Associations and the Accumulation of Preference

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This paper presents a theory of multi-alternative, multi-attribute preferential choice. It is assumed that the associations between an attribute and an available alternative impact the attribute's accessibility. The values of highly accessible attributes are more likely to be aggregated into preferences. Altering the choice task by adding new alternatives or by increasing the salience of preexisting alternatives can change the accessibility of the underlying attributes and subsequently bias choice. This mechanism is formalized by use of a preference accumulation decision process, embedded in a feed-forward neural network. The resulting model provides a unitary explanation for a large range of choice-set-dependent behaviors, including context effects, alignability effects, and less is more effects. The model also generates a gain–loss asymmetry relative to the reference point, without explicit loss aversion. This asymmetry accounts for all of the reference-dependent anomalies explained by loss aversion, as well as reference-dependent phenomena not captured by loss aversion.

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Associative processes play a fundamental role in human judgment and decision making. They perform simple computations, in parallel, and can provide quick responses without considerable deliberation (Kahneman & Frederick, 2002; Morewedge & Kahneman, 2010; Sloman, 1996; Strack & Deutsch, 2004). Although their speed and simplicity benefit the decision maker in many settings, these processes can also generate suboptimal decisions. In particular, associative processes exhibit a strong dependence on task-based cues. Altering the decision task in a logically irrelevant manner can alter the accessibility of these cues and subsequently bias the decision maker's response.

This type of task-dependent behavior is particularly well studied in the domain of preferential choice. Preferences are strongly influenced by available, yet irrelevant, alternatives. Adding, removing, or otherwise changing these alternatives can alter the decision maker's preferences and lead to a range of choice reversals (Huber, Payne, & Puto, 1982; Simonson, 1989). These reversals can also be generated by changing the salience of the alternatives in the choice set. Highly salient alternatives, such as reference points, are more likely to be chosen, and they can also affect the choice shares of other available options (Tversky & Kahneman, 1991).

In this paper, I propose a model of associative-value-based decision making that can be used to study these types of task dependence. This model assumes that the available choice alternatives bias the accessibility of attributes on the basis of their associative connections with these attributes. The affective values of attributes are subsequently accumulated into preferences. Attributes that are highly accessible have a larger weight in the accumulation process.

Additionally, this model assumes that the associative connection between a choice alternative and an attribute is proportional to the presence of the attribute in the alternative. In particular, a choice alternative is strongly associated with an attribute if the alternative has a large amount of the attribute. Attributes present in extreme quantities in some alternatives, attributes present in many alternatives, and attributes present in especially salient alternatives are more accessible relative to their competitors.

These assumptions imply that adding or removing irrelevant alternatives or altering the salience of alternatives can influence the accessibility of underlying attributes and subsequently bias their accumulation into preferences. This can potentially generate choice reversals as well as other choice-set-dependent or reference-dependent behaviors.

To explore these implications formally, in this paper I model the decision process with a connectionist network (Glöckner & Betsch, 2008; Guo & Holyoak, 2002; Roe, Busemeyer, & Townsend, 2001; Usher & McClelland, 2004; Usher & Zakay, 1993). This representation embeds the associative relationship between the choice task and attribute accessibility, within a stochastic sequential accumulation framework commonly used to model the dynamics of the preferential decision process (Bogacz, Usher, Zhang, & McClelland, 2007; Busemeyer & Johnson, 2004; Busemeyer & Townsend, 1993; Diederich, 1997, 2003; Johnson & Busemeyer, 2005; Krajbich, Armel, & Rangel, 2010; Milosavljevic, Malmaud, Huth, Koch, & Rangel, 2010; Rangel & Hare, 2010; Roe et al., 2001; Usher & McClelland, 2004). The associative connectivity in the proposed model generates a dependence between the choice task and the expected preferences of the decision maker, and

sequential accumulation leads to correlations between preferences for similar options. Together, these two processes can provide a comprehensive explanation for a wide range of choice-set-dependent and reference-dependent behaviors observed in multi-attribute, multi-alternative choice.

Task Dependence in Multi-Attribute Choice

Consider the task of choosing a restaurant for lunch. This task is often quite difficult. Even when choosing between only a few alternatives, there are still a large number of attributes that must be considered: How far should I walk? How much should I pay? Should I eat something healthy, or should I eat something tasty? Making correct decisions in this setting requires not only a mechanism with which to compare the values of various alternatives but also a way to select the attributes on which these alternatives will be evaluated. It is in complex settings like this that scholars of decision making have documented a number of behavioral anomalies. These anomalies relate to the dependence of the choice on the choice task itself, and they can be taxonomized into two categories: choice set dependence and reference dependence.

Choice Set Dependence

Choice set dependence relates to the relationship between the alternatives available in the choice task and the outcome of the choice task itself. Although theories of optimality dictate that adding or removing irrelevant alternatives should not affect the decision maker's choices (Luce, 1959; Sen, 1971), a number of researchers have documented systematic reversals of choice as novel, but irrelevant, alternatives are added to the choice set.

For example, the *asymmetric dominance effect* predicts that the relative choice shares of two alternatives can be reversed by the addition of a third alternative that is strongly dominated by one of the two initial alternatives (Huber et al., 1982). In the choice between an expensive but proximate restaurant, *x*, and a distant but reasonably priced restaurant, *y*, the addition of a third alternative that is both further than and more expensive than *x*, but still closer than *y*, will increase the choice share of *x* relative to *y*. This is because the new alternative will be dominated by *x* on every attribute but will still be an improvement over *y* on one attribute.

A similar change in choice shares can be obtained by adding new alternatives that make one of the previous alternatives seem like a compromise (Simonson, 1989). In particular, the *compromise effect* predicts that adding an extremely proximate, yet extremely expensive restaurant to the choice set will make *x* seem like a medium, compromise alternative in the choice set. This will increase its choice probability relative to *y*.

A third type of choice reversal can be obtained by adding an alternative that is similar to one of the initial alternatives in the choice set (Tversky, 1972). The *similarity effect* predicts that adding a restaurant that is almost equally expensive and equally proximate to *x* will reduce *x*'s choice share more than that of *y*.

The three *context effects* presented thus far are perhaps the most studied choice set effects in decision making. There are also, however, many other behavioral anomalies that can be categorized under choice set dependence. For example, a number of researchers have discovered that attributes common to multiple alternatives are more likely to be attended to relative to attributes unique to an alternative (Markman & Medin, 1995; Slovic & MacPhillamy, 1974; Zhang & Markman, 2001). Thus, adding or removing choice alternatives can alter the *alignability* of the alternatives in the choice set, the attributes that are attended to, and subsequently bias choice. Consider, for example, the choice between a sandwich shop, *x*, specializing in fairly healthy sandwiches, and a soup stand, *y*, offering even healthier soups. When they are evaluated separately, it is possible that *x* is rated as the more desirable alternative. When they are evaluated jointly, however, the overlapping attribute, healthiness, will get a higher weight. This can reverse the decision maker's choices, leading to *y* being chosen out of the two-alternative set.

A similar choice reversal is obtained when trivial attributes are added to an alternative. Consider the choice between two comparable restaurants, *x* and *y*. The first offers a free dessert along with a standard meal, whereas the latter does not offer the free dessert. If the decision maker is not particularly fond of dessert, the free dessert is a trivial attribute. In this setting, the *less is more effect* (Hsee, 1998; List, 2002; Simonson, Carmon, & O'Curry, 1994) predicts that *x* will have a lower desirability than *y* when evaluated separately. In the joint choice set, however, *x*, the dominant alternative, will be chosen.

Reference Dependence

Another type of task dependence relates to changes in choice shares caused by reference points: alternatives that are especially salient relative to other alternatives in the choice set. The best known reference-dependent phenomenon is the *endowment effect*. According to this effect, reference points (such as endowments) are generally selected over their competitors (Birnbaum & Stegner, 1979; Thaler, 1980; Tversky & Kahneman, 1991). Using the above example, restaurants that the decision maker has always frequented will be more desirable than novel restaurants: If *x* is the reference point, it will be chosen over a new alternative *y*.

A further anomaly regarding the endowment effect is that it reverses when choice alternatives are undesirable (Bhatia & Turan, 2012; Brenner, Rottenstreich, Sood, & Bilgin, 2007). In the unfortunate setting where the decision maker's reference restaurant, *x*, is particularly unpleasant to eat at, the *negative endowment effect* predicts that the decision maker will most likely switch to a new undesirable restaurant, restaurant *y*.

Reference dependence has also been documented in settings with more than two alternatives. The *improvements versus trade-off effect*, for example, finds that choice alternatives that are strict improvements over the reference point are more likely to be selected compared to alternatives that involve trade-offs from the reference point (Herne, 1998; Tversky & Kahneman, 1991). A restaurant *x* that is both less expensive and less distant than the reference point will be chosen over a restaurant *y* that is either more expensive and less distant or more distant and less expensive than the reference point.

A related finding is the *advantages and disadvantages effect*, according to which small trade-offs from the reference point are preferred over large trade-offs from the reference point (Tversky & Kahneman, 1991). This effect predicts that a reference point that is extremely proximate but extremely expensive will bias choices in favor of restaurant *x*, a close and expensive option, relative to restaurant *y*, a nonexpensive but distant option.

Models of Task Dependence

The above two sections have briefly outlined a number of types of task-dependent choice behaviors. These behaviors have been very well studied, and they are associated with many boundary conditions and exceptions (discussed subsequently in this paper). These behaviors have also been the subject of much theoretical inquiry. Tversky (1972), for example, has proposed the elimination by aspects model to capture the similarity effect, and Huber et al. (1982) have attempted to explain the asymmetric dominance effect using range-dependent attribute weighting. Choplin and Hummel (2005); Dhar and Glazer (1996); Wedell and Pettibone (1996); Pettibone and Wedell (2000); and Soltani, De Martino, and Camerer (2012) have shown that the asymmetric dominance effect can be explained by contrast-based shifts in attribute valuation (see also Pettibone & Wedell, 2007). Simonson (1989) has argued that justifiability can account for both the asymmetric dominance effect and the compromise effect. In contrast, Roe et al. (2001) have attempted to explain the similarity, asymmetric dominance, and compromise effects using similarity-based interactions between option preference, and Tversky and Simonson (1993) and Usher and McClelland (2004) have argued that these effects can be attributed to dimensional loss aversion. Schneider, Oppenheimer, and Detre (2011) have proposed a voting-based account of the three context effects, and Guo and Holyoak (2002) have explained the asymmetric dominance and similarity effects using bidirectional relationships between preferences and attributes. Tversky and Kahneman's (1991) model of loss aversion has been shown to capture the endowment, improvements versus trade-offs, and advantages and disadvantages effects. Finally, Busemeyer and Johnson (2004) have demonstrated that similarity-based correlations capture the improvements versus trade-offs and advantages and disadvantages effects, and Johnson and Busemeyer (2005) have shown that biased comparison processes can explain the endowment effect.

The models proposed by Roe et al. (2001); Guo and Holyoak (2002); Usher and McClelland (2004); and Johnson and Busemeyer (2005) can be classified as dynamic models, which explicitly represent the change in preference over the time course of the decision process, whereas the remaining models are static models that provide only absolute preference levels and choice probabilities for the available options. A dynamic account is important because these effects change as a function of deliberation time. Although both dynamic and static models provide a number of important insights regarding task dependence, none of them have attempted to capture the entire range of choice-set-dependent and reference-dependent behaviors observed in preferential choice. The next few sections, however, outline a framework that not only is able to explain all of these behaviors but also shows how they can be seen as by-products of task-based associative connectivity, a simple cognitive mechanism that helps ensure that the attributes relevant to the decision task are the ones that are attended to and aggregated.

The Decision Process

Value-based decision making is a dynamic and stochastic process that involves a representation of the choice task; the sampling and aggregation of valenced information regarding the choice task; and, finally, a rule for stopping aggregation and making a selection

of (or commitment to) one or more of the alternatives available in the choice task (see, e.g., Busemeyer & Townsend, 1993). A full model of this process must specify how the available alternatives determine the information that is sampled, as well as the mechanisms through which this information is aggregated into a decision.

Attributes, Associations, and Accessibility

Each alternative can be decomposed into a set of attributes. It is on the basis of these attributes that an alternative is evaluated and subsequently chosen or rejected. An alternative can contain varying, nonnegative amounts of an attribute. Large positive amounts correspond to attributes that are highly present in that alternative. Zero amounts correspond to attributes that are not present in, unrelated to, or not defined for the alternative.

The decision maker is assumed to consider the attributes that are the most accessible in any given decision problem. Accessibility is a broad concept that subsumes notions of salience for externally provided stimuli and retrievability and activation strength for memories and other mental objects (see, e.g., Kahneman, 2003, or Weber & Johnson, 2006, for a discussion of accessibility in preferential choice). In this paper it is assumed that the accessibility of an attribute depends primarily on its associative connections with the alternatives in the choice set (Kahneman, 2003; Kahneman & Frederick, 2002; Morewedge & Kahneman, 2010). Because the associative connections between two representations generally correspond to the strength of their relationship or frequency of co-occurrence, it is further assumed that the associative connection between an alternative and an attribute is equal to the amount of the attribute in the alternative. Attributes form a distributed representation of the choice alternatives, and attributes present in extreme amounts in some alternatives, present in multiple alternatives, or present in especially salient alternatives have a higher accessibility. Note that this is similar to Guo and Holyoak's (2002) model, in which the preference states of various alternatives affect the accessibility of the attributes, based on the amount of the attributes in the alternatives.

Affective Values and Preference Accumulation

Each attribute has an affective value, or valence for an alternative. This value depends on the total amount of this attribute in the alternative, and it ultimately influences the decision maker's approach or avoidance towards the alternative (Loewenstein & Lerner, 2003). Considerable evidence on the neuroscience of value-based decision making suggests that the brain does indeed hold explicit representations of the reward values of various attributes and outcomes (Kable & Glimcher 2009; Rangel, Camerer, & Montague, 2008). Choices are determined by the aggregation of these stored valuations into preferences. This aggregation is assumed to be accomplished by accumulators. Information accumulation through sequential sampling has been used to model decision making in a variety of domains (see, e.g., Ratcliff & Smith, 2004, for a review), and it is considered to be a biologically realistic approach to studying the processes underlying preferential choice (Basten, Biele, Heekeren, & Fieback, 2010; Busemeyer, Jessup, Johnson, & Townsend, 2006; Gold & Shadlen, 2007; Hare, Schultz, Camerer, O'Doherty, & Rangel, 2011; Lim, O'Doherty, & Rangel, 2011; Philiastides, Biele, & Heekeren, 2010). Sequen-

tial sampling can also generate optimal speed–accuracy trade-offs for a range of hypothesis tests (Bogacz, Brown, Moehlis, Holmes, & Cohen, 2006).

Information accumulation through sequential sampling was initially applied to value-based decision making in decision field theory (DFT; Busemeyer & Townsend, 1993). Subsequent work has adopted this perspective and has used it to model a range of decision-making phenomena (Bogacz et al., 2007; Busemeyer & Johnson, 2004; Diederich, 1997, 2003; Johnson & Busemeyer, 2005; Krajbich et al., 2010; Milosavljevic et al., 2010; Roe et al., 2001; Usher, Elhalal, & McClelland, 2008; Usher & McClelland, 2004). The associative accumulation model extends this line of research by specifying the determinants of attribute-value sampling in the preference accumulation process. It assumes that the affective values of highly accessible attributes are more likely to be sampled and accumulated relative to the affective values of inaccessible attributes. The relationship of attribute sampling and weighting with attribute accessibility implies that the decision maker's preferences depend strongly on the choice task. Changing the choice task by adding, removing, or altering the salience of certain alternatives can bias the accumulation of preferences and subsequently reverse choice.

The Associative Accumulation Model

We can represent each choice alternative i as a vector of M different attributes $\boldsymbol{x_i} = (x_{i1}, x_{i2} \ldots x_{iM})$. Any particular choice set can be represented as $X = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots \boldsymbol{x_N}\}$, where N is the total number of alternatives being considered. Here, x_{ij} is a scalar that represents the amount of attribute j in choice alternative i. x_{ij} is assumed to be greater than or equal to zero.

Each attribute has an affective value. The affective value of attribute j in alternative i can be written as $V_{ij} = V_j(x_{ij})$. $V_j(\cdot)$ is a function that is nonnegative and increasing in x_{ij} for positive attributes ("goods") and is nonpositive and decreasing in x_{ij} for negative attributes ("bads").[1] If attribute j is mediocre on, not present in, unrelated to, or undefined on alternative i, then x_{ij} is zero and V_{ij} is also subsequently assumed to be zero. In this paper, it will be assumed that $V_j(\cdot)$ is concave for all attributes.

Based on the discussion above, the association between a choice alternative i and an attribute j can be written as x_{ij}. This definition implies that choice alternatives with high amounts of an attribute will have stronger associations with that attribute. Note that modeling associations between attributes and alternatives in this manner does not require any free parameters. Associative connections are determined entirely by the attribute amounts, which serve as inputs in every decision-making model. A more realistic approach would represent the association between attribute j and choice alternative i as $\alpha_j \cdot x_{ij}$ for some parameter value α_j. This would allow attributes expressed in different units (e.g., dollars or minutes) to be comparable, with α parameters serving as *exchange rates* across attributes. Such an approach is also necessary for quantitative data fitting. For simplicity, however, in this paper only the simple case where $\alpha_j = 1$ for all attributes is considered.

The proposed framework can be instantiated in a simple three-layered neural network, as shown in Figure 1. The first layer corresponds to task representation and consists of nodes representing all the choice alternatives that may or may not be available in the decision task. Perceptual systems send constant inputs into this

layer, giving available alternatives and salient nonavailable alternatives a positive activation. The activation value for alternative i is represented as s_i. Additionally, especially salient choice alternatives such as reference points are assumed to receive stronger inputs, leading to higher activation values, relative to their competitors. Nonsalient alternatives are not activated, have $s_i = 0$, and are subsequently ignored.[2]

The second layer represents all the attributes that may or may not be relevant to the decision. The relative activation value of a node in this layer corresponds to the accessibility of its corresponding attribute. The connection between a node i in the choice representation layer and a node j in the attribute layer is determined by the association between alternative i and attribute j. This, as defined above, is x_{ij}. The inputs to an attribute from a particular alternative are equal to this value, weighted by the activation, or salience, of the alternative. These inputs are constant across time. Additionally, every attribute is assumed to have a linear activation function, with a nonnegative constant input, a_0, identical across all attributes. a_0 serves to moderate the strength of the proposed associative biases. For very low values of a_0, these biases are extremely strong, and as a_0 gets larger, these biases disappear. With this structure, the activation value of attribute j, at any time period, can be written as

$$a_j = a_0 + \sum_{i=1}^{N} s_i \cdot x_{ij} \qquad (1)$$

Finally, the third layer represents preferences. The activations of the nodes in this layer capture the preference states for the various alternatives. The most preferred alternatives have the highest activations, whereas less preferred alternatives have lower activations. In this paper it is assumed that the decision maker stochastically attends to one attribute in each time period and adds its affective values in each of the choice alternatives to the nodes in the preference accumulation layer (as in Busemeyer & Townsend, 1993). The affective value of an attribute j in any given alternative i is V_{ij}. The probability of the attribute being attended to is determined by the attribute's accessibility. This can be written as

$$w_j = \frac{a_j}{\sum_{l=1}^{M} a_l} \qquad (2)$$

In addition to being dependent on the affective value inputs from the second layer, the activation of a preference node is dependent on its activation in the previous time period. In particular, the preference nodes are assumed to accumulate information over time. This accumulation is not perfect and is subject to decay

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[1] An example of such a function for a "good" attribute j would be $V_j(x_{ij}) = x_{ij}^{1/2}$. The analogous function for a "bad" attribute would be $V_j(x_{ij}) = -(x_{ij})^{1/2}$. Note that both are defined for positive values of x_{ij}. The former is increasing in x_{ij}, whereas the latter is decreasing in x_{ij}.

[2] There are many determinants of alternative salience. Two particularly relevant determinants involve exogenous factors, such as the status of the alternative as a reference point, and endogenous factors, such as the extent of the presence of the alternative in the choice set. Choice alternatives that are not present in the choice set and are not reference points are generally not salient and can be assumed to be ignored. Alternatives that are present in the choice set for some amount of time but are removed prior to choice can be assumed to have a positive but weak level of salience (this is discussed in more detail in subsequent sections).
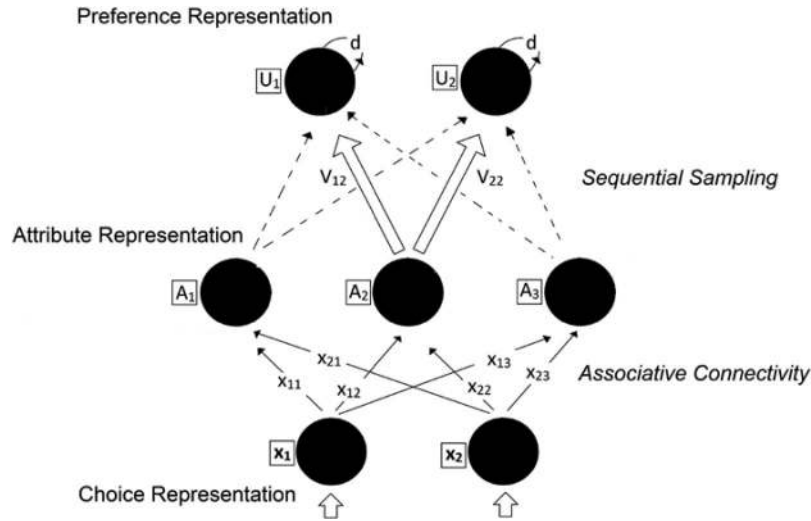
Figure 1. The associative accumulation model consists of three layers with feed-forward connectivity. These layers correspond to choice, attribute, and preference representation, respectively. Connections between the first and second layers are determined by the associations between alternatives and attributes (solid arrows), and connections between the second and third layers are determined by the values of the attributes in the alternatives (dashed arrows). Attributes are sampled sequentially over time, and in this figure, attribute 2 is being sampled (outlined arrows).

or leakage, as well as noise. Decay is captured by a parameter $d > 0$, which is identical across choice alternatives. The noise term ε is zero mean and normally distributed with standard deviation σ, which is also identical for all choice alternatives. Finally, preference nodes are assumed to have a linear activation function. If attribute j is sampled at time t, the preference state for alternative i at time t can be written as

$$P_i(t) = d \cdot P_i(t - 1) + V_{ij} + \varepsilon_i(t - 1) \tag{3}$$

Lastly, a decision rule that determines both the alternative that is chosen and the time at which this decision is made can be defined. In this paper it is assumed that in settings where the decision maker is free to deliberate as long as he or she wants, prior to committing the choice, decisions are made when the activity of a preference node crosses a threshold, Q. Alternatively, in settings where there is an externally controlled stopping time, T, decisions are made based on whichever alternative has the highest preference state at the specified time.

Associative Accumulation and Task Dependence

Theories of rational decision making require choice consistency. For deterministic decision theoretic models, consistency implies an independence of choice from any irrelevant alternative in the choice set (see, e.g., Sen, 1971). The stochastic analogue of this requirement is Luce's choice axiom, which states that the ratio of choice probabilities between two alternatives should be independent of any other alternatives in the choice set (Luce, 1959). Luce's choice axiom also implies a weaker property, regularity, according to which adding new alternatives to a choice set should not increase the choice probability of any of the initial alternatives. Additionally, both deterministic and stochastic theories of rational

choice assume an independence of choice from other task-related factors, such as reference points.

The associative accumulation model violates these requirements. Decision makers are both choice set and reference dependent. The two psychological mechanisms responsible for these violations are the associative connections between choice set and attribute representations and the stochastic sequential accumulation of attribute values. For the purposes of this paper, these two mechanisms can be best understood in terms of their respective effects on the first two central moments of the distribution of the decision maker's preferences.

Associations and Expected Preferences

The primary mechanism generating task dependence, in the associative accumulation model, is the associative connectivity itself. Associations between choice alternatives and attributes determine the accessibility of the attributes (i.e., their decision weights) and subsequently the preferences of the decision maker. Altering the decision task affects these weights and can change the decision maker's choice behavior.

Earlier theories of decision making do allow for a relationship between the available choice alternatives and attribute weights. This relationship generally depends on range effects, according to which judged attribute differences vary based on the composition of the available choice set (Parducci, 1965). Adding a novel choice alternative that extends the range of values on one attribute dimension can reduce the perceived differences on that attribute, subsequently reducing that attribute's weight in the decision task. Although this mechanism can generate some of the observed context effects (Huber et al., 1982; Wedell, 1991), several researchers have also found the opposite occurrence; namely, an

increase in attribute weighting following an increase in its range (see Pettibone & Wedell, 2007, for a review).

In the associative accumulation model, in contrast to earlier theories, the weight of an attribute (w_j) is proportional to the relative presence of the attribute in the available choice alternatives. Subsequently adding or increasing the salience of choice alternatives that contain large amounts of a particular attribute will increase that attribute's weight in the decision task. If a new choice alternative, x_k, is added to a choice set, or if the salience of a preexisting alternative x_k is increased (by making it a reference point), it can be shown that the weight of attribute j in the new setting increases if and only if[3]

$$\frac{x_{kj}}{\sum_{l=1}^{M} x_{kl}} > w_j \qquad (4)$$

If the proportion of attribute j in x_k is greater than the weight of j in the initial decision task (w_j), adding x_k or increasing the salience of x_k will increase the weighting of attribute j. In the simple case where the core alternatives are equally salient and symmetrically distributed in the choice space (resulting in $w_j = 1/M$ for all j), adding or increasing the salience of x_k will necessarily increase the weight of its strongest attributes.

Increases in the weight of an attribute can alter the expected preferences of all the alternatives in the choice set. To explore this property, consider Equation 5. Equation 5 represents the expected inputs from the attribute layer into the preference accumulation layer for choice alternative i. As these expectations remain constant across time, U_i can be used to infer the relative expected preference states for the choice alternatives, for all times. If $U_i > U_{i'}$, alternative i will have a higher expected preference state than alternative i' throughout the decision process. Subsequently, increasing U_i or reducing $U_{i'}$ generates a higher relative choice probability for alternative i, keeping any higher moment changes constant.

$$U_i = \sum_{j=1}^{M} w_j \cdot V_{ij} \qquad (5)$$

Equation 5 can be used to specify the impact of attribute weights on expected preferences. In particular, if a new choice alternative x_k is added to a choice set, or if the salience of a preexisting alternative x_k is increased, and the attribute weights in the new task are represented as w_j' and the expected inputs to alternative i in the new task are represented as U_i', then

$$U_i' - U_i = \sum_{j=1}^{M} (w_j' - w_j) \cdot V_{ij} \qquad (6)$$

Likewise, the impact of changing x_k on two choice alternatives, x_i and $x_{i'}$, can be written as

$$(U_{i'}' - U_i') - (U_{i'} - U_i) = \sum_{j=1}^{M} (w_j' - w_j) \cdot (V_{i'j} - V_{ij}) \qquad (7)$$

Equations 6 and 7 show that altering a decision task so as to increase the relative inputs to an attribute will increase the expected preferences for all choice alternatives that are highly valued on that attribute. Likewise, the difference in the expected preferences between two prior alternatives will increase if attributes on which they differ increase in their relative amounts. In the simple case where the initial alternatives are equally salient and symmetrically distributed in the choice space, adding or increasing the salience of x_k will disproportionately increase the expected preferences of alternatives whose strongest attributes are the same as those of x_k.

The dependence of expected preference on the choice set generates violations of both Luce's choice axiom and weaker axioms such as regularity. These violations have been documented empirically and are explored in more detail in subsequent sections. This relationship can create other effects as well. Optimal decision making requires that the expected inputs to the preference accumulation layer should be proportional to the total values of the choice alternatives themselves. The associative mechanisms proposed in this paper, however, overweigh attributes present in large quantities in the choice set. In the simple binary choice case, these attribute-weighting biases can generate a preference for extreme choice alternatives over more moderately distributed alternatives.

Consider, for example, the choice between $x_1 = (5, 5)$ and $x_2 = (10, 0)$. Because x_2 is extremely strong on attribute 1, whereas x_1 is evenly split between attributes 1 and 2, the associative accumulation model predicts that w_1, the weight on attribute 1, should be greater than w_2, the weight on attribute 2. If this bias is strong enough, then it is possible for x_2 to have a higher expected preference than x_1, even if $V_j(\cdot)$ are concave functions that would, in the absence of the associative bias, support alternative x_1 over x_2.

The preference for extreme alternatives created by the associative mechanism in the proposed model is illustrated in Figure 2. This figure shows curves corresponding to the sets of alternatives with the same expected inputs to the preference accumulation layer, as the alternative $x_1 = (5, 5)$, in a simple binary choice.[4] These three *indifference* curves correspond to different values of a_O in Equation 1. For any particular parameter value, all points on its corresponding indifference curve should have the same expected preference as x_1. Moreover, points above these indifference curves, which have higher inputs to the preference accumulation layer than x_1, should have a higher expected preference than x_1. Likewise, x_1 should have a higher expected preference than points below these indifference curves. The value functions used in this demonstration are concave, with $V_{ij} = x_{ij}^{1/2}$ for $j = 1, 2$. Hence, in the absence of any weighting bias, the indifference curves should be facing outward, with the set of alternatives preferred to x_1 being convex.

The associative bias is strongest for $a_O = 0$, and for this case, it is found that decision makers do prefer extreme alternatives such as (10, 0) over moderate alternatives such as (5, 5). At $a_O = \infty$ the associative bias is nonexistent, attribute weights are equal, and the

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[3] Let the salience of x_k be changed from s_k to s_k'. If $s_k = 0$ and $s_k' = 1$, this captures the setting in which x_k is added to the choice set. If $s_k = 1$ and $s_k' > 1$, this captures the setting in which x_k is made the reference point. The new weight on attribute j, w_j', will be greater than the old weight on attribute j if the following condition is satisfied: $w_j' = \frac{a_0 + \sum_{i=1}^{N} s_i \cdot x_{ij} + (s_k' - s_k) \cdot x_{kj}}{\sum_{l=1}^{M} [a_0 + \sum_{i=1}^{N} s_i \cdot x_{il}] + \sum_{l=1}^{M} [(s_k' - s_k) \cdot x_{kl}]} > w_j$. Some algebra shows that this is satisfied only if the inequality in Equation 4 is satisfied.

[4] More specifically, each curve consists of all choice alternatives x_k such that $U_1 = U_k$ in the choice set $\{x_1, x_k\}$, keeping $s_1 = s_k$.
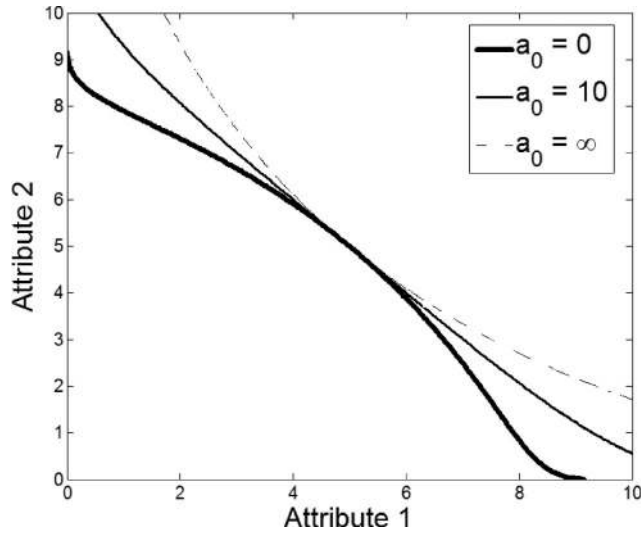
Figure 2. Illustration of extremity bias in the associative accumulation model. Each curve captures points x_k such that $U_k = U_I$ for $x_I = (5, 5)$. These points receive the same expected preference inputs as x_I and are thus equally preferred to x_I in binary choice.

decision maker exhibits the optimal preference for moderate alternatives over extreme alternatives. Decision makers also display the preference for moderate alternatives over extreme alternatives at the intermediate value $a_o = 10$. Here, as with $a_o = \infty$, the indifference curve is facing outward and the set of alternatives preferred to x_I is convex. Although this does resemble the extremeness-averse behavior observed with optimal preferences, the indifference curve for this parameter is flatter than its unbiased counterpart. Despite being extremeness averse, decision makers with association-biased attribute weights will have a higher expected preference for extreme alternatives than will optimal decision makers.

Sequential Accumulation and Preference Covariance

The expected preferences of choice alternatives are not enough to predict the relative choice probabilities of the available alternatives. The covariance between the preferences of two or more alternatives can affect choice as well. The mechanism responsible for covariance-related effects, in the associative accumulation model, is the sequential and stochastic accumulation of attributes. Decision makers accumulate the affective values of only one randomly chosen attribute in each time period. This implies that choice alternatives that have comparable distributions of attributes will have similar changes in their preference states across time. When one of their primary attributes is sampled, their preferences will increase together. If their attributes are not sampled, their preferences will decrease together. These positive correlations imply that if one of the alternatives is a highly preferred alternative at any given time, the other will be as well. As a result these two alternatives will directly compete with each other for choice, and they will detract from each other's choice probability more than they will detract from the choice probability of a third, uncorrelated or negatively correlated, alternative. Adding a new alternative—a decoy—to a choice set thus disproportionately reduces the

choice probability of alternatives similar to the decoy. Sequential accumulation, without the associative mechanism discussed above, produces similarity effects, which are sufficient to generate violations of Luce's choice axiom (but not of regularity).

Summary

This section has outlined two distinct causes of task dependence in the associative accumulation model. The first involves biased expected preferences, generated by the associative connection between choice and attribute representation. This mechanism implies that adding decoys or increasing the salience of preexisting alternatives will increase the weights on their strongest dimensions and subsequently increase the relative preference for other choice alternatives that are highly valuable on these dimensions. The other cause of task dependence relates to the covariance between preferences, generated by stochastic sequential value accumulation. This mechanism implies that adding decoys to the choice set will disproportionately reduce the choice share of alternatives similar to the decoy.

Intuitively, these two mechanisms have opposite effects on an alternative's choice shares, as the choice set is varied. A novel decoy, for example, will correlate and compete with alternatives to which it is similar. It will also, however, boost the expected preferences of these alternatives, as well as other alternatives that are strongest on dimensions on which the decoy itself is strongest. If the competitive effect of the decoy, caused by preference covariance, is greater than the boost in expected preferences, one should expect a reduction in the choice share of alternatives similar to the decoy. The opposite should be observed if attribute weights are highly sensitive to the decision task (as with low values of a_o) and exert a larger influence on expected preference and, subsequently, on choice shares.

The next two sections show how both these mechanisms can be used to understand the effect of choice sets and reference points on decisions. For a moderately powerful associative bias, generated by intermediate values of a_o, the associative accumulation model predicts the emergence of both asymmetric dominance and compromise effects (due to changes in expected preferences), as well as similarity effects (due to changes in preference covariance). Other types of choice set dependence involving the alignability effect and the less is more effect are also predicted by this framework. Finally, the proposed associative biases are able to explain the emergence of gain–loss asymmetries relative to reference points, as well as a wide range of other reference-dependent choice behaviors.

The effect of associative connectivity can also create a relative preference for extreme options in the choice set. Unlike the choice-set-dependent and reference-dependent anomalies generated by this associative mechanism, there is little empirical work directly testing the relationship between extremeness aversion and the distribution of attributes in the choice set. Some research does, however, suggest that preferences may be biased in the ways predicted by the proposed model. According to the alignability effect, attributes common to multiple options receive higher weights relative to those that are unique to one option. This leads to biased preferences for the alternatives dominant on the common attributes. In many cases, these dominant alternatives are extreme valued (with a lot of the common attribute but none of the unique

attribute contained in their competitor), and preferences for these alternatives can appear to be extremeness seeking. Shafir (1993) also found that choice options with many extreme attributes are more likely to be chosen than equally desirable options with moderate attributes.

Additionally, although extremeness aversion is very common, decision makers can display extremeness-seeking behavior in some domains. Goal-directed choice, for example, involves convex utility functions (Heath, Larrick, & Wu, 1999), and preferences regarding attributes such as price and money are less extremeness averse than preferences regarding attributes like quality (Simonson & Tversky, 1992). It is possible that goal-related attributes or attributes regarding money have disproportionately large effects on attention, generating extremeness-seeking behavior for alternatives composed of these attributes.

Finally, note that it is possible that associative connectivity and attribute activation are not linear. The linear forms assumed in this paper allow for mathematical tractability and ensure that no additional parameters are used to describe associative connectivity or activation. They also, however, imply that both associative strength and attribute activation are unbounded and increasing at a constant rate. A more complex model (with sigmoidal activation) would place stronger restrictions on preferences and would generate even less extremeness-seeking behavior.

Choice Set Dependence

This section explores the impact of changing the composition of the choice set on choice shares. Equations 4, 5, 6, and 7 are used to illustrate the impact of choice sets on attribute accessibility and expected preferences. Simulations (programs for which are available in online supplemental materials) are used to illustrate the impact of choice sets on preference covariance and the combined impact of changing expected preferences and preference covariance on choice shares. Simulations involve two or three attribute choice sets with either two or three alternatives. Unless otherwise specified, these alternatives are assumed to be equally salient ($s_i = 1$). In this paper it is also assumed that each attribute has the same valuation function, $V_{ij} = x_{ij}^{1/2}$, and that $d = 0.8$, $\sigma = 0.05$, and (unless otherwise specified) $a_o = 10$. Each simulation runs for a total time of $T = 100$, and the choice alternative with the highest preference state at $T = 100$ is the one that is selected.

Context Effects

Findings. Perhaps the most studied choice-set-dependent phenomena are the asymmetric dominance and compromise effects. As outlined in the first section of this paper, the asymmetric dominance effect (Huber et al., 1982; Huber & Puto, 1983) refers to the finding that the relative choice share of a particular alternative increases with the introduction of a novel alternative that it, but not its competitor, dominates. Likewise, the compromise effect (Simonson, 1989) refers to the finding that the relative choice share of a particular alternative increases with the addition of a novel, extreme option that makes the alternative appear as a compromise.

Consider Figure 3. The asymmetric dominance effect predicts that the addition of the dominated alternative d to the binary choice set $X = \{x_1, x_2\}$ will lead to an increased choice share of x_1
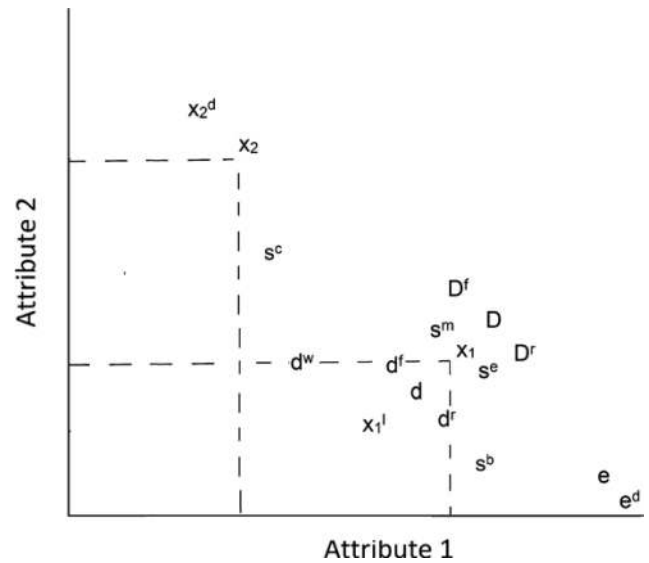

Figure 3. A graphical description of observed context effects. x_1 and x_2 are core options. The other points represent decoy locations that have been shown to bias the relative choice shares of the core options. For example, introducing asymmetrically dominated decoy d, which is dominated by x_1 but not by x_2, causes an increase in the choice share of x_1 relative to x_2.

relative to x_2. Likewise, the compromise effect predicts that the addition of the extreme alternative e to the binary choice set $X = \{x_1, x_2\}$ will also lead to an increased choice share of x_1 relative to x_2. Note that these effects pertain to the relative choice shares of x_1 and x_2. It is possible for these effects to emerge even if the total choice share of x_1 drops with the addition of d or e. If the choice share of x_1 increases (above its binary choice rate) with the addition of d or e, then regularity is violated (independence of irrelevant alternatives is violated as long as there are changes to the relative shares of x_1 and x_2).

A number of variables moderating the strength of these effects have been discovered. With regard to the asymmetric dominance effect, Huber et al. (1982) found that range decoys, which are dominated by the target on the competitor's primary dimension but not on the target's primary dimension, increase the target's share relative to frequency decoys, which are dominated by the target on the target's primary dimension but not on the competitor's primary dimension. In Figure 3, this implies that d^r will lead to a higher choice share of x_1 relative to x_2 than will d^f.

Introducing a nondominated decoy that is close to but nonetheless inferior to the target also increases the choice share of the target, creating an effect much like asymmetric dominance (Huber & Puto, 1983; Pettibone & Wedell, 2000). In Figure 3, this alternative is represented as i. As with d, adding i should increase the choice share of x_1 relative to x_2. Conversely, Dhar and Glazer (1996) found that the addition of a dominated decoy that is extremely weak on the target's primary dimension does not generate a significant asymmetric dominance effect. In Figure 3, this implies that a decoy such as d^w will not bias choice shares in favor of x_1 relative to x_2.

The compromise effect, too, has its moderators. In particular, Simonson and Tversky (1992) found that decreasing the value of the target alternative can reverse the compromise effect, leading to

the increased relative choice share of the initial extreme option (the competitor), once a novel extreme option (the decoy) has been added to the choice set. This led them to suggest that the compromise effect has two forms, one associated with the enhancement of a high-value compromise (the standard compromise effect) and one associated with the detraction of a low-valued compromise. In Figure 3, this implies that the addition of e to the core set $\{x_1', x_2\}$ should reverse the compromise effect and bias choice shares in favor of x_2 relative to x_1'.

More recently, Schneider et al. (2011) have shown that the compromise effect is also determined by distance of the extreme alternatives from the compromise alternative. In particular, extreme alternatives farther away from the compromise alternative lead to stronger compromise effects than do extreme alternatives closer to the compromise alternative. In Figure 3, this implies that the distant extreme alternative e^d and the distant competitor x_2^d should be associated with a stronger compromise effect than the proximate extreme alternative e and the standard competitor x_2. In particular, the increase in the choice share of x_1 relative to x_2^d when e^d is added should be higher than the increase in the choice share of x_1 relative to x_2 when e is added.

Choice from the three-alternative sets used in the above papers identifies the most valued choice alternative from the three-alternative set. Although this is enough to establish explicit choice reversals, this does not allow researchers to determine the order of preferences within the three-alternative sets. Understanding ordering is necessary to determine the processes responsible for these inconsistencies. Highhouse (1996) and Pettibone and Wedell (2000) explored this with regard to asymmetric dominance, and Usher et al. (2008) explored this with regard to the compromise effect. Both these sets of studies used phantom decoys, which are attractive choice alternatives included in the choice set initially but then removed immediately prior to choice. The resulting binary choice can be used to determine preference relations between the two nonchosen alternatives in the presence of the decoy.

With regard to asymmetric dominance, Highhouse (1996) and Pettibone and Wedell (2000) found that removing the dominating alternative from the choice set reveals a preference for the initially dominated alternative, or the target, over the nondominated alternative, or the competitor. This effect is strongest for range phantom decoys that dominate the target on its primary dimension and is weakest or nonexistent for frequency decoys that dominate the target on the competitor's primary dimension (Pettibone & Wedell, 2007). In Figure 3, this implies that adding D, D^r, and D^f will lead to a higher relative preference for x_1 over x_2 and that D^r should have a stronger overall effect than D^f.

With the compromise effect, Usher et al. (2008) found that removing an extreme alternative from the choice set, when this extreme alternative is chosen, reveals a preference for the compromise rather than the other extreme. This suggests that e should increase the relative preference of x_1 over x_2, even when e is the chosen alternative in the three-alternative choice set. Tsetsos, Chater, and Usher (2012) reported a moderator of this result. If the extreme option, e, is removed early on in the decision process, this effect disappears: The choice proportion of x_1 over x_2 in the three-option choice, with the early removal of x_3, is no different than the choice proportion of x_1 over x_2 in the two-option choice.

Another set of moderators relates to time constraints. Dhar, Nowlis, and Sherman (2000), for example, found that the compro-

mise effect reduces under time pressure. Pettibone (2012) replicated this result and also found that the asymmetric dominance effect reduces under time pressure. Additionally Pettibone (2012) noted that the asymmetric dominance effect emerges very early on in the decision process, whereas the compromise effect is nonexistent (and occasionally reversed) for low deliberation times.

A final context effect is the similarity effect (Tversky, 1972), which predicts that the addition of an alternative that is similar to the target should reduce its choice probability. In Figure 3, this implies that adding either the moderate similar decoy s^m or the extreme similar decoy s^e should reduce the choice probability of x_1 relative to x_2. Note that a closely related effect has also been documented in binary choice. In particular, Mellers and Biagini (1994) found that options that are strongest on the attributes on which the two options differ the most have a higher binary choice probability compared to equally desirable options that are relatively weak on these attributes. For example, in Figure 3, the binary similarity effect would predict that the choice probability of x_1 from the set $\{x_1, s^b\}$ is higher than the choice probability of x_2 from the set $\{x_2, s^b\}$ but that the choice probability of x_1 from the set $\{x_1, s^c\}$ is lower than the choice probability of x_2 from the set $\{x_2, s^c\}$. These inequalities can generate violations of strong stochastic transitivity (see Mellers & Biagini, 1994, for a discussion).

Explanation. There are two mechanisms with which the associative accumulation model can explain the above effects. Associative connectivity determines expected preferences, and sequential accumulation determines preference covariance. Let us first explore the relationship of these context effects with associations and expected preferences, independently of sequential accumulation preference covariance.

Expected preferences. As discussed earlier, the addition of a new alternative can alter the preferences for the preexisting alternatives based on the new alternative's attributes. Due to the associative mechanisms proposed in this model, the new alternative will increase the accessibility, or weight, of the attributes on which it is strongest. This will subsequently bias the sampling of these attributes and will disproportionately increase the expected preferences of the choice alternatives strongest on these attributes.

This mechanism can explain the asymmetric dominance and compromise effects, the phantom dominating and phantom extreme decoy effects, as well as effects related to impact of inferior nondominated decoys on choice probabilities. In all of these cases, the decoy alternative contains more of the target alternative's strongest attributes than the competitor's strongest attributes. This can also explain the relative effects of different types of decoys. Range-dominated decoys contain more of the target's strongest attributes than do frequency decoys or weak dominated decoys.

Equations 4, 5, 6, and 7 can be used to explore this intuition more formally. Take any two alternatives x_1 and x_2 in a two-attribute choice space, with any strictly increasing valuation functions. Let it be assumed that these alternatives do not dominate each other and that attribute 1 is x_1's most valuable attribute and attribute 2 is x_2's most valuable attribute. This implies that $V_{11} > V_{12}$ and $V_{22} > V_{21}$.

Now consider adding a third alternative, x_3, to this set to make a larger, three-alternative, set $X' = \{x_1, x_2, x_3\}$. In this simple, two-attribute setting, any increase in the weight of attribute 1 will necessarily lead to a higher preference for x_1, and any increase in weight of attribute 2 will necessarily lead to a higher preference for

x_2. In particular, $U_1' - U_2' > U_1 - U_2$ if and only if $w_1' > w_1$. From Equation 4 it is known that this happens if and only if x_3 has a higher proportion of attribute 1 than the weight on attribute 1 in the core set X.

This requirement is necessarily satisfied for any extreme decoy e and any inferior decoy i, for the conditions specified above, and for any parameters in this model. Both these decoys have more of attribute 1 than x_1 itself, which is itself the strongest alternative on that attribute in the core set. This condition also holds for the asymmetrically dominated decoy, d, and the dominating phantom decoy, D, in numerous scenarios. For example, this condition is necessarily satisfied if d and D lie on the same vector as x_1 and thus contain the same relative proportion of the two attributes as x_1. This condition is also necessarily satisfied if x_1 and x_2 are symmetric (i.e., $x_{11} = x_{22}$, and $x_{12} = x_{21}$) and d and D are, like the target x_1, stronger on attribute 1 than on attribute 2.

Equation 7 can also be used to model the impact of changing the decoy from d^f to d^r and from D^f to D^r. In particular, if it is assumed that d^f has the same total amount of attributes as d^r and that D^f has the same total amount of attributes as D^r, then the weight on attribute 1 with the range decoys is necessarily higher than with frequency decoys. This implies that the relative preference for x_1 is higher under the range decoys than the frequency decoys. The same principle explains why d^w has a diminished effect on choice. d^f has a higher proportion of attribute 1 than does d^w, implying that the weight on attribute 1 and subsequently the preference for x_1 is lower with d^w relative to the already weak d^f.

Note that the effect of these decoys on choice decreases with their salience. If it is assumed that the dominating phantom decoys, D, D^r, and D^f, and the extreme phantom decoy, e, are less salient than nonphantom decoys (i.e., options that are available for selection in the choice task), then their effect on the choice shares of the core options should also be less than that of these nonphantom decoys. This is because reduced salience reduces the effect of the decoy on attribute accessibility, subsequently decreasing the changes to the choice shares of x_1 and x_2 caused by the addition of x_3. If it is assumed that the salience of the decoy is proportional to the extent of its presence in the choice task, then this can easily explain the finding that the effect of e on the choice share of the core option is eliminated when e is removed early on in the decision process but not when it is removed later in the decision process.

Preference covariance. The proposed associative mechanisms provide a unitary explanation for many of the observed context effects. A complete account of context dependence, however, requires additional psychological details, such as stochastic sequential accumulation of attribute values. This mechanism generates correlations between the preferences of alternatives with comparable attribute distributions. Hence, introducing the similarity decoys, s^m and s^e, can reduce the choice shares of x_1 relative x_2, generating the similarity effect. Of course, because both the asymmetrically dominated and the extreme decoys are more similar to x_1 relative to x_2, a strong enough similarity effect can also eradicate the biases explained in the preceding paragraphs.

The associative accumulation model thus does not guarantee the simultaneous occurrence of all the context effects for all parameter values. That said, a number of different parameter values, including the ones listed at the start of this section, guarantee the simultaneous emergence of both the similarity and the asymmetric

dominance and compromise effects. This is illustrated in Figure 4, which plots the effects of different decoys (different positions of x_3) on the choice share of $x_1 = (7, 3)$ relative to $x_2 = (3, 7)$, for the parameter values listed at the start of this section. The shades capture the values of $C = C_3 - C_2$, where $C_3 = (N[x_1 \text{ chosen}] - N[x_2 \text{ chosen}])/(N[x_1 \text{ chosen}] + N[x_2 \text{ chosen}])$ in the presence of x_3, and $C_2 = N[x_1 \text{ chosen}] - N[x_2 \text{ chosen}])/(N[x_1 \text{ chosen}] + N[x_2 \text{ chosen}])$ in the absence of x_3. $N[x_1 \text{ chosen}]$ and $N[x_2 \text{ chosen}]$ are the proportion of times x_1 and x_2 are chosen in each set of simulations. Note that $C_2 = 0$ is expected for the binary choice set $\{x_1, x_2\}$, as the two core alternatives are symmetric on identical attributes. In the simulations, x_{31} and x_{32} change in intervals of 0.1, with each simulation run 500 times. The shade at the point (a, b) in the figure corresponds the value of C with $x_{31} = a$ and $x_{32} = b$. The diagonal line through x_1 and x_2 captures every point that has the same total attributes as x_1 and x_2. Finally, positive values of C correspond to a bias in favor of x_1 and are associated with darker shades on the grid, whereas negative values of C correspond to a bias in favor of x_2 and are associated with lighter shades on the grid. Values of x_3 for which C is especially large are black, values of x_3 for which C is especially small are white, and values of x_3 for which C is close to zero are gray.

The asymmetric dominance, compromise, and similarity effects can all be observed in Figure 4. Points lying directly below x_1 are black, showing that they boost the choice share of x_1, as predicted by the asymmetric dominance effect. Points at the extreme right-hand corner of the figure are similarly black, indicating the presence of the compromise effect. On the other hand, points that lie on the diagonal line, near x_1, are white, as predicted by the similarity effect. This figure also shows the emergence of related decoy effects, such as the range-frequency asymmetric dominance effects involving d^r and d^f, the inferior decoy effect involving i, and the weak asymmetric dominance effect involving d^w.

Although the asymmetric dominance effect emerges for all decoys dominated by x_1 that are close enough to x_1, Figure 4
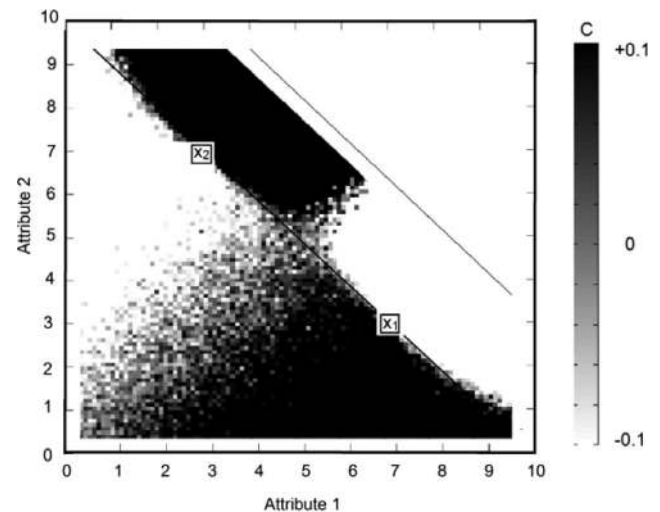

Figure 4. Choice shares of x_1 relative to x_2 at various decoy locations. The shade at any point (a, b) captures the bias C created in favor of x_1 relative to x_2 when $x_3 = (a, b)$ is added to the binary set $X = \{x_1, x_2\}$. Darker shades represent increases in choice shares of x_1, and lighter shades represent increases in choice shares of x_2.

shows that the compromise effect is somewhat more variable. Not all extreme decoys that make x_1 a compromise alternative boost its choice share. Those that are too close to x_1 generate the similarity effect. This trade-off is further illustrated in Figure 5. Here, $x_1 = (7, 3)$ and $x_2 = (3, 7)$. The horizontal axis represents different positions of the new choice alternative x_3, as it is moved along the line $x_{31} + x_{32} = 10$, which is the diagonal line through x_1 and x_2 in Figure 4. The vertical axis in Figure 5 captures the value of C, as used in Figure 4. C > 0 for decoys to the right of $x_1 = (7, 3)$ represents the compromise effect, and C < 0 for decoys near x_1 represents the similarity effect. This diagram involves 2,000 simulations for each choice set, with values of x_{31} and x_{32} changing in increments of 0.05. The three lines in this figure represent varying values of a_0, with $a_0 = 0$, $a_0 = 10$, and $a_0 = \infty$, respectively. Note that increasing a_0 decreases the impact of the associative bias. This reduces the incidence of the compromise effect, while increasing the incidence of the similarity effect. Not surprisingly, there is no compromise effect for $a_0 = \infty$, as attribute weights are equal in this setting. In contrast, the moderate similarity decoy does not generate the similarity effect for $a_0 = 0$. Rather, it increases the choice share of x_1. This happens because the increase to expected preferences for x_1 with the addition of s^m is stronger than the competitive effect of s^m. For $a_0 = 10$, the value used in the simulation in Figure 4, there is an emergence of the compromise effect for decoys more extreme than (8.5, 1.5) and the similarity effect for decoys between (6, 4) and (8.5, 1.5).

Also note that Figure 4 does not display phantom decoy effects. This is because the simulations used in Figure 4 assume that x_3 can be selected. Figure 6 displays a similar series of simulations, configured so that the decoy option x_3 is unavailable. As in Figure 4, x_{31} and x_{32} change in intervals of 0.1, with each simulation run 500 times. The shade at the point (a, b) in the figure corresponds the value of C (defined above) with $x_{31} = a$ and $x_{32} = b$. Finally,
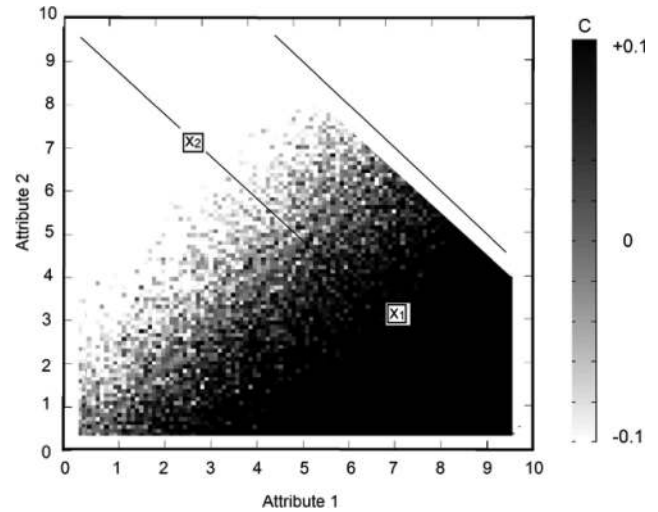
Figure 6. Choice shares of x_1 relative to x_2 at various decoy locations, when the decoy is unavailable. The shade at any point (a, b) captures the bias C created in favor of x_1 relative to x_2 when the phantom decoy $x_3 = (a, b)$ is added to the binary set $X = \{x_1, x_2\}$. Darker shades represent increases in choice shares of x_1, and lighter shades represent increases in choice shares of x_2.

positive values of C correspond to a bias in favor of x_1 and are associated with darker shades on the grid, whereas negative values of C correspond to a bias in favor of x_2 and are associated with lighter shades on the grid.

Many of the patterns observed in Figure 4 emerge in Figure 6 as well. Phantom dominated decoys and phantom extreme decoys bias choice shares in favor of the dominating and compromise core options, in the same way as their available counterparts. Unlike in Figure 4, however, dominating phantom decoys do not reduce the choice shares of the target. Dominating decoys in Figure 4 are available, and due to their attribute overlap with the dominated target, they disproportionately reduce the target's choice share, compared to the choice share of the nondominated competitor. The phantom dominating decoys in Figure 6 are, however, unavailable and thus do not have a competitive effect on the choice share of the target. Instead, they only alter attribute accessibility and expected preference in favor of the target, ultimately increasing its choice share relative to the nondominated competitor. This effect is enhanced for range phantom decoys relative to frequency phantom decoys (more formally discussed in the section above).

Phantom similarity decoys also have this property. These decoys reduced the choice share of the similar target, as shown in Figures 4 and 5. In Figure 6, however, they are unavailable, and the similarity effect does not emerge. Figures 4 and 6 thus highlight a key difference between phantom and nonphantom decoys: Choice set dependence caused by the effect of associative connectivity on expected preference emerges in the presence of both phantom and nonphantom decoys, whereas choice set dependence caused by preference covariance emerges only in the presence of nonphantom decoys.

The insights regarding the standard similarity effect can also be used to explain the binary similarity effect: Sequential attribute sampling affects preference covariance in such a way that alter-
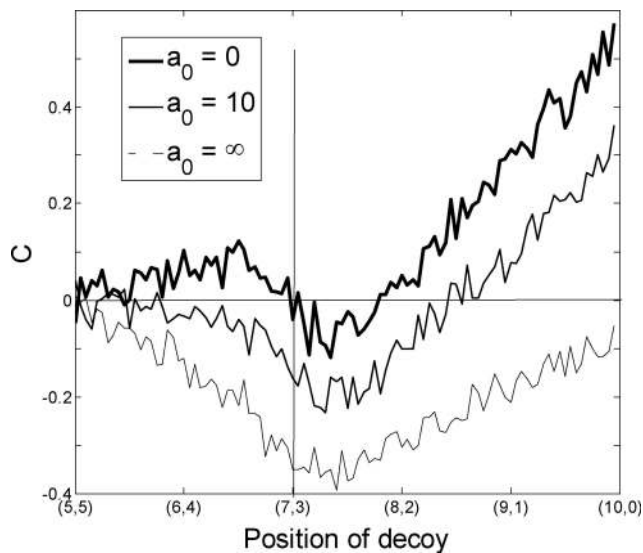

Figure 5. Strength of the similarity and compromise effects for various decoy locations and various values of a_0. $x_1 = (7, 3)$ and $x_2 = (3, 7)$; x_3 is varied along the horizontal axis. The vertical axis captures the bias C created in favor of x_1 relative to x_2 when x_3 is added to the binary set $X = \{x_1, x_2\}$.

natives that are stronger on similar attributes are, all else constant, less likely to be chosen than alternatives that are stronger on dissimilar attributes. In particular, when a similar attribute is sampled, the preferences for the available alternatives increase by similar amounts; however, when a dissimilar attribute is sampled, these preferences increase by dissimilar amounts. Thus, all else equal, fewer samples of the dissimilar attribute are necessary for the alternative strongest on this attribute to have a higher preference and subsequently be selected. This creates a choice bias in favor of the alternative strongest on the dissimilar attribute.

As an example, consider x_1 and x_2 defined above, as well as $x_3 = (7.1, 1)$ and $x_4 = (3.1, 5)$. Note that x_1 is strongest on the attribute on which x_1 and x_3 differ the most, whereas x_2 is strongest on the attribute on which x_2 and x_4 differ the most. When the associative accumulation model is simulated 2,000 times on these choice options (with the parameter configurations used above), x_1 is chosen from $\{x_1, x_3\}$ 99.0% of the time, x_2 is chosen from $\{x_2, x_3\}$ 75.6% of the time, x_1 is chosen from $\{x_1, x_4\}$ 84.6% of the time, and x_2 is chosen from $\{x_2, x_4\}$ 99.8% of the time.

Note that associative connectivity (and its effect on expected preference) does not generate this effect. However, as associative connectivity affects attribute sampling, which is the primary mechanism determining the binary similarity effect, it can moderate the strength of this effect. When both the available alternatives have large amounts of the dissimilar attribute and small amounts of the similar attribute, the proposed associative mechanism and its effect on expected preference work in the direction of preference covariance to amplify the binary similarity effect (the alternative strongest on the dissimilar attribute would be even more likely to be chosen). When the opposite is the case, the associative mechanism works against preference covariance to dilute the binary similarity effect. That said, the binary similarity effect documented above emerges both when the associative connectivity is disabled ($a_0 = \infty$) and when the associative connectivity is at its strongest ($a_0 = 0$), indicating that the binary similarity effect is robust to associative connectivity for the above choice options, in the proposed model.

Combined effects. Thus far, this paper has explored associative connectivity and sequential accumulation with the assumption that they have conflicting effects on choice. This section, however, analyzes the way these two mechanisms jointly explain the moderators of the compromise effect, as well as the dependence of the asymmetric dominance and compromise effects on time.

Consider the addition of an extreme decoy, e, to a binary choice set. In this case, both the decoy's and the target's primary attribute is attribute 1. If the target is an inferior alternative that is extremely weak on this attribute, such as x_1', then every time the decision maker samples this attribute, the preference for the decoy will increase substantially more than the preference for the target. The extreme decoy will ultimately capture almost all of the target's choice share, generating the detraction compromise effect.

The strength of the compromise effect thus depends on the position of the target. If the target is a sufficiently desirable alternative, the associative mechanisms in the proposed model will generate the compromise effect. If the target is sufficiently undesirable, sequential accumulation will cause the extreme decoy to compete with the target and generate a reversal of the compromise effect. Figure 7 demonstrates this with three simulations. It plots the magnitude of the compromise effect for varying x_1 and for
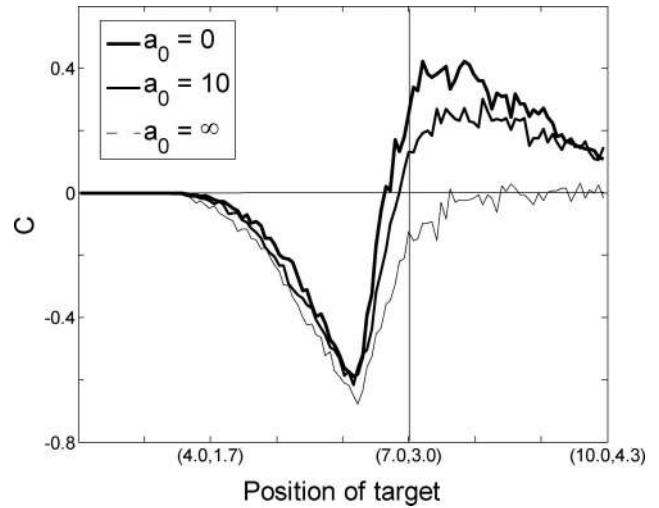

Figure 7. The detraction and enhancement compromise effects for various target (x_1) locations and various values of a_0. $x_2 = (3, 7)$ and $x_3 = (9, 1)$; x_1 is varied along the horizontal axis. The vertical axis captures the bias C created in favor of x_1 relative to x_2 when x_3 is added to the binary set $X = \{x_1, x_2\}$.

fixed $x_2 = (3, 7)$ and $x_3 = (9, 1)$. The horizontal axis corresponds to different target positions for $x_1 = (\gamma, \cdot 7, \gamma, \cdot 3)$, with γ changing in increments of 0.1 (with 2,000 simulations for each value of γ). This is equivalent to gradually increasing the magnitude of the target along the vector $(7, 3)$. The vertical axis captures the change in the choice share of x_1 relative to x_2 in the three-alternative versus the two-alternative choice set. This is equal to C, as described in the previous paragraphs. Positive values of C correspond to the compromise effect, whereas negative values correspond to the reversal of the compromise effect. As with prior figures, Figure 7 reports results for $a_0 = 0$, $a_0 = 10$, and $a_0 = \infty$.

As predicted, the detraction compromise effect does emerge for sufficiently undesirable targets, for all three values of a_0. The standard compromise effect also emerges for $a_0 = 0$ and $a_0 = 10$, as shown in the previous simulations. The compromise effect does not emerge for $a_0 = \infty$ (when the associative mechanism is inactive). Additionally C = 0 is observed for highly desirable and highly undesirable targets, as in these settings the target is either always or never chosen, regardless of the decoy's presence. This causes the compromise effect to have a nonmonotonic relationship with the target's desirability. In Figure 7, the compromise effect peaks near $x_1 = (8.0, 3.4)$.

Both the associative connectivity mechanism and the sequential accumulation mechanism can also be used to explain the dependence of the compromise effect on the distance of the extreme alternatives. As the decoy becomes more distant (i.e., more extreme, with a higher amount of attribute 1), the associative mechanism generates a stronger increase in the expected preferences of x_1, whereas the sequential accumulation mechanism leads to a reduction in the similarity effect. Additionally, as the competitor becomes more distant (more extreme but with a higher amount of attribute 2), the associative mechanism generates a stronger reduction in its expected preferences. Subsequently, the compromise effect is predicted to increase with an increase in the extremity of the competitor and the decoy.

This is verified in Figure 8, which presents the strength of the compromise effect as the competitor and the decoy increase in their distance from $x_1 = (7, 3)$. These alternatives move along the line $x_{i1} + x_{i2} = 10$ in increments of 0.1 for each attribute with 500 simulations at each value. Darker shades at any point indicate a stronger compromise effect (captured by the value C), for corresponding decoy and competitor locations. Note that shades are darkest at the top right corner of the diagram, which corresponds to distant decoys and competitors.

Finally, both the proposed mechanisms can be used to explain the dependence of the context effects on time. Associative connectivity is of course needed to generate the asymmetric dominance and compromise effects. Sequential accumulation generates the similarity effect and imposes a time dependence on all three effects. At earlier times in the decision process, when very few attributes have been sampled, extreme alternatives will be disproportionately desirable. The compromise effect should thus be reversed at these times, and it should grow as the deliberation time increases. The asymmetric dominance effect should also grow, as the difference in the expected preferences of the target and the competitor increases with deliberation time. At earlier time periods, when this difference is small, random noise will have a stronger impact, reducing the strength of the asymmetric dominance effect. This impact will decrease over the time course of the decision. Unlike the compromise effect, however, the asymmetric dominance effect is predicted to emerge at all time periods. Finally, the similarity effect is predicted to decrease over time. Preference correlations, caused by attribute overlap, decrease as more attributes are sampled. When only one or two attributes have been sampled, the preferences of similar options are nearly identical and the similarity effect is at its strongest. As more attributes are sampled, these preferences diverge and the similarity effect weakens. However, because the preferences of similar options are always more correlated than the preferences of dissimilar options, a (weak) similarity effect is guaranteed to emerge at all time periods.

Figure 9 plots the strength of the three context effects with time, in terms of C, as defined above. It sets $x_1 = (7, 3)$, $x_2 = (3, 7)$, $x_3 = (9, 1)$ for the compromise effect; $x_3 = (6.5, 2.5)$ for the asymmetric dominance effect; and $x_3 = (7.1, 2.9)$ for the similarity effect. Two thousand simulations are performed for each time period. Note that the compromise effect is actually reversed for earlier times, emerging only after $T = 10$. The asymmetric dominance effect, in contrast, always emerges, but grows with time. Finally, as predicted, the similarity effect also always emerges, but it reduces over time.

Alignability Effects

Findings.   A robust choice-set-dependent phenomenon relates to the alignability (Markman & Medin, 1995; Zhang & Markman, 1998, 2001), the comparability (Nowlis & Simonson, 1997), or the commonality (Kivetz & Simonson, 2000; Lipe & Salterio, 2000; Slovic & MacPhillamy, 1974) of the choice alternatives. According to the alignability effect, individuals place a higher weight on an attribute if it is common to multiple alternatives, relative to if it is unique to one alternative with no correspondence to the other alternatives. Altering a choice set to make an attribute the common attribute increases its weight, whereas altering a choice set to make an attribute unique decreases its weight. These changes to the weights of the attributes can lead to reversals between the joint and the separate evaluation of choice alternatives, as well to reversals in choices across different sets of alternatives.

Slovic and MacPhillamy (1974) first documented the alignability effect with regard to judgments. Subsequently Markman and Medin (1995) replicated this effect in the domain of choice, using subject-generated measures of alignability. Lipe and Salterio (2000) demonstrated the existence of the alignability effect in a naturalistic setting. Similar results were noted by Huber and Mc-Cann (1982), who found that individuals place a lower weight on attributes for which information is not available and cannot be compared relative to attributes whose information is known and can be compared.

Although this research has generally explored alignability effects across different equally sized choice sets, Nowlis and Simonson (1997) found similar differences between single-option evaluations and evaluations in binary sets. Unique attributes are relatively more important when alternatives are considered in isolation. In contrast, common attributes receive greater weight when alternatives are compared against each other jointly. Nowlis and Simonson also found that this particular setting can lead to reversed choices, if common attributes and unique attributes support different alternatives. Kivetz and Simonson (2000) discovered a similar type of reversal across multiple choice sets, when the common dimensions in the choice sets are varied. In particular, x_1 can be chosen over x_2 if it is strongest on the common attribute in a binary choice set consisting of these two alternatives. x_2 can likewise be chosen over x_3 in the choice set $\{x_2, x_3\}$. Finally, x_3 can be chosen over x_1 in $\{x_1, x_3\}$, generating a choice cycle. Lastly, Zhang and Markman (2001) noted that alignability-based choice reversals emerge only in low-motivation conditions. Alternatives that are strongest on the common dimension but are not the
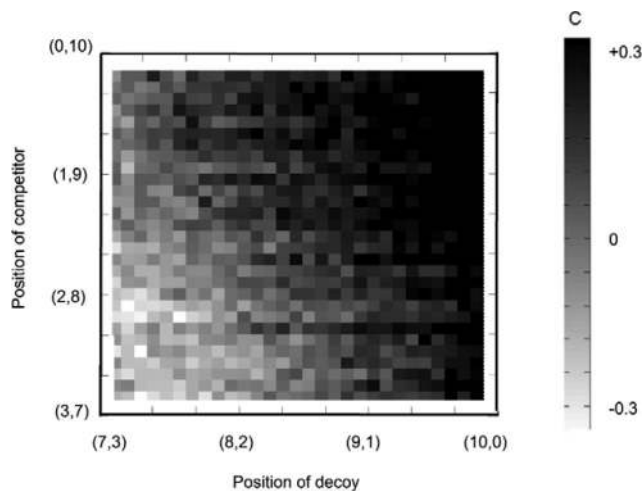

Figure 8.   The compromise effect as a function of decoy and competitor location. The decoy x_3 varies along the horizontal axis, and the competitor x_2 varies along the vertical axis. The shade at any point captures the bias C created in favor of $x_1 = (7, 3)$ relative to the corresponding x_2 when the corresponding x_3 is added to the binary set $X = \{x_1, x_2\}$. Darker shades correspond to higher choice shares of x_1, and lighter shades correspond to higher choice shares of x_2.
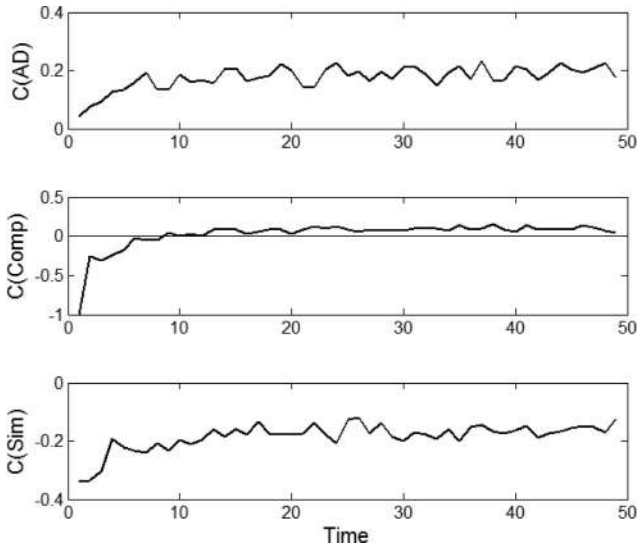
Figure 9. Asymmetric dominance (AD; top panel), compromise (Comp; middle panel), and similarity (Sim; bottom panel) effects over time. Here, $x_1 = (7, 3)$ and $x_2 = (3, 7)$ are the core options, and $x_3 = (6.5, 2.5)$, $x_3 = (9, 1)$, and $x_3 = (7.1, 2.9)$ are the decoy options for the asymmetric dominance, compromise, and similarity effects. The horizontal axis corresponds to the exogenously controlled deliberation time, and the vertical axis corresponds to the bias C created in favor of x_1 relative to x_2 when x_3 is added to the binary set $X = \{x_1, x_2\}$.

best overall alternatives are more likely to be chosen when decision makers are not motivated to make an accurate choice. However, when participants are involved in the experiment, the alignability of attributes does not influence which alternative is chosen.

Explanation. The associative accumulation model in this paper provides a simple explanation for the alignability effect. Common attributes are attributes that are contained in, and thus associated with, multiple alternatives, and unique attributes are attributes that are contained in, and thus associated with, only one alternative. Because the accessibility of an attribute is proportional to its total amount in the choice set, a particular attribute has a higher weight if it is contained in multiple alternatives, relative to if it is contained in only one alternative.

More formally, consider three choice alternatives x_1, x_2, and x_3, defined on three attributes $j = 1, 2, 3$, such that $x_{13} = x_{22} = x_{31} = 0$. An example of such a set of alternatives is $x_1 = (y, z, 0)$, $x_2 = (z, 0, y)$, and $x_3 = (0, y, z)$, where y and z are any positive scalars. If we write the choice sets as $X^{12} = \{x_1, x_2\}$, $X^{13} = \{x_1, x_3\}$, and $X^{23} = \{x_2, x_3\}$, and the weights on attribute j in these choice sets as w_j^{12}, w_j^{13}, and w_j^{23}, we can easily show that $w_1^{12} > w_1^{13}$, $w_1^{12} > w_1^{23}$, $w_2^{13} > w_2^{12}$, $w_2^{13} > w_2^{23}$, and $w_3^{23} > w_3^{12}$, $w_3^{23} > w_3^{13}$. In particular, attributes in the each binary choice set are more accessible and are weighted higher if they are common to the two choice alternatives compared to if they are unique to one alternative. This result holds for all values of a_O.

If it is assumed that individuals with low motivation sample fewer attributes and make their decisions more quickly than individuals in high-motivation conditions, then the associative accumulation model can also capture alignability-based choice reversals. Common attributes are more accessible and have a higher

probability of sampling relative to unique attributes. Early on during the decision process, when fewer attributes have been sampled, this increased probability leads to the increased choice share for the alternative that is dominant on the common attribute. If this alternative is not the most frequently chosen alternative in separate evaluation, this can lead to a choice reversal. As the decision progresses, however, the accumulation of affective values averages out, and the choice reversal disappears.

Figure 10 demonstrates this with a simulation. It sets $x_1 = (7, 3, 0)$. However, x_2 is set as $(3, 0, 8)$. The horizontal axis of the figure captures the length of deliberation time, and the vertical axis of the figure represents the difference in choice proportion for x_1 relative to x_2 in the binary choice set. This is $C = (N[x_1 \text{ chosen}] - N[x_2 \text{ chosen}])/(N[x_1 \text{ chosen}] + N[x_2 \text{ chosen}])$. Note that the expected preference state of x_2 in isolated choice set $\{x_2\}$ is higher than the expected state of x_1 in isolated choice set $\{x_1\}$. This is true for any time. Hence, x_2 should always be selected over x_1 in isolated evaluation. The choice share of x_2 is also higher than x_1 in the joint choice set $X = \{x_1, x_2\}$. This, however, only emerges after a sufficient period of time. If the decision maker is allowed to sample only one attribute, the alternative that dominates on the most accessible attribute—the common attribute—is the alternative that will be selected. Hence, x_1 is chosen over x_2 in the joint choice set, when $T = 1$. This process can also be used to generate a choice cycle with the alternatives $x_1 = (7, 3, 0)$, $x_2 = (3, 0, 7)$, and $x_3 = (0, 7, 3)$. Here, at $T = 1$, x_1 is chosen over x_2 in the choice set $\{x_1, x_2\}$, x_2 is chosen over x_3 in the choice set $\{x_2, x_3\}$, and x_3 is chosen over x_1 in the choice set $\{x_1, x_3\}$.

Less Is More Effects

Findings. A number of researchers have explored the impact of adding irrelevant or mediocre attributes on the preferences for a choice alternative. This work finds that adding such attributes to a
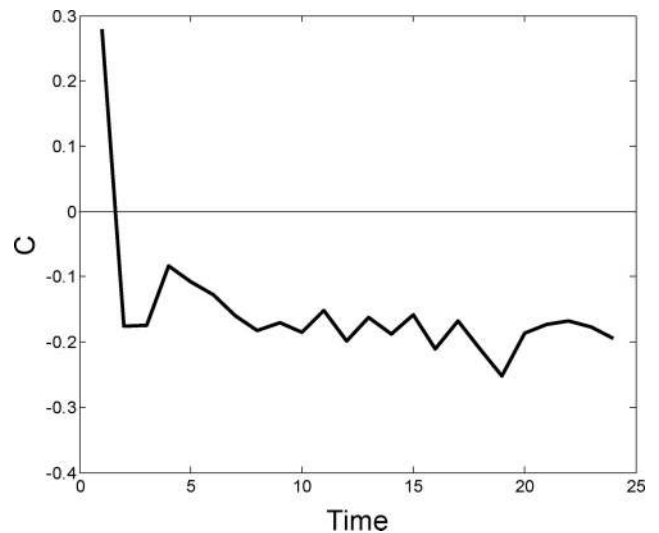

Figure 10. Alignability effects over time. Here, there is a three-attribute binary choice with $x_1 = (7, 3, 0)$ and $x_2 = (3, 0, 8)$. The horizontal axis corresponds to the exogenously controlled deliberation time, and the vertical axis corresponds to preference for x_1 relative to x_2 in the joint choice set $X = \{x_1, x_2\}$.

choice alternative decreases the overall evaluation of the alternative, even though these attributes are positively valued. Simonson et al. (1994) first explored this with regard to real promotions used in the marketplace. They found that adding trivial attributes—ones that the decision maker does not particularly value—to a choice alternative reduces the likelihood that that alternative is purchased. Hsee (1998) and List (2002) documented similar anomalies but with alternatives consisting of large groups of items (dinnerware and baseball cards, respectively). Both these papers found that increasing the size of these groups by adding low but positively valued items (e.g., cups or baseball cards in poorer condition) to the initial groups decreases their reported evaluation. This leads to the paradoxical less is more effect, in which clearly inferior alternatives appear more desirable than alternatives that dominate them. Importantly, however, this effect holds only in separate evaluation. When the two alternatives are compared jointly, in the same choice set, individuals always choose the dominant alternative.

Explanation. The associative accumulation model can capture this phenomenon using the relativism in attribute weighting. As the total weight on all the attributes in the choice set is held constant at one, increasing the amount of an attribute in an alternative increases the weight on the added attribute but also reduces the weight on other attributes. Adding an attribute to a choice alternative thus does not necessarily increase the alternative's expected preference. If the reduction in the weighting of other, more valuable attributes is stronger than the added value of the novel attribute, the choice alternative's expected preferences can drop. This is more likely when the added attribute is trivial or low valued.

Consider, for example, the evaluation of $x_1 = (x_{11}, x_{12})$. It can be assumed that attribute 1 is a regular attribute with $V_{11} = x_{11}^{1/2}$. Let it be assumed, however, that attribute 2 is a trivial and less valued attribute. With this assumption, one can write $V_{12} = \gamma \cdot x_{12}^{1/2}$ for $\gamma < 1$. Now, increasing the amount of x_{12} increases the total value $V_{11} + V_{12}$. This does not mean the expected inputs into the preference accumulation layer, $U_1 = w_1 \cdot V_{11} + w_2 \cdot V_{12}$, will also increase. If the attribute is trivial and γ is small enough, the increase in V_{12} will be offset by the reduction in w_1 and the lower weight on V_{11}. Figure 11 demonstrates this for $x_{11} = 7$ and varying values of x_{12} and γ. Note that U_1 is increasing in x_{12} for high values of γ. For $\gamma = 0.1$, however, increases to x_{12} actually reduce U_1. In fact, we can show that increasing x_{12} for $x_1 = (7, 3)$ will necessarily reduce U_1 for values of γ lower than 0.255.

These results, however, apply only in separate evaluation. Consider the joint evaluation choice set $X = \{x_1, x_2\}$. x_1 and x_2 are identical on their most valued attribute, attribute 1, but not on the trivial attribute, attribute 2. In particular, $x_{22} > x_{12}$, and $V_{11} = V_{21} > V_{22} > V_{12}$. Here, even though the weight on attribute 1 is lower in joint evaluation relative to the separate evaluation of these alternatives, the less is more effect disappears, and $U_2 > U_1$. This is because the primary mechanism behind the less is more effect is the differential weighting of the most valuable attribute across single-option choice sets. When the two alternatives are in the same set, however, this type of differential weighting is impossible. Ultimately, $U_2 = w_1 \cdot V_{21} + w_2 \cdot V_{22} > w_1 \cdot V_{11} + w_2 \cdot V_{12} = U_1$, as $V_{11} = V_{21}$ but $V_{22} > V_{12}$.
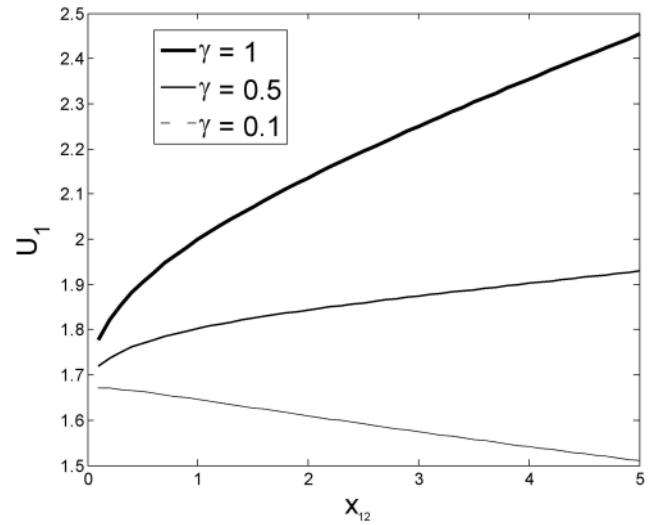

Figure 11. Expected inputs into the preference node for x_1 with $x_{11} = 7$ and varying amounts of x_{12}. Note that low values of γ correspond to settings where attribute 2 is trivial (positively but weakly desirable). Expected inputs are proportional to expected preference states over time.

Reference Dependence

I now explore the impact of changing the salience of alternatives in the choice set on choice shares. The effects of alternative salience operate entirely through the associative mechanism in the proposed model. As formalized in Equation 1, highly salient alternatives, such as reference points, exert a stronger effect on attribute accessibility than do less salient alternatives. This can alter expected preferences and reverse choice, as formalized in Equations 6 and 7. In the subsequent sections I explore this in more detail, finding that the associative accumulation model provides a unitary explanation for every reference-dependent behavioral anomaly without requiring loss aversion. The discussion in these sections is limited to the associative mechanism, as preference covariance does not change significantly as reference points are varied. The salience of alternatives that are not reference points is fixed at $s_i = 1$, whereas the salience of reference points is $s_i > 1$.

Reference Dependence Effects

Findings. Perhaps the best known reference dependence effects are the endowment effect and the status quo bias. The former refers to the finding that individuals are less inclined to give up an object once they have acquired it than they are to obtain it in the first place (Birnbaum & Stegner, 1979; Kahneman, Knetsch, & Thaler, 1990; Thaler, 1980), whereas the latter refers the finding that individuals often choose the status quo alternative over other alternatives (Knetsch, 1989; Samuelson & Zeckhauser, 1988). In Figure 12, both effects can be represented by the increased choice share for x_1 over x_2, if x_1 is the reference point, compared to if x_2 is the reference point.

Tversky and Kahneman (1991) noted the existence of two other reference dependence effects. The first refers to the apparent preference for improvements versus trade-offs. In particular, Tversky and Kahneman found that a reference point that is dominated
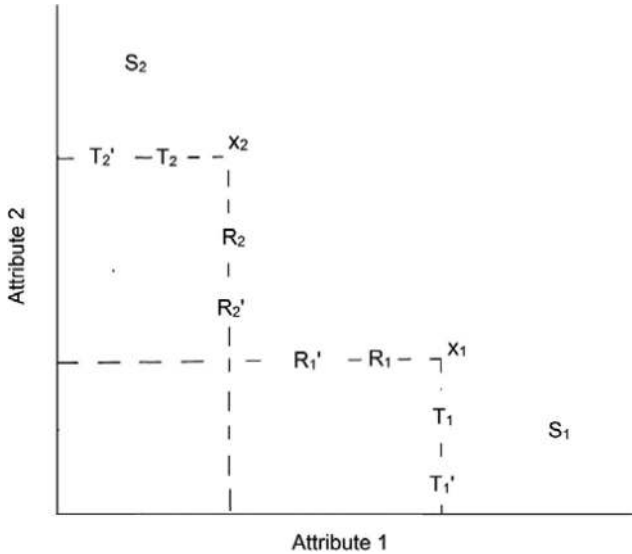
Figure 12. A graphical description of observed reference point effects. x_1 and x_2 are core options. The other points represent reference point locations that have been shown to bias the relative choice shares of the core options. For example, compared to reference point R_2, reference point R_1 causes an increase in the choice share of x_1 relative to x_2.

by an alternative leads to choices in favor of that alternative. In Figure 12, this implies that x_1 is more frequently chosen over x_2 when R_1 is the reference point, relative to when R_2 is the reference point.

The second reference point effect, noted by Tversky and Kahneman (1991), is titled the advantages and disadvantages effect. This effect refers to the apparent preference for a small gain and a small loss from the reference point over a large gain and a large loss from the reference point. In Figure 12, this effect implies that x_1 is more frequently chosen over x_2 when S_1 is the reference point, relative to when S_2 is the reference point.

Explanation. These three findings have generally been seen as evidence for reference-dependent loss aversion in preferential choice (Tversky & Kahneman, 1991; Usher & McClelland, 2004). However, the proposed associative mechanism can also explain these three effects. As outlined above, a salient reference point increases the accessibility of its component attributes. This biases the expected preferences for the other alternatives in the choice set, depending on whether they contain these attributes or not. Alternatives that are highly valuable on the reference point's primary attributes will have higher preferences relative to alternatives that have low values on the reference point's primary attributes.

Take any two alternatives x_1 and x_2 in a two-attribute choice space. Also assume that these two alternatives do not dominate each other and that attribute 1 is x_1's most valuable attribute and attribute 2 is x_2's most valuable attribute. This implies that $V_{11} > V_{12}$ and $V_{22} > V_{21}$. Now consider the task in which x_1 is the reference point and the corresponding task in which x_2 is the reference point. In the former case $s_1 > s_2 = 1$, whereas in the latter case $s_2 > s_1 = 1$. Because x_1 is stronger on attribute 1 than attribute 2 and x_2 is stronger on attribute 2 than attribute 1, Equation 4 implies that weight on attribute 1 when x_1 is the reference point is higher than when x_2 is the reference point.

Subsequently, Equation 7 implies that the preference node for x_1 receives higher inputs than that of x_2 when x_1 is the reference point relative to when x_2 is the reference point. If the expected inputs to x_1 and x_2 when x_i is the reference point are written as U_1^{xi} and U_2^{xi}, respectively, this means that $U_1^{x1} - U_2^{x1} > U_1^{x2} - U_2^{x2}$. Subsequently, x_1 is more likely to be selected when it is the reference point relative to when x_2 is the reference point.

Now consider the case where alternatives R_1 or R_2 are the reference points. Once again, because R_1 has more of attribute 1 and less of attribute 2 than R_2, $U_1^{R1} - U_2^{R1} > U_1^{R2} - U_2^{R2}$, giving the improvements versus trade-offs effect. The same argument holds when alternatives S_1 or S_2 are the reference points, giving the advantages and disadvantages effect.

Additional Reference Dependence Effects

Findings. A number of reference-dependent anomalies have been documented, in addition to the above phenomena. In particular, Herne (1998) found that the improvements versus trade-offs effect also exists for more extreme reference points that are dominated by the target on the competitor's primary dimension. Additionally, this effect increases as the reference point moves farther away from the target alternative. Herne (1998) also noted that the regular improvements versus trade-offs effect reduces as the reference alternative moves farther away from the target and closer to the competitor. In Figure 12 this implies that the choice share of x_1 relative to x_2 is higher if T_1 is the reference point, relative to if T_2 is the reference point. This difference increases for T_1' and T_2'. Additionally, the difference in the preferences for x_1 and x_2 is higher for the reference points R_1 and R_2, relative to the reference points R_1' and R_2'.

Finally, deliberation time effects have been documented for reference points. In particular, Ashby, Dickert, and Glöckner (2012) have found that the endowment effect grows with time. Although these results have been established with willingness-to-accept and willingness-to-pay measures, it is likely that they also hold for explicit choices between alternatives (as the two types of responses are based on a common underlying preference measure).

Explanation. The accessibility biases generating the standard reference-dependent phenomena, discussed in the section above, can also explain these effects. Herne (1998) documented an increased choice share for the target alternative as the reference points get stronger on the target's primary dimension or weaker on the competitor's primary dimension. Note that T_1' is weaker than T_1 on attribute 2, and R_1 is stronger than R_1' on attribute 1. Due to the associative mechanisms at play in the proposed model, this implies that the weighting bias in favor of x_1 relative to x_2 is higher for T_1' relative to T_1 and higher for R_1 relative to R_1'. This insight also holds for the relative impact of T_2' and T_2 and R_2 and R_2'.

The associative accumulation model can also explain why the endowment effect increases with deliberation time. This explanation is similar to that regarding the relationship of the asymmetric dominance effect with time: The associative mechanisms in the proposed model generate a difference in the expected preferences for the endowed reference point relative to those for the nonendowed competitor. These differences grow over time. The random noise in the preference accumulation layer, which is constant across time, subsequently has a smaller effect on choice shares later on in the decision process (when the preference difference is

large) relative to early in the decision process (when the preference difference is small).

Figure 13 explores this with a simulation, with the same set of parameters used in the choice set dependence section. The horizontal axis captures deliberation time (with 2,000 simulations for each time unit). The vertical axis captures the difference in the choice share of $x_1 = (7, 3)$ relative to $x_2 = (3, 7)$ when x_1 is the reference point relative to when x_2 is the reference point. This is defined using the value $C = C^{x1} - C^{x2}$, where $C^{xi} = (N[x_1 \text{ chosen}] - N[x_2 \text{ chosen}])/(N[x_1 \text{ chosen}] + N[x_2 \text{ chosen}])$ when x_i is the reference point. Varying values of s_i are used for the reference point, whereas a fixed value of $s_i = 1$ is used for the competitor.

As shown in Figure 13, the endowment effect does grow with time. Additionally increasing the salience of the reference point can also increase the endowment effect. Ultimately, the endowment effect is largest for long deliberation times with highly salient reference points.

The Negative Endowment Effect

Findings. One peculiar finding relating to reference dependence is the reversal of the endowment effect for negative goods. In particular, when given the choice among undesirable alternatives, endowing the decision maker with one of the alternatives makes him or her less likely to choose that alternative from the choice set. This violation of loss-averse reference dependence was first demonstrated by Brenner et al. (2007). Subsequently, Bhatia and Turan (2012) showed that individuals generally focus on the primary attributes of the endowed alternative. Because the endowed alternative in this scenario has negative values on its attributes, increased attention toward the endowed alternative's attributes relative to those of the nonendowed alternative leads to a reduced preference for the endowed alternative.
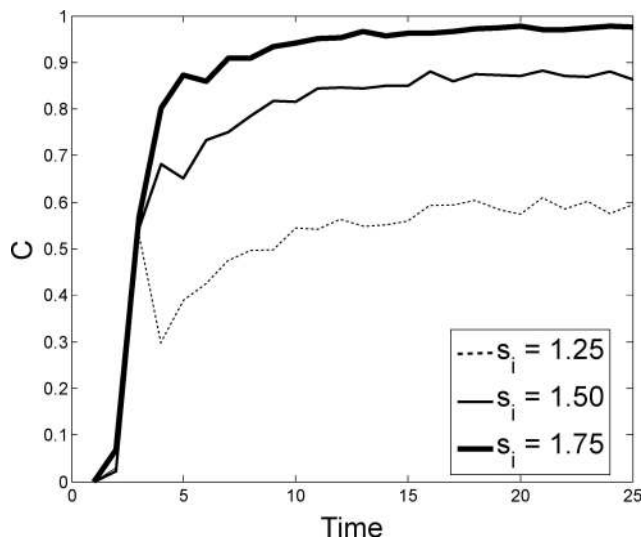

Figure 13. The endowment effect over time. Here, $x_1 = (7, 3)$, $x_2 = (3, 7)$, and s_i is the salience of the reference point. The salience of the competitor is kept constant at $s = 1$. The horizontal axis captures time, and the vertical axis captures the bias C in favor of x_1 relative to x_2 when x_1 is the reference point, compared to when x_2 is the reference point.

Explanation. The associative accumulation framework, discussed above, is able to capture this more rigorously. Particularly, let it again be assumed that the decision maker is presented with alternatives x_1 and x_2, with $x_{11} > x_{21} > 0$, $x_{22} > x_{12} > 0$, $x_{11} > x_{12} > 0$ and $x_{22} > x_{21} > 0$. Because these attributes are undesirable and V_{ij} are negative and decreasing in attribute amounts, $0 > V_{21} > V_{11}$ and $0 > V_{12} > V_{22}$.

As above, consider the task in which x_1 is the reference point and the parallel task in which x_2 is the reference point. In the former case, $s_1 > s_2 = 1$ generates a higher weight on attribute 1, x_1's primary attribute. In the latter case, $s_2 > s_1 = 1$ generates a higher weight on attribute 2, x_2's primary attribute. Note that this bias in accessibility is the same as in the standard, positive, endowment effect. Because $0 > V_{21} > V_{11}$ and $0 > V_{12} > V_{22}$, however, this leads to $U_1^{x2} - U_2^{x2} > U_1^{x1} - U_2^{x1}$. This means that the probability of choosing x_1 over x_2 necessarily decreases if x_1 is the undesirable reference point, compared to if x_2 is the undesirable reference point. This is a reversal of the endowment effect.[5]

Gain–Loss Asymmetry

These results are generated without assuming an explicit loss-averse asymmetry in the valuation of the attributes. However, an asymmetry in the valuation of gains and losses is often assumed to be a psychologically realistic aspect of human choice behavior (see, e.g., Novemsky & Kahneman, 2005). Individuals place a higher value on losing some amount of an attribute than they do on gaining the same amount. More generally, moving from one alternative to another, holding all else constant except for one attribute, has a stronger impact on preferences if this change is a reduction in the attribute, relative to if it is an increase in the attribute.

This property can be captured by the associative accumulation model using only the associative mechanism outlined above. Intuitively, the reference point (i.e., the endowment) in the attribute increase scenario has less of the altered attribute than the reference point in the attribute decrease scenario. This implies that the attentional weight on the changed attribute is lower in the attribute increase setting relative to the attribute decrease setting. Subsequently, the changes to the expected preferences from increasing the attribute are lower than the corresponding changes from decreasing the attribute. This generates the gain–loss asymmetry.

More formally, consider two choice alternatives, x_1 and x_2, such that $x_{11} > x_{21}$ but $x_{1j} = x_{2j}$ for all $j > 1$. This means that x_1 and x_2 are identical on every attribute except attribute 1, on which x_1 is the stronger alternative. Because these attributes are desirable, this implies that $V_{11} > V_{21}$, and $V_{1j} = V_{2j}$ for all $j > 1$. Now, assume that the decision maker is initially endowed with x_2 but then is given x_1 (a gain). In this case, x_2 is the reference point and subsequently the more salient alternative. Using Equation 6, one can write the change in the decision maker's expected inputs to the preference accumulation layer as $\Delta_{12}^{x2} = U_1^{x2} - U_2^{x2} = w_1^{x2}(V_{11} - V_{21})$. Note that all valuations of

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[5] This intuition can also explain findings that are not currently attributed to reference dependence. Dhar, Nowlis, and Sherman (1999), for example, found that increasing the salience of a desirable alternative increases that alternative's choice share relative to the competitor. In contrast, increasing the salience of an undesirable alternative reduces its choice share. These effects are shown to stem from an enhanced attention toward the focal alternative's attributes.

all other attribute cancel out, as they are identical across the alternatives. Similarly, consider the setting in which the decision maker is endowed with x_1 and then given x_2 (a loss). Here, $\Delta_{12}{}^{x1} = U_1{}^{x1} - U_2{}^{x1} = w_1{}^{x1} \cdot (V_{11} - V_{21})$. Now, because $x_{11} > x_{21}$, but $x_{1j} = x_{2j}$ for all $j > 1$, one has $w_1{}^{x1} > w_1{}^{x2}$. Additionally, $V_{11} > V_{21}$. This implies that $\Delta_{12}{}^{x1} - \Delta_{12}{}^{x2} = (w_1{}^{x1} - w_1{}^{x2}) \cdot (V_{11} - V_{21}) > 0$, and that the change in expected preferences when x_1 is the endowed alternative and the change is experienced as a loss is greater than when x_2 is the endowed alternative and the change is experienced as a gain.

Discussion

The associative accumulation model is heavily influenced by previous dynamic models of multi-attribute choice, based on the preference accumulation framework. These models involve the sequential sampling of attributes and also allow for the differential sampling of attributes. An early such model is multi-alternative decision field theory (MDFT; Roe et al., 2001). MDFT is a connectionist extension of decision field theory (DFT; Busemeyer & Townsend, 1993), a dynamic stochastic model of preference. Unlike its predecessor, MDFT assumes that preference accumulation involves distance-dependent inhibition, which can explain the asymmetric dominance and compromise effects (see Hotaling, Busemeyer, & Li, 2010, and Tsetsos, Chater, & Chater, 2010, for a recent debate on this mechanism). The similarity effect, as well as the binary similarity effect, is explained by sequential accumulation (just as it is in this paper). In addition to explaining these three context effects, MDFT can account for the improvements versus trade-offs and advantages and disadvantages reference point effects (Busemeyer & Johnson, 2004). Extending DFT by adding a biased adjustment process for willingness-to-accept and willingness-to-pay judgments can explain the endowment effect (Johnson & Busemeyer, 2005). DFT also provides a unified account of a number of decision-making effects not related to task dependence (see, e.g., Busemeyer & Diederich, 2002).

A second model is leaky competitive accumulation (LCA), proposed by Usher and McClelland (2004). The LCA model incorporates Tversky and Simonson's (1993) context-dependent dimensional loss aversion within a sequential accumulation connectionist network. In LCA, dimensional loss aversion is able to account for the asymmetric dominance and compromise effects as well as Tversky and Kahneman's (1991) three reference point effects, whereas attentional switching can capture the similarity effect.

A Model of the Entire Decision Process

Although these theories present a significant advancement in the understanding of choice set and reference point effects, they encounter an important limitation. They model only the preference accumulation process. They do not explore the psychological structures involved in representing the choice task or the linkages between these structures and the processes responsible for attribute representation and attentional selectivity. Both these theories, for example, assume that the weights of the attributes present in the choice alternatives are higher than those of attributes absent from and irrelevant to the choice task. Although this is a justifiable assumption, a complete model of the decision process must be able to explain not only how sampled attributes are accumulated into

preferences but also how the choice task determines the attributes that are sampled.

The associative accumulation model provides a formal account of the entire decision process, from choice task representation, to attribute sampling, to preference accumulation. It does so by embedding the preference accumulation process used in MDFT and LCA in a simple associative network, with feed-forward connectivity between choice alternatives and their component attributes. The learning and the network dynamics of associative networks have been extremely well studied and have been used to explain behavior in a number of psychological domains.[6] Within the proposed framework, the effect of associative connectivity on attribute sampling is mathematically tractable, and simple derivations can outline the conditions under which novel choice alternatives or reference points will bias the relative expected preferences of the available alternatives. If this associative mechanism was embedded in an accumulation process without sequential sampling, analytical results regarding choice probabilities and response times could also be obtained (the bias in expected inputs caused by the proposed associative mechanism could, for example, be represented as a change in the drift rate in a standard diffusion model).

Memory and Inference

Choice set and reference-dependent effects in the associative accumulation model emerge due to a mechanism considered to play a large role in intuitive judgment, reasoning, and social judgment (Evans, 2008; Kahneman & Frederick, 2002; Morewedge & Kahneman, 2010; Sloman, 1996; Strack & Deutsch, 2004). In all these domains, associative processing generates quick and effortless judgments, albeit ones that are vulnerable to a number of systematic biases. The associative accumulation model is, as such, a direct extension of research on the broader psychological processes underlying judgment and decision making, rather than a unique, specialized theory, formulated to explain a narrow set of choice behavior.

An implication of this generality is that the associative accumulation model can also make predictions outside the domain of value-based choice. For example, it predicts that choice set effects documented in multi-attribute value-based decision making should also emerge in other psychological domains that involve the sequential aggregation of cue-based information. Indeed, Maylor and Roberts (2007) documented the asymmetric dominance and similarity effects in episodic memory judgments, and Trueblood (2012) documented the asymmetric dominance, compromise, and similarity effects in inference. Such effects cannot be captured by models such as LCA, unless one oddly assumes that memory and inference involve loss aversion.

Note that the context effects discussed in this paper have recently been found in perceptual decisions (Trueblood, Brown, Heathcote, & Busemeyer, in press) and in preferential choices involving evidence presented sequentially over time (Tsetsos et al., 2012). This suggests two things: Either associative processes are also involved in these alternate domains (affecting attention to

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[6] Note that neural networks are not essential for modeling the proposed process. The mathematical structure presented in this section can be captured with alternate cognitive frameworks as well.

sensory data or biasing the recall of evidence immediately prior to the decision), or the proposed associative mechanisms are not alone in generating context dependence. It is likely that both these possibilities are true.

Choice Process

The descriptive power of the associative accumulation model extends beyond choice behavior. This model is also able to provide a comprehensive account of a range of process-level findings regarding attribute attention. Fiske (1980), for example, showed that attention is biased toward attributes present in extreme quantities in available alternatives. Similarly, Gentner and Markman (1994) and Zhang and Markman (1998) noted that attributes common to multiple alternatives are more likely to be listed and recalled, compared to attributes present in only one alternative.

A large number of researchers have also discovered that reference points bias the decision maker's attention toward the attributes that they contain (Ashby et al., 2012; Bhatia & Turan, 2012; Carmon & Ariely, 2000; Johnson, Häubl, & Keinan, 2007; Nayakankuppam and Mishra, 2005; Pachur & Scheibehenne, 2012; Willemsen, Böckenholt, & Johnson, 2011). These findings suggest that salient alternatives such as reference points act as primes, increasing the accessibility of attributes that they are associated with and subsequently affecting choice.

The observed interplay of attribute attention with attribute extremity, attribute commonality, and choice alternative salience corresponds to the three main properties of associative connectivity that allow that allow the associative accumulation model to capture the choice-set-dependent and reference-dependent effects discussed in this paper. These findings are thus more than just support for the descriptive power of the model; they provide strong evidence validating the fundamental assumptions of the model itself. Further process-level data (involving, for example, subject reports of estimated attribute weights) can allow for additional tests of the associative accumulation model.

Conclusion

Research on decision making has used biases in the accessibility of underlying attributes to explain a range of choice behavior. This includes choices driven by exogenous influences on attribute accessibility, such as primes (Mandel & Johnson, 2002), response modes (Tversky, Sattath, & Slovic, 1988), and anchors (Chapman & Johnson, 1999; Strack & Mussweiler, 1997), as well as choices driven by endogenous influences on attribute accessibility, such as preference feedback (Glöckner & Betsch, 2008; Simon, Snow, & Read, 2004) and deliberative heuristic search strategies (Glöckner & Betsch, 2008; Kahneman & Frederick, 2002). Biases in the accessibility (or weighting) of attributes and outcomes have also been used to model risky decision making (Birnbaum, 2008) and to taxonomize a range of heuristic rules (Shah & Oppenheimer, 2008). Finally, association-based accessibility plays a very important role in intuitive judgment, and numerous judgment-related phenomena have been attributed to the extensive dependence of attribute attention on attribute associations with the decision task (Evans, 2008; Kahneman, 2003; Kahneman & Frederick, 2002; Sloman, 1996; Strack & Deutsch, 2004).

This paper shows that accessibility biases generated by the associative connectivity between choice alternatives and their component attributes can also predict a large range of choice-set-dependent phenomena. This includes not only the context effects discussed in prior theoretical work but also alignability effects and less is more effects. Additionally, the associative mechanism in the proposed model is able to account for all of the findings regarding reference-dependent choice, without assuming an explicit loss-averse valuation function. In fact, a gain–loss asymmetry emerges implicitly from associative connectivity, and losses in the associative accumulation model loom larger than corresponding gains. Finally, the descriptive power of the proposed framework extends beyond choice behavior. Associations are shown to provide a simple account of the process-level findings regarding attention, memory, and other mechanisms involved in preferential choice.

This paper also presents a model of the *entire* decision process, one that specifies not only how information is accumulated into a decision but also how choice alternatives and attributes are represented and how this representation influences the information that is sampled. As such, it adds an important detail to previous models of preference accumulation. Although highly simplified, this model can easily be extended to incorporate other exogenous or endogenous determinants of attribute accessibility, such as primes, response modes, preference feedback, deliberate search, and cognitive control. This model can also be used to explore learning in preferential choice, both with regard to valenced rewards, as with standard reinforcement learning models, and with regard to spatiotemporal contingencies, as with models of associative memory. The associative accumulation model, in this sense, is a step toward an architecture of value-based choice; a broad, cognitive system that can be used to formalize the diverse psychological processes underlying preferential decision making, without sacrificing quantitative rigor, psychological and physiological realism, or descriptive validity.

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Correction to Doumas, Hummel, and Sandhofer (2008)

In the article "A theory of the discovery and predication of relational concepts" by Leonidas A. A. Doumas, John E. Hummel, and Catherine M. Sandhofer (*Psychological Review*, 2008, Vol. 115, No. 1, pp. 1–43), there are errors in the text, which are clarified below.

In Figure 4, the RB unit (rectangle) labeled "causer+gravity" should be connected to the causer PO (triangle) and a PO representing "gravity", and not to the PO representing "sun".

On page 11, in the 3rd paragraph under the Mapping subheading, the reference to Equation A13 in the Appendix should have been a reference to Equation A9.

Equation A9 in the Appendix should have read:

$$M_i = \sum_j a_j \left(3w_{ij} - Max(Map(i)) - Max(Map(j)) \right)$$

Also in the Appendix, the first sentence following Equation A12 should have read:

"where j is RB units to which PO unit i is connected, SEM_i is the semantic input to unit i, M_i is the mapping input to unit i, k is all PO units in the recipient that are not connected to the same RB (or RBs if unit i is connected to multiple RBs) as i, l is all other P units in the recipient currently in child mode that are not connected to the same RB (or RBs) as i, m is PO units connected to the same RB (or RBs) as i, and n is RB units in the recipient to which unit i is not connected (input from j is only included on phase sets beyond the first)."