Astronomical imaging by filtered weighted-shift-and-add technique

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The weighted-shift-and-add speckle imaging technique is analyzed using simple assumptions. The end product is shown to be a convolution of the object with a typical point-spread function (psf) that is similar in shape to the telescope psf and depends marginally on the speckle psf. A filter can be applied to each data frame before locating the maxima, either to identify the speckle locations (matched filter) or to estimate the instantaneous atmospheric psf (Wiener filter). Preliminary results show the power of the technique when applied to photon-limited data and to extended objects.

1. INTRODUCTION

Recent efforts in the field of stellar speckle interferometry[1-4] have been directed toward achieving true images of the observed objects. A great deal has been done to reconstruct the Fourier phases of these objects in order to obtain the true image. Approaches known as the shift-and-add techniques try to retain the phases as they appear in the original specklegram.

Bates and Cady[4] realized that at least one Fourier-plane phase is easy to find: that corresponding to the strongest intensity in the specklegram frame. If the displacement of this point is known, then the frame can be shifted to place this maximum at its center. Adding many such shifted frames will yield the average intensity around the brightest spots in all the specklegrams. This is known as the shift-and-add (SAA) technique. Lynds, Worden, and Harvey[5] (LWH) locate not just the absolute maximum in each frame but also the brightest local maxima (after some initial smoothing). A set of weighted delta functions is created, corresponding to the coordinates and intensities of these maxima. A cross correlation of this set with the original frame shifts the brightest local maxima to the center, producing a result similar to the SAA technique, though with higher efficiency.

The seeing-calibrated weighted-shift-and-add[1] (WSA) is a natural continuation of these two methods. Similar to the LWH method, it finds all the local maxima. A set of delta functions is created, multiplied (or weighted) by the intensities of the corresponding maxima to create a set of impulses. Each frame is correlated in the Fourier plane with its set of impulses, and the correlations are averaged. Finally, the average cross spectrum is deconvolved by the average power spectrum of the impulses in order to reduce the atmospheric-seeing effects on the final result.

All realizations of the SAA technique have one problem in common: they rely strongly on finding local maxima and on the assumption that these are equivalent to speckle locations. Shot noise, atmospheric effects, and object morphology can all invalidate this assumption.

Because of the effect of photon statistics, the number of maxima can be much larger than the number of speckles. If the intensity of the object is low, there are some speckles that will contain only one photon, with no structure information in them. If the object has a wide maximum, then both Poisson noise and atmospheric phase fluctuations might distort its image and create spurious local maxima. Finally, the object could be multiple peaked, which can be mistaken as multiple speckled, with the final result resembling an autocorrelation instead of an image. Bates[4] calls this "ghosting."

As a remedy, we use a filter that smooths out each speckle and at the same time defines its location. The best filter should be close to the mean speckle itself: a matched filter. Since the mean speckle is initially unknown, a crude guess is used to locate filtered speckle maxima. These are then used to produce a better mean-speckle estimate by SAA. The procedure is iterated until the mean speckle converges.

We find that the iterative speckle estimate is not the optimum matched filter. The most suitable filter must suppress the variable background created by coalescing speckles in a large speckle cloud as well as smooth the single-photon-event noise. Thus we combine the mean speckle with a bandpass filter into a Wiener filter. Local speckle maxima are thus enhanced, whereas single photons are discriminated against by using a comparison low-pass-filtered frame. The combined process, speckle identification and WSA, can be carried out in the image plane or in the Fourier plane. We have experimented in both domains.

2. MATHEMATICAL FORMULATION OF WEIGHTED SHIFT-AND-ADD

We first summarize the WSA formalism of Christou et al.[1] Let us write the kth quasi-instantaneous image $i_k(\mathbf{x})$ as the Poisson realization of the convolution of the object $o(\mathbf{x})$ with the instantaneous speckle point-spread function $s_k(\mathbf{x})$. For ease of understanding, we choose to define the Poisson noise as the difference between the realization and the convolution, i.e.,

$$n_k(\mathbf{x}) = i_k(\mathbf{x}) - o(\mathbf{x}) * s_k(\mathbf{x}). \tag{1}$$

When detecting this image, we must assume a detector point-spread function $d(\mathbf{x})$, which is not necessarily point-like; specifically, single photons create typical splotches when they are recorded. We revise the definition of the image to be

$$i_k(\mathbf{x}) = d(\mathbf{x}) * [o(\mathbf{x}) * s_k(\mathbf{x}) + n_k(\mathbf{x})]. \tag{2}$$

We create an impulse frame from $i_k(\mathbf{x})$ by substituting delta functions for local maxima only. We assume that each is weighted by the corresponding maximum. This nonlinear process can be viewed as a crude estimate $a_k(\mathbf{x})$ of the instantaneous speckle point-spread function $s_k(\mathbf{x})$ because the grainy appearance of the speckle pattern can be attributed chiefly to the atmosphere. At the same time, this estimate also has a portion that can be attributed to maxima in the noise $n_k(\mathbf{x})$. This algorithm, which singles out maxima only, assumes that the object $o(\mathbf{x})$ has a single, sharp peak. The following sections will deal with the problems of extended objects and how to minimize their influence on the speckle estimate.

The next step is to apply a Fourier transform to the specklegram and the impulse frame. Denoting all quantities in the Fourier plane in upper-case letters and using nondimensional units $\mathbf{u} = \mathbf{f}/\lambda$, we have

$$I_k(\mathbf{u}) = D(\mathbf{u})[O(\mathbf{u})S_k(\mathbf{u}) + N_k(\mathbf{u})] \tag{3}$$

for the image transform and $A_k(\mathbf{u})$ for the transform of the impulse frame $a_k(\mathbf{x})$. Now we calculate the cross spectrum of these two quantities:

$$C_k(\mathbf{u}) = D(\mathbf{u})[O(\mathbf{u})S_k(\mathbf{u}) + N_k(\mathbf{u})]A_k^*(\mathbf{u}), \tag{4}$$

and create the impulse power spectrum:

$$R_k(\mathbf{u}) = A_k(\mathbf{u})A_k^*(\mathbf{u}) = |A_k(\mathbf{u})|^2. \tag{5}$$

The cross spectrum and the power spectrum are accumulated for a large number of frames, yielding averages $C(\mathbf{u}) = \langle C_k(\mathbf{u})\rangle$, $R(\mathbf{u}) = \langle R_k(\mathbf{u})\rangle$. To get the final WSA result $W(\mathbf{u})$, we divide these two quantities:

$$W(\mathbf{u}) = \frac{C(\mathbf{u})}{R(\mathbf{u})} = \frac{D(\mathbf{u})[O(\mathbf{u})\langle S(\mathbf{u})A^*(\mathbf{u})\rangle + \langle N(\mathbf{u})A^*(\mathbf{u})\rangle]}{\langle |(\mathbf{u})|^2 \rangle}. \tag{6}$$

We now see the importance of having a good estimate of the atmosphere. If this is really the case, then $A(\mathbf{u})$ does not contain any terms that correlate with the photon noise, and the last term in the numerator can be considered negligible. Therefore,

$$W(\mathbf{u}) \cong D(\mathbf{u})O(\mathbf{u}) \frac{\langle S(\mathbf{u})A^*(\mathbf{u})\rangle}{\langle |A(\mathbf{u})|^2 \rangle}. \tag{7}$$

To find out more about the averages involved in Eqs. (6) and (7), let us write the speckle point spread function (psf) as[6]

$$s_k(\mathbf{x}) = |\mathrm{FT}[\psi_k(\mathbf{u})P(\mathbf{u})]|^2, \tag{8}$$

where $P(\mathbf{u})$ is the telescope aperture function and $\psi(\mathbf{u})$ is the atmospheric transfer function. FT[ ] stands for a Fourier transform. Let us assume that our estimate $a_k(\mathbf{x})$ is actually equal to the instantaneous psf $s_k(\mathbf{x})$ deconvolved by the telescope transfer function

$$a_k(\mathbf{x}) = |\mathrm{FT}[\psi_k(\mathbf{u})P(\mathbf{u})]|^2 / |\mathrm{FT}[P(\mathbf{u})]|^2, \tag{9}$$

where / denotes deconvolution (division in the Fourier space). This assumption would be exact in the case when the speckles can be represented simply as a double convolution of the object, the telescope psf, and a set of delta functions (which in turn represent the atmosphere). This description is adequate at high frequencies [see Eqs. (22) below]. Replacing each maximum by a delta function amounts to a crude deconvolution, similar to that of the CLEAN algorithm. The numerator in expression (7) is the average cross spectrum of the Fourier transforms of Eqs. (8) and (9). Following the reviews and definitions of Roddier and Dainty,[6]

$$\langle S(\mathbf{u})A^*(\mathbf{u})\rangle = \frac{\left\langle \iint d\mathbf{v}d\mathbf{v}'\psi(\mathbf{v})\psi^*(\mathbf{v}+\mathbf{u})\psi^*(\mathbf{v}')\psi(\mathbf{v}'+\mathbf{u})P(\mathbf{v})P^*(\mathbf{v}+\mathbf{u})P^*(\mathbf{v}')P(\mathbf{v}'+\mathbf{u}) \right\rangle}{\mathbf{S}^{-1}\int d\mathbf{v}P(\mathbf{v})P^*(\mathbf{v}+\mathbf{u})}$$

$$= \frac{\mathbf{S}^{-2}\iint d\mathbf{u}'M(\mathbf{u},\mathbf{u}')H(\mathbf{u},\mathbf{u}')}{\mathbf{S}^{-1}\int d\mathbf{v}P(\mathbf{v})P^*(\mathbf{v}+\mathbf{u})}$$

$$= \frac{P_s(\mathbf{u})}{T(\mathbf{u})}, \tag{10}$$

where $M(\mathbf{u}, \mathbf{u}')$ is the fourth-order moment of the atmospheric wave-front fluctuations; $P_s(\mathbf{u})$ is the speckle transfer function; $H(\mathbf{u}, \mathbf{u}')$ is the fourth-order correlation of the telescope; $T(\mathbf{u})$ is the second-order correlation of the telescope or the telescope transfer function; and $\mathbf{S}$ is the pupil area. Factoring the integral into a product of two integrals is possible since $M(\mathbf{u}, \mathbf{u}')$ is a function of the frequency difference $\mathbf{u}' = \mathbf{v}' - \mathbf{v}$.

The denominator in expression (7) is the average power spectrum of the Fourier transform of Eq. (9):

$$\langle |A(\mathbf{u})|^2 \rangle = \frac{\left\langle \iint d\mathbf{v}d\mathbf{v}'\psi(\mathbf{v})\psi^*(\mathbf{v} + \mathbf{u})\psi^*(\mathbf{v}')\psi(\mathbf{v}' + \mathbf{u})P(\mathbf{v})P^*(\mathbf{v} + \mathbf{u})P^*(\mathbf{v}')P(\mathbf{v}' + \mathbf{u}) \right\rangle}{\mathbf{S}^{-2} \iint d\mathbf{v}d\mathbf{v}'P(\mathbf{v})P^*(\mathbf{v} + \mathbf{u})P^*(\mathbf{v}')P(\mathbf{v}' + \mathbf{u})}$$

$$= \frac{\mathbf{S}^{-2} \int d\mathbf{u}'M(\mathbf{u}, \mathbf{u}')H(\mathbf{u}, \mathbf{u}')}{\mathbf{S}^{-2} \int d\mathbf{u}'H(\mathbf{u}, \mathbf{u}')}$$

$$= \frac{P_s(\mathbf{u})}{T^2(\mathbf{u})}. \tag{11}$$

Substituting Eqs. (10) and (11) into expression (7), we get

$$W(\mathbf{u}) \cong D(\mathbf{u})O(\mathbf{u})T(\mathbf{u}). \tag{12}$$

To evaluate this expression further, we follow previous work[7,8] and assume that the kth frame is composed of a set of N_k photons at positions $\mathbf{x}_n$, each producing the detector psf $d(\mathbf{x})$:

$$i_k(\mathbf{x}) = d(\mathbf{x}) * \sum_{n=1}^{N_k} \delta(\mathbf{x} - \mathbf{x}_n). \tag{13}$$

The estimate of the speckle psf, $a_k(\mathbf{x})$, is composed of a different set of impulses, which we model to be the realization of a point process representing the atmosphere. Because of the digital form of our data, we consider every impulse to be the sum of all delta functions within the same pixel. Unfortunately, our estimate is contaminated by the Poisson process, which creates some spurious local maxima. Here we have L_k speckles and K_k contaminating photons:

$$a_k(\mathbf{x}) = \sum_{l=1}^{L_k} \delta(\mathbf{x} - \mathbf{x}_l) + \sum_{j=1}^{K_k} \delta(\mathbf{x} - \mathbf{x}_j). \tag{14}$$

In this way we model our data $i_k(\mathbf{x})$ as a Poisson process of rate $\lambda_k(\mathbf{x})$ proportional to the ideal image $o(\mathbf{x}) * s_k(\mathbf{x})$. Our estimate of the atmospheric speckles $a_k(\mathbf{x})$ is modeled as the sum of two processes. The first is an unknown point process with rate $\mu_k(\mathbf{x})$ proportional to the atmospheric psf (Fried's "short exposure"[6]). In the cases when the atmospheric turbulence is low, the object is extended, or the light level is low, the variance that is due to the photon noise can be comparably large. The Poisson process is then assumed to be independent of the atmospheric process.[9] Thus it is written on the right-hand side of Eq. (14) as an additive term of rate $\lambda_k(\mathbf{x})$. Next we calculate the expectation value of the cross spectrum of the Fourier transforms of the data and the estimate [Eq. (4)] with respect to coordinates:

$$\mathbf{E}_{nlj}[C_k(\mathbf{u})] = D(\mathbf{u}) \left\{ \sum_{n=1}^{N_k} \sum_{l=1}^{L_k} \mathbf{E}_{nl}[\exp - 2\pi i\mathbf{u} \cdot (\mathbf{x}_n - \mathbf{x}_l')] \right.$$

$$\left. + \sum_{n=1}^{N_k} \sum_{j=1}^{K_k} \mathbf{E}_{nj}[\exp - 2\pi i\mathbf{u} \cdot (\mathbf{x}_n - \mathbf{x}_j)] \right\}. \tag{15}$$

The total number of elements in the first summation is N_kL_k, almost all of which are not unity, as they originate from different, independent processes (Poisson and atmo-

spheric). The number of elements in the second summation is N_kK_k, of which K_k are unity (when $\mathbf{x}_n = \mathbf{x}_j$). For the first summation the probability of an event's occurring at $\mathbf{x}_n$ is $\lambda_k(\mathbf{x}_n)/\int \lambda_k(\mathbf{x})d\mathbf{x}$, where the integral is over the frame. The probability of a speckle's occurring at $\mathbf{x}_l$ is similarly $\mu_k(\mathbf{x}_l)/\int \mu_k(\mathbf{x})d\mathbf{x}$. The joint probability of the two is their product, these being independent point processes, yielding an average cross term

$$\frac{\int \lambda_k(\mathbf{x})\exp - 2\pi i\mathbf{u} \cdot \mathbf{x}d\mathbf{x} \int \mu_k(\mathbf{x})\exp 2\pi i\mathbf{u} \cdot \mathbf{x}d\mathbf{x}}{\int \lambda_k(\mathbf{x})d\mathbf{x} \int \mu_k(\mathbf{x})d\mathbf{x}}$$

$$= \frac{\Lambda_k(\mathbf{u})M_k{}^*(\mathbf{u})}{\Lambda_k(0)M_k{}^*(0)}, \tag{16}$$

where we assume that $\Lambda(\mathbf{u})$ and $M(\mathbf{u})$, the Fourier transforms of $\lambda(\mathbf{x})$ and $\mu(\mathbf{x})$, are Hermitian. The second summation yields a similar term. From Eqs. (15) and (16) we have

$$\mathbf{E}_{nlj}[C_k(\mathbf{u})] = D(\mathbf{u}) \left[K_k + (K_kN_k - K_k) \frac{|\Lambda_k(\mathbf{u})|^2}{|\Lambda_k(0)|^2} \right.$$

$$\left. + L_kN_k \frac{\Lambda_k(\mathbf{u})M_k{}^*(\mathbf{u})}{\Lambda_k(0)M_k{}^*(0)} \right]. \tag{17}$$

Now we average over the Poisson statistics of the photons and the atmospheric statistics of the speckles,

$$\mathbf{E}_{nlj,\mathrm{NL}}[C_k(\mathbf{u})] = D(\mathbf{u}) \left[K + (KN - K) \frac{\langle |\Lambda(\mathbf{u})|^2 \rangle}{\langle |\Lambda(0)|^2 \rangle} \right.$$

$$\left. + LN \frac{\langle \Lambda(\mathbf{u})M^*(\mathbf{u}) \rangle}{\langle \Lambda(0)M^*(0) \rangle} \right], \tag{18}$$

where L and N are the average numbers of speckles and photons per frame and K is the corresponding number of coincidences between the estimate and the photons. Similarly, we get for our estimate for the atmosphere $a_k(\mathbf{x})$ an average power spectrum

$$\mathbf{E}_{nl,\mathrm{NL}}[R_k(\mathbf{u})] = L + L^{(2)} \frac{\langle |M(\mathbf{u})|^2 \rangle}{\langle |M(0)|^2 \rangle} + K + K^{(2)} \frac{\langle |\Lambda(\mathbf{u})|^2 \rangle}{\langle |\Lambda(0)|^2 \rangle}$$

$$+ KL \frac{\langle \Lambda(\mathbf{u})M^*(\mathbf{u}) \rangle}{\langle \Lambda(0)M^*(0) \rangle}, \tag{19}$$

where $L^{(2)} = \mathbf{E}[L_k(L_k - 1)]$ is the second factorial moment of

the speckles and $K^{(2)} = \mathbf{E}[K_k(K_k - 1)]$ is the second factorial moment of the photons. Next we substitute Eqs. (4), (5), (10), and (11) for the cross spectrum and power spectrum into Eqs. (18) and (19) and divide to get

$$W(\mathbf{u}) = D(\mathbf{u}) \frac{LNO(\mathbf{u})P_s(\mathbf{u})/T(\mathbf{u}) + K(N-1)|O(\mathbf{u})|^2 P_s(\mathbf{u}) + K}{L^{(2)} P_s(\mathbf{u})/T^2(\mathbf{u}) + K^{(2)}|O(\mathbf{u})|^2 P_s(\mathbf{u}) + KLO(\mathbf{u})P_s(\mathbf{u})/T(\mathbf{u}) + K + L}, \quad (20)$$

where we have made use of the standard speckle interferometry power spectrum[8] $|O(\mathbf{u})|^2 P_s(\mathbf{u})$ for the second term in Eq. (18) and the fourth term in Eq. (19). If we assume that the number of cross terms between the specklegram and the atmospheric estimate is small, i.e., $N \gg K$, $L \gg K$, and that $L^{(2)} \simeq L^2$, $K^{(2)} \simeq K^2$, we have

$$W(\mathbf{u}) \simeq (N/L)D(\mathbf{u})O(\mathbf{u})Q(\mathbf{u}), \quad (21a)$$

where

$$Q(\mathbf{u}) = \frac{T(u)P_s(\mathbf{u})}{P_s(\mathbf{u}) + T^2(\mathbf{u})/L} \quad (21b)$$

is the WSA transfer function. We observe that, under the Gaussian approximation,[6] the speckle transfer $P_s(\mathbf{u})$ can be written as

$$P_s(\mathbf{u}) \simeq B^2(\mathbf{u})T^2(\mathbf{u}) + T(\mathbf{u})/L, \quad (22a)$$

where the number of speckles is defined as[6]

$$L = 2.3(D/r_0)^2 = \mathbf{S} \iint d\mathbf{u} B^2(\mathbf{u}). \quad (22b)$$

$B(\mathbf{u})$ is the atmospheric second-order moment, and r_0 is Fried's seeing parameter. Equations (22) are a good approximation under bad-seeing conditions, especially at low and high frequencies. Applying them to Eqs. (21), we get for the WSA transfer function

$$Q(\mathbf{u}) \simeq \begin{cases} T(\mathbf{u}), & u \ll r_0/\lambda \\ \dfrac{T(\mathbf{u})}{T(\mathbf{u}) + 1}, & u \gg r_0/\lambda \end{cases}. \quad (23)$$

This calculation was done under the assumption that the atmospheric estimate contained no maxima due to Poisson noise. This is justified when the noise is negligible, either because the signal was strong or because it was smoothed down by filtering. One should also bear in mind that with most conventional telescopes $T(\mathbf{u}) \ll 1$ for $u \gg r_0/\lambda$, and the WSA transfer function will tend to the telescope transfer function at both high and low frequencies.

3. MATCHED-FILTER APPROACH

The problem to address is how to locate speckles of approximately the same shape but of different intensity in a noisy background, even if this shape might not be single peaked. If there is no knowledge of this shape, the answer is Wiener filtering; if there is full knowledge, matched filtering should be applied. Fortunately, in most cases prior information about the object is available, usually through conventional speckle interferometry, which provides the object autocorrelation. This enables us to use matched filtering, which is more efficient than Wiener filtering.[10] The initial estimate provides a filter that is correlated with the speckle frame. A peak-finding algorithm detects all maxima above a thresh-

old, and the corresponding speckles are shifted to the center (by any of the SAA methods described above). Averaging over a few frames, we have a better estimate for the object, which can be modified into a matched filter for the next frames. After a few such iterations most of the refinements to the estimate are in the finer details.[11,12]

Suppose that we have an image of our object $o(\mathbf{x})$ with an additive noise $n(\mathbf{x})$ whose power spectrum is $P_n(\mathbf{u})$ (we ignore the telescope psf for the time being). We now apply a filter to the noisy frame and require that the signal-to-noise power ratio be maximal at the output of the filter. As a result we get for the filter[10]

$$F_m(\mathbf{u}) = C \exp(- 2\pi i u)\mathbf{x}O^*(\mathbf{u})/P_n(\mathbf{u}), \quad (24)$$

where C is an arbitrary constant. In the simple case of white noise, where $P_n(\mathbf{u}) = N^2$, we can set $C = P_n$ and get, in reference to some arbitrary point $\mathbf{x}_0$,

$$F_m(\mathbf{u}) = O^*(\mathbf{u})\exp - 2\pi i u \cdot \mathbf{x}_0. \quad (25)$$

Transforming back, the filter will be $o(\mathbf{x}_0 - \mathbf{x})$, which is the object reversed with respect to the point $\mathbf{x}_0$.

Going through the assumptions leading to the above derivation, we find some departures from our case. The main ones are the following: (1) the exact shape of the object is not known, (2) speckle power is variable and can be below noise level, (3) the white-noise spectrum cannot describe properly the combination of the speckle and Poisson processes, (4) the low-pass filtering inherent in the application of the matched filter hinders detection of close speckles, especially for larger objects. None of these departures invalidates the process, and we describe what has been done in order to reduce their effect.

We have used our algorithm with different initial guesses in order to test its sensitivity to the fact that the object shape is not really known. We found that for a rather small number of iterations and for an initial guess not smaller than the object (whose size can be inferred from the standard speckle autocorrelation), the process always converged toward the same answer. Specifically, a Gaussian bell of the approximate size always seems to be a proper initial guess (Fig. 1).

The problem of speckles of different power seems to be easy to solve. After a matched filter is applied, all the local maxima above the noise have to be detected. To that end, a constant threshold is set just above the noise level. Since in the case of speckle frames the background is higher in the center, we chose to utilize a variable threshold, which depends on the intensity around the speckles. At the same time that the frame is passed through a matched filter, it is also passed through a smoothing filter. The highly smoothed version of the frame then serves as a threshold for the filtered frame, and only maxima above it are considered as speckles.

The two last problems regarding the statistics of the atmospheric distortions and the detection process seem to be more severe, and the partial theory above does not suffice to solve them. An alternative is to describe a speckle pattern in the same formalism as for laser speckle, with the obvious
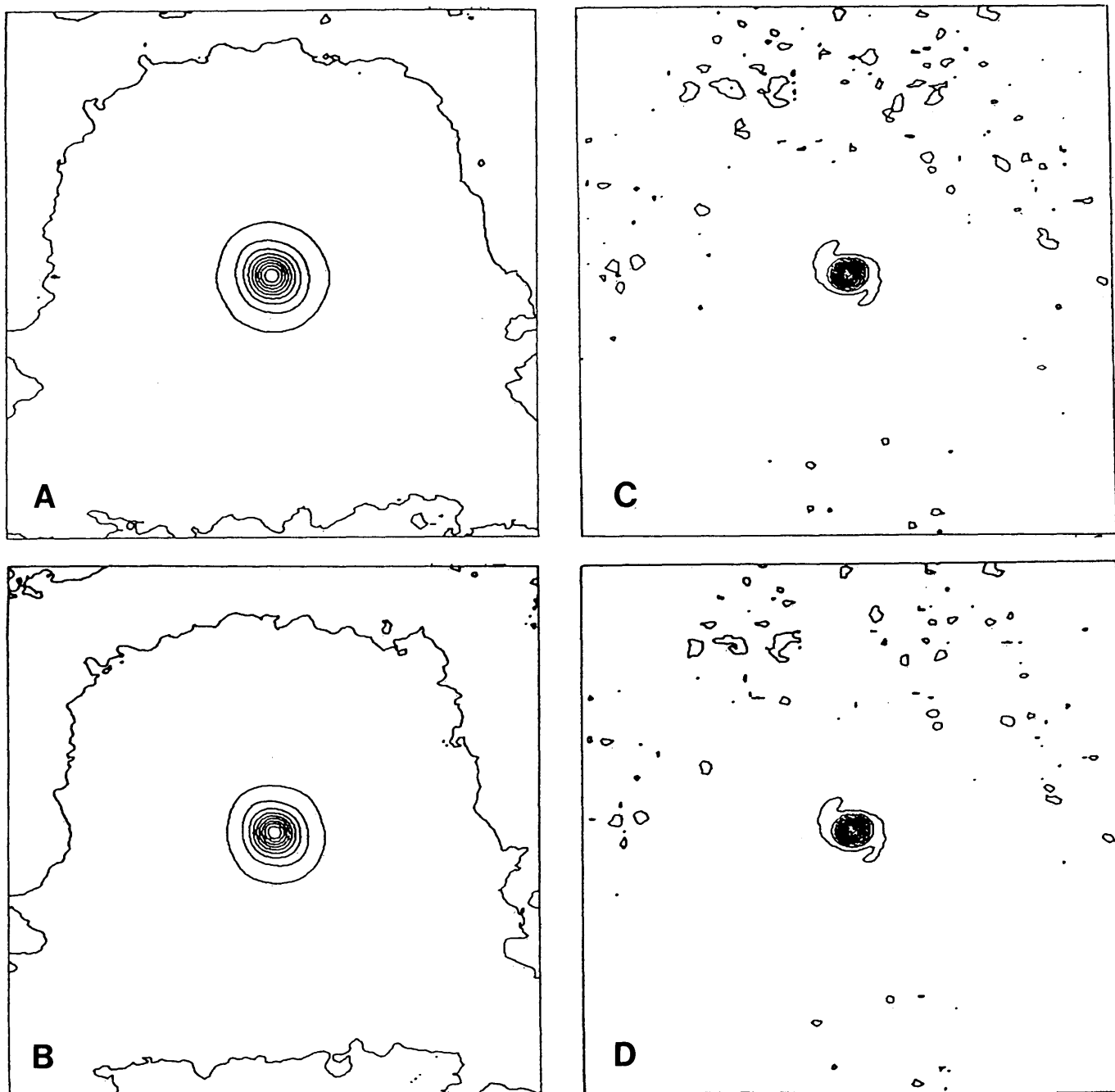
Fig. 1.   A, B, *Alpha Orionis* and C, D, *Gamma Orionis* for two apodizations applied to the matched filter.   A, C, 28 pixels FWHM.   B, D, 52 pixels FWHM.   Frame size is 128 × 128.   Observations were made with a 3.8-m telescope at 650 ± 1 nm.   The image of *Gamma Orionis* is unresolved.   The images of *Alpha Orionis* are different by 10% in size (50 frames for each image).

changes for coherence and aperture size accounted for.[13,14] The drawback of this description is that it assumes stationarity of the speckles, true only in the middle of the speckle cloud.   As mentioned earlier, we model the specklegram as a triple convolution of the object with the telescope (Fig. 2) and with a set of delta functions representing the atmosphere.   In the different realizations of SAA an effort is made to proceed in the reverse direction.   An estimate of a subset of the (strongest) delta functions is produced, the original specklegram is deconvolved by it, and the result is averaged over many frames.[1]   From this average it is possible to remove the atmospheric-seeing background (due to

the incompleteness of the subset of the estimate) and the central photon spike (due to the Poisson statistics).

Despite the fact that it is possible to remove the seeing background and the photon spike from the final image, it is preferrable to devise a matched filter that would give the right answer directly.   Since some information is available about the power spectra of the Poisson noise and the seeing, we incorporate it into the matched filter while performing the iterative data reduction.   In the Fourier plane, the main effect of the seeing background is in the very low frequencies (what is called the "long exposure" [6]), whereas the Poisson noise exists at all frequencies.   Thus the total noise power
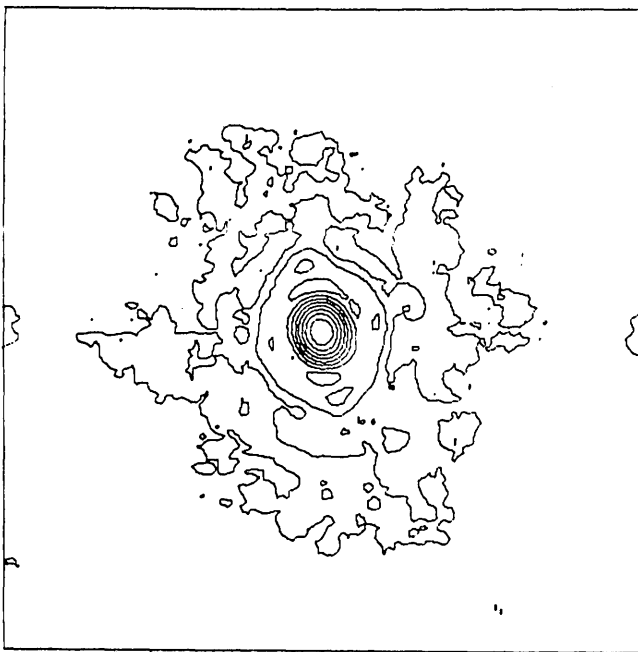
Fig. 2.   An image of an unresolved star (*Zeta Aquaril* A) with the 2.28-m telescope at 750 ± 10 nm, showing the (aberrated) telescope-detector psf.   Logarithmic intensity scale.   Processed from 100 frames, about 100 photons each.   All figures are with North down, East to the right.

spectrum [Eq. (24)] is accentuated at the very low frequencies and constant elsewhere.   Dividing the object transform by this power spectrum is equivalent to multiplying it by its reciprocal, i.e., a bandpass filter.

The combination of a bandpass filter with the object produces an optimum matched filter, which is now applied to the specklegram.   The bandpass filter, convolved with the object, partially solves both problems (3) and (4) discussed earlier.   It smooths out single-photon events below the threshold level, so that they are not regarded as speckles.   At the same time it sharpens the wide autocorrelations between the speckles and the object, and as a result the efficiency of detecting speckle maxima increases.

4.   WIENER FILTER APPROACH

The other approach possible is not to locate the speckles (using the assumption that each one is a replica of the object) but to find the original distribution of the atmosphere, apply some nonlinear process (such as maximum finding) and, continue from there on as with the standard WSA.   As mentioned in Section 3, this calls for a Wiener filter, since there is no prior knowledge of the atmospheric distribution and its shape is varying from one realization to the other.   Our estimate is now achieved by applying a Wiener filter $F_w(\mathbf{u})$ to the current image transform:

$$A_h(\mathbf{u}) = F_w(\mathbf{u})I_h(\mathbf{u})$$
$$= F_w(\mathbf{u})D(\mathbf{u})[O(\mathbf{u})S_h(\mathbf{u}) + N(\mathbf{u})]. \qquad (26)$$

Actually, we also apply here the nonlinear process of maximum singling in the image plane described above.   [If only linear processes were applied, we would get $F_w(\mathbf{u})$ as the end product.]   We form the least-squares difference e between the speckle psf and the estimate

$$\mathbf{e} = \left\langle \int d\mathbf{x}|s(\mathbf{x}) - a(\mathbf{x})|^2 \right\rangle = \left\langle \int d\mathbf{u}|S(\mathbf{u}) - A(\mathbf{u})|^2 \right\rangle \qquad (27)$$

using Parseval's theorem.   Combining Eq. (26) with Eq. (27), and assuming that the cross spectrum of the atmosphere and the noise average out, we arrive at the required filter[10,15]

$$F_w(\mathbf{u}) = \frac{O(\mathbf{u})^*P_s(\mathbf{u})}{D(\mathbf{u})[|O(\mathbf{u})|^2P_s(\mathbf{u}) + P_n(\mathbf{u})]}, \qquad (28)$$

where $P_s(\mathbf{u})$ and $P_n(\mathbf{u})$ are the speckle and noise power spectra, respectively.   We make use of the value of the average image power spectrum calculated elsewhere[7,8]

$$P_i(\mathbf{u}) = |D(\mathbf{u})|^2[|O(\mathbf{u})|^2P_s(\mathbf{u}) + P_n(\mathbf{u})] \qquad (29)$$

to get the final filter

$$F_w(\mathbf{u}) = \frac{[D(\mathbf{u})O(\mathbf{u})]^*P_s(\mathbf{u})}{P_i(\mathbf{u})}. \qquad (30)$$

This is much like the intuitive filter that was mentioned before in combination with the matched filter.   It tells us that by using iterative estimates of the object, the image power spectrum, and the speckle power spectrum, we can generate a valid Wiener filter.   The speckle power spectrum and the detector transfer function can be achieved before the calculation by observing a point source under similar atmospheric conditions with the same detector.   If the atmospheric conditions are different, then the ratio of the reference speckle power spectrum $P_s(\mathbf{u})$ to the actual one will differ mainly in the $u = r_0/\lambda$ regime.[16]   At higher frequencies this ratio will tend toward the ratio of the numbers of speckles for the two cases.   So unless the object has a great information content at low frequencies, it is safe to use a reference speckle power spectrum.   If not, a theoretical estimate can be made from the image power spectrum, which is actually the matched-filter method discussed above.

5.   APPLICATION AND RESULTS

The data processed were produced on the Kitt Peak National Observatory 3.8-m telescope and the Steward Observatory 2.3-m telescope.   The detector used was the Steward Observatory speckle camera,[17] with television frames digitized to a 128×128 format with 8-bit integers.   Some of the data were photon limited (see Figs. 1–3).

The first reductions were done in image space, in an approach similar to the simple SAA.   An estimate of the object, usually a Gaussian, was slid along and multiplied with the frame (image-plane correlation).   At the same time a smoothing function (a wider Gaussian) was correlated with the frame.   Every local maximum that was higher in the filtered frame than in the smoothed frame was counted as a speckle, and the corresponding part in the original specklegram was added to the running sum.   With bright objects, where the photon noise is negligible, the result appeared after only one frame.   The computational efficiency of this method was rather low, especially for larger objects, which demanded many multiplications.   The advantage was that only one full frame resided in memory at any time.

The process was then repeated in the Fourier plane (Fig. 4).   The cycle for each frame is as follows: (a) read frame in, (b) transform the frame, (c) multiply the frame transform
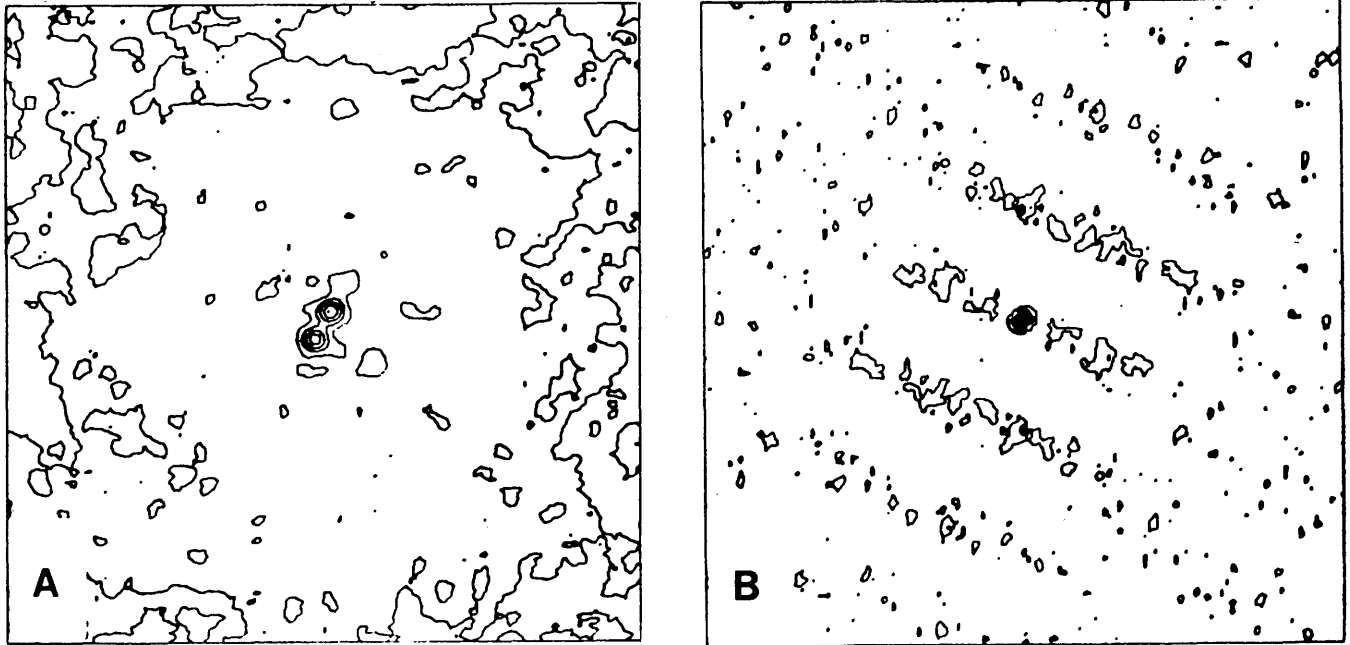
Fig. 3. A, The binary star Capella (*Alpha Aurigae*, 3.8-m telescope, 550 ± 10 nm), with two sidelobes (at the top and bottom), created by double detections of single speckles. B, The impulse power spectrum for the same data, showing fringes at the 0.5% level produced by this effect. No apodization was applied to the matched filter (20 frames).

with the current matched/Wiener filter estimate, (d) multiply the frame transform with the smoothing (low-pass) filter, (e) inverse transform the filtered frame, (f) inverse transform the smoothed frame, (g) locate all the peaks in the filtered frame above the smoothed one and create a frame with corresponding impulses in it, (h) transform the impulse frame, (i) multiply the frame transform by the impulses transform and add to running sum, (j) add impulse power spectrum to running sum. This process is similar to the WSA[1] process, which requires two Fourier transforms per frame. The matched-filter process includes the extra steps (c)–(f), which imply two more transforms per frame. It turns out that the time required per 128 × 128 frame is about 6 sec, compared with 4 with WSA (the computer was a Data General MV10000). For larger frames, a constant threshold (as opposed to a variable one) can be applied, saving one Fourier transform per frame [step (e)].

Once every 1,000–10,000 detected speckles, depending on the noise in the data and the expected object shape, we update the filter estimate. This is done by (1) dividing the average frame-impulse cross spectrum by the average impulse power spectrum to get the current speckle image, (2) reducing the photon noise by estimating it beyond the diffraction limit ("despiking"[1]) and apodizing the result outside that limit, (3) inverse transforming this average image, (4) apodizing the result to get rid of noise in the wings of the image, (5) transforming the apodized image, and (6) multiplying by the bandpass filter to give the optimum matched or Wiener filter. The apodization is usually not necessary, except for the first iterations when the noise at the outskirts of the image can correlate with the specklegrams to produce false maxima. Also, for multiple-peaked objects, which tend to have artificial sidelobes, an apodization can deter positive feedback, which leads to growth of these sidelobes (Fig. 3). Care should be taken when applying apodization,
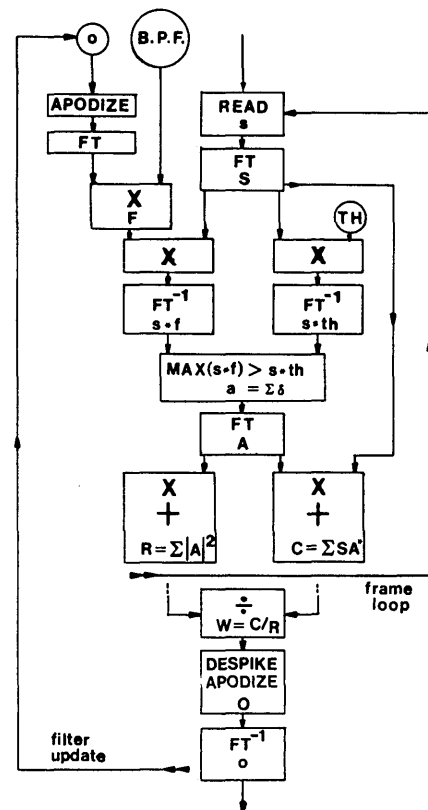

Fig. 4. General flow chart for filtered WSA (refer to the text for explanation). The frame loop is temporarily broken and the filter updated when enough speckles have been counted. Capital letters signify Fourier space quantities, FT is a Fourier transform, s is the current frame, TH and F are the threshold and matched filters, a is the impulse frame, o is the image estimate, and B.P.F. is the bandpass filter.

since the apodizing bell should be larger than the object extent (Fig. 1).

The main deficiencies of the current implementation of the matched/Wiener-filter method are the following: (1) complicated calculations requiring much computer time are necessary, (2) for extended objects, too few speckles are found, which lead to an extended seeing background,[12,13] (3) for multiple-peak objects, great care must be taken when building the filter lest the filtered frame have more than one maximum per speckle and thus lose the advantage over simple SAA methods.

ACKNOWLEDGMENTS

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