs on consumption-portfolio decisions and asset prices in a dynamic *general equilibrium* economy with a financial market that has a single-period bond and two risky stocks, one of which incurs the transaction cost. Our model has multiple investors with stochastic labor income, heterogeneous beliefs, and heterogeneous Epstein-Zin-Weil utility functions. The transaction cost gives rise to endogenous variations in liquidity. We show how equilibrium in this incomplete-markets economy can be characterized and solved for in a recursive fashion. We have three main findings. One, costs for trading a stock lead to a substantial reduction in the trading volume of that stock, but have only a small effect on the trading volume of the other stock and the bond. Two, even in the presence of stochastic labor income and heterogeneous beliefs, transaction costs have only a small effect on the consumption decisions of investors, and hence, on equity risk premia and the liquidity premium. Three, the effects of transaction costs on quantities such as the liquidity premium are overestimated in partial equilibrium relative to general equilibrium.

Keywords: Liquidity premium, incomplete markets, portfolio choice, heterogeneous agents.
JEL: G11, G12

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1 Introduction

Constantinides (1986) shows that the effect of transaction costs on the liquidity premium is one order of magnitude smaller than the transaction cost, because investors adjust optimally their frequency and volume of trade. On the other hand, Lynch and Tan (2011) find that if investors have shocks to their labor income, then the liquidity premium can be of the same order of magnitude as transaction costs. However, the conclusion of the Lynch and Tan (2011, p. 1364) article states:

> "One important limitation of our analysis is that it is a partial equilibrium analysis. Therefore, it says nothing about how transaction costs affect equilibrium prices by limiting the ability of investors to share risk. More work is needed to understand how transaction costs affect prices and returns in a general equilibrium setting."

Our objective is to fill this gap. In particular, we address the following questions: How do investors change their consumption and portfolio decisions in the presence of proportional transaction costs in general equilibrium? What are the effects of these optimal trading and consumption decisions of individuals on asset prices and risk premia? How do transaction costs affect the liquidity of the assets, and what is the magnitude of the resulting liquidity premium, which is defined to be the difference in the expected returns of the low-liquidity and high-liquidity assets? How do stochastic labor income and heterogeneous beliefs influence the equilibrium effects of transaction costs? How sensitive are the answers to these questions to whether the analysis is undertaken in general or partial equilibrium, and to the particular functional form assumed for individual utility functions?

To answer these questions, we construct a general-equilibrium model with investors who have Epstein and Zin (1989) and Weil (1990) utility functions and who may differ with respect to their preferences, beliefs, and their idiosyncratic labor income (which is uncorrelated to the payouts on financial assets). We consider a financial market in which the traded securities consist of a single-period discount bond and two risky stocks, of which one incurs an exogenously-specified transaction cost.[1] This transaction cost gives rise to

[1]For models where the transaction cost is determined endogenously, see the literature on market microstructure, which is reviewed in Biais, Glosten, and Spatt (2005).

endogenous illiquidity; that is, periods where investors choose not to trade a particular asset. We show how the problem of identifying equilibrium in this economy can be solved in a recursive fashion even in the presence of transaction costs, which make markets incomplete.

Our key findings are summarized below. As one would expect, the main effect of transaction costs is on portfolio turnover. In the presence of transaction costs, investors reduce substantially the frequency with which they trade Stock 1, which incurs a transaction cost. Typically, for a transaction cost of 1%, the average trading volume in this stock drops by more than 50%. Trading in the bond also drops, but by a smaller proportion, whereas trading in Stock 2 changes negligibly. Moreover, in about 85% of all states, investors decide not to trade Stock 1, rendering the stock illiquid. Consequently, investors alter their consumption and portfolio choices: the consumption of each investor is more highly correlated to her endowment, because the more risk tolerant investor reduces the riskiness of her portfolio, while the more risk averse investor increases the riskiness of her portfolio. Asset prices respond to these changes in asset demands, but because investors can optimize their trading decisions in the presence of transaction costs, and because the changes in demands of the two investors offset each other, the net impact on expected returns is small, and the impact on volatility of stock returns is essentially zero.

We also study the effect of transaction costs on the *liquidity premium*. The presence of a transaction cost for trading Stock 1 renders it less liquid than the other stock and gives rise to a liquidity premium. Our key finding is that the liquidity premium in general equilibrium is much smaller than in partial equilibrium. In a model with deterministic labor income, a 1% transaction cost for trading one of the stocks generates a liquidity premium of only 0.14%; with stochastic labor income, this increases to only 0.23%. One limitation of these models where the only motive for trade is risk sharing is that the trading volume in stocks is substantially lower than what we observe empirically. However, even when we introduce differences in beliefs so that the trading volume is closer to its empirical estimate, a 1% transaction cost generates a liquidity premium of only 0.16%.

On the other hand, in partial-equilibrium where asset returns are fixed exogenously,[2] the liquidity premium is 0.32%, instead of the 0.14% in general equilibrium. The reason

[2]We choose the moments of asset returns to match those from our general equilibrium economy; this is explained in greater detail in Section 5.2.

for the smaller liquidity premium in general equilibrium is that risk sharing between the heterogeneous investors reduces the impact of the transaction costs (that is, the transaction cost has opposite effects on the demand for assets by the two investors, so the net effect is small). Moreover, because the prices of *all* assets change in general equilibrium due to change in the pricing kernel, the changes in the prices of other assets absorb some of the effect of the transaction costs on Stock 1. We find that to get a liquidity premium that is of the same order of magnitude as the transaction cost, one has to assume a partial-equilibrium setting, a ratio of financial wealth to labor income that is very low (around 1.25, compared to its empirical value of about 5 for U.S. data), and a transaction cost function of the kind assumed by Lynch and Tan (2011), where an investor is required to finance consumption from the sale of financial assets rather than from labor income.

Our paper makes two major contributions. One, we contribute to the asset-pricing literature by extending the results of existing models examining the effects of transaction costs on liquidity to a dynamic general equilibrium setting (where the interest rate is also endogenous) with investors who have Epstein-Zin-Weil utility functions with different preference parameters, differences in beliefs, stochastic labor income, and financial markets are incomplete. Moreover, as discussed above, our extension to a general equilibrium setting leads to economic insights that are different from those in partial equilibrium settings, such as the ones considered in Jang, Koo, Liu, and Loewenstein (2007) and Lynch and Tan (2011), where there is a single investor, prices are given exogenously, and the interest rate is assumed to be constant. Our analysis of the empirical implications of the model shows that the dispersion in reservation prices across investors contributes to explaining the liquidity premium, even when this dispersion is instrumented by a number of observable macroeconomic and financial variables. This finding confirms the intuition of previous research (see, for example, Huang (2003) and Vayanos and Wang (2013)) that the need for liquidity and rebalancing is one of the key determinants of the cross-sectional differences in expected stock returns of stocks with varying liquidity levels.

Two, we demonstrate how to identify in a recursive fashion the equilibrium in this economy with heterogeneous investors who have Epstein-Zin-Weil utility functions when financial markets are incomplete. There are two problems that arise in identifying the equilibrium when markets are incomplete. The first is that one can no longer use the "central-planner's approach" to identify the equilibrium, which, in a complete-market setting, can be done conveniently in two steps: first, allocate consumption optimally across investors, and then, determine the asset prices and investors' portfolio policies that support this allocation. In contrast, with incomplete markets, one must solve *simultaneously* for consumption and portfolio policies because the consumption allocation that one chooses must lie in the span of the traded assets, making it difficult to implement a recursive scheme. The second problem is that in the presence of transaction costs, the problem of each investor is path dependent: the endogenous holding period of each investor depends not just on exogenous state variables, but also on the current portfolio holdings of the investor and time-varying liquidity. This is in contrast to models with an exogenous holding period, such as the one in Acharya and Pedersen (2005, p. 379), who write:

> "Perhaps the strongest assumption is that investors need to sell all their securities after one period (when they die). In a more general setting with endogenous holding periods, deriving a general equilibrium with time-varying liquidity is an onerous task."

We show how this "onerous task" can be handled, and hence, our recursive solution method can be applied to study other problems in general equilibrium with incomplete markets. For example, it allows us to extend to incomplete markets with transaction costs the analysis of Dumas, Uppal, and Wang (2000), who show how to characterize equilibrium in a setting with multiple heterogeneous investors with recursive utility in complete markets.

The rest of the paper is organized as follows. In Section 2, we discuss the existing literature that is related to our work. In Section 3, we describe the general model. In Section 4, we characterize the equilibrium and explain how it can be described by a system of path-independent backward-only equations instead of a system of backward-forward equations. In Section 5, we analyze the quantitative effect of transaction costs on consumption and portfolio choices, asset returns, the equity risk premium, and the liquidity premium. In Section 6, we examine the empirical implications of our model. We conclude in Section 7.

2 Related Literature

Our work is related to several strands of the literature, and our focus in this section is to explain the relation of our work to the existing literature; for comprehensive surveys of existing work on portfolio choice and asset pricing with transaction costs and illiquidity, see the review papers by Amihud, Mendelson, and Pedersen (2005), and Vayanos and Wang (2011, 2012, 2013).

The first strand of the related literature consists of *general-equilibrium models with incomplete markets and transaction costs.* Buss and Dumas (2013) is the paper that is closest to our work. Just like our work, this paper also studies a model of a general equilibrium economy with transaction costs. However, its focus is on the microstructure effects of transaction costs, and so it considers a model with only a single risky asset. In contrast, the focus of our work is on the effect of transaction costs on the cross section of asset returns, and so we consider a model with multiple risky assets; in addition, we allow for idiosyncratic labor income and our investors have Epstein-Zin-Weil utility rather than power utility. On the technical side, both papers use the insights in Dumas and Lyasoff (2012) to solve the model; however, we solve the primal problem, whereas Buss and Dumas (2013) solve the dual problem.

Heaton and Lucas (1996) also consider a general equilibrium model, but with a quadratic transaction cost for trading the stock.[3] In their model, heterogeneity across investors arises because of idiosyncratic labor income shocks. They find that the model can produce a sizable equity premium only if transaction costs are large or the assumed quantity of tradable assets is limited and borrowing constraints are binding. In contrast to Heaton and Lucas (1996), our model has proportional transaction costs that leads to states where investors choose not to trade the stock with the transaction cost, investors are heterogeneous with respect to labor income, beliefs, and preferences, which are described Epstein-Zin-Weil utility rather than power utility, and our focus is on the liquidity premium rather than the equity premium.

[3] Heaton and Lucas (1996, their Equation (19)) also consider a specification where the transaction cost function is quadratic for small transaction and linear for larger transaction. Other models with quadratic transaction costs include Grinold (2006) and Garleânu and Pedersen (2013).

Longstaff (2009) considers a model with extreme illiquidity of an asset, that is, the asset cannot be traded at all (at any cost) during an exogenously specified interval of time. He studies the asset-pricing implications of this illiquidity when there are two investors with logarithmic utility who differ only in their rates of subjective time preference, and finds that the effect of this kind of illiquidity on portfolio choice and asset prices can be substantial. In contrast, in our model illiquidity arises endogenously as a result of the transaction cost (that is, investors choose whether or not to trade an asset), investors are heterogeneous with respect to preferences and also labor income, and investors have Epstein-Zin-Weil utility functions.

A second strand of the literature consists of *models with transaction costs in which stock prices are determined in equilibrium but interest rates are exogenous.* For example, Vayanos (1998) considers an overlapping-generations model with multiple stocks and also finds that transaction costs have a small impact on prices.[4] One of the strengths of his paper is that the model has a closed-form solution, which can be used to obtain several interesting insights. However, to obtain a closed-form solution, several restrictive assumptions need to be made. For instance, the interest rate is assumed to be exogenous and constant, which can have an important bearing on results as shown by Loewenstein and Willard (2006), and investors have exponential utility functions, which do not allow for the study of wealth effects.[5] Acharya and Pedersen (2005) also consider an overlapping-generations model where investors have exponential utility and the interest rate is exogenous, as is the holding period; they use this model to motivate empirical tests of the liquidity-adjusted CAPM to study how a security's expected return depends on its expected liquidity as well as on the covariances of its own return and liquidity with the market return and market liquidity, thereby extending the work of Pástor and Stambaugh (2003), who study the relation between a security's expected return and the covariance of this return with market liquidity.[6] In contrast to Vayanos (1998) and Acharya and Pedersen (2005), we allow for

[4]Vayanos and Vila (1999) also study the effects of transaction costs in a model with no labor income but two assets, *both* of which are risk-free, but where one asset has transaction costs while the other does not. Huang (2003) studies the liquidity premium in a model in which both assets are also risk-free, but one is liquid and the other is not, and investors face exogenous liquidity shocks that force them to liquidate their holdings and exit the market.

[5]Some of the other assumptions are: dividends follow an Ornstein-Uhlenbeck, so they are normal, instead of being lognormal; and, risk aversion is increasing with age. A consequence of these assumptions is that stock prices are linear in dividends, and the stock holdings of investors are deterministic.

[6]Another paper with exponential utility and an exogenous risk-free rate is Gârleanu (2009), who studies a model of portfolio choice and asset pricing in illiquid markets, that is, where immediate trading may not

an endogenous interest rate, recursive utility functions, investors who can be heterogeneous with respect to their endowments, preferences, and beliefs, and the holding period in our model is endogenous.

Lo, Mamaysky, and Wang (2004) consider a setting with *fixed* transaction costs and *high-frequency transaction needs.* The motivation for trading in the model is heterogeneous nontraded (labor) income, which in aggregate sums to zero; that is, there is no aggregate risk. They find that the effect of fixed transaction costs in such a setting is larger, and of the same order as the transaction costs. In contrast to their model, we allow for multiple risky assets, proportional transaction costs, and investors who have Epstein-Zin-Weil utility functions.

The third strand of the literature consists of *partial equilibrium models that study the effects of transaction costs.* Amihud and Mendelson (1986) consider a single-period model in which investors are risk-neutral and must exit the market at which time they sell stock to newly arriving investors; they find that the excess return on a stock equals the product of the asset's turnover and the proportional transaction cost. Constantinides (1986) shows that if the investor chooses optimally the decision to trade, then the effect of transaction costs on asset prices is much smaller than that suggested by Amihud and Mendelson.

However, there are also partial-equilibrium models that find that the effect of transaction costs can be large. Jang, Koo, Liu, and Loewenstein (2007) demonstrate in a model with an exogenously-specified time-varying investment opportunity set that transaction costs can have a first-order effect on the liquidity premium.[7] Lynch and Tan (2011) also find that the liquidity premium is of the same order of magnitude as transaction costs in their model if investors have stochastic labor income, and the ratio of financial wealth to income is very low. This strand of the literature also includes partial-equilibrium models that focus on studying the effect of transaction costs on portfolio policies.[8] In particular, these papers

always be possible. He then extends the model to allow for transaction costs, and uses this extension to explain the relation between results in the literature on transaction costs and that on illiquidity.

[7]In our model, the investment opportunity set varies endogenously as the share of wealth between the two agents fluctuates randomly; the effect of this stochastic distribution of wealth on asset prices has been studied in the complete-market models of Chan and Kogan (2002), where investors have power utility, and Bhamra and Uppal (2013), where investor's utility exhibits habit.

[8]This includes the work in Davis and Norman (1990), Dumas and Luciano (1991), Gennotte and Jung (1994), Bertsimas and Lo (1998), Schroder (1998), Balduzzi and Lynch (1999), Lynch and Balduzzi (2000), Liu and Loewenstein (2002), Liu (2004), and Garleânu and Pedersen (2013). See also Ang, Papanikolaou,

identify the "region of no-trade" where, because of transaction costs, an investor finds it optimal not to rebalance her portfolio even though asset prices have changed. We, too, identify the *joint* region of no trade across the two agents that arises in *equilibrium*, but in contrast to these papers, our investors have recursive utility, asset prices are endogenous, and the changes in the investment-opportunity set arise endogenously.

Our work is related also to two strands of the literature studying models *without* transaction costs. The first strand consists of general-equilibrium models with *incomplete markets* but without transaction costs.[9] In contrast, in our model the restriction on asset holdings comes from the transaction cost, which, in addition to the labor income, is another reason for markets being incomplete. The second strand studies general equilibrium models with *heterogeneous investors but complete financial markets*.[10] In contrast to these models, we allow for heterogeneous investors with Epstein-Zin-Weil utility, and markets that are incomplete because of the transaction cost and also the "trees" in our model include, in addition to payouts on two stocks, two streams of idiosyncratic labor income, so that the total number of "trees" is four, while the number of traded assets is three.

and Westerfield (2013), who consider a model where certain assets cannot be traded at all for intervals of uncertain duration.

[9]One set of models in this strand studies investors with time-additive utility who are constrained or prohibited from holding some of the financial assets: see, for example, Basak and Cuoco (1998), Gromb and Vayanos (2002), Pavlova and Rigobon (2008), Garleânu and Pedersen (2011), and especially Dumas and Lyasoff (2012), who propose an elegant solution method that is recursive. Chabakauri (2013) studies asset prices in a model with *multiple* trees and heterogeneous investors who have power utility, but extends the analysis to allow for portfolio constraints. Guvenen (2009) extends these models to allow for Epstein-Zin-Weil preferences. A second set of models within this strand considers a setting where the source of market incompleteness is idiosyncratic labor income: see, for example, Lucas (1994), Telmer (1993), and Krusell and Smith (1998) who examine asset prices in a model with investors who have time-additive utility functions and transitory idiosyncratic income shocks and find that market incompleteness has only a small effect on equilibrium prices. On the other hand, Mankiw (1986), Constantinides and Duffie (1996), and Krueger and Lustig (2010) allow for permanent idiosyncratic shocks and identify the condition under which market incompleteness will have a substantial effect on equilibrium prices. Gomes and Michaelides (2005) extends these models further to allow for recursive utility functions.

[10]This includes models with a single endowment process, and complete markets, such as Dumas (1989), Dumas, Kurshev, and Uppal (2009), and Bhamra and Uppal (2013), and models where investors have recursive utility, as in Dumas, Uppal, and Wang (2000) and Dumas and Uppal (2001). This strand of the literature also includes general-equilibrium models with *multiple* endowment processes and time-additive preferences. Models with more than one "tree" include, for instance, Menzly, Santos, and Veronesi (2004), Cochrane, Longstaff, and Santa Clara (2008), and Santos and Veronesi (2006), who assume that there is a representative investor with logarithmic utility; Pavlova and Rigobon (2007), who assume logarithmic investors with heterogeneous preferences over the multiple goods; Buraschi, Trojani, and Vedolin (2013), Martin (2013), Chen and Joslin (2012), and Ehling and Heyerdahl-Larsen (2012) who assume power utility.

3 The Model

In this section, we describe the features of the model we study. In our model, there is a single consumption good. Time is assumed to be discrete, denoted by t, with the first date being $t = 0$ and the terminal date being $t = T$. In the rest of this section, we give the details of the model.

3.1 Uncertainty

We assume that there are multiple sources of uncertainty, with the number of sources of uncertainty denoted by M. Uncertainty is represented by a σ-algebra $\mathcal{F}$ on the set of states Ω. The filtration $\mathbb{F}$ denotes the collection of σ-algebras $\mathcal{F}_t$ such that $\mathcal{F}_t \in \mathcal{F}_s, \forall s > t$, with the standard assumptions that $\mathcal{F}_0 = \{\emptyset, \Omega\}$ and $\mathcal{F}_T = \mathcal{F}$. In addition to time being discrete, we will also assume that the set of states is finite, and so the filtration can be represented by a tree, with each node on the tree representing a particular state of nature, $\omega(t, s)$. The probability measure on this space is represented by $P : \mathcal{F} \to [0, 1]$ with the usual properties that $P(\emptyset) = 0, P(\Omega) = 1$, and for a set of disjoint events $A_i \in \mathcal{F}$ we have that $P(\cup_i A_i) = \sum_i P(A_i)$.

In our implementation of the model, we will assume that uncertainty is generated by an M-dimensional multinomial process, as described in He (1990), which is an extension of the binomial process that is typically used for pricing options in a discrete-time and discrete-state framework (see Cox, Ross, and Rubinstein (1979)).[11]

3.2 Financial Assets

We assume that there are $N + 1$ assets that are traded in financial markets, indexed by $n = \{0, 1, \ldots, N\}$, with the consumption good being the numeraire. The first asset is a one-period discount bond in zero net supply. The other N assets are stocks assumed to be in unit supply with payouts $d(n, t)$, which are assumed to be $\mathcal{F}_t$ measurable. These payouts may be correlated. The ex-dividend price of each asset n as perceived by investor k at date t, $S(n, k, t)$, is determined in equilibrium; note that in the presence of transaction costs,

[11] Given that we allow for incomplete financial markets, the exact process used to generate uncertainty could be more general; for instance, we could specify a process that, in the limit, has jumps.

investors may choose *not* to trade a particular asset at a particular date, in which case they need not agree on the price of this asset: $S(n,k,t) \neq S(n,k',t)$. The ex-divided price on the terminal date for these assets is zero. The number of units of a particular asset n held by investor k at date t is denoted by $\theta(n,k,t)$.

3.3 Transaction Costs

We assume that investors pay a proportional cost for trading financial assets. The transaction cost at t depends on the value of the assets being traded.[12] We denote this transaction cost by

$$\tau\big(\theta(n,k,t),\theta(n,k,t-1)\big) \equiv \big|\theta(n,k,t)-\theta(n,k,t-1)\big| \times S(n,k,t) \times \kappa(n), \tag{1}$$

where $\theta(n,k,t)$ denotes the number of shares of asset n held by investor k at date t, and $\kappa(n)$ is the proportional transaction cost for trading asset n.[13] We assume that the transaction cost is a deadweight cost for making a transaction, and hence, this amount does not flow to any investor.[14]

3.4 Labor Income

The labor income of Investor k is denoted by $Y(k,t)$. We adopt the same process as in Lynch and Tan (2011), who, following Carroll (1996, 1997), specify the logarithm of labor income, $\log Y(k,t) = y(k,t)$, to have both permanent and temporary components:

$$y(k,t) = y^P(k,t) + \varepsilon(k,t), \tag{2}$$

$$y^P(k,t) = y^P(k,t-1) + \bar{g}(k) + u(k,t), \tag{3}$$

where ε_t and u_t are uncorrelated i.i.d. processes that have normal distributions, and $\bar{g}(k)$ is a constant representing the average growth rate for the labor income of Investor k.[15]

[12] We could also consider the case where the transaction cost depends on the number of shares being traded, which is the specification studied in Vayanos (1998).

[13] While in our model transaction costs differ only across assets, they could differ also across investors and over time.

[14] We also study the case where the transaction cost paid by each investor is added back to the investor's consumption *after* the investor has made her consumption and portfolio decisions; this eliminates any wealth effects of the deadweight loss arising from transaction costs.

[15] As mentioned above, we use the approach in He (1990) to generate the multinomial process that matches the distributions we wish to model.

Just as in Lynch and Tan (2011), we also turn off the temporary component because, when calibrated to data, the temporary component has a negligible impact on liquidity premia. Thus, throughout our analysis, we consider the case in which $y(k,t) = y^P(k,t)$ and $\varepsilon(k,t) = 0$ for all t. Moreover, labor income is assumed to be uncorrelated to the payouts on the risky financial assets.

3.5 Preferences

In our model, we will allow for $k = \{1, \ldots, K\}$ investors who have preferences that are of the Kreps and Porteus (1978) type. These utility functions nest the more standard time-separable utility functions, and have the well-known advantage that the risk aversion parameter, which drives the desire to smooth consumption across states of nature, is distinct from the elasticity of intertemporal substitution (EIS) parameter, which drives the desire to smooth consumption over time. We adopt the Epstein and Zin (1989) and Weil (1990) specification of this utility function, in which lifetime utility $V(k,t)$ is defined recursively:

$$V(k,t) = \left[(1-\beta_k)\, c(k,t)^{1-\frac{1}{\psi_k}} + \beta_k E_t \left[V(k,t+1)^{1-\gamma_k} \right]^{\frac{1}{\phi_k}} \right]^{\frac{\phi_k}{1-\gamma_k}}. \tag{4}$$

In the above specification, E_t denotes the conditional expectation at t, $c(k,t) > 0$ is the consumption of investor k at date t in state $\omega(t,s)$,[16] β_k is the subjective rate of time preference, $\gamma_k > 0$ is the coefficient of relative risk aversion, $\psi_k > 0$ is the elasticity of intertemporal substitution, and $\phi_k = \frac{1-\gamma_k}{1-1/\psi_k}$. The above specification reduces to the case of constant relative risk aversion if $\phi_k = 1$, which occurs when $\psi_k = 1/\gamma_k$. The index k for the preference parameters β_k, γ_k, and ψ_k indicates that the investors may differ along all three dimensions of their utility functions.

4 Characterization of Equilibrium

In this section, we first describe the optimization problem of each investor. We then impose market clearing to obtain a characterization of equilibrium, which is given in terms of a backward-forward system of equations. Finally, we explain how this backward-forward system of equations can be transformed into a recursive (backward-only) system of equations.

[16] To simplify notation, we do not write explicitly the dependence on the state $\omega(t,s)$.

4.1 The Optimization Problem of Each Investor

The objective of each Investor k is to maximize lifetime utility given in (4) by choosing consumption, $c(k,t)$, and the portfolio positions in each of the financial assets, $\theta(n,k,t), n = \{0,1,\ldots,N\}$. This optimization is subject to a dynamic budget constraint:

$$c(k,t) + \sum_{n=0}^{N} \theta(n,k,t) S(n,k,t) + \sum_{n=0}^{N} \tau\big(\theta(n,k,t), \theta(n,k,t-1)\big) \leq \qquad (5)$$

$$Y(k,t) + \sum_{n=0}^{N} \theta(n,k,t-1)\Big(S(n,k,t) + d(n,t)\Big),$$

where the left-hand side of the above equation is the amount of wealth allocated to consumption and the purchase of assets at date t, and the right-hand is the sum of labor income and the value, using the prices prevailing at date t, of shares that were purchased at date $t-1$, and the "payouts" received at t from these assets; this sum can be interpreted as the investor's wealth at time t.[17] We assume that each investor is endowed with some shares of the risky assets at the start of the economy.

Thus, the Lagrangian for the utility function in (4) that is to be maximized subject to the budget constraint in (5) is:

$$\begin{aligned}
\mathcal{L}(k,t) \quad = \quad & \sup_{c(k,t),\theta(n,k,t)} \inf_{\lambda(k,t)} \left[(1-\beta_k)\, c(k,t)^{1-\frac{1}{\psi_k}} + \beta_k E_t \left[V(k,t+1)^{1-\gamma_k}\right]^{\frac{1}{\phi_k}}\right]^{\frac{\phi_k}{1-\gamma_k}} \qquad (6) \\
& + \lambda(k,t) \left[Y(k,t) + \sum_{n=0}^{N} \theta(n,k,t-1)\Big(S(n,k,t) + d(n,t)\Big) \right. \\
& \left. - c(k,t) - \sum_{n=0}^{N} \theta(n,k,t) S(n,k,t) - \sum_{n=0}^{N} \tau\left(\theta(n,k,t), \theta(n,k,t-1)\right) \right],
\end{aligned}$$

[17]Note that in the above formulation we have not imposed constraints on short selling or borrowing; if one wished, constraints on portfolio positions could be imposed on the trading strategy of the investor.

where $\lambda(k,t)$ is the Lagrange multiplier for the budget constraint. Based on the above Lagrangian, the first-order conditions with respect to $\lambda(k,t)$ and $\theta(n,k,t)$ are:

$$
\begin{aligned}
0 \;=\; & Y(k,t) + \sum_{n=0}^{N} \theta(n,k,t-1)\Big(S(n,k,t) + d(n,t)\Big) \\
& - c(k,t) - \sum_{n=0}^{N} \theta(n,k,t) S(n,k,t) - \sum_{n=0}^{N} \tau\Big(\theta(n,k,t), \theta(n,k,t-1)\Big),
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
0 \;=\; & \frac{\partial V(k,t)}{\partial c(k,t)} \left[S(n,k,t) + \frac{\partial \tau\Big(\theta(n,k,t), \theta(n,k,t-1)\Big)}{\partial \theta(n,k,t)} \right] \\
& - E_t \left[\frac{\partial V(k,t+1)}{\partial c(k,t+1)} \left(S(n,k,t+1) + d(n,t+1) - \frac{\partial \tau\Big(\theta(n,k,t+1), \theta(n,k,t)\Big)}{\partial \theta(n,k,t)} \right) \right],
\end{aligned}
\tag{8}
$$

where we have used the first-order condition for consumption to substitute in the marginal utility of consumption for $\lambda(k,t)$. Equation (7) is the budget constraint that the optimal consumption and portfolio policies must satisfy and equation (8) equates the benefit from holding the stock versus selling the stock, net of transaction costs. Thus, the solution to the problem of maximizing the lifetime utility in (4) subject to the budget constraint in (5) is characterized by the system of equations given in (7) and (8), which must hold for each investor at each date and state.

4.2 Market-Clearing Conditions

In the economy we are considering, there are financial markets for the one-period risk-free bond and the N risky securities, and a commodity market for the consumption good. The market-clearing conditions for the financial assets are that the aggregate supply must equal aggregate demand,

$$
\theta^{ss}(n) = 1 = \sum_{k=1}^{K} \theta(n,k,t), \;\; \forall \; n = \{1, \ldots, N\},
\tag{9}
$$

where the $\theta^{ss}(n)$ is the aggregate supply of asset n, which we have assumed is equal to 1. For the one-period risk-free bond that is in zero net supply, we have that $\theta^{ss}(0) = 0$.

Finally, aggregate payouts from the stocks and labor income should be equal to aggregate consumption and transaction costs:[18]

$$0 = \left(\sum_{k=1}^{K} Y(k,t) + \sum_{n=0}^{N} d(n,t) \right) - \left(\sum_{k=1}^{K} c(k,t) + \sum_{k=1}^{K} \sum_{n=1}^{N} \tau \Big(\theta(n,k,t), \theta(n,k,t-1) \Big) \right). \quad (10)$$

4.3 Equilibrium in the Economy

Equilibrium in this economy is defined as a set of consumption policies, $c(k,t)$, and portfolio policies, $\theta(n,k,t)$, along with the resulting price processes for the financial assets, $S(n,k,t)$, such that the consumption policy of each investor maximizes her lifetime utility, that this consumption policy is financed by the optimal portfolio policy, and markets for financial assets and the consumption good clear.

4.4 Solving for the Equilibrium

When financial markets are *complete*, one can separate the task of identifying the equilibrium into two distinct steps by exploiting the condition that investors can achieve perfect risk sharing. This condition can be used to first identify the optimal allocation of aggregate consumption across investors ("central planner's problem"). Then, in the second step, using these consumption choices we can determine asset prices and also the portfolio policy of each investor that supports this allocation.

In contrast, when financial markets are *incomplete*, one cannot separate the task of identifying the equilibrium into two steps because the consumption allocation one chooses must lie in the span of traded assets. Instead, one must solve for the consumption and portfolio policies *simultaneously*. In principle, one can identify the equilibrium by solving simultaneously the set of nonlinear first-order conditions for the investors in (7) and (8), along with the market-clearing conditions in (9), for all the states across all dates — an approach proposed in Cuoco and He (2001). The problem in implementing this approach is that the number of equations grows exponentially with the number of periods, so that a recursive approach would be preferable.

[18] Note that we use the consumption good as the numeraire, and therefore, its price is equal to unity. This also implies that the final market-clearing condition in (10) is satisfied simply because of Walras' law.

However, there are two problems in solving the system of equations in (7)–(9) recursively in a general-equilibrium setting where markets are not complete because of the labor income. The first problem is that the current consumption and portfolio choices depend on the prices of assets, which from Equation (8) we see depend on *future* consumption. But, in a general-equilibrium setting, when the investor attempts to solve for the optimal consumption and portfolio policies at date t, asset prices need to adjust in order for markets to clear. However, if one were solving the system of equations backward, these prices could *not* adjust because they depend on future consumption, which has already been determined in the previous step. Thus, to solve these equations, one would need to iterate backwards *and* forwards until the equations for all the nodes on the tree are satisfied. Dumas and Lyasoff (2012) address this problem by proposing a "time-shift" whereby at date t one solves for the optimal portfolio for date t but the optimal consumption for date $t+1$, instead of the optimal consumption for date t. Exploiting this insight allows one to write the system of equations so that it is recursive; this "time shift" is described in detail in Appendix A.

A second set of problems arise because of transaction costs. First, one needs to include also the portfolio composition of the investor as an endogenous state variable, because the choice of the optimal portfolio at date t now depends also on the composition of the portfolio at date $t-1$. Second, whether a particular security is traded or not at a given node is determined endogenously. Specifically, if investors choose to trade each of the assets, then they will agree on the prices of these assets. However, if investors find it optimal not to trade some of the assets, then investors will disagree on the prices of the assets that are not traded at that node. Consequently, the system of equations characterizing the solution, depends on whether or not investors choose to trade all assets or only some of the assets.[19]

To address these problems, we note that the past portfolio holdings enter the system of equations only through condition (8), as a first partial derivative of the transaction cost function $\tau(\cdot)$ with respect to the current portfolio investment:

$$\frac{\partial \tau\Big(\theta(n,k,t), \theta(n,k,t-1)\Big)}{\partial \theta(n,k,t)}.$$

[19] For a discussion of the challenges in solving dynamic optimization problems with "occasionally-binding constraints" see Christiano and Fisher (2000).

Under the assumption that the transaction costs are a constant proportion $\kappa(n)$ of the value of an asset n being traded, we observe that there are only three possibilities for this derivative: it is equal to zero when an investor decides not to trade; it is equal to $\kappa(n) \times S(n, k, t)$ when the investor decides to increase the position in the asset; or, it is equal to $-\kappa(n) \times S(n, k, t)$ when the investor sells the asset. Consequently, all the $\theta(n, k, t-1)$ values for which the investor decides to buy (sell) an asset at time t result in the same solution $\theta(n, k, t)$ and $S(n, k, t)$ for a given value of current consumption $c(k, t)$; all other values of past portfolio holdings will result in no trading at t. In other words, instead of solving the problem over the undetermined wide grid of portfolio holdings at $t-1$, we can solve it first for the two trading decisions—sell or buy—at time t; that is, over the two values of the derivative of the transaction cost function.

The solution to the problem in which we decide whether or not to trade provides us with the bounds of the *no-trade region*—the region for which the portfolio investment from $t-1$ to t does not change. Within the bounds of the no-trade region, we solve the system of equations explicitly restricting current portfolio holdings to be equal to the past portfolio holdings $\theta(n, k, t-1)$, using the past portfolio holdings within the no-trade region as an endogenous state variable.[20] That is, within this no-trade bounds one does not need to solve for the new portfolio weight; at the same time, investors can disagree on the prices of the traded assets, and therefore, we can eliminate the condition that there is a price at which the market for this asset clears.

Using the "time-shift" and the insights described above, one can solve for the equilibrium recursively. Finally, after solving the dynamic program recursively up to time $t = 0$, we undertake a single "forward step" to determine the equilibrium quantities for each state of nature that satisfy the initial conditions. Additional details of this recursive solution method are given in Appendix A.

5 Quantitative Analysis of the Model

In this section, we evaluate the effect of transaction costs on consumption and portfolio decisions, and on liquidity and equity risk premia by undertaking a quantitative analysis

[20]The range of this endogenous state variable is predetermined by the upper and lower boundary.

of the model described in the previous section. Our analysis is divided into the following subsections. In Section 5.1, we explain our modeling assumptions and choice of parameter values. Then, in order to pin down precisely the effects of transaction costs in an economy in which agents receive stochastic labor income and have heterogeneous preferences and beliefs, we start our analysis by studying the simplest possible model — a model with homogeneous investors and deterministic labor income with no transaction costs — and add only one feature at a time to this simple model. In Section 5.2, we examine individual consumption and portfolio choices, and equilibrium asset prices for the case of *zero* transaction costs. In Section 5.3, we study the effect of transaction costs for the case of deterministic labor income. In Section 5.4, we extend the model to allow for stochastic labor income, and in Section 5.5, we extend the model to allow for differences in beliefs. We conclude in Section 5.6 by examining how our results would change with the choice of utility function, the values specified for various parameters, the level of transaction costs, and if the effect of transaction costs was evaluated in partial rather than in general equilibrium.

5.1 Modeling Choices and Parameter Values

In this section, we describe our modeling choices and the parameter values we use for the "base case" of our model. Our goal is not to match the magnitude of particular moments in the data, but rather to work with parameter values that are reasonable.

We consider the case of an economy with two investors and a financial market with a single-period, risk-free bond and two risky equities, of which one incurs a transaction cost and the other does not; we label the stock that incurs a transaction cost as "Stock 1" and the other stock, which can be traded without cost, as "Stock 2". We consider values of the transaction cost for Stock 1 that range from 0 to 2%.

The two stocks in the model are claims to two exogenous payout processes ("trees"). We assume that the level of initial payout for each stock is 0.45,[21] the expected growth rate of each payout stream is $\mu_1 = \mu_2 = 0.01$, and the volatility is $\sigma_1 = \sigma_2 = 0.0705$. We assume that the correlation between the payouts on the two stocks is $\rho = 0.20$. The prices of the two stocks and the moments of stock returns will be determined in equilibrium; in particular,

[21]We set initial payouts and labor income to be such that per capita consumption at the initial date is equal to one.

the correlation between stock returns (as opposed to payouts) will also be determined in equilibrium. We assume that each investor has an initial endowment of half a share of each of the two stocks, and zero bonds.

In addition to these shares, we assume that each investor is endowed also with labor income. The parameters for the labor income processes are assumed to be the same for both investors (but the realizations differ). The level of initial labor income for each investor is 0.55. The initial levels of payouts for the two stocks, 0.45, and the initial level of labor income, 0.55, are chosen to give a ratio of financial wealth to annual labor income that is around 6.50,[22] which is close to the estimate of the Federal Reserve Bank of Cleveland for the period 1980–2011. Based on Carroll (1996, 1997), we assume that the drift of the labor income process for each investor is $\bar{g}_1 = \bar{g}_2 = 0.02$, and the volatility of the labor income process for each investor is $\sigma_{u,1} = \sigma_{u,2} = 0.08$. We also assume zero correlations between the two labor income processes, and between the payout processes on the stocks and the labor-income processes. The above processes for corporate payouts and labor income imply that the mean of the growth rate of aggregate consumption is 1.8% while its volatility is 4.13%; Mehra and Presecott (1985) estimate the mean of the growth rate of aggregate consumption to be 1.83% and its volatility to be 3.57%.

We assume that the two investors have Epstein-Zin-Weil utility functions with the same subjective impatience factor of 0.975, the same elasticity of intertemporal substitution of 1.50, a risk aversion of $\gamma_1 = 8$ for Investor 1 and a risk aversion of $\gamma_2 = 14$ for Investor 2, and that the two investors maximize their lifetime utility of consumption over 10 periods; that is, t ranges from 0 to $T = 10$. Note that the only difference between the preferences of investors is in their relative risk aversion; all other preference parameters, and also the moments for the labor-income processes, are specified to be the same so as to not confound the effect of transaction costs.

The above parameter values, summarized in Table 1, are chosen so that the moments of asset returns are close to their empirical counterparts. Campbell (1999) reports that

[22]To compute the wealth-income ratio for a particular investor at $t = 0$, in the numerator we have the number of shares for the first stock multiplied by the initial level of payout and the stock price, plus the number of shares for the second stock multiplied by the initial level of payout and the price of the second stock, and in the denominator we have the initial labor income. For example, if the prices of the two stocks are 3.1281, then the initial wealth-income ratio is: $\frac{0.50(0.45+3.1281)+0.50(0.45+3.1281)}{0.55} = 6.50$.

for U.S. stock returns covering the period 1890-1991, the per annum risk-free rate is about 0.0194, the equity risk premium is 0.0544, the stock-return volatility is 0.1940, and the Sharpe ratio is 0.3200. For the parameter values described above, and interpreting equity to be a levered claim to corporate payouts, as in Abel (1999) with a leverage ratio of 2.74, we obtain a per annum risk-free rate of 0.0201, an equity risk premium for the market of 0.0408, stock-return volatility for the market of 0.1533, and a market Sharpe ratio of 0.2660.[23] Note that these quantities are for the market as a whole, rather than for the two stocks individually, and that they are computed as the average over the first five periods.

5.2 Equilibrium in the Absence of Transaction Costs

In this section, we first consider a model in which both agents are identical and the labor income they receive is deterministic and then we extend the model so that the two agents have different levels of relative risk aversion.[24] Results for each version of the model are reported in columns of tables. In each table, we label columns with the solution to a general-equilibrium model as GEnm, where n is the table number and m is the column number in that table. The moments that we report for consumption and also for asset returns in this tables and the ones that follow are the time-series averages from $t = 0$ to $t = 5$ and we ignore the last five dates in order to limit the effect of the finite horizon; that is, other than the portfolio holdings which are for $t = 0$, all quantities reported in the tables are averages over the first five periods.

The equilibrium where the two investors receive a deterministic labor income and have the *same risk aversion*, which we set equal to 10.18, the harmonic mean of $\gamma_1 = 8$ and $\gamma_2 = 14$, is reported in Table 2. We see from column "GE21" of Table 2 that both investors have the same average volatility of consumption growth, 0.0246. Both investors have also the same portfolio: zero holdings of the bond and 0.50 holdings of each share. In the financial market, there is zero turnover of each of the assets, and, in the absence of transaction costs, the liquidity premium is zero.

[23]These quantities are reported in the column labeled GE25 in Table 2, and discussed in greater detail below.

[24]The reason why we introduce labor income even when examining these simpler models is that stochastic labor income has two effects: one, it introduces a new source of risk; two, it introduces also a new source of income. We add to our model at this preliminary stage *deterministic* labor income in order to abstract away from the income effect. Consequently, when we introduce stochastic labor income, there will not be any income effect, and thus, we will be able to focus on the effect arising solely from the change in risk.

Next, we consider the equilibrium in which investors *differ in their risk aversion*: $\gamma_1 = 8$ and $\gamma_2 = 14$, but labor income is deterministic and there are zero transaction costs. The equilibrium quantities for this case are given in column "GE22" of Table 2. Because the investors engage in risk-sharing (over the entire horizon), Investor 1, who has a lower risk aversion is willing to tolerate higher consumption volatility, and so the average volatility of consumption growth for this investor increases from 0.0246 to 0.0309, while that for Investor 2 decreases from 0.0246 to 0.0183. This risk-sharing is reflected also in the asset holdings: Investor 1 holds a negative position in bonds; with -0.2665 of her wealth invested in bonds, and uses this to *increase* (lever up) her position in both stocks from 0.50 to 0.6332. Investor 2, in contrast, has 0.2691 of her wealth invested in bonds and *reduces* her holding in the two stocks from 0.50 to 0.3655. These changes in asset demands, arising because of heterogeneity in the risk aversion of the two investors, lead to only marginal changes in asset returns, which are reported in the table.

5.3 Equilibrium in the Presence of Transaction Costs

We now examine the effect of transaction costs on the equilibrium for the economy described above, in which the two investors differ in their risk aversion ($\gamma_1 = 8$ and $\gamma_2 = 14$) and receive deterministic labor income. The third column of Table 2, labeled "GE23," gives the equilibrium quantities for a transaction cost of 1%. The presence of the transaction cost reduces the amount of risk sharing between the two investors. This can be seen by comparing the consumption growth volatility for the case without ("GE22") and with ("GE23") transaction costs, and noting that in the presence of transaction costs, there is a smaller difference in the volatility of the consumption growth rate of the two investors; that is, in the presence of transaction costs, the consumption of each investor is more highly correlated with that investor's income because of a decrease in risk sharing.

This reduction in risk sharing is reflected also in portfolio positions: the presence of transaction costs reduces the amount of borrowing/lending activity and also brings the holding of shares in Stock 1 closer to the initial endowment level of 0.50. However, both investors use Stock 2 as a substitute for Stock 1, and *increase* slightly, relative to the case with zero transaction costs, the deviation in their holdings of these shares compared to their initial endowment of 0.50.

Transaction costs also lead to a substantial reduction in the turnover of Stock 1, which drops by more than 70% from 0.0288 to only 0.0079. Turnover of the bond also drops, but by about 33%, while the turnover of Stock 2 stays about the same.

The changes described above in the demands of the two investors for bonds and stocks are reflected in asset returns. The decrease in demand for bonds leads to the risk-free interest rate increasing from 0.0341 to 0.0342. There is also an increase in the expected return of Stock 1 from 0.0667 to 0.0682, whereas Stock 2 experiences a smaller increase in its expected return, from 0.0667 to 0.0668 – induced by changes in the pricing kernel. Along with the increase in expected returns of the two stocks, there is a very small change in the average volatility of stock returns.

The liquidity premium, measured as the difference in the expected returns of the stocks with and without the transaction cost, is only 14 basis points (0.0014), which is one order of magnitude smaller than the transaction cost of 100 basis points (0.0100). The transaction cost has a negligible effect on the equity risk premium: the equity risk premium for Stock 1 increases slightly from 0.0326 to 0.0340, while the equity risk premium for Stock 2 does not change (the increase in expected return on Stock 2 being offset exactly by the increase in the risk-free rate). The small change in the equity risk premium is similar to the finding in Heaton and Lucas (1996), when transaction costs are introduced only in the stock market.[25]

In summary, transaction costs on a stock lead to a substantial decrease in the turnover of that stock, but have a smaller effect on the turnover of bonds, and may lead to a small increase in the turnover of the stock that does not incur a transaction cost because investors use this stock as a substitute for the stock with a transaction cost. Other than that, the effect of transaction costs on the consumption and portfolio decisions of investors is quite small in general equilibrium. There are two reasons for this: first, at the optimum, the

[25] Heaton and Lucas (1996) report that: "For example, if transactions costs are introduced only in the stock market, then agents trade primarily in the bond market and by this means effectively smooth transitory income shocks. In this case transactions costs have little effect on required rates of return. When transactions costs are also introduced in the bond market in the form of a wedge between the borrowing and lending rate, then the equilibrium lending rate falls. With a binding borrowing constraint or a large wedge between the borrowing and lending rate, a transactions cost in the stock market can produce an equity premium of about half of the observed value."

indirect effect of an exogenous change in parameters is small; in our context, the direct effect of a transaction cost on turnover is large, but other than that, the indirect effect of transaction costs on other quantities and prices is small. Second, as explained in Gârleanu (2009), when there are heterogeneous investors, transaction costs may have opposite effects on these investors, for example, transaction costs may increase one investor's demand for stocks but decrease that of the other investor, so that the net effect is small. Because transaction costs have only a small effect on consumption, and consequently on the pricing kernel, they change only slightly the risk-free rate, expected stock returns, and volatility of stock returns. Thus, transaction costs give rise to a small liquidity premium and lead to only a small increase in the equity risk premium for the stock with the transaction cost and almost no change in the equity risk premium of the other stock.

5.4 Equilibrium with Stochastic Labor Income

In the section above, the liquidity premium of 14 basis points that we identified is substantially smaller than the transaction cost of 100 basis points. Lynch and Tan (2011) show, in a partial-equilibrium model, that in the presence of stochastic labor income the liquidity premium is much larger and of about the same magnitude as the transaction cost. Their intuition is that if an investor receives labor income that is stochastic instead of deterministic, then this will increase the investor's desire to trade financial assets to offset the shocks to labor income, and so if there is a cost for trading financial assets, the liquidity premium will increase. In this section, we extend to a general equilibrium setting the analysis by Lynch and Tan of the effect of stochastic labor income in the presence of transaction costs.

In column "GE24" of Table 2 we report the results for the case where labor income is stochastic, but there are no transaction costs, and in column "GE25" we report the results for a 1% transaction cost, which we refer to as our "*base case*" in the rest of the paper. Comparing the results in these two columns, we see that the effect of a cost for trading Stock 1 in the presence of stochastic labor income is very similar to that when labor income was assumed to be deterministic. The key result is that, as shown in Lynch and Tan (2011), the liquidity premium with stochastic labor income is higher: 23 basis points instead of the 14 basis points we had with deterministic labor income. However, even though the liquidity premium has increased, it is still quite small relative to the magnitude of the transaction cost of 100 basis points.

Imposing a transaction cost on Stock 1 has two effects on the economy. One, it restricts risk-sharing between the two investors, and hence, leads to a change in the pricing kernel. But, it also imposes a deadweight loss, because we have assumed so far that the transaction cost is lost to the economy. In the column labeled "GE26" of Table 2, we consider how the results described above would change if the transaction costs paid by investors each period were *refunded* to them at the end of the period so that there is no deadweight loss, although agents account for the transaction cost when making their decisions. Comparing "GE25" to "GE26," we see that the refund leads to almost no change in the choices of individual investors. The refund of the transaction cost does have a small effect on expected returns. However, all *relative* quantities are the same as they were with a transaction cost that was not refunded: the liquidity premium is still 23 basis points, and the equity risk premia on the two stocks are exactly the same.

In summary, the inferences about the effect of transaction costs on consumption and portfolio decisions and asset prices are similar with stochastic labor income as they were with deterministic labor income, with the only change being that the liquidity premium is larger under stochastic labor income. However, in terms of magnitude, the liquidity premium of 23 basis points for a transaction cost of 100 basis points implies that in general equilibrium the liquidity premium is still less than one-fourth of the transaction cost.

5.5 Equilibrium with Differences in Beliefs

One limitation of the model that we have studied above is that the volume of trade in equity markets is much smaller than that observed empirically. We measure trading volume as the number of shares traded. Given that we have one share outstanding for each stock, this gives us the total (percentage) turnover; for example, a trading volume of 0.25 means that we trade 25% of total market cap per year. In our model, we find that turnover in both stocks is 0.0407 in the absence of transaction costs (see column "GE24" of Table 2), and when we introduce a transaction cost of 1% for trading Stock 1, its turnover drops to 0.0177 (see column "GE25" of Table 2). However, these turnover numbers are much smaller than the 0.3222 turnover for the portfolios held by individual investors in the U.S. that is reported by Griffin, Harris, and Topaloglu (2003).[26]

Given that the liquidity premium depends on the desire of investors to trade, one might wonder whether the small liquidity premium generated in our model is an artifact of the low turnover in stocks. In order to address this concern, we extend our model to allow for differences in beliefs between the two agents about the mean growth rate of payouts for the two stocks. We model these differences in beliefs by changing the subjective probabilities of the two agents: we assume that the first investor, who is relatively more risk tolerant, is optimistic and believes that the growth rates for the two payout processes are 2% above their true values, while the second investor, who is relatively more risk averse, is pessimistic and believes that the two payout processes are 2% below their true values.[27] Thus, the two investors now have a second reason to trade: in addition to trading because of a desire to share risk, they trade also because the first agent is relatively optimistic about the future, while the second investor is relatively pessimistic. Consequently, turnover increases by about 5 times for each of the two stocks: from 0.0407 to 0.2157.

The results for the model with differences in beliefs are reported in Table 3. To make it easier to understand the effect of differences in beliefs, in the first two columns of this table we reproduce the results from Table 2, where the two investors have the same beliefs, and in the last two columns of Table 3, we report the results for the case of heterogeneous beliefs, first for zero transaction costs, and then for a cost of 1% for trading Stock 1. Comparing first the results in the column "GE31," which are for the case with zero transaction costs and homogeneous beliefs, with those in column "GE33" which are for the case with zero transaction costs but heterogeneous beliefs, we see that there is a much greater divergence in the portfolios positions of the two investors: Investor 1 increases leverage and uses the borrowed funds to take large positions in the two stocks. These changes are reflected in small changes in the moments of equilibrium asset returns.

[26]The Federal Reserve Bank of St. Louis reports that the stock-market turnover ratio (value traded/capitalization) for the world, including inter-dealer trade, is about 15% in 2012; this is reported on the page `http://research.stlouisfed.org/fred2/series/DDEM01WA156NWDB?cid=33194`.

[27]To get a sense of these magnitudes, recall that the standard error of the mean growth rates is the volatility of the growth rate divided by the square root of the length of the estimation period. If the volatility of the growth rate of payouts is 7%, and the estimation period is 9 years, then one standard error equals 7%/3, which is 2.33%; if the estimation period is 49 years, then one standard error equals 7%/7, which is 1%.

Next, we study the effect of transaction costs in the presence of heterogeneous beliefs (and stochastic labor income). To do this, we compare the results in the column "GE33," which are for the case with zero transaction costs and heterogeneous beliefs, with those in column "GE34" for the case with a 1% transaction cost and heterogeneous beliefs. As before, transaction costs lead both investors to reduce the magnitude of their portfolio positions, though the effect is not large. Turnover of Stock 1, which incurs the transaction cost, drops from 0.2157 to 0.1690, while turnover of Stock 2 increases slightly from 0.2157 to 0.2210, because the investors use this as a substitute for trading in the other stock. The 100 basis points transaction cost gives rise to a liquidity premium, but the size of the liquidity premium is only 17 basis points. The main reason why this liquidity premium is so small, despite the substantial increase in turnover, is because in general equilibrium the differences in beliefs affect the expected returns of *both* stocks through the pricing kernel channel: the expected return of Stock 1 increases by 68 basis points from 0.0513 in the absence of transaction costs to 0.0581 with transaction costs, but the expected return of Stock 2 also increases by 51 basis points from 0.0513 to 0.0564, so that the gap between the expected returns on Stock 1 and Stock 2 is only 17 basis points.

5.6 Robustness Tests

In this section, we examine the robustness of our results to our choice of utility function, the values specified for various parameters, our decision to undertake the analysis in general equilibrium rather than partial equilibrium, and the level of the transaction cost.

5.6.1 Sensitivity of Results to Choice of Utility Function

In the analysis reported in the sections above, we have assumed that investors have Epstein-Zin-Weil utility functions. The advantage of using an Epstein-Zin-Weil utility function is well understood: it allows one to distinguish between the parameter for relative risk aversion and the parameter for elasticity of intertemporal substitution. In the literature on transaction costs, most of the papers use power utility (see, for example, Lynch and Tan (2011)), exponential utility (see, for example, Vayanos (1998) and Acharya and Pedersen (2005)), or logarithmic utility (Longstaff (2001, 2009)). Power utility has the limitation that there is a single parameter that controls both relative risk aversion and elasticity of intertemporal substitution, exponential utility has the disadvantage that it does not account for wealth effects, and under logarithmic utility even investors with a multiperiod horizon behave in a myopic fashion because the substitution and income effects offset each other exactly.

In Table 4, we present the results for our general equilibrium model in the presence of a cost of 100 basis points for trading Stock 1 and stochastic labor income and homogeneous beliefs for different utility functions. In the first column labeled "GE41", we report our "base case" where investors have Epstein-Zin-Weil utility. In the next three columns, we consider the cases with power, logarithmic, and exponential utility.

Comparing the case for power utility in the column titled "GE42" (where the two investors have relative risk aversions of $\gamma_1 = 8$ and $\gamma_2 = 14$) to that for the "base case" with Epstein-Zin-Weil utility in "GE41" (where investors again have risk aversions of $\gamma_1 = 8$ and $\gamma_2 = 14$, but both have the same elasticity of intertemporal substitution of 1.50), we see that there are only small differences in the consumption and portfolio choices of individual investors. The liquidity premium with power utility is about the same as with Epstein-Zin-Weil utility: 24 basis points instead of 23 basis points. The equity risk premia on the two stocks are also similar under these two kinds of utility functions. As one would expect, the main difference is in the risk-free rate, which with power utility is 0.0863, while with Epstein-Zin-Weil utility is a much more reasonable 0.0201.

Comparing the case for logarithmic utility in the column titled "GE43" (where both investors now have relative risk aversions equal to one) to that for the "base case" with Epstein-Zin-Weil utility, we see that the two investors hold the same portfolio at $t = 0$ and there is very little turnover over the entire horizon, with zero turnover in Stock 1 and a small amount of turnover in Stock 2 financed by trading the bond. The equity risk premia on the two stocks are much smaller with logarithmic utility (0.0038), and the risk-free rate is still too high at 0.0421. The liquidity premium with logarithmic utility is zero for the parameter values we consider, but one needs to recognize that this is not a perfect comparison, because in the case of logarithmic utility for *both* investors, they both have exactly the same risk aversion, and so the desire for risk-sharing arises only because of different realizations of labor income.

Finally, comparing the case for exponential utility in the column labeled "GE44" (where the two investors now have absolute risk aversion of 8 and 14) to that for the "base case" with Epstein-Zin-Weil utility, we see that the two investors have similar consumption volatility under both utility functions and their portfolio positions are also similar. While the equity risk premia and Sharpe ratios are similar under both utility functions, the risk-free rate with exponential utility is again too high at 0.0938, while the liquidity premium is 13 basis points compared to the 23 basis points with Epstein-Zin-Weil utility.

We conclude from our comparison across utility functions that the liquidity premium is no higher under power, logarithmic and exponential utility than it was with Epstein-Zin-Weil utility, while the fit of the moments of asset returns is worse with these other utility functions.

5.6.2 Sensitivity of Results to Choice of Parameter Values

We now evaluate how sensitive our results are to other parameter values. In our analysis of the model, we have undertaken a large number of experiments with respect to the parameters of the model; the variations we consider are listed in the last column of Table 1. In order to conserve space, we discuss only the experiments which one might think are likely to yield insights that are different from the "base case" we have considered; the results of these experiments are reported in Table 5, with the first column containing results for the base case.

In the first experiment, we consider the case where there is a higher spread between the relative risk aversion (RRA) of the two investors: instead of $\gamma_1 = 8$ and $\gamma_2 = 14$, we change this to $\gamma_1 = 7.25$ and $\gamma_2 = 17$, so that the spread increases but the harmonic mean of the risk aversions stays the same. In this case, the volatility of consumption growth of Investor 1 increases and that of Investor 2 decreases, because of the increase in the desire to share risk; this is reflected in a larger position in bonds, and greater divergence between the two investors in their portfolio holdings, and slightly higher turnover. However, the effect on the equity premia and return volatilities is small, and the liquidity premium increases to 30 basis points from 23 basis points for the base case.

Throughout our analysis, we have assumed that the EIS of the two investors was equal to 1.50. When we studied the case of power utility functions in Table 4 with relative risk aversions of the two investors being $\gamma_1 = 8$ and $\gamma_2 = 14$, this was equivalent to studying the case of $\mathrm{EIS}_1 = 1/8$ and $\mathrm{EIS}_2 = 1/14$, and we saw that a lower EIS did not have a big bearing on the results (other than on the interest rate). We now study the case of EIS $= 0.75$ for both investors. We find that a change in the value of the EIS parameter does not have a substantial effect on any of the quantities we report. In particular, the liquidity premium is the same as it was for the "base case."

We also consider the case of higher growth rate of payouts for Stock 1. Our motivation for considering this case is that if Stock 1 were more attractive, then investors may wish to trade this asset even if it incurs a transaction cost. However, because we are considering a general equilibrium setting, where the expected returns of assets are determined endogenously depending on their risk, and because the two stocks have the same volatility, the resulting difference in *expected stock returns* is much smaller than the difference in the growth rates of the payouts for the two stocks. As a result, there is very little difference in the behavior of investors and asset prices when Stock 1 has a higher payout drift. In particular, the liquidity premium with the higher payout drift is essentially the same as its value for the "base case."

The last case we report is for the setting where the ratio of financial wealth to labor income is much lower than that in the "base case." In the "base case," the wealth-to-income ratio is about 6.3; now, we increase the initial level of labor income from 0.55 to 0.75 and reduce that of corporate payouts from 0.45 to 0.25, so that the wealth-to-income ratio is about 2.6. The motivation for considering this case is that, if the relative share of labor income is higher, then the demand for smoothing the shocks from labor income will increase the desire to trade financial assets, and hence, the liquidity premium. We find that there is indeed an increase in consumption volatilities of the two investors. However, there is a change in the expected returns of *both* stocks, and consequently, the liquidity premium does not change at all relative to the base case.

From the above experiments, we conclude that in our general equilibrium model, a 1% transaction cost is unlikely to generate a liquidity premium that is of the same order of magnitude as the transaction cost.

5.6.3 The Liquidity Premium in Partial Equilibrium

We have already observed from Table 2 that the liquidity premium for our "base-case" general-equilibrium model is 23 basis points, and we observe from Tables 3 and 4 that the liquidity premium is of similar magnitude when we consider variations of the model and parameter values. In light of these results, we now study the liquidity premium in various *partial-equilibrium* models in order to evaluate whether it is larger than in general equilibrium and of the same magnitude as the transaction costs.

We consider three partial-equilibrium models, each corresponding to one of the three general-equilibrium models we have analyzed above: the first with deterministic labor income, the second with stochastic labor income, and the third with differences in beliefs (in addition to stochastic labor income). In each of the three partial-equilibrium models that we examine, we study the liquidity premium for an investor with risk aversion of $\gamma = 8$ who faces a transaction cost of 1% for trading Stock 1, but we fix asset returns to be the ones that would prevail in the corresponding general-equilibrium economy, in which the two investors have risk aversions of $\gamma_1 = 8$ and $\gamma_2 = 14$ and there are zero transaction costs.[28]

For all three settings that we study, we find that the liquidity premium is larger in partial equilibrium than it is in general equilibrium: for the model with deterministic labor income, the liquidity premium in partial equilibrium is 32 basis points compared to 14 basis points in general equilibrium; for the model with stochastic labor income, the liquidity premium in partial equilibrium is 30 basis points compared to 23 basis points in general equilibrium; and, for the model with differences in beliefs and stochastic labor income, the liquidity premium in partial equilibrium is 33 basis points compared to 16 basis points in general equilibrium.

[28] In the experiment that we considered above with a refund of the transaction cost, we allowed the pricing kernel to change, while eliminating the deadweight loss arising from the payment of the transaction cost. The partial-equilibrium experiment can be considered to be one where we keep fixed the pricing kernel, and examine the effect of the deadweight transaction costs on the liquidity premium. Note that the results of the partial equilibrium models are similar whether one fixes asset returns to match those from a general equilibrium model with transaction costs that are 0% or 1%.

The reason why the liquidity premium is larger in partial equilibrium is because the pricing kernel is exogenous, and therefore, the prices of other assets do not change at all when the cost for trading Stock 1 is introduced; in particular, the expected return on Stock 2 does not change at all when a cost for trading Stock 1 is introduced. Moreover, in partial equilibrium the investor does not have the opportunity to share risk with other investors.

The above analysis shows that the liquidity premium is higher in partial equilibrium than in general equilibrium. However, the magnitude of the liquidity premium is still smaller than that of the transaction cost. In order to understand how one can get a liquidity premium that is of about the same magnitude as the one obtained by Lynch and Tan (2011), we consider a large range of variations of the partial-equilibrium model. For example, we consider a model where we increase the mean growth rate for payouts on the stock that incurs transaction costs; we find that this does not increase the liquidity premium. We also consider a model where the wealth-income ratio is much lower, and we find that this, too, has little effect on the liquidity premium.

Finally, we consider the case where instead of using our transaction cost function in (1), we use the transaction cost function used in Lynch and Tan (2011):

$$\left| \theta_t - \theta_{t-1} \times \frac{\frac{W_t}{Y_{t-1}}}{\frac{W_t}{Y_{t-1}} + \exp(g_t)} \right| \times \kappa. \tag{11}$$

The above transaction-cost function implies that in each period an investor first sells assets from his portfolio to finance his consumption and, then, invests the labor income back into the assets.[29] In contrast to the specification of the transaction-cost function in (11), in our specification of the transaction-cost function in (1), an investor is allowed to consume out of the labor income she receives each period first, and only pays the cost κ for changing her portfolio holdings. We find that when the share of labor income is high, implying that the wealth-income ratio is low, then the transaction cost function in (11) has bite: for example, if the wealth-income ratio is 1.25, then for a transaction cost of 100 basis points the liquidity

[29]Note, the above transaction cost function requires a positive wealth income ratio $\frac{W_t}{Y_{t-1}}$ at each date; similar to Lynch and Tan (2011), we achieve this by imposing borrowing and short-sale constraints for the assets.

premium is 51 basis points, which is of the same order of magnitude as the one obtained by Lynch and Tan (2011).[30]

From the above analysis, we conclude that in a general-equilibrium setting where the moments of asset returns are close to their empirically-observed values, it is difficult to get a liquidity premium that is of the same order of magnitude as the transaction costs. In partial equilibrium, one can obtain a liquidity premium that is half as large as the transaction cost, if the ratio of financial wealth to labor income is sufficiently low in the model and one uses the transaction-cost function in (11). There are, however, two problems with setting the wealth-income ratio to be so low: first, this low wealth-income ratio is far from its empirical estimate; and second, setting the wealth-income ratio to be so low implies that the moments of asset returns are very different from those observed empirically. We conclude that if one sets the ratio of wealth to income to be close to its empirical estimate, then even in partial equilibrium the liquidity premium is much smaller in magnitude than the transaction cost.

5.6.4 Sensitivity of Results to Level of Transaction Costs

Above, we have discussed a number of models for a variety of parameter values but always for a transaction cost level of 1%. We now illustrate, in Figures 1–3, the portfolio holdings, turnovers, and liquidity premia for transaction costs ranging from 0% to 2% for our main setups.

Each plot has three lines. The blue line with no markers is for the general-equilibrium model with two investors who have relative risk aversion of $\gamma_1 = 8$ and $\gamma_2 = 14$, EIS = 1.50, and receive labor income that is deterministic. The red line with square markers is for the "base case" general-equilibrium model with two investors who have relative risk aversion of $\gamma_1 = 8$ and $\gamma_2 = 14$, EIS = 1.50, and receive labor income that is stochastic. The black line with triangle markers is for the partial-equilibrium case where there is a single investor with relative risk aversion of $\gamma = 8$ and EIS = 1.50, with prices taken from the "base case" with zero transaction cost.

[30]Note that Lynch and Tan (2011) report their results for different levels of financial wealth to *monthly* income; thus, to obtain the ratio of wealth to monthly income from our wealth-to-annual-income ratio one needs to divide the income in the denominator of this ratio by 12, which is equivalent to multiplying the ratio of wealth to annual income by 12; for example, the ratio of wealth to annual income of 5 is equivalent to a ratio of wealth to monthly income of 60. Alternatively, the monthly wealth-income ratio of 1 in Lynch and Tan is equivalent to 1/12 in annual terms.

Figure 1 has three panels showing changes in Investor 1's *relative* holdings of the bond, of Stock 1, and of Stock 2 (relative to the case of zero transaction cost). We see from the top panel of this figure that, except for the case of partial equilibrium, there is a monotonic decline in the holding of bonds as transaction costs increase. The reason for the decline in bond holdings is because as the level of transaction costs increases, the magnitude of risk-sharing between the two agents declines, and so fewer bonds are required to finance the lower volume of stock trading between the two investors. From the middle panel of Figure 2, we see that there is a substantial monotone decline in the holdings of Share 1 as the cost of trading this stock increases. The bottom panel of the figure shows the number of shares held of Stock 2: we see that for all three cases, there is first an increase in the number of shares of Stock 2 that are held, followed by a leveling off as the transaction cost increases.

The shift in demands for assets that is described in the previous paragraph is reflected in the turnovers of these assets. Figure 2 has three panels showing the average turnover for the three assets: the bond, Stock 1, and Stock 2. The top panel shows that, as the transaction cost increases from 0 to 2%, the relative turnover of the bond (relative to the case of zero transaction costs) decreases monotonically and markedly in the general-equilibrium cases, but the decrease is small for the partial-equilibrium case. The middle panel shows that the decline in the relative turnover of Stock 1, which incurs the transaction cost, is also monotonic, and much more substantial. The bottom panel shows that the turnover for Stock 2 behaves very differently in partial-equilibrium than in general equilibrium. For the case of partial equilibrium, the turnover for Stock 2 increases, but in general equilibrium with a deterministic labor income turnover is flat, while with stochastic labor income turnover declines.

Figure 3 shows the *absolute* level of the liquidity premium in the top panel, and the liquidity premium *relative* to the transaction cost in the bottom panel. From the top panel, we see that the liquidity premium is increasing and concave in transaction costs, which is reminiscent of the result in Amihud and Mendelson (1986), although in their paper the liquidity premium is plotted against the bid-ask spread rather than transaction costs. Note also that the liquidity premium is greater for the case of partial-equilibrium than for the general-equilibrium cases considered. The lower panel shows the interesting result that the ratio of the liquidity premium to the transaction cost peaks at a transaction cost of about 0.50%, and flattens out after a transaction cost of about 1%. Even at the peak, the ratio of the liquidity premium to the transaction cost is 0.34 for the case of partial equilibrium, and smaller for the general-equilibrium cases considered. The flattening out of these lines suggests that even at a higher level of transaction cost, the ratio of the liquidity premium to transaction cost is unlikely to be high.

6 Empirical Implications of the Model

An attractive feature of our framework is that the introduction of an exogenously-specified transaction cost in a general equilibrium model generates endogenous illiquidity, which is stochastic. In Section 5 above, we have examined the implications of this illiquidity for a variety of quantities in the model; in particular, we have investigated how the liquidity premium is affected by the parameters of the model, such as the degree of heterogeneity between agents, the wealth-to-income ratio, the moments of the processes for payouts and labor income, etc. In this section, we take the perspective of an econometrician. Given that the effect of liquidity on expected returns has already been studied extensively in the literature (see, for instance, Amihud and Mendelson (1986), Pástor and Stambaugh (2003), and Acharya and Pedersen (2005)), we focus on the liquidity premium: we ask what an econometrician would find if she or he were to study the relation implied by our model between the liquidity premium and various observable quantities.

The liquidity premium in our model depends on the desire of the two investors to trade and the illiquidity induced by the presence of transaction costs. If investors were homogeneous, and therefore, had no desire to trade, then the liquidity premium would be zero; on the other hand, if investors were heterogeneous because of differences in risk aversion, beliefs, or realized labor income, then they would like to trade. The decision on whether to trade an asset in a particular state of nature depends on the relative valuation of that security by the two investors. In each state of nature, after receiving labor income and payouts on their financial portfolio, each investor has a reservation price for an asset. The two investors trade an asset if the difference in their reservation prices of the asset exceeds the joint cost of trading (that is, the cost of buying and selling) that asset. In those states where the dispersion in relative reservation prices is not large enough to offset the transaction cost, there is no trade. Thus, the dispersion in reservation prices, and hence the desire to trade, increases with heterogeneity across investors.[31]

To estimate the relation between the liquidity premium and the dispersion in reservation prices one could, in principle, regress the liquidity premium on the dispersion in reservation prices, expressed in relative terms with respect to the mid-point of the reservation price. However, in the empirical data one does not observe dispersion in reservation prices for an asset. So, we utilize our model to find quantities that can be used as instrumental variables for the unobserved dispersion in reservation prices of Stock 1 across investors.

There are two groups of variables that one could use as instruments: (i) macroeconomic variables, such as aggregate consumption, volatility of consumption or consumption growth, and consumption dispersion across investors, and (ii) financial variables, such as aggregate dividends, equity risk premium, expected stock returns, volatility of stock returns, price-dividend ratio, and the risk-free rate.[32] One needs to be careful in selecting the variables for each regression in order to avoid multicollinearity issues: because many of these variables are determined endogenously in equilibrium, they can be highly correlated; this is especially true for the financial variables, and we use only one financial variables at a time in our regression tests.

We study the relation between the liquidity premium for Stock 1 and the "dispersion in the reservation price of Stock 1" based on the "cross-sectional data" across different states of nature from the solution of our model. We get this data by solving the "base case" of our model (whose solution is in GE24); in addition to this model where the transaction cost is 100 basis points, we also solve this model for transaction costs of 50, 150, and 200 basis points. Then, we collect the exogenous and endogenous variables across all states of nature at $t = 4$, to ensure that our results are not affected by the finite horizon.[33] There are

[31]The quantity "dispersion in reservation prices" in our model has the same interpretation as the σ_z variable in Vayanos and Wang (2013).

[32]The construction of most of these variables is self-explanatory from their names, except for *consumption dispersion*, which we measure as the ratio of the consumption of the first investor relative to aggregate consumption.

[33]The results of our analysis are robust to the choice of date that we choose; for instance, the results are very similar when we undertake the analysis using fewer data points from the solution at $t = 3$.

625 states for each model at $t = 4$,[34] and given that we solve the model for four levels of transaction costs, we get a total of 2500 data points.

In order to establish that the liquidity premium is indeed related to the dispersion in reservation prices, we regress cross-sectionally (where the cross-section is defined by the states of nature at a given date) the liquidity premium on the "Dispersion in reservation price of Stock 1." We find that the coefficient is positive with a p-value smaller than 0.01 and the R^2 value is 89.57%; these results are reported in Table 6, in the column labeled "Regression 1."

Next, in "Regression 2" and "Regression 3" we use a number of instrumental variables to proxy for the "Dispersion of reservation price," and we then regress the liquidity premium on the instrument. We use the standard two-stage least squares procedure for these regressions. For "Regression 2" we use three macroeconomic variables as instruments, which are aggregate consumption, volatility of consumption growth, and consumption dispersion across investors, and one financial variable — "equity risk premium of Stock 1." We see from the results reported in the column labeled "Regression 2" that the instruments work well: the coefficient on the "Dispersion in reservation price of stock 1" has the same sign and is of similar magnitude as the one in Regression 1, its p-value is still smaller than 0.01, and the R^2 value is 89.97%, slightly higher than in the original "Regression 1".[35] In "Regression 3" we use the same set of instruments as in "Regression 2," but add to the set the "level of transaction costs." Adding the level of transaction costs (though this would be difficult to observe with high precision in the empirical data) improves the fit further, and gives an R^2 value of 91.64%. The coefficient for the instrumented variable goes down from 0.11 to 0.03, but it stays significant with a p-value smaller than 0.01.

We conclude from this empirical exercise that "dispersion in reservation price" indeed contributes to explaining the liquidity premium in our model, even when estimated from a projection on macroeconomic and financial variables. It confirms the intuition of previous research (see, for example, the articles described in the review by Vayanos and Wang (2013))

[34] To see why there are 625 states at $t = 4$, note that when using the approach in He (1990) to model four stochastic processes (two for stocks and two for labor incomes), we need five nodes at each date; so, after four dates, the number of states is $5^4 = 625$.

[35] Most regressors in the first-stage regressions are significant, except for the aggregate consumption growth volatility.

that the need for liquidity and rebalancing is one of the key determinants of the cross-sectional differences in expected stock returns of stocks with varying liquidity levels. Our results also demonstrate that if one were undertaking this exercise using real data, it would be useful to know the level of transaction costs associated with each asset, though one would have to use a proxy for transaction costs such as the observable (but endogenous) bid-ask spread.

7 Conclusion

In this paper, we develop a method that allows us to obtain asset prices in a general equilibrium economy with multiple investors who are *heterogeneous* and face proportional costs for trading financial assets. The investors in our model have Epstein-Zin-Weil utility functions and can be heterogeneous with respect to their endowments, beliefs, and all three characteristics of their utility functions—time preference, risk aversion, and elasticity of intertemporal substitution. The securities traded in the financial market include a one-period discount bond and *multiple* risky stocks. Our method allows us to identify the equilibrium in a recursive fashion even in the presence of transaction costs and stochastic idiosyncratic labor income, which make markets incomplete.

We use our model to study the effect of transaction costs on the consumption and portfolio choices of investors, and the resulting impact on asset prices, equity premia, and the liquidity premium. This allows us to answer the questions we had posed at the start of the paper. We find that in general equilibrium, transaction costs have only a small effect on risk-sharing between the two investors and their ability to smooth consumption optimally over time. The only substantial effect of transaction costs is to reduce the turnover of financial assets. As a result of this less frequent rebalancing of portfolios, investing in financial assets is more risky than in the absence of transaction costs. Consequently, transaction costs also lead to a reduction in demand for the bond and the stock with the transaction cost. Because the change in net demand (across the two types of investors) for financial assets is small, transaction costs have only a small effect on the equity risk premia, volatility of stock returns, and Sharpe ratio.

The change in demand for the two stocks described above is reflected also in the liquidity premium. We find that for the version of our general-equilibrium model where the investor has only deterministic labor income, a transaction cost of 100 basis points results in a liquidity premium of only 14 basis points. If we assume that the labor income is stochastic, then the liquidity premium increases to 23 basis points, but it is still much smaller than the transaction cost. The magnitude of the liquidity premium does not depend on whether the transaction cost is treated as a deadweight cost or it is refunded to the investors at the end of each period. The liquidity premium increases from 14 basis points to about 32 basis points if one assumes a partial-equilibrium setting, where prices are fixed exogenously. However, even in partial equilibrium, to get a liquidity premium that is of the same magnitude as transaction costs, one needs to reduce substantially the ratio of financial wealth to labor income and to assume a transaction-cost function that requires an investor to finance consumption from selling financial assets while using labor income to purchase financial assets. We also analyze the empirical implications of our model and find that the dispersion in reservation prices across investors contributes to explaining the liquidity premium, even when this dispersion is instrumented by a projection on macroeconomic and financial variables. This finding confirms the intuition of previous research that the need for liquidity and rebalancing is one of the key determinants of the cross-sectional differences in expected stock returns of the stocks with varying liquidity levels.

We conclude that the insight in Constantinides (1986), that the effect of transaction costs on asset prices is small, is correct when evaluated in general-equilibrium, even in the presence of stochastic labor income that is large relative to the share of financial wealth, and this conclusion is robust to a wide set of variations of the general-equilibrium model.

A Details of the Recursive Solution Method

In this appendix, we explain more precisely the recursive solution method (dynamic program) described in the text.[36] Observe that the *unknowns* to be solved for are the choice variables consumption and investments of each investor, and the prices of the available assets. For each choice variable, there is a corresponding first-order condition, and the prices of the assets are pinned down by the market-clearing conditions. Finally, the Lagrange multiplier is identified by the budget constraint. As explained above, one can substitute out the Lagrange multiplier, so that the system of equations to be solved consists of Equations (7), (8), (9), which we reproduce below for convenience: the budget constraint for Investor k

$$\begin{aligned} 0 &= Y(k,t) + \sum_{n=0}^{N} \theta(n,k,t-1)\Big(S(n,k,t) + d(n,t)\Big) \\ &\quad - c(k,t) - \sum_{n=0}^{N} \theta(n,k,t)S(n,k,t) - \sum_{n=0}^{N} \tau\Big(\theta(n,k,t), \theta(n,k,t-1)\Big), \end{aligned} \tag{A1}$$

the first-order conditions for optimal portfolio investment

$$\begin{aligned} 0 &= \frac{\partial V(k,t)}{\partial c(k,t)} \left[S(n,k,t) + \frac{\partial \tau\Big(\theta(n,k,t), \theta(n,k,t-1)\Big)}{\partial \theta(n,k,t)} \right] \\ &\quad - E_t \left[\frac{\partial V(k,t+1)}{\partial c(k,t+1)} \left(S(n,k,t+1) + d(n,t+1) - \frac{\partial \tau\Big(\theta(n,k,t+1), \theta(n,k,t)\Big)}{\partial \theta(n,k,t)} \right) \right], \end{aligned} \tag{A2}$$

and the market clearing conditions

$$\theta^{ss}(n) = 1 = \sum_{k=1}^{K} \theta(n,k,t), \quad \forall\ n = \{1, \ldots, N\}, \tag{A3}$$

where $\theta^{ss}(n)$ is the aggregate supply of asset n, which we have assumed is equal to 1, and for the one-period risk-free bond that is in zero net supply, $\theta^{ss}(0) = 0$. In the recursive formulation, at date t we "time-shift" forward to date $t+1$ equations (A1), while leaving (A2) and (A3) as it is. Below, we describe the particular set of equations to be solved at (i) date $T-1$, (ii) the dates before the terminal date, $t < T$, and (iii) the initial date 0.

[36]We show in the next appendix that the principle of the dynamic programming applies to the problem with transaction costs, that is, the maximization goal of Investor k at time 0 is achieved if and only if the value function of the recursive problem is maximized at all times and states. In particular, we show that the first-order conditions of the dynamic program are equivalent to the first order conditions (A1) and (A2).

The set of equations are always solved for all possible combinations of the endogenous state variables. Dumas and Lyasoff (2012) show that in a general-equilibrium setting using as the endogenous state variable the consumption share of the first investor instead of individual wealth has two advantages: one, it is a variable that is bounded, and two, the recursive problem becomes path-independent, so that the decision tree, when optimizing recursively, is recombining. In our setup, because of transaction costs, the consumptions of the two investors do not add up to aggregate endowment and labor income; thus, the state variables need to consist of the consumptions of both investors. The previous period's stock holdings serve as a third endogenous state variable within the no-trade region.

At the terminal date T, there are no decisions to be made. So, the recursive scheme starts at the penultimate date, $T-1$. We solve for the optimal portfolio at that date, $\theta(n, k, T-1)$, and optimal consumption at date T, $c(k, T)$; and once we have these quantities, we use (A2) to identify the equilibrium asset prices at $T-1$, $S(n, k, T-1)$. The recursive optimization problem of each investor at $T-1$ is written as a function of the state variables, which consist of the portfolio holdings of Investor k coming into this period, $\theta(n, k, T-2)$, and the consumptions of both investors at date $T-1$, $c(1, T-1)$ and $c(2, T-1)$.

For any date $t < T$, the recursive system to be solved is the same as that described for date $T-1$: we solve for $\theta(n, k, t)$, $c(k, t+1)$, and $S(n, k, t)$ using Equation (A2) and (A3) for date t and the time-shifted Equations (A1). And, at each date t, the recursive optimization problem of each investor is written as a function of the state variables, which consist of the portfolio holdings of Investor k coming into this period, $\theta(n, k, t-1)$, and the consumptions of both investors at that date, $c(1, t)$ and $c(2, t)$.

At date $t = 0$, we solve for $\theta(n, k, 0)$, $S(n, k, 0)$, and $c(k, 1)$ using Equation (A2) and (A3) for date $t = 0$ and the time-shifted Equations (A1) for date $t = 1$. The recursive optimization problem of each investor is written as a function of the state variables, which consist of the portfolio holdings of Investor k coming into this period (the holdings with which the investor was initially endowed), which we label $\theta(n, k, -1)$, and the consumptions of both investors at that date, $c(1, 0)$ and $c(2, 0)$. This leaves us with the unknown $c(k, 0)$, which can be determined using the only set of equations we have not used so far: the budget constraints for the two investors for date $t = 0$ given in Equation (A1).

Once we have the optimal $c(k, 0)$, we can identify the solution by moving forward through the entire tree from date 0 to T using as the starting value the $c(k, 0)$ that we have determined. Thus, only a *single* forward step is required to obtain the solution.

B The Proofs for Dynamic Programming

B.1 The Derivations of the First-Order Conditions

Note, for ease of exposition we omit the subscript k for Investor k and concentrate on the case without labor income for a single asset with transaction costs and, thus, also omit the subscript n. The case with several assets subject to transaction costs follows accordingly. Below we show that the first-order conditions derived from the recursive utility function and from the indirect utility function in the dynamic programming formulation are the same.

B.1.1 Global Solution

We have to solve at *each time t simultaneously* the following problem:

$$\sup_{c(t),\theta(t)} V(t) = \sup_{c(t),\theta(t)} \left[(1-\beta)\, c(t)^{1-\frac{1}{\psi}} + \beta E_t \left[V(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}},$$

subject to for all points in time t:

$$c(t) + \theta(t) \cdot S(t) + \tau\Big(\theta(t), \theta(t-1)\Big) = \theta(t-1) \cdot \Big(S(t) + d(t)\Big). \tag{B1}$$

Form the Lagrangian with $\xi(t)$ as the multiplier for the budget constraint:

$$\begin{aligned} \mathcal{L}(t) &= \left[(1-\beta)\, c(t)^{1-\frac{1}{\psi}} + \beta E_t \left[V(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}} \\ &\quad + \xi(t) \cdot \Bigg(\theta(t-1) \cdot \Big(S(t) + d(t)\Big) - c(t) - \theta(t) \cdot S(t) - \tau\Big(\theta(t), \theta(t-1)\Big)\Bigg), \end{aligned}$$

and obtain the first-order conditions:

$$\begin{aligned} 0 = \frac{\partial \mathcal{L}(t)}{\partial c(t)} &= \frac{\phi}{1-\gamma} \cdot \left[(1-\beta)\, c(t)^{1-\frac{1}{\psi}} + \beta E_t \left[V(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}-1} \\ &\quad \times (1-\beta) \cdot \left(1 - \frac{1}{\psi}\right) \cdot c(t)^{-\frac{1}{\psi}} - \xi(t) \\ &= V(t)^{\frac{1}{\psi}} \cdot c(t)^{-\frac{1}{\psi}} \cdot (1-\beta) - \xi(t); \end{aligned}$$

$$0 = \frac{\partial \mathcal{L}(t)}{\partial \xi(t)} = \theta(t-1) \cdot \Big(S(t) + d(t)\Big) - c(t) - \theta(t) \cdot S(t) - \tau\Big(\theta(t), \theta(t-1)\Big).$$

$$
\begin{aligned}
0=\frac{\partial\mathcal{L}(t)}{\partial\theta(t)} &= \frac{\phi}{1-\gamma}\cdot\left[(1-\beta)\,c(t)^{1-\frac{1}{\psi}}+\beta E_t\left[V(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}-1}\cdot\frac{\beta}{\phi}\cdot E_t\left[V(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}-1} \\
&\quad\times(1-\gamma)\cdot E_t\left[V(t+1)^{-\gamma}\frac{\partial V(t+1)}{\partial\theta(t)}\right]-\xi(t)\cdot\left(S(t)+\frac{\tau\Big(\theta(t),\theta(t-1)\Big)}{\partial\theta(t)}\right) \\
&= V(t)^{\frac{1}{\psi}}\cdot\beta\cdot E_t\left[V(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}-1}\cdot E_t\left[V(t+1)^{-\gamma}\frac{\partial V(t+1)}{\partial\theta(t)}\right] \\
&\quad-\xi(t)\cdot\left(S(t)+\frac{\tau\Big(\theta(t),\theta(t-1)\Big)}{\partial\theta(t)}\right). \qquad \text{(B2)}
\end{aligned}
$$

Now use the result that:

$$
\begin{aligned}
\frac{\partial V(t+1)}{\partial\theta(t)} &= \frac{\partial V(t+1)}{\partial c(t+1)}\cdot\frac{\partial c(t+1)}{\partial\theta(t)} \\
&= \left[V(t+1)^{\frac{1}{\psi}}\cdot c(t+1)^{-\frac{1}{\psi}}\cdot(1-\beta)\right]\cdot\left[S(t+1)+d(t+1)-\frac{\partial\tau\Big(\theta(t+1),\theta(t)\Big)}{\partial\theta(t)}\right],
\end{aligned}
$$

and plug into (B2), to get:

$$
\begin{aligned}
&V(t)^{\frac{1}{\psi}}\cdot\beta\cdot E_t\left[V(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}-1}\cdot \\
&\qquad E_t\left[V(t+1)^{-\gamma+\frac{1}{\psi}}\cdot c(t+1)^{-\frac{1}{\psi}}\cdot(1-\beta)\cdot\left[S(t+1)+d(t+1)-\frac{\partial\tau\Big(\theta(t+1),\theta(t)\Big)}{\partial\theta(t)}\right]\right] \\
&= \xi(t)\cdot\left(S(t)+\frac{\tau\Big(\theta(t),\theta(t-1)\Big)}{\partial\theta(t)}\right).
\end{aligned}
$$

B.1.2 Dynamic Programming Solution

Define the value function recursively as:

$$
\begin{aligned}
J(T) &= \sup_{\{c(T)\}}(1-\beta)\,c(T) \\
J(t) &= \sup_{\{c(t)\}}\left[(1-\beta)\,c(t)^{1-\frac{1}{\psi}}+\beta E_t\left[J(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}},
\end{aligned}
$$

and write the problem as follows

$$
J(t)=\sup_{c(t),\theta(t)}\left[(1-\beta)\,c(t)^{1-\frac{1}{\psi}}+\beta E_t\left[J(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}},
$$

subject to (B1). The Lagrangian:

$$
\begin{aligned}
\mathcal{L}(t) &= \left[(1-\beta)\, c(t)^{1-\frac{1}{\psi}} + \beta E_t\left[J(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}} \\
&\quad + \xi(t)\cdot\left(\theta(t-1)\cdot\Big(S(t)+d(t)\Big) - c(t) - \theta(t)\cdot S(t) - \tau\Big(\theta(t),\theta(t-1)\Big)\right)
\end{aligned}
$$

with first-order conditions

$$
\begin{aligned}
0 = \frac{\partial\mathcal{L}(t)}{\partial c(t)} &= \frac{\phi}{1-\gamma}\cdot\left[(1-\beta)\, c(t)^{1-\frac{1}{\psi}} + \beta E_t\left[J(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}-1} \\
&\quad \times (1-\beta)\cdot\left(1-\frac{1}{\psi}\right)\cdot c(t)^{-\frac{1}{\psi}} - \xi(t) \\
&= J(t)^{\frac{1}{\psi}}\cdot c(t)^{-\frac{1}{\psi}}\cdot(1-\beta) - \xi(t); \\
0 = \frac{\partial\mathcal{L}_t}{\partial\xi(t)} &= \theta(t-1)\cdot\Big(S(t)+d(t)\Big) - c(t) - \theta(t)\cdot S(t); \\
0 = \frac{\partial\mathcal{L}_t}{\partial\theta(t)} &= \frac{\phi}{1-\gamma}\cdot\left[(1-\beta)\, c(t)^{1-\frac{1}{\psi}} + \beta E_t\left[J(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}-1}\cdot\frac{\beta}{\phi}\cdot E_t\left[J(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}-1} \\
&\quad \cdot(1-\gamma)\cdot E_t\left[J(t+1)^{-\gamma}\frac{\partial J(t+1)}{\partial\theta(t)}\right] - \xi(t)\cdot\left(S(t)+\frac{\tau\Big(\theta(t),\theta(t-1)\Big)}{\partial\theta(t)}\right) \\
&= J(t)^{\frac{1}{\psi}}\cdot\beta\cdot E_t\left[J(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}-1}\cdot E_t\left[J(t+1)^{-\gamma}\frac{\partial J(t+1)}{\partial\theta(t)}\right] \\
&\quad - \xi(t)\cdot\left(S(t)+\frac{\tau\Big(\theta(t),\theta(t-1)\Big)}{\partial\theta(t)}\right).
\end{aligned}
\tag{B3}
$$

The envelope theorem gives us:

$$
\frac{\partial J(t)}{\partial\theta(t-1)} = \frac{\partial\mathcal{L}(t)}{\partial\theta(t-1)} = \xi(t)\cdot\left(S(t)+d(t)-\frac{\partial\tau\Big(\theta(t),\theta(t-1)\Big)}{\partial\theta(t-1)}\right),
$$

which we can plug into (B3) to get:

$$
\begin{aligned}
& J(t)^{\frac{1}{\psi}}\cdot\beta\cdot E_t\left[J(t+1)^{1-\gamma}\right]^{\frac{1}{\phi}-1}\cdot \\
&\qquad E_t\left[J(t+1)^{-\gamma+\frac{1}{\psi}}\cdot c(t+1)^{-\frac{1}{\psi}}\cdot(1-\beta)\cdot\left[S(t+1)+d(t+1)-\frac{\partial\tau\Big(\theta(t+1),\theta(t)\Big)}{\partial\theta(t)}\right]\right] \\
= &\qquad \xi(t)\cdot\left(S(t)+\frac{\tau\Big(\theta(t),\theta(t-1)\Big)}{\partial\theta(t)}\right).
\end{aligned}
$$

The first-order conditions for the global case and the recursive case are the same, except that in the global formulation the utility function $V(t)$ shows up, and in the recursive formulation the value function $J(t)$ shows up. However, at the optimum, when we solve the global case for all t, the utility function and the value function are the same.

Table 1: Parameter Values

In this table, we report the parameters used for the quantitative assessment of the model. These parameters are for the horizon of the model, the levels of transaction costs considered, the payout processes for the two stocks, the labor-income processes, the utility function (subjective time discount, elasticity of intertemporal substitution, and risk aversion, including absolute risk aversion for the case where we consider exponential utility), and the differences in beliefs.

Parameter	Symbol	Base case value	Range considered
Horizon			
Number of periods in the model (each period is one year)	T	10	10
Parameter for transaction cost			
Rate of proportional transaction cost	κ	0.01	0.00–0.04
Parameters for the dividend processes			
Level of initial dividends for Stocks 1 and 2		0.45	0.25–0.45
Drift of dividend process for Stock 1	μ_1	0.01	0.01–0.03
Drift of dividend process for Stock 2	μ_2	0.01	0.01
Volatility of dividend process for Stock 1	σ_1	0.0705	0.0705
Volatility of dividend process for Stock 2	σ_2	0.0705	0.0705
Correlation between dividends of Stock 1 and 2	ρ	0.20	0.20
Leverage ratio (as in Abel (1999))	—	2.75	2.75
Parameters for the labor-income processes			
Level of initial labor income for Investors 1 and 2		0.55	0.55–0.75
Drift of labor income process for Investor 1	$\bar{g}_1$	0.02	0–0.02
Drift of labor income process for Investor 2	$\bar{g}_2$	0.02	0–0.02
Volatility of labor income process for Investor 1	$\sigma_{u,1}$	0.08	0.08
Volatility of labor income process for Investor 2	$\sigma_{u,2}$	0.08	0.08
Correlation between labor income process of Investors 1 and 2	—	0.0	0.0
Correlation between labor income processes and dividends	—	0.0	0.0
Parameters for the utility functions			
Time discount factor for Investors 1 and 2	β_1, β_2	0.975	0.975
Elasticity of intertemporal substitution of Investor 1	ψ_1	1.50	0.75–1.50
Elasticity of intertemporal substitution of Investor 2	ψ_2	1.50	0.75–1.50
Relative risk aversion of Investor 1	γ_1	8.00	7.25–8.00
Relative risk aversion of Investor 2	γ_2	14.00	14.00–17.00
Absolute risk aversion (for exponential utility) of Investor 1	—	8.00	8.00
Absolute risk aversion (for exponential utility) of Investor 2	—	14.00	14.00
Parameters for beliefs			
Bias in beliefs of Investor 1 about drift of dividends for both stocks	—	0.0	+0.02
Bias in beliefs of Investor 2 about drift of dividends for both stocks	—	0.0	−0.02

Table 2: Equilibrium with Transaction Costs and Stochastic Labor Income

This table reports the consumption and portfolio choices of the two agents and the resulting implications for asset returns for a variety of general-equilibrium models, which are described below. GE21: investors are *homogeneous* with relative risk aversion of $\gamma_1 = \gamma_2 = 10.18$ and zero transaction costs. GE22: investors are *heterogenous* with $\gamma_1 = 8$, $\gamma_2 = 14$ and zero transaction costs. GE23: investors are heterogenous with $\gamma_1 = 8$, $\gamma_2 = 14$ and there is a *transaction cost* of 1% (100 basis points) for trading Stock 1. GE24: investors are heterogenous with $\gamma_1 = 8$, $\gamma_2 = 14$, and labor income is now *stochastic*, but there is no transaction cost. GE25: investors are heterogenous with $\gamma_1 = 8$, $\gamma_2 = 14$, labor income is stochastic and there is a 1% transaction cost. GE26: investors are heterogenous with $\gamma_1 = 8$, $\gamma_2 = 14$, labor income is stochastic, and there is a 1% transaction cost that is *refunded* to the two agents at the end of each period.

	Variations of the Model					
Setup	GE21	GE22	GE23	GE24	GE25	GE26
RRA	10.18	$\{8, 14\}$	$\{8, 14\}$	$\{8, 14\}$	$\{8, 14\}$	$\{8, 14\}$
Labor Income	Det	Det	Det	Stoch	Stoch	Stoch
TC	0%	0%	1%	0%	1%	1% (refund)
Beliefs	Homog	Homog	Homog	Homog	Homog	Homog
Consumption gr mean, aggr	0.0186	0.0186	0.0189	0.0186	0.0193	0.0186
Consumption gr volatility, aggr	0.0246	0.0246	0.0246	0.0413	0.0413	0.0413
Risk-free rate	0.0341	0.0341	0.0342	0.0198	0.0201	0.0196
Expected return, market	0.0668	0.0668	0.0676	0.0594	0.0609	0.0604
Equity premium, market	0.0327	0.0327	0.0334	0.0396	0.0408	0.0408
Sharpe ratio, market	0.2055	0.2052	0.2098	0.2583	0.2660	0.2661
Consumption gr volatility, Investor 1	0.0246	0.0309	0.0290	0.0569	0.0561	0.0561
Consumption gr volatility, Investor 2	0.0246	0.0183	0.0204	0.0500	0.0510	0.0509
Bond investment, Investor 1,%	0.0000	-0.2665	-0.1774	-0.3206	-0.2526	-0.2523
Bond investment, Investor 2,%	0.0000	0.2691	0.1788	0.3192	0.2506	0.2504
Stock 1 investment, Investor 1,%	0.5000	0.6332	0.5395	0.6603	0.5905	0.5905
Stock 1 investment, Investor 2,%	0.5000	0.3655	0.4650	0.3404	0.4129	0.4130
Stock 2 investment, Investor 1,%	0.5000	0.6332	0.6379	0.6603	0.6620	0.6619
Stock 2 investment, Investor 2,%	0.5000	0.3655	0.3617	0.3404	0.3402	0.3403
Turnover, Bond	0.0000	0.2794	0.1870	0.3617	0.2743	0.2734
Turnover, Stock 1	0.0000	0.0288	0.0079	0.0407	0.0177	0.0177
Turnover, Stock 2	0.0000	0.0288	0.0288	0.0407	0.0407	0.0405
Expected return, Stock 1	0.0667	0.0667	0.0682	0.0592	0.0619	0.0614
Expected return, Stock 2	0.0667	0.0667	0.0668	0.0592	0.0596	0.0592
Equity premium, Stock 1	0.0326	0.0326	0.0340	0.0395	0.0418	0.0418
Equity premium, Stock 2	0.0326	0.0326	0.0326	0.0395	0.0395	0.0395
Liquidity premium	0.0000	0.0000	0.0014	0.0000	0.0023	0.0023
Volatility, Stock 1	0.1982	0.1983	0.1981	0.1920	0.1918	0.1918
Volatility, Stock 2	0.1982	0.1983	0.1984	0.1920	0.1921	0.1920
Stock return correlation	0.2875	0.2882	0.2879	0.2730	0.2723	0.2724
Sharpe ratio, Stock 1	0.1646	0.1644	0.1718	0.2057	0.2178	0.2179
Sharpe ratio, Stock 2	0.1646	0.1644	0.1643	0.2057	0.2057	0.2058
Wealth-income ratio, Investor 1	6.2346	6.3144	6.0858	6.6475	6.3221	6.3253
Wealth-income ratio, Investor 2	6.2346	6.1553	5.7761	6.5469	6.0324	6.0325

Table 3: Equilibrium with Transaction Costs and Differences in Beliefs

This table reports the consumption and portfolio choices of the two agents and the resulting implications for asset returns for four general-equilibrium models, which are described below. The first two models are from the previous table, where the two investors have homogeneous beliefs, and are reproduced here to make it easier to compare with the models with heterogeneous beliefs: GE31: investors have relative risk aversions of $\gamma_1 = 8$ and $\gamma_2 = 14$, labor income is stochastic, but there is no transaction cost and agents have homogeneous beliefs; this is the same as GE24 in Table 2. GE35: investors have relative risk aversions of $\gamma_1 = 8$ and $\gamma_2 = 14$, labor income is stochastic and there is a 1% transaction cost, but agents have homogeneous beliefs; this is the same as GE25 in Table 2. The two models with heterogeneous beliefs are given in the last two columns, with "GE33" being without transaction costs and GE34 with a 1% transaction cost for Stock 1.

	Variations of the Model			
Setup	GE31	GE32	GE33	GE34
RRA	{8,14}	{8,14}	{8,14}	{8,14}
Labor Income	Stoch	Stoch	Stoch	Stoch
TC	0%	1%	0%	1%
Beliefs	Homog	Homog	Heter	Heter
Consumption gr mean, aggr	0.0186	0.0193	0.0186	0.0252
Consumption gr volatility, aggr	0.0413	0.0413	0.0413	0.0415
Risk-free rate	0.0198	0.0201	0.0190	0.0240
Expected return, market	0.0594	0.0609	0.0515	0.0573
Equity premium, market	0.0396	0.0408	0.0325	0.0333
Sharpe ratio, market	0.2583	0.2660	0.2049	0.2084
Consumption gr volatility, Investor 1	0.0569	0.0561	0.0869	0.0855
Consumption gr volatility, Investor 2	0.0500	0.0510	0.0546	0.0540
Bond investment, Investor 1,%	-0.3206	-0.2526	-1.7828	-1.7068
Bond investment, Investor 2,%	0.3192	0.2506	1.8751	1.7920
Stock 1 investment, Investor 1,%	0.6603	0.5905	1.3914	1.2983
Stock 1 investment, Investor 2,%	0.3404	0.4129	-0.4376	-0.3393
Stock 2 investment, Investor 1,%	0.6603	0.6620	1.3914	1.4085
Stock 2 investment, Investor 2,%	0.3404	0.3402	-0.4376	-0.4544
Turnover, Bond	0.3617	0.2743	2.0404	1.9185
Turnover, Stock 1	0.0407	0.0177	0.2157	0.1690
Turnover, Stock 2	0.0407	0.0407	0.2157	0.2210
Expected return, Stock 1	0.0592	0.0619	0.0513	0.0581
Expected return, Stock 2	0.0592	0.0596	0.0513	0.0564
Equity premium, Stock 1	0.0395	0.0418	0.0323	0.0341
Equity premium, Stock 2	0.0395	0.0395	0.0323	0.0324
Liquidity premium	0.0000	0.0023	0.0000	0.0017
Volatility, Stock 1	0.1920	0.1918	0.1958	0.1993
Volatility, Stock 2	0.1920	0.1921	0.1958	0.1964
Stock return correlation	0.2730	0.2723	0.3053	0.3035
Sharpe ratio, Stock 1	0.2057	0.2178	0.1651	0.1709
Sharpe ratio, Stock 2	0.2057	0.2057	0.1651	0.1650
Wealth-income ratio, Investor 1	6.6475	6.3221	7.1260	6.2247
Wealth-income ratio, Investor 2	6.5469	6.0324	6.2617	5.9129

Table 4: Sensitivity of Results to Different Utility Functions

We report the results for four general-equilibrium (GE) models in this table, where each model is based on a different utility function. The first is with Epstein-Zin-Weil utility that is our "base case," and the other three are with power, logarithmic, and exponential utility functions. All four cases are with a transaction cost of 1% and with stochastic labor income but homogenous beliefs. GE41: the two investors are heterogenous with Epstein-Zin-Weil utility with relative risk aversions of $\text{RRA}_1 = 8$ and $\text{RRA}_2 = 14$, and elasticities of intertemporal substitution of $\text{EIS}_1 = \text{EIS}_2 = 1.50$. GE42: investors are heterogenous with power utility with $\text{RRA}_1 = 1/\text{EIS}_1 = 8$ and $\text{RRA}_2 = 1/\text{EIS}_2 = 14$. GE43: investors are homogeneous with logarithmic utility with $\text{RRA} = 1/\text{EIS} = 1$ for both agents. GE44: investors are heterogenous with exponential utility with absolute risk aversion $\text{ARA}_1 = 8$ and $\text{ARA}_2 = 14$.

	Variations of the Model			
Setup	GE41	GE42	GE43	GE44
Preferences	EZ	CRRA	LOG	CARA
Risk Aversion	{8,14}	{8,14}	{1,1}	{8,14}
Labor Income	Stoch	Stoch	Stoch	Stoch
TC	1%	1%	1%	1%
Beliefs	Homog	Homog	Homog	Homog
Consumption gr mean, aggr	0.0193	0.0192	0.0186	0.0191
Consumption gr volatility, aggr	0.0413	0.0413	0.0413	0.0413
Risk-free rate	0.0201	0.0863	0.0421	0.0938
Expected return, market	0.0609	0.1300	0.0459	0.1443
Equity premium, market	0.0408	0.0437	0.0038	0.0505
Sharpe ratio, market	0.2660	0.2629	0.0238	0.2895
Consumption gr volatility, Investor 1	0.0561	0.0567	0.0541	0.0569
Consumption gr volatility, Investor 2	0.0510	0.0509	0.0541	0.0498
Bond investment, Investor 1,%	-0.2526	-0.2763	0.0000	-0.2619
Bond investment, Investor 2,%	0.2506	0.2776	0.0000	0.2630
Stock 1 investment, Investor 1,%	0.5905	0.5982	0.5000	0.5885
Stock 1 investment, Investor 2,%	0.4129	0.4044	0.5000	0.4153
Stock 2 investment, Investor 1,%	0.6620	0.6781	0.5000	0.6734
Stock 2 investment, Investor 2,%	0.3402	0.3218	0.5000	0.3262
Turnover, Bond	0.2743	0.2428	0.0345	0.2097
Turnover, Stock 1	0.0177	0.0198	0.0000	0.0180
Turnover, Stock 2	0.0407	0.0451	0.0103	0.0438
Expected return, Stock 1	0.0619	0.1311	0.0458	0.1448
Expected return, Stock 2	0.0596	0.1287	0.0458	0.1436
Equity premium, Stock 1	0.0418	0.0448	0.0038	0.0510
Equity premium, Stock 2	0.0395	0.0423	0.0038	0.0497
Liquidity premium	0.0023	0.0024	0.0000	0.0013
Volatility, Stock 1	0.1918	0.2068	0.2000	0.2139
Volatility, Stock 2	0.1921	0.2071	0.2000	0.2140
Stock return correlation	0.2723	0.2846	0.2661	0.3250
Sharpe ratio, Stock 1	0.2178	0.2164	0.0190	0.2384
Sharpe ratio, Stock 2	0.2057	0.2043	0.0189	0.2323
Wealth-income ratio, Investor 1	6.3221	5.1369	6.3670	5.0259
Wealth-income ratio, Investor 2	6.0324	4.8670	6.3670	4.9610

Table 5: Sensitivity of Results to Parameter Values

In this table, we evaluate the sensitivity of our results to various parameter values. All the cases are for investors with homogeneous beliefs who have Epstein-Zin-Weil utility functions with heterogeneous relative risk aversion and receive stochastic labor income. The first is our "base case," and the remaining cases are with changes to various parameter values. GE51: this is our "*base case*," where investors are heterogenous with $\gamma_1 = 8$, $\gamma_2 = 14$. GE52: compared to the dispersion in risk aversion of $\gamma_1 = 8$ and $\gamma_2 = 14$ in the "base case," there is a *higher dispersion in risk aversion* with RRA $\gamma_1 = 7.25$, $\gamma_2 = 17$ (so that the harmonic mean of relative risk aversion is the same as it was in the "base case"). GE53: compared to the "base case" EIS of 1.50, there is a *lower EIS* of 0.75 for both agents. GE54: compared to the "base case" where the drift of the payout on each stock is 0.01, there is now a *higher drift of 0.03 for the payout on Stock 1*, which is the stock that incurs the transaction cost. GE55: compared to the "base case" where the share of labor income is 0.55 (and the resulting wealth-to-income ratio is about 6.4), in this case the share of labor income is 0.75 (with the resulting wealth-to-income ratio being about 2.6).

	Variations of the Model				
Setup	GE51	GE52	GE53	GE54	GE55
RRA	{8,14}	{7.25,17}	{8,14}	{8,14}	{8,14}
Labor Income	Stoch	Stoch	Stoch	Stoch	Stoch
TC	1%	1%	1%	1%	1%
Beliefs	Homog	Homog	Homog	Homog	Homog
Variation	None	High RRA	Low EIS	High Div	High Labor
	Base Case	spread	of 0.75	drift of 3%	share of 0.75
Consumption gr mean, aggr	0.0193	0.0199	0.0193	0.0285	0.0219
Consumption gr volatility, aggr	0.0413	0.0413	0.0413	0.0414	0.0475
Risk-free rate	0.0201	0.0204	0.0247	0.0266	0.0183
Expected return, market	0.0609	0.0612	0.0656	0.0696	0.0440
Equity premium, market	0.0408	0.0408	0.0409	0.0430	0.0257
Sharpe ratio, market	0.2660	0.2664	0.2657	0.2814	0.1702
Consumption gr volatility, Investor 1	0.0561	0.0582	0.0563	0.0559	0.0663
Consumption gr volatility, Investor 2	0.0510	0.0498	0.0512	0.0497	0.0656
Bond investment, Investor 1,%	-0.2526	-0.4081	-0.2570	-0.2576	-0.2421
Bond investment, Investor 2,%	0.2506	0.4040	0.2551	0.2567	0.2328
Stock 1 investment, Investor 1,%	0.5905	0.6719	0.5922	0.5957	0.5592
Stock 1 investment, Investor 2,%	0.4129	0.3316	0.4112	0.4074	0.4467
Stock 2 investment, Investor 1,%	0.6620	0.7362	0.6647	0.6620	0.6829
Stock 2 investment, Investor 2,%	0.3402	0.2672	0.3374	0.3396	0.3251
Turnover, Bond	0.2743	0.4427	0.2758	0.2984	0.1459
Turnover, Stock 1	0.0177	0.0339	0.0180	0.0189	0.0095
Turnover, Stock 2	0.0407	0.0555	0.0414	0.0405	0.0465
Expected return, Stock 1	0.0619	0.0626	0.0666	0.0706	0.0451
Expected return, Stock 2	0.0596	0.0596	0.0643	0.0684	0.0428
Equity premium, Stock 1	0.0418	0.0422	0.0419	0.0440	0.0268
Equity premium, Stock 2	0.0395	0.0392	0.0397	0.0417	0.0245
Liquidity premium	0.0023	0.0030	0.0023	0.0022	0.0023
Volatility, Stock 1	0.1918	0.1917	0.1928	0.1916	0.1909
Volatility, Stock 2	0.1921	0.1922	0.1930	0.1919	0.1911
Stock return correlation	0.2723	0.2721	0.2727	0.2666	0.2495
Sharpe ratio, Stock 1	0.2178	0.2201	0.2176	0.2294	0.1405
Sharpe ratio, Stock 2	0.2057	0.2040	0.2055	0.2176	0.1281
Wealth-income ratio, Investor 1	6.6244	6.3185	6.2230	6.9309	2.6536
Wealth-income ratio, Investor 2	6.5537	6.0042	5.9387	6.6019	2.6094

Table 6: Liquidity Premium and Dispersion in Reservation Price of Stock 1

In this table, we study the relation between the liquidity premium and the dispersion in the reservation price of Stock 1, which is the stock that incurs a transaction cost. In Regression 1, we regress cross-sectionally (where the cross-section is defined by the states of nature at a given date) the liquidity premium on the dispersion in the reservation price of Stock 1. In "Regression 2" and "Regression 3" we use a number of instrumental variables to proxy for the "dispersion of reservation price," and regress the liquidity premium on the instrumented value. We use the standard two-stage least squares procedure to conduct these two regressions. For "Regression 2," we use three macroeconomic variables, namely, aggregate consumption, volatility of consumption growth, and consumption dispersion across investors, and one financial variable—"equity risk premium of stock 1". For "Regression 3," we use the same set of instruments as in "Regression 2," but add to the set the "level of transaction costs."

Coefficient	Regression 1	Regression 2	Regression 3
		Instrumented Variable	
Dispersion in reservation price of stock 1	0.1143	0.1103	0.0306
(p-value)	0.0000	0.0000	0.0000
		Instrumental Variables	
Intercept	0.0006	-0.0008	-0.0074
(p-value)	0.0000	0.0000	0.0000
Level of transaction costs	–	–	0.1037
(p-value)	–	–	0.0000
Consumption, aggr	–	0.0002	0.0002
(p-value)	–	0.0000	0.0000
Consumption gr volatility, aggr	–	0.0001	0.0003
(p-value)	–	0.2745	0.0005
Equity premium of stock 1	–	0.0166	0.0487
(p-value)	–	0.0000	0.0000
Consumption dispersion	–	0.0007	0.0111
(p-value)	–	0.0067	0.0000
$\bar{R}^2, \%$	89.57	89.97	91.64

Figure 1: Investor 1's Holdings of the Bond and the Two Stocks at $t = 0$

The three panels of this figure show the *change* in Investor 1's holding of Stock 1, Stock 2, and the bond at $t = 0$, as the transaction cost changes from 0% to 2%. Each plot has three lines. The blue line with no markers is for the general-equilibrium model with two investors who have relative risk aversions of $\gamma_1 = 8$ and $\gamma_2 = 14$, EIS $= 1.50$, and receive labor income that is deterministic. The red line with square markers is for the "base case" general-equilibrium model with two investors who have relative risk aversions of $\gamma_1 = 8$ and $\gamma_2 = 14$, EIS $= 1.50$, and receive labor income that is stochastic. The black line with triangle markers is for the partial-equilibrium case where there is a single investor with relative risk aversion of $\gamma = 8$ and EIS $= 1.50$, with prices taken from the "base case" with zero transaction cost.

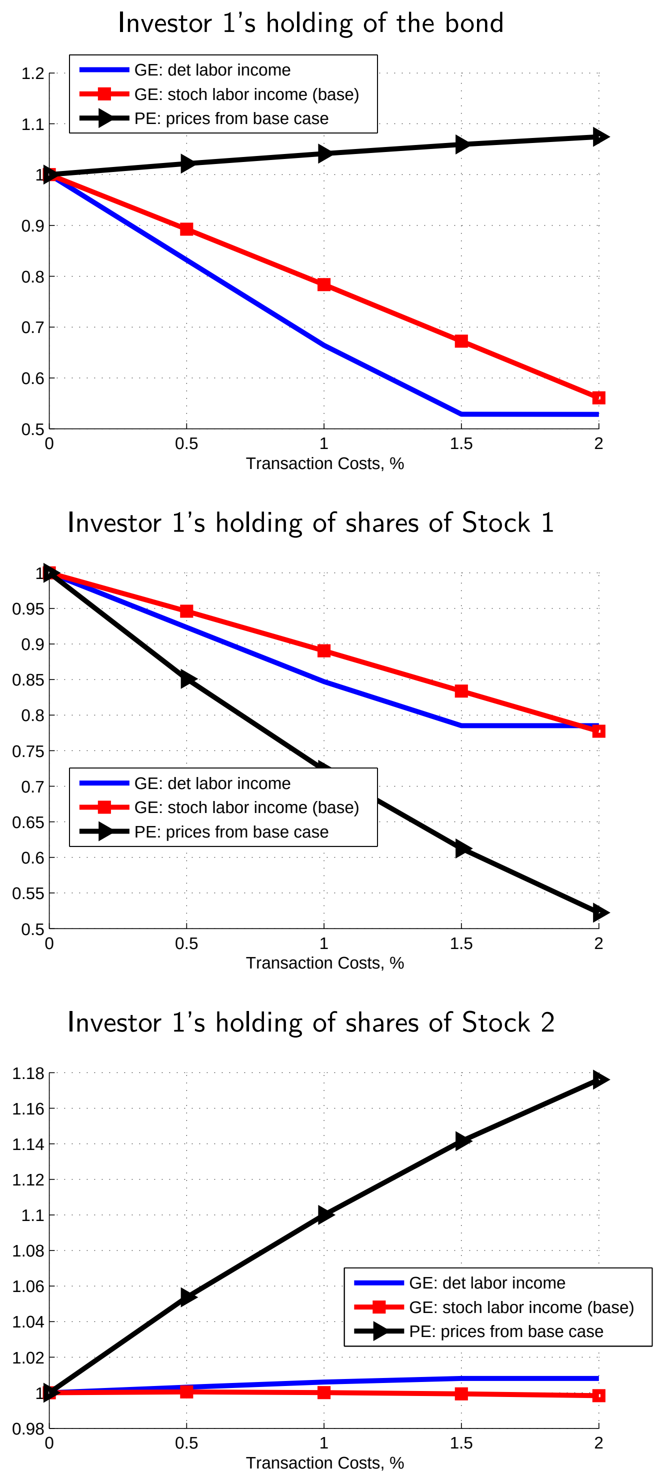


Figure 2: Average Turnover of Bond and Stocks

The three panels of this figure show the *change* in the turnovers of the bond, Stock 1, and Stock 2, averaged over the dates $t = 0$ to $t = 5$, as the transaction cost changes from 0% to 2%. Each plot has three lines. The blue line with no markers is for the general-equilibrium model with two investors who have relative risk aversions of $\gamma_1 = 8$ and $\gamma_2 = 14$, EIS = 1.50, and receive labor income that is deterministic. The red line with square markers is for the "base case" general-equilibrium model with two investors who have relative risk aversions of $\gamma_1 = 8$ and $\gamma_2 = 14$, EIS = 1.50, and receive labor income that is stochastic. The black line with triangle markers is for the partial-equilibrium case where there is a single investor with relative risk aversion of $\gamma = 8$ and EIS = 1.50, with prices taken from the "base case" with zero transaction cost.

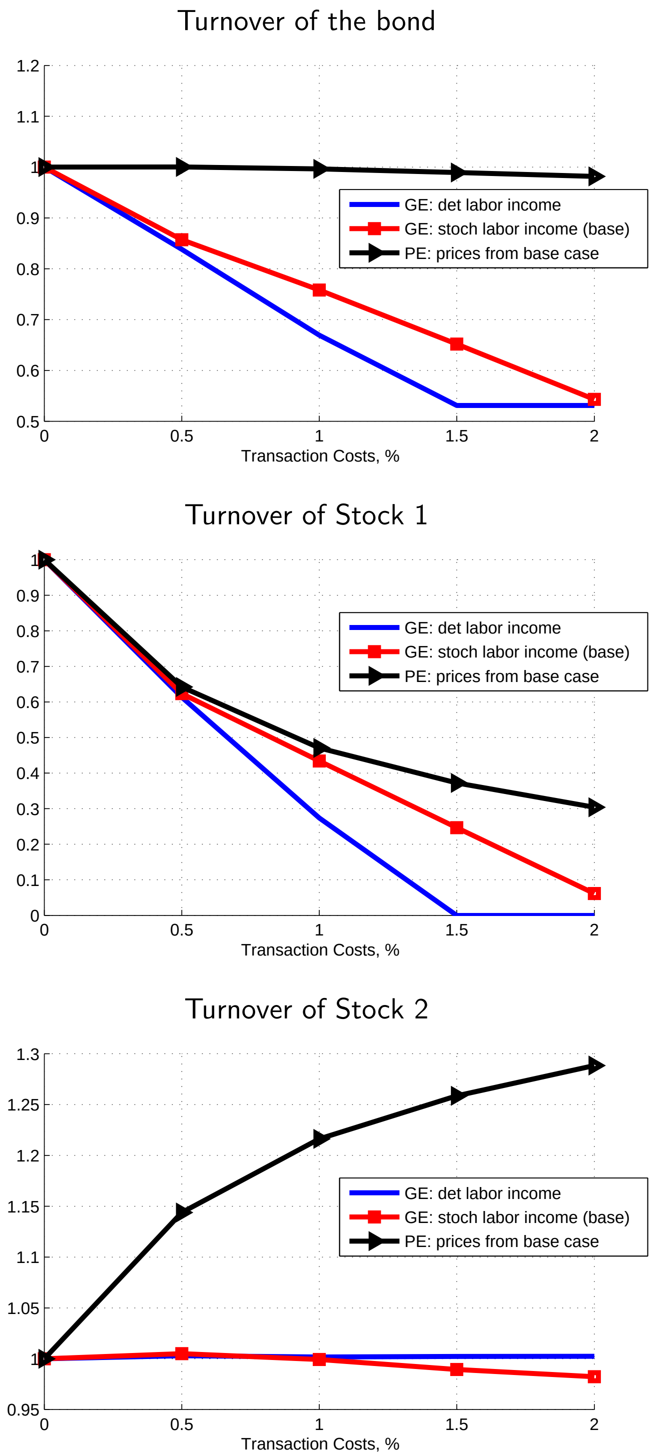
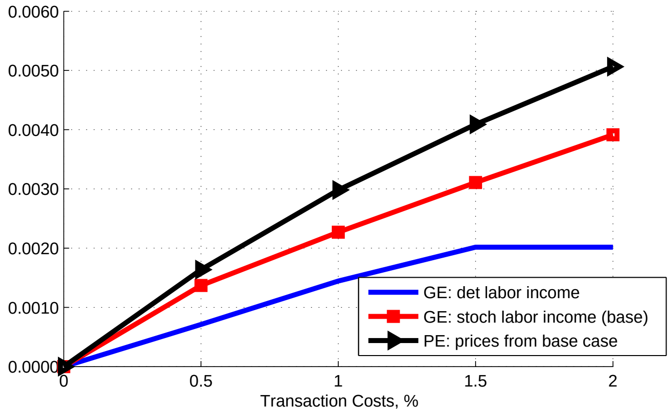


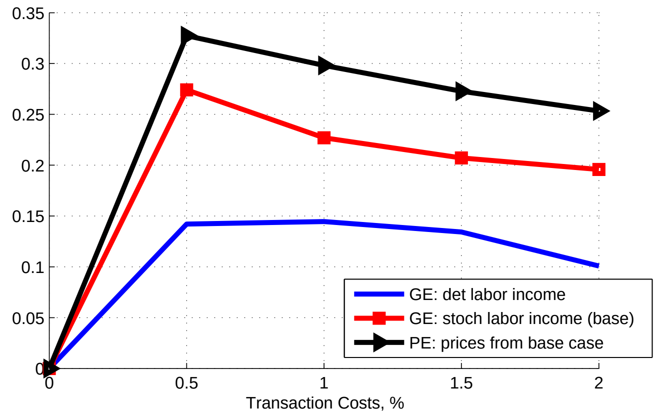
Figure 3: Liquidity Premium

The top panel of this figure shows the *absolute* level of the liquidity premium as the transaction cost ranges from 0% to 2%. The bottom panel shows the level of the liquidity premium *relative* to the level of the transaction cost. Each plot has three lines. The blue line with no markers is for the general-equilibrium model with two investors who have relative risk aversions of $\gamma_1 = 8$ and $\gamma_2 = 14$, EIS $= 1.50$, and receive labor income that is deterministic. The red line with square markers is for the "base case" general-equilibrium model with two investors who have relative risk aversions of $\gamma_1 = 8$ and $\gamma_2 = 14$, EIS $= 1.50$, and receive labor income that is stochastic. The black line with triangle markers is for the partial-equilibrium case where there is a single investor with relative risk aversion of $\gamma = 8$ and EIS $= 1.50$, with prices taken from the "base case" with zero transaction cost.

The liquidity premium in *absolute* terms



The liquidity premium *relative* to the transaction cost



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