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Assignment of Eigenvalues in a Disc D (c, r) of Complex Plane with Application of the Gerschgorin Theorem

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Abstract: This paper is concerned with the problem of designing discrete-time control systems with closed-loop eigenvalues in a prescribed region of stability. First, we obtain a state feedback matrix which assigns all the eigenvalues to zero and then by elementary similarity operations and using the Gerschgorin theorem we find a state feedback which assigns the eigenvalues inside a circle with center c and radius r. This new algorithm can also be used for the placement of closed-loop eigenvalues in a specified disc in z-plane and can be employed for large-scale discrete-time linear control systems. Some illustrative examples are presented to show the advantages of this new technique.

Key words: Discrete-time control systems . state feedback matrix . localization of eigenvalues . disc . large-scale systems . gerschgorin theorem

INTRODUCTION

In many applications, mere stability of the controlled object is not enough and it is required that the poles of the closed-loop system should lie in a certain restricted region of stability. Several design methods have been reported which utilize the LQ technique to achieve the desired pole allocation Amin [1] derived an improved result in which the optimality of the closed-loop system is assured. Furuta and Kim [2] obtained a method for assigning the closed loop poles in a specified disk based on gain and phase margins which is named -stability margin. They considered the case, when the perturbations are unknown gains as a diagonal form. Yuan and Achenie and Jiang [3] addressed the problem of linear quadratic regulator (LQR) synthesis with regional closed-loop pole constraints. Figueroa and Romagnoli [4] presented a method for designing controllers which attempt to place the roots of a characteristic polynomial of an uncertain system inside some prescribed regions. The analysis is based on the transfer function of a characteristic polynomial. Chou [5] described another pole assignment method with a spectral radius and proposed a pulse transfer function. The procedure is simple, but it is used only for checking the positions of closed loop poles, not for designing the controller. Benner *et al*. [6] presented the method for partial stabilization of large-scale discrete-time linear control systems. Recently, Grammont and Largillier [7] employed an approach to localize matrix eigenvalues in the sense that they build a sufficiently small neighborhood for each eigenvalue (or for a cluster).

A well-known desired region for discrete systems is a disc D (c, r) centered at (c, 0) with radius *r*, in which $|c|+r<1$, as shown in Fig. 1. In this paper, the aim is to present a method for localization of eigenvalues in small specified region of complex plane by state feedback control for large-scale discrete-time linear control systems.

PROBLEM STATEMENT

The problem of localization of eigenvalues in a small specified region has been the subject of many investigators in the last decade [6, 7]. Consider a controllable linear time-invariant system defined by the state equation

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

or its discrete-time version

$$x(t+1) = Ax(t) + Bu(t) \quad (2)$$

where $x(t)\in\Re^n$, $u(t)\in\Re^m$ and the matrices A and B are real constant matrices of dimensions $n\times n$ and $n\times m$ respectively, with rank (B) = m. The aim of eigenvalue assignment in a specified region is to design a state feedback controller, K, producing a closed-loop system with a satisfactory response by shifting controllable

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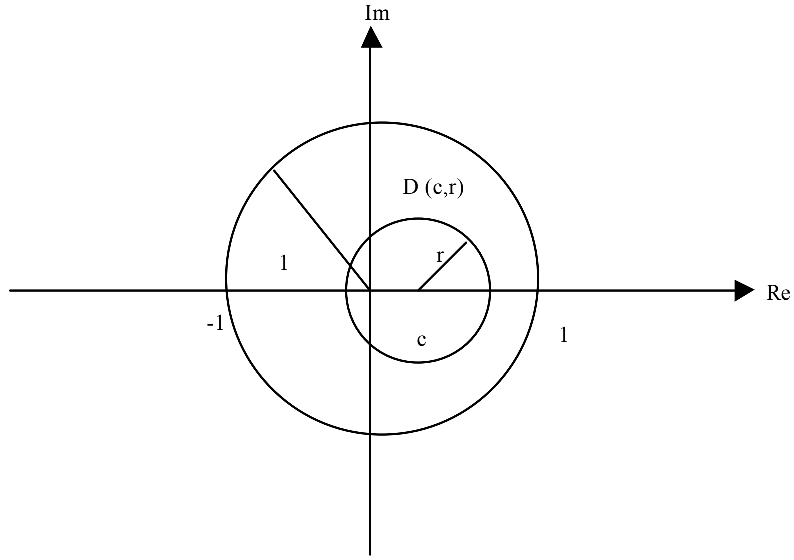

Fig. 1: A specified disc D (c, r)

poles from undesirable to desirable locations. Karbassi and Bell [8, 9], have introduced an algorithm for obtaining an explicit parametric controller matrix K by performing similarity operations on the controllable pair (B, A). In fact, K is chosen such that the closed-loop system eigenvalues

$$\Gamma = A + BK \tag{3}$$

lie in the self conjugate eigenvalue spectrum $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$. Recently, Karbassi and Tehrani [10] extended the previous results as to obtain an explicit formula involving nonlinear parameters in the control law. The stabilization problem consists in finding a feedback matrix $K \in \Re^{m\times n}$ such that the input $u_k = Kx_k$, k = 0, 1, 2,… yields a stable closed loop system

$$x_{k+1} = (A + BK)x_k = \Gamma x_k, \quad k = 0,1,2,... \tag{4}$$

In case the spectrum (or set of eigenvalues) of the closed-loop matrix, denoted by $\Lambda(\Gamma)$, is contained in the open unit disk we say that Γ is (Schur) stable or convergent (in other words, $|\lambda_i|<1$ for all $\lambda_i \in \Lambda(\Gamma)$). The stabilization problem arises in control problem such as, the computation of an initial approximate solution in Newton's method for solving discrete-time algebraic Riccati equations, simple synthesis methods to design controllers. Large-scale problems occur whenever the linear system results from some sort of a partial differential equation or from delay systems. There, the number of states is often a couple of thousands.

The stabilization problem can in principal be solved as a pole assignment problem. Pole assignment methods compute a feedback matrix such that the closed-loop matrix of system (3) has a specified spectrum. In this paper, we present an efficient approach for localization of eigenvalues in small specified regions for large-scale linear discrete-time systems. Our assignment procedure is composed of two stages. We first obtain a primary state feedback matrix F_p which assigns all the eigenvalues of closed-loop system to zero, then produce a state feedback matrix K which assigns all the closed-loop system eigenvalues in a small specified disc or discs.

SYNTHESIS

Consider the state transformation

$$x(t) = T\,\tilde{x}(t) \tag{5}$$

where T can be obtained by elementary similarity operations as described in [8]. In this way, $\tilde{A} = T^{-1}AT$ and $\tilde{B} = T^{-1}B$ are in a compact canonical form known as vector companion form:

$$\tilde{A} = \begin{bmatrix} G_0 \\ \hline I_{n-m} \;\vdots\; 0_{n-m\times m} \end{bmatrix} \tilde{B} = \begin{bmatrix} B_0 \\ \hline 0_{n-m\times m} \end{bmatrix} \tag{6}$$

Here G_0 is an m×n matrix and B_0 is an m×m upper triangular matrix. Note that if the Kronecker invariants of the pair (B, A) are regular, then $\tilde{A}$ and $\tilde{B}$ are always in the above form [8]. In the case of irregular Kronecker invariants, some rows of I_{n-m} in $\tilde{A}$ are displaced [9]. It may also be concluded that if the vector companion form of $\tilde{A}$ obtained from similarity operations has the above structure, then the Kronecker invariants associated with the pair (B, A) are regular [8].

The state feedback matrix which assigns all the eigenvalues to zero, for the transformed pair$(\tilde{B},\tilde{A})$, is then chosen as

$$u = -B_0^{-1}G_0\tilde{x} = \tilde{F}\tilde{x} \tag{7}$$

which results in the primary state feedback matrix for the pair (B, A) defined as

$$F_p = \tilde{F}T^{-1} \tag{8}$$

The transformed closed-loop matrix $\tilde{\Gamma}_0 = \tilde{A} + \tilde{B}\tilde{F}$ assumes a compact Jordan form with zero eigenvalues

$$\tilde{\Gamma}_0 = \left[\begin{array}{c} 0_{m\times n} \\ \hline I_{n-m} \;\vdots\; 0_{n-m\times m} \end{array}\right] \tag{9}$$

Then consider the n×n matrix H with form:

$$H = \begin{bmatrix} H_{11} & H_{12} & H_{13} & \cdots & H_{1r} & H_{1r+1} \\ H_{21} & H_{22} & 0 & \cdots & 0 & 0 \\ 0 & H_{32} & H_{33} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & H_{r\,r-1} & H_{rr} & 0 \\ 0 & 0 & 0 & 0 & H_{r+1r}\; 0_{s\times m-s} & H_{r+1r+1} \end{bmatrix} \tag{10}$$

where H_{1j}, j = 1,…,r are m×m matrices and H_{1r+1} is an m×s matrix and H_{ij}, i = 2,…, r, j = 1,…, r and H_{ii}, i = 2,…r are diagonal m×m matrices and H_{r+1r}, H_{r+1r+1} are diagonal s×s matrices, i.e,

$$H_{1J} = \begin{bmatrix} h_{1,(j-1)m+1} & h_{1,(j-1)m+2} & \cdots & h_{1,jm} \\ h_{2,(j-1)m+1} & h_{2,(j-1)m+2} & \cdots & h_{2,jm} \\ \vdots & \vdots & \vdots & \vdots \\ h_{m,(j-1)m+1} & h_{m,(j-1)m+2} & \cdots & h_{m,jm} \end{bmatrix} \quad j = 1,\ldots,r \tag{11}$$

$$H_{1r+1} = \begin{bmatrix} h_{1,rm+1} & h_{1,rm+2} & \cdots & h_{1,rm+s} \\ h_{2,rm+1} & h_{2,rm+2} & \cdots & h_{2,rm+s} \\ \vdots & \vdots & \vdots & \vdots \\ h_{m,rm+1} & h_{m,rm+2} & \cdots & h_{m,rm+s} \end{bmatrix} \tag{12}$$

$$H_{ii} = \begin{bmatrix} h_{(i-1)m+1,(i-1)m+1} & 0 & \cdots & 0 \\ 0 & h_{(i-1)m+2,(i-1)m+2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{im,im} \end{bmatrix} \quad i = 2,\ldots,r \tag{13}$$

$$H_{r+1,r+1} = \begin{bmatrix} h_{rm+1,rm+1} & 0 & \cdots & 0 \\ 0 & h_{rm+2,rm+2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{rm+s,rm+s} \end{bmatrix} \tag{14}$$

$$H_{i,i+1} = \begin{bmatrix} h_{(i-1)m+1,(i-2)m+1} & 0 & \cdots & 0 \\ 0 & h_{(i-1)m+2,(i-2)m+2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{im,(i-1)m} \end{bmatrix} \quad i = 2,\ldots,r \tag{15}$$

$$H_{r+1,r} = \begin{bmatrix} h_{rm+1,(r-1)m+1} & 0 & \cdots & 0 \\ 0 & h_{rm+2,(r-1)m+2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{rm+s,(r-1)m+s} \end{bmatrix} \tag{16}$$

We chose the elements of principal diameter H_{11} and H_{ii}, i = 2,3,…,r+1inside interval c – r, c+r) by:

$$h_{ii} = \pm r{*}\text{random}(0,1) + c \qquad i = 1,\ldots,n \tag{17}$$

where $h_{ii}\neq 0$ and chose the elements $H_{i,i-1}$, i = 2,…,r+1 by :

$$h_{m+i,i} = \frac{1}{\alpha}\{\min(\left\| c-r \right| - \left| h_{ii} \right\|, \left\| c+r \right| - \left| h_{ii} \right\|)\} \quad i = 1,\ldots,n-m \tag{18}$$

where h_{ii} is the corresponding diameter element of column ith of matrix H and α is arbitrary such that $|1/\alpha|<1$ and $h_{m+1,i}\neq 0$.

Now we take the nonzero elements of remainder H_{1i}, i = 2,…,r+1 such that:

$$\sum_{\substack{i=1 \\ i\neq j}}^{n} \left| h_{ij} \right| \leq \min(\left\| c-r \right| - \left| h_{jj} \right\|, \left\| c+r \right| - \left| h_{jj} \right\|) \tag{19}$$

Then $\tilde{H}$ can be obtained from H by performing elementary similarity operations [7] on H. The matrix $\tilde{H}$ thus obtained will be in primary vector companion form such that

$$\tilde{H} = \begin{bmatrix} H_0 \\ I_{n-m} \;,\; 0_{n-m,m} \end{bmatrix} \tag{20}$$

where H_0 is an m×n matrix.

Because of similarity operation, the eigenvalues of the matrix $\tilde{H}$ are the same as the eigenvalues of H. Now the feedback matrix of the pair $(\tilde{A},\tilde{B})$ is defined by:

$$\tilde{K} = \tilde{F} + B_0^{-1}H_0 = B_0^{-1}(-G_0 + H_0) \tag{21}$$

Theorem: The state feedback matrix $\tilde{K}$ assigns the eigenvalues of closed-loop matrix $\tilde{\Gamma} = \tilde{A} + \tilde{B}\tilde{K}$ inside a circle with center c and radius r.

Proof: Let

$$\tilde{\Gamma} = \tilde{A} + \tilde{B}\tilde{K} = \begin{bmatrix} & G_0 & \\ I_{n-m} & & 0_{n-m,m} \end{bmatrix} + \begin{bmatrix} B_0 \\ 0_{n-m,m} \end{bmatrix} \begin{bmatrix} B_0^{-1}(-G_0 + H_0) \end{bmatrix} \quad (22)$$

$$\tilde{\Gamma} = \begin{bmatrix} & G_0 - B_0 B_0^{-1} G_0 + B_0 B_0^{-1} H_0 & \\ I_{n-m} & & 0_{n-m,m} \end{bmatrix} = \begin{bmatrix} & H_0 & \\ I_{n-m} & & 0_{n-m,m} \end{bmatrix} \quad (23)$$

Clearly $\tilde{\Gamma} = \tilde{H}$, since $\tilde{H}$ is similar to the matrix H and the eigenvalues of matrix $\tilde{H}$ are the same as that of matrix H and elementary similarity operations do not change the eigenvalues, according to the Gerschgorin theorem in the columns matrix H, we define C_j, j = 1,…,n a circle with center h_{jj} and radius

$$r_j = \sum_{\substack{i=1 \\ i \neq j}}^{n} \left| h_{ij} \right|$$

The definition the matrix H involve that all C_j, j = 1,…,n fall inside a circle with center c and radius r, thus $\bigcup_{j=1}^{n} C_j$ is inside the disc D (c, r). Therefore the eigenvalues of matrix H are inside he disc D (c, r). Then the eigenvalues of closed-loop matrix $\tilde{\Gamma} = \tilde{A} + \tilde{B}\tilde{K}$ also fall inside a circle with center c and radius r.

Remark: Since $\tilde{K}$ assigns the eigenvalues of the closed-loop matrix $\tilde{\Gamma} = \tilde{A} + \tilde{B}\tilde{K}$ inside a circle with center c and radius r it is obvious that the state feedback controller matrix, $K = \tilde{K}T^{-1} = B_0^{-1}(-G_0 + H_0)T^{-1}$ also assigns the eigenvalues of the closed-loop matrix Γ = A+BK inside a circle with center c and radius r too.

An algorithm for assignment of eigenvalues in a disc D (c, r)

In this section we first give an algorithm for finding a state feedback matrix which assigns zero eigenvalues to the closed-loop system. Then we determine a gain matrix which assigns the closed-loop eigenvalues in a circle with real center c and radius r.

Input: The controllable pair (A, B), the primary state feedback F_p, B_0^{-1} and T^{-1} which are calculated by the algorithm proposed by Karbassi and Bell [8, 9], the real valued center c and radius r of the target circle.

Step 1: Construct the block diagonal matrix H in the form (10), for assigning eigenvalues in a circle with the real valued circle c and radius r we choose $h_{ii} = \pm r * \text{random}(0,1) + c$ for i = 1, 2,…,n where $h_{ii} \neq 0$ i = 1,…,n

$$h_{m+i,i} = \frac{1}{\alpha}\{\min(\left|\left|c-r\right| - \left|h_{ii}\right|\right|, \left|\left|c+r\right| - \left|h_{ii}\right|\right|)\}$$

for i = 1,2,…,n-m where $h_{m+1,i} \neq 0$ and $|1/\alpha| < 1$

We choose the nonzero elements of column jth, j = 1,…,n such that:

$$\sum_{\substack{i=1 \\ i \neq j}}^{n} \left|h_{ij}\right| \leq \min(\left|\left|c-r\right| - \left|h_{jj}\right|\right|, \left|\left|c+r\right| - \left|h_{jj}\right|\right|) \quad j = 1,\ldots,n$$

Step 2: Transform H to primary vector companion form $\tilde{H}$ as in (20) using elementary similarity operations as specified in [8].

Step 3: Now compute $K = F_p + B_0^{-1} H_0 T^{-1}$ the required state feedback matrix

ILLUSTRATIVE EXAMPLES

Example 1: Consider a discrete-time system given by

x (t+1) = Ax(t)+Bu(t)

Where A and B are random matrices as follows:

$$A = \begin{bmatrix} 9 & 8 & 8 & 9 \\ 2 & 7 & 4 & 7 \\ 6 & 4 & 6 & 1 \\ 4 & 0 & 7 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 9 & 0 \\ 9 & 3 \\ 4 & 8 \\ 8 & 0 \end{bmatrix}$$

The open-loop eigenvalues of the system are found to be as follows: {21.4707, 1.1178±4.5139i, 2.2938}

It is desired to obtain a state feedback controller witch assigns the closed-loop eigenvalues in the disc D (-0.2, 0.3), We perform the above algorithm with α = 10. First, the primary state feedback matrix which locates all the eigenvalues of the closed-loop system to the origin of the complex plane is found to be:

$$F_p = \begin{bmatrix} -0.1607 & -0.4627 & -0.4682 & -0.7418 \\ -2.0522 & -0.9423 & -1.2194 & -0.1640 \end{bmatrix}$$

By using the algorithm, the state feedback matrix we obtain is:

$$K = \begin{bmatrix} -0.1589 & -0.4784 & -0.4611 & -0.7500 \\ -2.1438 & -0.8449 & -1.3188 & -0.1171 \end{bmatrix}$$

The closed-loop eigenvalues are: {-0.0021, -0.2561±0.0092i,-0.1512}, all of which are inside the disc D (-0.2, 0.3).

Example 2: Consider a large discrete-time system given by x (t+1) = Ax(t)+Bu(t)
where A and B are randomly generated with n = 10 and m = 6.

$$A = \begin{bmatrix} 9 & 6 & 0 & 0 & 8 & 1 & 4 & 7 & 7 & 1 \\ 2 & 7 & 3 & 7 & 0 & 6 & 8 & 3 & 9 & 0 \\ 6 & 9 & 8 & 4 & 6 & 3 & 8 & 8 & 5 & 8 \\ 4 & 7 & 0 & 9 & 3 & 5 & 6 & 5 & 8 & 1 \\ 8 & 1 & 1 & 4 & 8 & 1 & 8 & 3 & 1 & 2 \\ 7 & 4 & 2 & 4 & 5 & 6 & 6 & 7 & 9 & 6 \\ 4 & 9 & 1 & 8 & 7 & 3 & 3 & 5 & 2 & 2 \\ 0 & 9 & 6 & 5 & 4 & 8 & 2 & 4 & 2 & 4 \\ 8 & 4 & 2 & 2 & 3 & 8 & 3 & 6 & 8 & 0 \\ 4 & 8 & 1 & 6 & 1 & 5 & 5 & 6 & 7 & 9 \end{bmatrix} B = \begin{bmatrix} 5 & 2 & 4 & 2 & 6 & 4 \\ 4 & 3 & 3 & 6 & 2 & 0 \\ 5 & 7 & 8 & 3 & 8 & 0 \\ 3 & 6 & 0 & 9 & 6 & 3 \\ 4 & 4 & 7 & 7 & 1 & 0 \\ 2 & 5 & 9 & 4 & 2 & 3 \\ 5 & 7 & 9 & 7 & 6 & 6 \\ 7 & 0 & 7 & 2 & 6 & 0 \\ 5 & 6 & 4 & 4 & 3 & 0 \\ 6 & 0 & 4 & 9 & 5 & 6 \end{bmatrix}$$

The open loop eigenvalues are: {7.6677±5.2510i, 3.9208±0.6461i, -2.2069±1.6298i, -1.2318, -3.9311, 10.9532, 46.4465}

which are widely spread in the complex plane. In order to locate them in small discs inside the unit circle, we employ the above algorithm step by step and chose $\alpha = 100$. First, the primary state feedback matrix which locates all the eigenvalues of the closed-loop system to the origin of the complex plane is found to be:

$$F_p = \begin{bmatrix} 1.4865 & -0.5547 & -0.4419 & 0.3630 & 1.2907 & 0.9842 & 1.7726 & 1.2960 & 3.1981 & 0.5510 \\ -0.9816 & 0.7597 & 1.4466 & -0.0136 & -2.5560 & 1.4522 & 0.4522 & 0.8020 & 1.7824 & 3.5008 \\ -2.1839 & 0.5442 & 0.1709 & 0.3888 & -1.8402 & -0.6415 & -1.3561 & -1.2853 & -1.8854 & -0.6550 \\ 3.0370 & -2.7784 & -3.4762 & -1.8626 & 7.3590 & -5.8022 & -0.3900 & -1.8946 & -5.3964 & -6.6379 \\ -2.3524 & 0.6276 & 1.4215 & 0.1697 & -5.2022 & 3.6383 & -2.0661 & 0.1840 & 1.4889 & 2.7943 \\ 0.6951 & -0.5421 & 0.4148 & -0.3413 & 1.4439 & -2.3124 & 1.8808 & -0.5139 & -1.3333 & -0.4415 \end{bmatrix}$$

Now we consider the following different cases:

a) It is desired to locate the closed-loop eigenvalues inside the unit circle centered at origin. By using the algorithm, the state feedback matrix we obtain would be:

$$K = \begin{bmatrix} 1.3677 & -0.5498 & -0.4314 & 0.3507 & 1.0324 & 0.9896 & 1.8708 & 1.3300 & 3.3051 & 0.7243 \\ -0.9863 & 0.7427 & 1.5808 & 0.0161 & -2.6969 & 1.6707 & 0.1811 & 0.8594 & 1.5988 & 3.7565 \\ -2.0035 & 0.4943 & 0.1204 & 0.4396 & -1.7066 & -0.6107 & -1.3921 & -1.2272 & -1.9633 & -0.7423 \\ 2.8519 & -2.8267 & -3.5311 & -1.9467 & 7.1805 & -5.6905 & -0.3573 & -1.8914 & -5.1682 & -6.5178 \\ -2.1349 & 0.6142 & 1.3835 & 0.2780 & -5.0338 & 3.5955 & -2.0849 & 0.2157 & 1.3051 & 2.6373 \\ 0.7983 & -0.4743 & 0.5137 & -0.3688 & 1.6459 & -2.4838 & 1.8728 & -0.5620 & -1.4714 & -0.5170 \end{bmatrix}$$

The closed-loop eigenvalues are {-0.0464±0.2182i, 0.8747, -0.9505, 0.6838, -0.7770, 0.6368, 0.4227, 0.1476, -0.1156} clearly all are inside the unit circle.

b) In this case, we find the state feedback matrix which assigns the closed-loop eigenvalues in the disc D (0.6, 0.2). By using the algorithm, the state feedback matrix we obtain would be:

$$K = \begin{bmatrix} 1.4720 & -0.5420 & -0.4008 & 0.1275 & 1.3466 & 0.4759 & 2.1575 & 1.2437 & 3.3304 & 0.6142 \\ -0.7866 & 0.7387 & 1.3256 & 0.0501 & -2.3021 & 1.2374 & 0.6798 & 0.7984 & 1.7609 & 3.1993 \\ -2.0614 & 0.4590 & 0.0972 & 0.4599 & -1.7638 & -0.2516 & -1.6376 & -1.2082 & -1.9089 & -0.6898 \\ 2.5381 & -2.6416 & -3.2510 & -1.9229 & 6.8329 & -5.4821 & -0.5860 & -1.9354 & -5.1848 & -6.1922 \\ -2.0237 & 0.6094 & 1.3652 & 0.3408 & -4.9501 & 3.5863 & -2.1053 & 0.2323 & 1.2055 & 2.5557 \\ 0.6475 & -0.5810 & 0.3595 & -0.3310 & 1.3399 & -2.2427 & 1.9033 & -0.4861 & -1.2585 & -0.4023 \end{bmatrix}$$

The closed-loop eigenvalues are {0.6555±0.0060i, 0.5151±0.0232i, 0.795, 0.4088, 0.6337, 0.7569, 0.5538, 0.4122} all of which are inside the disc D (0.6, 0.2).

CONCLUSION

A simple algorithm has been given for localization of eigenvalues in small specified regions of complex plane by state feedback control. This method has been achieved by implementing properties of vector companion forms. The merit of this approach is that it can be achieved by elementary similarity operations and the Gereschgorin theorem which is significantly simpler to realize computationally than the existing methods. Also this method can be used for large-scale discrete-time linear control systems as well. It is claimed that the transformations obtained by similarity operations reduce accuracy of the computations [6], however, other methods such as LQR methods [3] are more complicated.

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