Mon. Not. R. astr. Soc. (1973) **165**, 403–411

ASTEROID NUTATION ANGLES

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(Communicated by T. Gold)

(Received 1973 June 25)

SUMMARY

Approximate expressions are derived for (a) the characteristic time τ that it takes to align the rotation axis of an asteroid with its body axes, and (b) the average expected value of the nutation angle $\bar{\alpha}$. Alignment occurs because of internal energy dissipation: a portion of the stored strain energy, due to the bending stresses caused by the gyroscopic torque and due to the strains associated with the displacement of the centrifugal bulge during the wobble motion itself, is lost during each wobble period. For an asteroid of radius r, spinning with an angular velocity ω, $\tau = \mu Q/(\rho K_3{}^2 r^2 \omega^3)$, where μ is the asteroid rigidity, Q is its quality factor and ρ its density. $K_3{}^2$ is a shape factor which is about 10^{-2} for nearly spherical bodies and $10^{-1}\,H^2$ for non-spherical bodies with oblateness H. For reasonable parameter choices $\tau \approx 5\times 10^4\, T^3/(rK_3)^2$ yr with T the rotation period in hours and r in kilometres.

The asteroids, which are represented by a power law mass distribution, continually collide with one another, producing misalignment. A statistical treatment of the transferred angular momentum permits a rate of misalignment to be computed. From this and the alignment rate, $\bar{\alpha}$ is found to $\sim r^{-2\cdot 6}\omega^{-2\cdot 5}$ and to be very small for almost all observed asteroids, thus explaining why asteroid lightcurves are observed singly periodic. Small irregular asteroids, such as Geographos and 1971 F, are suggested to be the most likely candidates to be seen precessing.

1. INTRODUCTION

Asteroid brightnesses are known to vary with time and are believed to result primarily from an irregular asteroid shape rather than variable surface properties (Gehrels 1970). Taking into account the fact that the period of an asteroid's light curve is one-half the asteroid's rotation period, the mean rotation period for asteroids is about 9 hr (McAdoo & Burns 1973a).

Usually only a single period of oscillation is observed and this means that the asteroids are in pure spin (i.e. no precession) about their principal axes of maximum moment of inertia, henceforth called the C axis. Such a conclusion is surprising because collisions are continually occurring which misalign an asteroid's rotation axis from the C axis; thus precession should be seen even if asteroids are not left with a substantial nutation angle following their original accumulation. Prendergast (1958) and Burns (1971) have argued independently that pure spin can result only if internal energy dissipation is causing alignment. A knowledge of the time scale over which this alignment occurs is crucial for determining the importance of collisions amongst asteroids (McAdoo & Burns 1973a) and in arguments of asteroid origin (Kopal 1970).

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In this presentation we will give an approximate approach for determining the characteristic axial alignment time scales of precessing asteroids in terms of various asteroid parameters, such as asteroid size, rotation rate and material properties. This time scale will be combined with a misalignment time scale which is based on the angular momentum transferred between a population of asteroids having a power law mass distribution. The resulting ordinary differential equation will be solved and its long time solution gives an average expected value for the nutation angle and, hence, the properties of an asteroid that should be most easily observed to be precessing.

2. ASTEROID ALIGNMENT

Alignment caused by internal energy dissipation

An asteroid which does not undergo collisions will ultimately spin about its major principal axis because this is the minimum energy state for rotation with conservation of angular momentum; any loss of energy causes the body to move closer to this state (Lamy & Burns 1972). One way of approximating the characteristic alignment time is to compare the excess energy E_W stored in a precessing body to the energy ΔE lost during each wobble period (Stacey 1969).

We consider an oblate body with moments of inertia (A, A, C) about the fixed body axes, having a dynamical oblateness $H = (C-A)/C$ and rotating with an angular velocity $\boldsymbol{\omega} = (d_1, d_2, d_3)\,\omega$, where the d's are direction cosines relative to the body axes. The total rotational kinetic energy is

$$E_{\text{total}} = \tfrac{1}{2}[A(d_1{}^2+d_2{}^2)+C\,d_3{}^2]\,\omega^2, \tag{1}$$

which is to be compared to the minimum rotational energy

$$E_{\text{min}} = \tfrac{1}{2}C\omega_0{}^2. \tag{2}$$

The final angular velocity ω_0 can be computed from conservation of angular momentum

$$C\omega_0 = [A^2(d_1{}^2+d_2{}^2)+C^2\,d_3{}^2]^{1/2}\,\omega. \tag{3}$$

The excess energy in the wobble motion of an oblate spheroid is from (1), (2) and (3)

$$E_W = E_{\text{total}}-E_{\text{min}} = \tfrac{1}{2}AH\omega^2\alpha^2 \tag{4}$$

in terms of the nutation (or wobble) angle $\alpha = (d_1{}^2+d_2{}^2)^{1/2}$.

The number of precession periods needed to damp the angle is

$$n_\alpha = 2E_W/\Delta E. \tag{5}$$

The coefficient of two is present because the energy is proportional to the square of the wobble amplitude whereas we are computing an amplitude damping time ($n_\alpha = 2n_E$).

The energy loss per cycle is defined by

$$\Delta E = 2\pi E/Q \tag{6}$$

in which Q is the material's quality factor. E is the part of the total stored strain energy of the body which oscillates during the wobble motion; it is found by summing over the entire body the strain energy which varies in each element because the energy lost by inelastic effects ultimately comes from the wobble energy.

Equations (4), (5) and (6) give

$$n_\alpha = AHQ\omega^2\alpha^2/(2\pi E) \tag{7}$$

and the alignment time is just (7) times the wobble period $P = 2\pi/H\omega$. P is computed from rigid body dynamics since elastic effects are not important in lengthening the free precession period of asteroids. Hence the characteristic alignment time is

$$\tau = AQ\omega\alpha^2/E. \tag{8}$$

The problem thus reduces to a computation of the strain energy induced by the wobble motion. The strain energy of interest is the shear strain energy since the quality factor Q for shear motions in the Earth, and presumably in rocky asteroids, is much smaller than the compressional Q. The shear strain energy per unit volume is $\frac{1}{2}\mu\gamma^2$, where μ is the shear modulus and γ is the shear strain. It is accurate enough for our purposes, however, to take the shear strain and the normal strain to be of the same order as can be shown by a Mohr's circle construction.

Strain energy computations

Bending. One can approximate the strain energy stored in the bending produced during the wobble motion (see Lyttleton & Singer 1964) by computing the same quantity when the asteroid is considered to be a slender beam. The normal bending stress σ, produced in a beam which is supporting a bending moment N is

$$\sigma = Ny/I, \tag{9}$$

where, for symmetrical cross-sections, y is the distance from the symmetry plane of the beam and I is the area moment of inertia of the beam's cross-section about the symmetry plane.

The maximum bending stress is experienced at the outer boundary ($y = \pm h$) where the maximum normal strain is

$$\epsilon_0 = hN/(YI) \tag{10}$$

with Y being Young's modulus. The average value of the maximum strain taken over the whole body is roughly $\epsilon_0/2$.

The bending moment felt by a freely precessing body is just the gyroscopic torque which can be found by differentiating (4), the excess energy stored in the wobble motion for an oblate spheroid,

$$N = -dE_{\mathrm{W}}/d\alpha = -AH\alpha\omega^2. \tag{11}$$

Thus to order of magnitude the average shear strain is given by

$$\gamma = K_1 hAH\alpha\omega^2/(\mu I), \tag{12}$$

where μ is the shear modulus, which approximately equals Y, and K_1, which is about $\frac{1}{4}$, accounts for the ratio between the average and maximum strains. Now $h \sim r$, $A \sim \rho r^5$ and $I \sim r^4$, where r is a mean radius and ρ is mass density; so

$$\gamma \sim K_1\rho\alpha H\omega^2 r^2/\mu. \tag{13}$$

Therefore the total strain energy produced by bending which oscillates during a wobble is

$$E_b \sim \mu\gamma^2 r^3 \sim K_1^2(\rho\alpha H\omega^2)^2 r^7/\mu. \tag{14}$$

Bulge. The rotation of the asteroid produces a centrifugal bulge centred around the plane normal to the instantaneous rotation axis. This bulge is superimposed on the shape bulge H, which is maintained by the asteroid's rigidity. The centrifugal bulge, and its associated strains, wobble back and forth relative to the body as the rotation axis $\boldsymbol{\omega}$ moves through the body during a wobble period.

The size of the rotation bulge is easily approximated. The coefficient J_2 of the second order (oblateness) term in the gravitational expansion is for a rotating, fluid body, merely the ratio of the perturbing rotation potential to the point mass gravitational potential if we ignore the effect of the self-potential. The dynamical oblateness H_f produced by the rotation is then (Kaula 1968)

$$H_f \approx J_2 M r^2/3C \approx \omega^2 r^5/3GC, \tag{15}$$

where G is the universal gravitational constant and M is the asteroid's mass. Since the asteroid has considerable rigidity, its bulge is smaller:

$$H' \sim kH_f, \tag{16}$$

where k is the second-order Love number (Jeffreys 1970). For a homogeneous elastic sphere

$$k = \frac{3/2}{1+19\mu/(2g\rho r)} \approx 4\pi G\rho^2 r^2/19\mu; \tag{17}$$

g is the surface gravitational acceleration. The right-hand approximation in (17) is valid for small bodies ($r \ll 10^4$ km) and is true for all asteroids.

Combining (15), (16) and (17), and taking the body to be a uniform sphere, shows that the dynamical oblateness produced by the rotation is

$$H' \sim (5/38)\,\rho\omega^2 r^2/\mu. \tag{18}$$

We consider that the strains associated directly with this bulge are zero, that is to say, the body has relaxed so that no strain energy is stored in the rotation bulge when there is zero wobble. We assume that only the relatively rapid motion of bulge through the angle α produces strain energy. This assumption means the following solution for stored strain energy is a lower bound solution while the computed alignment times are upper bounds.

When the rotation axis moves through α, the distance to a surface point from the centre of the asteroid changes by Δr. It is easily shown that associated with this change is a radial strain which has a maximum value during a wobble period of

$$\epsilon \sim \Delta r/r \tag{19}$$

and that for a bulge of oblateness H' superimposed on an already oblate body

$$\Delta r \sim H'\alpha r. \tag{20}$$

Hence the maximum strain is from (18), (19) and (20)

$$\epsilon \sim (5/38)\,\rho\alpha\omega^2 r^2/\mu \tag{21}$$

and the total strain energy over the whole body which varies because of the motion of the centrifugal bulge during the precession is

$$E_c \approx K_2^2(\rho\alpha\omega^2)^2 r^7/\mu, \tag{22}$$

where K_2^2 is a numerical coefficient on the order of 10^{-2}. Note that (14) and (22) are of identical form except that in the bending energy result K_2^2 is replaced by the shape oblateness $K_1^2H^2$. It is entirely reasonable that E_b should include H whereas E_c does not since E_b is caused by a gyroscopic torque which is proportional to H. On the other hand, E_c, while requiring a non-zero H so that a wobble occurs, is not affected by the body shape to first order.

We can combine (14) and (22) to finally give

$$E = K_3^2(\rho\alpha\omega^2)^2r^7/\mu \tag{23}$$

where K_3^2 is typically of order 10^{-2} for bodies which are not particularly oblate and of order $10^{-1}\,H^2$ for very irregular bodies.

Alignment times for typical asteroids

From (8) and (23) the characteristic alignment time for an asteroid of mean radius r rotating with angular velocity ω, is

$$\tau \sim \mu Q/(\rho K_3^2r^2\omega^3), \tag{24}$$

where Q is the quality factor of the asteroid material while ρ is its mass density and μ its rigidity.

Typical values for the parameters in (24) are μ: 3×10^{11} to 3×10^{12} dyne-cm^{-2}, Q: 100 to 1000, K_3^2: 0·01 to 0·1 and ρ: 3 to 5 g-cm^{-3}. Choosing the most extreme cases, we find that $4\times10^4\,T^3/r^2$ yr is the quickest alignment time while $6\times10^7\,T^3/r^2$ yr is the longest; T is the rotation period in hours and r is the mean radius in kilometres. Therefore, except in the most unusual circumstances, τ is always substantially less than the age of the solar system for visible asteroids.

Shown in Table I are computed alignment times for various asteroid classes: Vesta (large, regular), Hektor (large, irregular), Icarus (small, regular) and Geographos (small, irregular). The computed values are based on the following mean

TABLE I

Asteroid (No.)	r (km)	T (hr)	H	K_3^2	τ (yr)
Vesta (4)	200	10·7	0·1	0·01	$1{\cdot}4\times10^5$
Hektor (624)	40	6·9	0·8	0·05	$2{\cdot}0\times10^5$
Icarus (1566)	1	2·3	0·1	0·01	$6{\cdot}0\times10^7$
Geographos (1620)	4	5·2	0·8	0·05	$5{\cdot}0\times10^6$

parameter choices: $\mu = 10^{12}$ dyne-cm^{-2}, $Q = 300$ and $\rho = 4$ g-cm^{-3} or $\tau \approx 5\times10^4\,T^3/(rK_3)^2$. The radii used in the calculation come from the relation shown in Allen (1963) and are based on the maximum observed absolute magnitude (McAdoo & Burns 1973a). The rotation periods are tabulated by McAdoo & Burns (1973a) while the K_3^2 are estimated using the computed H from lightcurve amplitudes.

The tabulated alignment times are somewhat smaller than would be expected for a random sample of asteroids because the chosen asteroids spin relatively fast

(McAdoo & Burns 1973a). This is particularly true in the case of Icarus, which has the smallest known rotation period for an asteroid; if it were to be spinning with the mean rotation period for all asteroids, its alignment time would extend to 4×10^9 yr. These results as well as those found through another approximate scheme by McAdoo & Burns (1973b) give quite short alignment times and, in those cases which can be tested, similar parameter dependences.

3. MISALIGNMENT DUE TO COLLISIONS

We consider the angular momentum transferred to an asteroid of mass M and radius r by impacts of other (smaller) bodies. The method to be employed is that used by Safronov (1969) in estimating the random component of rotation acquired by a planet during its accumulation.

During a collision with a mass m_i, which is moving with a relative velocity v_i at an impact parameter l_i, the asteroid acquires a random component of angular momentum with magnitude $\Delta K_i = m_i v_i l_i$, if m_i is retained by the asteroid (i.e. there is no mass ejection following impact). If there is preferential ejection in the direction of v_i, then ΔK_i is somewhat smaller whereas if the ejection is a ' backfire ' type, then ΔK_i is greater. In general

$$\Delta K_i = \zeta m_i v_i l_i \tag{25}$$

where $\zeta \sim o(1)$.

To find the result of random impacts of many bodies, we must sum the squares of ΔK_i

$$\Delta K^2 = \sum_i \Delta K_i{}^2. \tag{26}$$

At the typical impact velocities of $v_i \approx 5 \times 10^5$ cm s^{-1} (Dohnanyi 1971), the trajectories of bodies approaching an asteroid are essentially unchanged by the asteroid's gravity field. Hence, denoting the pericentre collision radius by r_i,

$$\overline{l_i{}^2} = \overline{r_i{}^2(1+2GM/[v_i{}^2 r_i])} \approx \overline{r_i{}^2} = \tfrac{1}{2}r^2 \tag{27}$$

and thus

$$\Delta K^2 \approx \zeta^2 v^2 r^2 \sum_i n_i m_i{}^2/2, \tag{28}$$

where n_i is the number of bodies of mass m_i which collide with the asteroid and v is the average relative velocity before encounter.

We assume an inverse power law for the distribution function of the masses of the impacting bodies

$$n(m) = Cm^{-q}. \tag{29}$$

Then the increase of ΔK^2 is

$$\Delta K^2 = \frac{\zeta^2}{2} v^2 r^2 \pi r^2 vt \int_0^{m_1} Cm^{2-q}\, dm = \frac{\zeta^2 \pi C}{2(3-q)} v^3 r^4 m_1{}^{3-q} t, \tag{30}$$

where the upper limit of integration m_1 is the mass of the largest body which collides with the asteroid in time t. m_1 can be approximately determined by requiring that the probability of a collision between the asteroid and a body larger than m_1 during its lifetime t_f is a reasonable fraction β of unity, for example, one-half:

$$\pi r^2 v t_f \int_{m_1}^{m_m} C m^{-q}\, dm \approx \pi C r^2 v t_f m_1^{1-q}/(q-1) = \beta \sim \tfrac{1}{2} \qquad (31)$$

since, as we shall see, $m_m \gg m_1$ and $q > 1$. To assure that disintegration of the target asteroid has not occurred we guarantee as well that

$$m_1 < \xi M \quad (\xi \sim 10^{-2}) \qquad (32)$$

is satisfied.

For small ΔK, the change in the direction of the axis of rotation is

$$\Delta\alpha \approx (\Delta K/K) \sin \psi, \qquad (33)$$

where $K \approx \frac{2}{5}\omega M r^2$; and ψ is the angle between $\mathbf{K}$ and $\mathbf{\Delta K}$. For randomly directed vectors $\mathbf{\Delta K}$ we have $\overline{\sin^2 \psi} = \frac{2}{3}$.

We obtain then from (30), (31) and (33)

$$\Delta\alpha^2 = \frac{25}{12} \frac{\zeta^2 \pi C}{(3-q)} \frac{v^3}{M^2 \omega^2} \left[\frac{\pi C r^2 v t_f}{\beta(q-1)}\right]^{3-q/q-1} t = At. \qquad (34)$$

This gives the rate of increase of α^2 as a result of collisions

$$(d\alpha^2/dt)_{\text{col.}} = A. \qquad (35)$$

This increase is opposed by the dissipation of the precession energy which causes

$$(d\alpha^2/\alpha^2\, dt)_{\text{pre.}} = -2/\tau, \qquad (36)$$

where τ is given by (24).

Thus the change of α^2 with time due to both effects is described by the equation

$$d\alpha^2/dt + 2\alpha^2/\tau = A \qquad (37)$$

which has a solution

$$\alpha^2 = A\tau/2 + (\alpha_0^2 - A\tau/2)\, e^{-2t/\tau}, \qquad (38)$$

where α_0 is the initial orientation. Now for all cases of interest, $\tau \ll t$ and so

$$\bar{\alpha}^2 \approx A\tau/2 \qquad (39)$$

or

$$\bar{\alpha}^2 = \frac{225}{384} \frac{\zeta^2 C \mu Q v^3}{(3-q)\, \pi K_3{}^2 \rho^3 \omega^5 r^8} \left[\frac{\pi C r^2 v t_f}{\beta(q-1)}\right]^{3-q/q-1} \qquad (40)$$

using (24).

From Dohnanyi (1971) and Burns (1971) we have that $C = 1{\cdot}7 \times 10^{-22}$ in cgs units and $q = 1{\cdot}837$ and thus $\bar{\alpha} \sim r^{-2{\cdot}61} \omega^{-2{\cdot}5}$. Shown in Table II are the average expected values for the nutation angle as a function of asteroid mean radius; they are computed from (40) with $v = 5 \times 10^5$ cm s^{-1}, $\rho = 4{\cdot}0$ g cm^{-3}, $Q = 300$, $K_3{}^2 = 0{\cdot}01$, $\mu = 10^{12}$ dyne cm^{-2}, $\zeta = 1$ and $\omega = 1{\cdot}9 \times 10^{-4}$ rad s^{-1}. The last corresponds to the mean rotation period for all asteroids and may be an under-estimate for irregular or small asteroids (McAdoo & Burns 1973a).

Also given in Table II is m_1/M, which is seen to satisfy criteria (33) in all cases. For comparison we also list the values of the nutation angle (a) when there is no

Table II

Radius r (km)		1	10	10^2
α	(°)	$\geqslant 360$		
	(radians)	32	0·08	2×10^{-4}
$\alpha_{\text{no damping}}$	(°)	$\geqslant 360$	$\geqslant 360$	
	(radians)	440	10	0·25
m_1/M		$5\cdot 6\times 10^{-3}$	$1\cdot 3\times 10^{-3}$	$3\cdot 2\times 10^{-4}$
$\Delta\alpha_{m1}$		$\geqslant 360$	$\geqslant 360$	
		300	7·1	0·17

damping mechanism ($\alpha_{\text{no damping}}$), and (b) the increase in $\alpha(\Delta\alpha_{m1})$ caused by the collision of the largest impacting body at v and r. We see from each of these cases that collisions transfer considerable angular momentum, substantiating a previous empirical conclusion of McAdoo & Burns (1973a). These latter values also indicate that the obliquities of all asteroids less than 100 km in radius are determined by collisions with other asteroids.

It is apparent from Table II that a wobble could be easily identified only for asteroids smaller than 10 km. Very few small asteroids have known rotation properties: of the 64 asteroids with determined properties only five are 10 km or smaller. The reason then that wobble has not been observed is that almost all asteroids with known lightcurves are so large that internal energy dissipation causes them to be nearly aligned; it is not a result of their mode of origin as was suggested by Kopal (1970).

We consider further the five smallest asteroids in order to suggest which should be observed more thoroughly. They have the following computed expected values for their nutation angles in radians: 433 Eros, 0·03; 1566 Icarus, 5·0; 1620 Geographos, 0·45; 1931 PH, 0·63; and 1971 F, 4·5. Of the five, only Eros, the one whose precession is too small to be seen, has been observed at more than one opposition. Since Icarus is so nearly spherical and since the data for 1931 PH are only photographic, we suggest that 1620 Geographos and, particularly, 1971 F should be observed in more detail. Their wobble periods, and hence the beat period in their lightcurves, should be within an order of magnitude of their spin period (Burns 1971).

4. CONCLUSIONS

We have shown that internal energy dissipation causes relatively rapid alignment of the rotation axis of a spinning asteroid with the angular momentum axis. This rapid alignment means that collisions between asteroids are not usually effective enough to produce an observable precession for a large asteroid; typical computed average expected nutation angles are quite small for most asteroids whose lightcurves have been determined, thus explaining why asteroid lightcurves are seen to be singly periodic. The results show that the observed axial alignment is a phenomenon which has been true for a long time but which is continually being reinforced and, thus, in contrast to the conclusion of Kopal (1970), the current alignment says nothing about the alignment of asteroids at their origin. Furthermore, the present-day asteroid obliquities bear little relation to those at origin except perhaps for the largest asteroids.

The results further point out that it may be possible to observe a precessing asteroid; such an asteroid would be small (with a radius of a few kilometres) and spinning slowly. While irregular asteroids are predicted to have a somewhat smaller nutation angle, it is likely that an irregular asteroid would be easier to detect as precessing because the observations themselves are simpler to make and because a large shape irregularity is probably indicative of a major collision (McAdoo & Burns 1973a). In particular, we suggest that further observations be made of 1620 Geographos and 1971 F in order to experimentally verify the theory developed here.

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REFERENCES

Allen, C. W., 1963. *Astrophysical quantities*, second edition, Athlone Press, London.

Burns, J. A., 1971. The alignment of asteroid rotation, in *Physical studies of minor planets*, ed. T. Gehrels, NASA-SP267, Washington, 257.

Dohnanyi, J. S., 1971. Fragmentation and distribution of asteroids, in *Physical studies of minor planets*, ed. T. Gehrels, NASA-SP267, Washington, 263.

Gehrels, T., 1970. Photometry of asteroids, in *Surfaces and interiors of planets and satellites*, ed. A. Dollfus, Academic Press, London, 319.

Jeffreys, H., 1970. *The Earth*, fifth edition, Cambridge University Press.

Kaula, W. M., 1968. *An introduction to planetary physics*, John Wiley & Sons, New York.

Kopal, Z., 1970. The axial rotation of asteroids, *Astrophys. Space Sci.*, **6**, 33.

Lamy, P. L. & Burns, J. A., 1972. A geometrical approach to the motion of a moment-free rigid body having internal energy dissipation, *Am. J. Phys.*, **40**, 441.

Lyttleton, R. A. & Singer, S. F., 1964. Dynamical considerations relating to the West Ford experiment, in *Torques and attitude sensing in Earth satellites*, ed. S. F. Singer, Academic Press, New York, 107.

McAdoo, D. C. & Burns, J. A., 1973a. Further evidence for collisions among asteroids, *Icarus*, **18**, 285.

McAdoo, D. C. & Burns, J. A., 1973b. Approximate axial alignment times for spinning bodies, *Icarus*, in press.

Prendergast, K. H., 1958. The effects of imperfect elasticity in problems of celestial mechanics, *Astr. J.*, **63**, 412.

Safronov, V. S., 1969. *Evolution of the protoplanetary cloud and formation of the Earth and the planets*, Nauka, Moscow.

Stacey, F. D., 1969. *Physics of the Earth*, John Wiley & Sons, New York.