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PAPER

Astronomical measurements and constraints on the variability of fundamental constants

E. García-Berro · J. Isern · Y. A. Kubyshin

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Abstract We review the present status of how astronomical observations allow to constrain the evolution of fundamental constants of nature. The main observational constraints on the variation of the gravitational constant, on the fine structure constant and on the proton-to-electron mass ratio are reviewed. We also elaborate on some theoretical schemes which naturally lead to such variations.

Keywords Gravitation · Cosmology: miscellaneous · Cosmology: theory · Cosmology: early Universe

E. García-Berro (✉)
Departament de Física Aplicada, Universitat Politècnica de Catalunya,
Escola Politècnica Superior de Castelldefels, Avda. del Canal Olímpic s/n,
08860 Castelldefels, Spain
e-mail: garcia@fa.upc.edu

J. Isern
Institut de Ciències de l'Espai, CSIC, Campus de la UAB,
Facultat de Ciències, Torre C-5, 08193 Bellaterra, Spain
e-mail: isern@ieec.fcr.es

Y. A. Kubyshin
Institut de Tècniques Energètiques, Universitat Politècnica de Catalunya,
Avda. Diagonal, 647, Edif. PC, 08028 Barcelona, Spain

Y. A. Kubyshin
Skobeltsyn Institute of Nuclear Physics, Moscow State University,
119992 Moscow, Russia
e-mail: iouri.koubychine@upc.edu

1 Introduction

All the laws of nature establishing relations between various dynamical characteristics involve parameters (or factors) which are assumed to be independent of the time or of the space–time location and are thus regarded as *constants of nature*. For example, the gravitational interaction contains the gravitational constant, G. In fact, the statement about the constancy of the "constants" is just a hypothesis, though quite an important one. It is a part of the Copernican principle—see a thorough discussion of this issue in Uzan (2003)—and is crucial for comparing and reproducing experiments. However, it is not senseless to question its validity. In particular, one may conceive that the constant G is in fact a smooth and slow varying function $G(t, \mathbf{r})$. This may be understood as a way to modify the standard physics framework or, equivalently, a way to introduce new elements of the physics beyond the now widely accepted Standard Model of electroweak and strong interactions.

The issue of variation of the physical constants was first addressed by Dirac (1937) who formulated it within the framework of his Large Number Hypothesis. We quote him literally: "very large and very small dimensionless universal constants cannot be pure mathematical numbers and should rather be considered as variable parameters characterizing the state of the Universe". He basically considered the couplings

$$\alpha \equiv \frac{e^2}{\hbar c} \simeq \frac{1}{137.036}, \tag{1}$$

which characterizes the strength of the electromagnetic interaction,

$$\alpha_{\rm G} \equiv \frac{G m_{\rm p}^2}{\hbar c} = \frac{m_{\rm p}^2}{M_{\rm Pl}^2} \approx 5.9 \times 10^{-39}, \tag{2}$$

which characterizes the strength of the gravitational interaction—where $M_{\rm Pl} = \sqrt{\hbar c/G}$ is the Planck mass, the constant which sets the energy scale of the gravitational interaction—and

$$\alpha_{\rm W} \equiv \frac{G_{\rm F} m_{\rm p}^2 c}{\hbar^3} \approx 1.03 \times 10^{-5} \tag{3}$$

being $G_{\rm F}$ the Fermi constant—characterizing the strength of the weak force—and their combinations

$$\delta \equiv \frac{H_0 \hbar}{m_{\rm p} c^2} \sim 2h \times 10^{-42} \quad \text{and} \quad \epsilon \equiv \frac{G \rho_0}{H_0^2} \sim 5h^{-2} \times 10^{-4}, \tag{4}$$

where $H_0 \simeq 70 {\rm kms^{-1}Mpc^{-1}}$ is the present day Hubble constant, $h = H_0/70$ ${\rm kms^{-1}Mpc^{-1}}$ and ρ_0 is the actual density of the Universe, and asked which of them may vary with time. He also noticed that the relative magnitude of the electrostatic and gravitational forces between a proton and an electron is

$$\frac{\alpha_G}{\mu\alpha} = \frac{G m_p m_e}{e^2} \sim 3.7 \times 10^{-40} \tag{5}$$

and the inverse number of times an electron has orbited around a proton during the age of the Universe, (the age of the Universe in atomic time):

$$\frac{H_0 e^2}{m_e c^3} = 4\pi\alpha\mu\delta \sim 2.4 \times 10^{-40} \tag{6}$$

are of the same order. This prompted Dirac to assume that $\delta \propto H_0$ and α_G may vary with time as $\propto 1/t$.

Since then, and for more than six decades, physicists and astronomers have been interested in devising new methods and techniques to detect and measure any hypothetical time variation of the fundamental constants or, at least, to obtain upper bounds on their variation. Most of them rely on astronomical observations, although a few come from terrestrial or geological experiments. The reason for this is quite obvious. Astronomy provides a very long time baseline and, thus, even small variations of the fundamental couplings can become prominent if large look-back times are used. On the contrary, the precision of the astronomical techniques is usually hampered by the fact that long exposures on large telescopes are usually needed. In subsequent sections we will summarize the most successful methods used so far for detecting any hypothetical variation of the fundamental constants.

The issue of the level of "fundamentality" of a given coupling or parameter is not a simple question. The status of a coupling depends on the theory considered, namely whether it is effective or "fundamental" (microscopic). Some of the "fundamental" couplings of a large scale (effective) theory remain to be fundamental in the underlying microscopic theory, others do not. A pragmatic approach is to fix a theoretical framework characterized by a set of known parameters and then pose questions like why the parameters have the values they do really have and whether they are constants. Thus, questioning the constancy of fundamental parameters is essentially trying to understand a more fundamental theory behind. A discussion of this issue of fundamentality of couplings can be found in Duff et al. (2002). Moreover, it is important to realize that the only meaningful question whatsoever is, in fact, to ask about the variation of a *dimensionless* parameter, like the fine structure constant α, or the proton-to-electron mass ratio $\mu \equiv m_p/m_e$, or the dimensionless parameter α_G defined in Eq. (3), since only in this case the measurement of the variation is independent of the choice of the system of units and of the choice of standard rulers and clocks.

In this review by "variation of constants" we *will not mean* the change of couplings with increasing energy transfer in particle processes, that is, the running of couplings in the renormalization group context. Instead, here we will be concerned with the variation of couplings—like the fine structure constant, the gravitational constant or the gauge coupling of the electroweak theory, amongst others—and mass parameters—like for example the proton-to-electron mass ratio—with time during the cosmological evolution in the low-energy limit. Of course, such variations on Solar System or geological time scales are indeed very small. However, there are at least two reasons for considering this effect. Firstly, there are nowadays a few puzzling astrophysical observations which challenge theorists. Clearly, one of them is that there exist some indications that some constants of nature may have had a different value in past epochs. This was first proposed in Drinkwater and Webb (1998) and Dzuba et al. (1999) and later discussed at length in a series of papers (Webb et al. 1999, 2001; Murphy et al. 2001a,b, 2003; Ivanchik et al. 2002, 2005). These results have nevertheless been challenged by similar (and more recent) analyses carried out in Chand et al. (2004) and Srianand et al. (2004) which will be described in the following sections. The question is now whether or not these observational results (if reliable) can really be interpreted as a sign for the need of new physics beyond the Standard Model. Secondly, there are a number of theoretical models which contain built-in mechanisms which allow for such variations.

Another point of concern is how terrestrial and astronomical constraints compare with each other. The question is if the local variations are representative of the cosmological ones. This point has been recently studied in Shaw and Barrow (2005) and it has been found that the Solar System and terrestrial constraints can help under certain conditions to track the cosmological variations for a very general set of models. This is the reason why in the following subsections we will also summarize the terrestrial bounds to the variation of the fundamental constants.

Finally, and before entering into details, we would like to point out that anthropic considerations allow only a certain interval of admissible values for some of the constants, though do not tell whether they are varying or not. For example, for the fine structure constant such interval is limited by

$$\frac{1}{170} < \alpha < \frac{1}{80}, \tag{7}$$

where the lower bound comes from the requirement that the Grand Unification takes place at the scale $M_{\rm GUT} < M_{\rm Pl}$, and the upper one from the requirement that the proton lifetime $\tau_{\rm p}$ exceeds the age of the Universe $t_{\rm U}$ (Rozental 1988). With the present value of α we have for these two quantities

$$\tau_{\rm p} \sim \frac{1}{\alpha^2}\frac{\hbar}{m_{\rm p}c^2}e^{\frac{1}{\alpha}} \sim 10^{32}\ \text{year}, \quad t_{\rm U} = \frac{c}{H_0} \sim 10^{17}\ \text{s}, \tag{8}$$

which provide the bounds quoted above.

In the present paper, we review the theoretical and observational aspects concerning the hypothetical time variations of the fine structure constant, α, of the gravitational constant, G, and of the proton-to-electron mass ratio, μ.

The reason for such a selection is that these parameters are regarded as truly fundamental in most of the theoretical frameworks. Additionally, the issue of their possible variations has attracted much attention lately and many interesting results, both theoretical and observational, on this subject have been recently obtained. The variation of other parameters, like the couplings of the weak and strong interactions or the proton gyromagnetic factor, are beyond scope of the paper—see, for example, Uzan (2003) for a recent review concerning these constants.

The plan of the paper is the following. In Sect. 2 we review the most classical theoretical approaches which can be used to formally describe the variation of fundamental constants. We first discuss the general features of these models, whereas in the rest of the section we review the Bekenstein–Sandvik–Barrow–Magueijo models, the string inspired models with a runaway dilaton, three classes of models with extra dimensions and quintessence models. The interested reader can skip this section in a first reading of this review if looking for a particular result on the variation of a given fundamental constant. We nevertheless consider it important to briefly discuss the most basic theoretical framework. The experimental bounds on the time variation of the fine structure constant, of the gravitational constant, and of the proton-to-electron mass ratio are reviewed, respectively, in Sects. 3, 4 and 5. A discussion of how the theoretical predictions of the models compare with the observational data is also presented in Sect. 6. Finally, in Sect. 7 we elaborate our conclusions and we discuss future prospects.

2 Theoretical foundations

2.1 General features

Within an effective four-dimensional field theory the only consistent way to make Lagrangian parameters time dependent is through promoting them into functions of a dynamical scalar field, ϕ. Hence, α, G, and other effective couplings should be understood as functions $\alpha(\phi)$ and $G(\phi)$, respectively, of the scalar field. Then, the value of, say, the fine structure constant can be considered as $\alpha_0 \equiv \alpha(\phi)|_{\phi=\langle\phi\rangle}$, where $\langle\phi\rangle$ is the vacuum expectation value of the scalar field. It turns out that string models and theories with extra dimensions contain built-in mechanisms for the variation of (effective) constants in four dimensions. In the vicinity of $\phi = \langle\phi\rangle$ the function $\alpha(\phi)$ can be written as $\alpha(\phi) = \alpha_0 + \lambda\varphi/M_{\rm Pl}$, where $\varphi = \phi - \langle\phi\rangle$ and λ is some constant. For instance, the variation of the fine structure constant $\Delta\alpha$ is related to the corresponding variation of the scalar field as

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha(\phi) - \alpha_0}{\alpha_0} = \frac{\lambda}{\alpha_0}\frac{\Delta\phi}{M_{\rm Pl}}. \tag{9}$$

Such models have two general features (Dvali and Zaldarriaga 2002). Firstly, the mass of the scalar field driving the change of α is very small: $m_\varphi \sim H_0 \sim 10^{-33}$ eV. Secondly, since from the analysis of Murphy et al. (2001a) the maximum hypothetical variation of the fine structure constant is $|\Delta\alpha/\alpha| \sim 10^{-5}$ in the best of the cases, from the previous equation one immediately obtains

$$\left|\lambda \frac{\Delta\varphi}{M_{\rm Pl}}\right| \sim 10^{-7} \quad \text{within} \quad \Delta t \sim H_0^{-1}. \tag{10}$$

With all these considerations in mind in the rest of the section we briefly summarize the most well-studied theoretical frameworks which predict a variation of the fundamental couplings.

2.2 The Jordan–Brans–Dicke theory

The Jordan–Brans–Dicke theory (Jordan 1949; Brans and Dicke 1961) is the most classical example of an alternative theory of gravitation, and represents a self-consistent framework for modeling a possible variation of the gravitational constant, G. In these theories the gravitational interaction is mediated not only by the usual metric tensor field of General Relativity, but also by an additional scalar field. This class of theories was first considered by Jordan (1949) and later developed by Brans and Dicke (1961) for the case of a constant coupling between the scalar and gravitational field. These pioneering papers were followed by more detailed studies and generalizations—see, for instance, Bergmann et al. (1968), Nordtvedt (1970), Will (1993), Barrow and Parsons (1997) and **?**. Within this theory the action is written as

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-\gamma} \left[\phi \mathcal{R} + \frac{w_{\rm BD}(\phi)}{\phi} g^{\mu\nu} \phi_{;\mu} \phi_{;\nu} + 16\pi L_{\rm m} \right]. \tag{11}$$

In this expression $\mathcal{R} = g^{\mu\nu} R_{\mu\nu}$ is the Ricci scalar of the Einstein metric, $g^{\mu\nu}$, γ is the determinant of the metric, ϕ is the scalar field, $w_{\rm BD}(\phi)$ is the coupling function and $L_{\rm m}$ is the Lagrangian of matter fields. In the limit $w_{\rm BD} \to \infty$ the theory reduces to General Relativity. Within this theory the relation between the scalar field ϕ and the gravitational constant G in the weak-field limit is the following:

$$G = \phi^{-1} \frac{4 + 2w_{\rm BD}}{3 + 2w_{\rm BD}} \tag{12}$$

(Brans and Dicke 1961) and the variation of the gravitational constant is given by the cosmological evolution of the scalar field which, in turn, is determined by the specific cosmological scenario. The coupling function $w_{\rm BD}(\phi)$ is unconstrained by the theory itself and its choice determines the cosmological model and the form of the weak-field limit. The interested reader can find some cosmological solutions (and the corresponding variation of G) according to this theory in, for example, Barrow and Parsons (1997) and Will (1993), and references therein.

2.3 The Bekenstein–Sandvik–Barrow–Magueijo model

The Bekenstein–Sandvik–Barrow–Magueijo model (Barrow et al. 2002a,b; Sandvik et al. 2002) is a generalization of the Jordan–Brans–Dicke model. It includes two scalar fields, the Jordan–Brans–Dicke field ϕ and the "dielectric" field ψ, and is described by the action:

$$S = \int d^4x\sqrt{-\gamma}\left[\phi\mathcal{R} - w_{\mathrm{BD}}\frac{\phi_{;\mu}\phi^{;\mu}}{\phi} + 16\pi\left(L_{\mathrm{m}} + e^{-4\psi}L_{\mathrm{em}} - \frac{w}{2}\psi_{;\mu}\psi^{;\mu}\right)\right], \tag{13}$$

where, as before, $\mathcal{R}$ is the scalar curvature and L_{m} and L_{em} are the matter and electromagnetic Lagrangians, respectively. Then the gravitational and fine structure couplings are given by $G \propto 1/\phi$ and $\alpha = \alpha_0 e^{2\psi}$, respectively. The scale of new physics is set by w_{BD} satisfying the bounds $10\,\mathrm{MeV} \leq w_{\mathrm{BD}} \leq M_{\mathrm{Pl}}$. It can be shown that within this model the product $G\alpha$ remains constant for the dust era. Another important characteristic of this model is that α cannot display oscillatory behavior in time in a Friedmann universe of any curvature.

2.4 String inspired models

Within the theoretical framework of string models it is shown that the low-energy action, which includes string-loop effects, is of the form:

$$S = \int d^4x\sqrt{-\gamma}\left[B_g(\phi)\mathcal{R} - B_\phi(\phi)(\nabla\phi)^2 - \frac{1}{4}B_F(\phi)F^2 + \cdots\right], \tag{14}$$

where ϕ is the dilaton field and the coefficient functions behave as

$$B_i(\phi) = e^{-\phi} + a_0^{(i)} + a_1^{(i)}e^{\phi} + a_2^{(i)}e^{2\phi} + \cdots \tag{15}$$

for $g_s^2 = e^{\phi} \to 0$. Here $i = g, \phi, F$. It has been shown (Damour and Polyakov 1994) that at certain values $\phi = \phi_{\mathrm{m}}$, which are fixed points of the theory, the dilaton decouples from the matter. One of these fixed points (Veneziano 2002) can be $\phi_{\mathrm{m}} = +\infty$ (the so–called runaway dilaton). In this case $B_i(\phi)|_{\phi\to+\infty} = C_i + \mathcal{O}\left(e^{-\phi}\right) \to C_i$. In this approach all the couplings (including, for instance, α and G) are related to the *same* scalar field ϕ. Thus, in the Einstein frame, where we denote the scalar field as φ, the fine structure coupling and the hadronic coupling are given by:

$$\alpha(\varphi) = \alpha(\infty)\left[1 - b_F e^{-\kappa\varphi}\right], \tag{16}$$

$$\alpha_{\rm h}(\varphi) \approx 40 b_F e^{-\kappa\varphi}, \tag{17}$$

where b_F is some constant. Since in string-inspired models the variations of the different couplings are given by the change of the same dilaton field ϕ correlated variations of various couplings are predicted. Consequently, these correlations can be potentially used to obtain insights into the underlying theory.

It turns out that the correlation of the variations of constants is a rather generic feature which takes place, for example, in theories of Grand Unification (GUT) and models with extra dimensions. Thus, in the $SU(5)$ GUT the three couplings of the Standard Model $\alpha_1, \alpha_2, \alpha_3$ at the scale $M_Z \approx 100\,\text{GeV}$ are related to the unique fundamental constant $\alpha_{\rm GUT}$ by the following 1–loop renormalization group relations:

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{\rm GUT}} + \frac{b_i}{2\pi}\ln\frac{M_Z}{M_{\rm GUT}}, \quad i = 1, 2, 3, \tag{18}$$

where $M_{\rm GUT} \approx 3 \times 10^{16}$ GeV is the scale of Grand Unification, and b_i are some constants calculated in this theory. In particular, the fine structure constant at the electroweak scale is given by

$$\alpha^{-1}(M_Z) = \frac{5}{3}\alpha_1^{-1}(M_Z) + \alpha_2^{-1}(M_Z) \approx 127.9 \tag{19}$$

If the GUT constant $\alpha_{\rm GUT}$ varies with time then the couplings of the Standard Model experience the corresponding correlated variations. Such scenario with the scales M_Z and $M_{\rm GUT}$ kept constant was studied in Langacker et al. (2002), where it was shown that the fine structure constant, the strong gauge coupling $\alpha_{\rm strong}$ and the scale of quantum chromodynamics $\Lambda_{\rm QCD}$ vary according to:

$$\begin{aligned} \frac{\Delta\alpha}{\alpha} &\approx 0.49\frac{\Delta\alpha_{\rm GUT}}{\alpha_{\rm GUT}}, \\ \frac{\Delta\alpha_{\rm strong}}{\alpha_{\rm strong}} &\equiv \frac{\Delta\alpha_3}{\alpha_3} \approx 5.8\frac{\Delta\alpha}{\alpha}, \\ \frac{\Delta\Lambda_{\rm QCD}}{\Lambda_{\rm QCD}} &\approx 34\frac{\Delta\alpha}{\alpha}. \end{aligned} \tag{20}$$

2.5 Theories with extra dimensions

Models with extra dimensions incorporate a natural mechanism for the space and time variation of the fundamental constants. This kind of models was apparently studied for the first time in Forgacs and Horvath (1979a,b) and later on in a number of papers—see, for instance, the pioneering works of Chodos and Detweiler (1980), Marciano (1984) and Barrow (1987). A large amount of studies on this subject has been done since then. Within this approach the interactions are described by a fundamental theory formulated in the $(4 + d)$–dimensional space–time with d compact extra dimensions and a metric of the form:

$$ds^2 = -dt^2 + a^2(t) \sum_{i,j=1}^{3} \hat{\gamma}_{ij} dx^i dx^j + R^2(t) \sum_{m,n=4}^{d+3} \hat{\gamma}_{mn} dy^m dy^n. \quad (21)$$

It is first important to realize that the four-dimensional theory appears as the result of the dimensional reduction of the multidimensional theory. Moreover, the parameters in four dimensions are determined by both a set of a few new constants of the multidimensional theory and by the size, R, of the space of extra dimensions—see, for instance, Salam and Strathdee (1982) and Duff et al. (1986). Additionally, the multidimensional constants are assumed to be genuinely fundamental and, consequently, do not vary with time. On the other hand, in the astrophysical context it is quite natural to assume that R is a function of time, $R(t)$, very much in the same way as the scale factor $a(t)$ of the three-dimensional space is. Hence, the dynamical solutions for $a(t)$ and $R(t)$ are determined by the parameters of a cosmological scenario which provides the evolution of the Universe. The variation of the scale factors with t gives rise to the time variations of the parameters, like the gravitational constant G and the fine structure constant α, of the effective four-dimensional theory which subsequently are themselves correlated. We will discuss the correlated variations of G and α in three types of models with extra dimensions following closely Lorén-Aguilar et al. (2003).

2.5.1 Kaluza–Klein theories

Kaluza (1921) and, independently, Klein (1926) formulated the essential elements of the multidimensional approach used to describe the fundamental interactions which was later called the Kaluza–Klein approach. They considered the equations for gravity in a five-dimensional space time $M^4 \times S^1$ and showed that the sector of zero modes of the dimensionally reduced theory includes the classical four-dimensional gravity and the classical Maxwell theory with the electromagnetic potential given by the (4μ)-components of the multidimensional metric tensor, $\hat{\gamma}$. Later on this construction was generalized (de Witt 1964; Rayski 1965a,b) to more complicated compact spaces of extra dimensions K_d. Within the Kaluza–Klein approach, the action of the multidimensional theory is quite generally given by

$$S = \int d^{4+d}\hat{x}\sqrt{-\hat{\gamma}}\, \frac{1}{16\pi G_{(4+d)}} \mathcal{R}^{(4+d)}, \quad (22)$$

where $\mathcal{R}^{(4+d)}$ is the scalar curvature in the $(4+d)$-dimensional space-time and $G_{(4+d)}$ is the multidimensional gravitational parameter, which is assumed to be constant.

To obtain the four-dimensional effective theory, the $\mu\nu$-components of the metric tensor $g_{\mu\nu}$ are first identified as the four-dimensional metric tensor. In a second step, certain combinations of the rest of the components are identified as the gauge and scalar fields. Later on, the mode expansion of all these fields is performed. The coefficients of the expansion only depend on the coordinates of the reduced space–time, x^μ, and are interpreted as four-dimensional fields. In general, there is an infinite number of them. Here we are interested in the sector corresponding to the leading terms of the mode expansion. Its action is given by

$$S_{\rm LO} = \int d^4x\sqrt{-\gamma}\left[\frac{1}{16\pi G}\mathcal{R}^{(4)} + \sum_i \frac{1}{4g_i^2}{\rm Tr}F^{(i)}_{\mu\nu}F^{(i)\mu\nu}\right], \tag{23}$$

where $G \equiv G_{(4)}$ is the four-dimensional gravitational constant. The parameters $g_i \equiv g_{(4)i}$ are the gauge couplings, and the index i labels the simple subgroups of the gauge group. The scalar fields usually give highly nonlinear interaction terms and are coupled nonminimally to the gravitational and gauge fields. They are supposed to be frozen out and their contribution is neglected. A simple calculation gives the following expressions for the couplings of the four-dimensional theory in terms of $G_{(4+d)}$:

$$G = \frac{G_{(4+d)}}{V_d}, \tag{24}$$

$$g_i^2 = \kappa_i \frac{G_{(4+d)}}{R^2 V_d}, \tag{25}$$

where $V_d(t) \propto R^d(t)$ is the volume of the space of extra dimensions and κ_i are coefficients which depend on the isometry group of K_d. Assuming that the dimensionally reduced theory includes the electrodynamics, the fine structure constant α is given by a linear combination of g_i^2, the specific relation depending on the model, namely on the gauge group and on the scheme of the spontaneous symmetry breaking. It is easy to show using the previous equations that the time variation of the fine structure and of the gravitational constants are related by

$$\frac{\dot\alpha}{\alpha} = \frac{d+2}{d}\frac{\dot G}{G}. \tag{26}$$

2.5.2 Einstein–Yang–Mills theories

These are theories on the $(4+d)$-dimensional space–time $M_{(4)} \times K_d$ that include gravity and the Yang–Mills field with the action:

$$S = \int d^{4+d}\hat{x}\sqrt{-\hat{\gamma}}\left[\frac{1}{16\pi G_{(4+d)}}\mathcal{R}^{(4+d)} + \frac{1}{4g^2_{(4+d)}}\mathrm{Tr}\hat{F}_{MN}\hat{F}^{MN}\right], \quad (27)$$

where, as above, $G_{(4+d)}$ is the multidimensional gravitational constant, and $g_{(4+d)}$ is the multidimensional gauge coupling. Both are supposed not to depend on time and, thus, are genuine constants. The dimensionally reduced theory includes the classical theory of gravitation, the four-dimensional gauge fields and the scalar fields. The explicit form of the dimensionally reduced theory depends on the topology and geometry of the space of extra dimensions and on the multidimensional gauge group. The best studied case is that in which K_d is a homogeneous space (Witten 1997). For this particular case (Manton 1979) the four-dimensional couplings are given by

$$G = \frac{G_{(4+d)}}{V_d}, \quad (28)$$

$$\alpha = \frac{\alpha_{(4+d)}}{V_d}. \quad (29)$$

From these expressions we obtain the following relation between the time variations of G and α:

$$\frac{\dot{\alpha}}{\alpha} = \frac{\dot{G}}{G}. \quad (30)$$

2.5.3 Randall–Sundrum–type models with gauge fields in the bulk

Another interesting multidimensional setting motivated by the string and the M-theories was first proposed and studied in Randall and Sundrum (1999a,b). The model is formulated in the five-dimensional space $M^4 \times S^1/Z_2$ with the space of extra dimensions compactified to the orbifold S^1/Z_2. There are two three-branes located at the fixed points of the orbifold, which play the role of the macroscopic three-dimensional spaces. In the initial version of the model only gravity was assumed to propagate in the five-dimensional bulk, whereas the fields of the Standard Model were localized on the branes (Randall and Sundrum 1999b). If the tensions of the branes are fine tuned to be equal in absolute value, but of the opposite sign, then the model possesses a background solution for the metric with the exponential warp factor. This assumes that the visible brane—that is, the brane we live on—is the negative tension brane. Due to this feature the model provides an elegant geometric solution to the hierarchy problem and predicts new interesting physical effects which may be eventually observed in the current and future collider experiments. The form of the relation between G and $G_{(4+d)}$ depends on the coordinates on the visible brane.

It has been recently argued (Lorén-Aguilar et al. 2003) that the relationship which provides the correct expression for the time variation of G is obtained in the Galilean coordinates on the visible brane (Boos et al. 2002) and is given by

$$G = \frac{k}{16\pi M^3} \frac{1}{e^{2k\pi R(t)} - 1} \approx \frac{k}{16\pi M^3} e^{-2k\pi R(t)}, \tag{31}$$

where M is the fundamental mass scale and k is a parameter with dimensions of mass. This parameter is, in fact, related to the tension of the branes and to the five-dimensional cosmological constant. Within this model it can be shown that the time variation of the gravitational constant is given by

$$\frac{\dot{G}}{G} = -2\pi kR \frac{\dot{R}}{R} \frac{1}{1 - e^{-2k\pi R}} \approx -2\pi kR \frac{\dot{R}}{R}. \tag{32}$$

However, since the fields of the Standard Model are localized on the brane and do not depend on R this scenario does not explain the variation of the fine structure constant. To describe this effect one has to consider bulk gauge and fermionic fields. Such a model has been studied, for instance, in Davoudiasl et al. (2001). A simple calculation shows that

$$\alpha \propto \frac{g^2_{(5)}}{R}. \tag{33}$$

Hence, for this particular class of models the variation of α and of the gravitational constant are also correlated:

$$\frac{\dot{\alpha}}{\alpha} = \frac{1}{2\pi kR} \frac{\dot{G}}{G} \frac{1}{1 - e^{-2k\pi R}} \approx \frac{1}{2\pi kR} \frac{\dot{G}}{G}. \tag{34}$$

2.6 Quintessence theories

It has recently been suggested that the Universe is dominated by some form of homogeneously distributed dark energy. One of the interesting candidates for this is quintessence–the energy density of a slowly-evolving scalar field, called "cosmon" (Wetterich 1988a,b; Ratra and Peebles 1988). In this theoretical framework the fundamental physical couplings depend on the expectation value of the scalar field. Hence, its variation in the course of cosmological evolution leads to the time variation of the coupling constants. The size of the effect generally depends on the mass scale of unification and is not yet known. Since all couplings and mass ratios depend on the same cosmon field, another generic feature of such models is a correlation between variations of various constants and the appearance of composition-dependent gravity-like forces.

The field dependence of couplings is usually modeled by an action of the type:

$$S = \int d^4x\sqrt{-\gamma}\left[\frac{1}{12}f^2(\chi)\chi^2\mathcal{R} + \frac{1}{2}Z_\chi(\chi)\partial_\mu(\chi)\partial_\nu(\chi)g^{\mu\nu} + \frac{1}{4}Z_F(\chi)F_{\mu\nu}F^{\mu\nu} + \cdots\right], \tag{35}$$

where χ is the scalar cosmon field, $f(\chi)$ is a coupling function, $F_{\mu\nu}$ describes the gauge field and the dots account for a cosmon potential and for matter and its couplings. The first term is responsible for the dependence of the gravitational constant on the scalar field, in particular the Planck mass is given by $M_{\mathrm{Pl}}^2 = 4\pi f^2\chi^2/3$. The third term is the kinetic term for the gauge fields coupled to the cosmon field and gives rise to the variation of the gauge couplings, in particular of the fine structure constant. Some recent studies (Watterich 2003a,b) have shown that this framework may provide a good theoretical description of a cosmological variation of the fundamental constants consistent with Big Bang nucleosynthesis.

3 Observational constraints on the variation of α

We start by describing the methods used to measure any hypothetical variation of the fine structure constant, α. As shown below some of the most frequently used methods involve spectroscopic measurements. The reason is quite obvious: the fine structure constant plays a key role in electronic atomic transitions. Hence, by comparing the intrinsic properties of the spectra of local and distant sources we can assess whether α has varied over cosmological timescales or not. As discussed later, spectroscopic methods are also used to set upper bounds to the variation of the proton-to-electron mass ratio, μ. In fact, depending on which transitions are chosen a combination of both is probed. We will come back to this issue later in Sect. 5.

3.1 Terrestrial constraints

Although these methods are somewhat beyond the scope of the present review we consider worth mentioning them since they provide a natural framework to which the astronomical techniques can be compared to and, moreover, set the astronomical measurements in the proper context. On the other hand, the reader should be aware that most of these measurements only provide constraints on the present-day rate of variation of the fundamental constants, and it is quite possible that while α could have varied at larger look-back times, it has not varied recently.

Terrestrial limits on the variation of the fine structure constant come firstly from atomic clocks (Prestage et al. 1995; Marion et al. 2003; Bize et al. 2003; Fischer et al. 2004; Peik et al. 2004). The basic idea underlying most of these methods is to compare the rates between clocks based on hyperfine transitions in atoms with different atomic numbers Z. In particular, H-maser, Cs, Rb, Yb and Hg^+ clocks have a different dependence on α via relativistic contributions of order Z^2. Recent clock comparisons have improved the laboratory limits on a time variation of α by a large factor, yielding

$$\frac{\dot{\alpha}}{\alpha} \leq (-0.3 \pm 2.0) \times 10^{-15} \text{ year}^{-1}. \quad (36)$$

One of the most stringent terrestrial upper bounds on the variation of α is given by the Oklo natural nuclear reactor. It is a prehistorical natural fission reactor that operated about $t \approx 2 \times 10^9$ years ago ($z \approx 0.15$) during $(2.3 \pm 0.7) \times 10^5$ years in the Oklo uranium mine in Gabon, West Africa. This phenomenon, first discovered by the French Comissariat à l'Énergie Atomique in 1972, consists in an abnormally low relative concentration of the isotopes of Samarium—the isotopic ratio of the ^{149}Sm and ^{147}Sm isotopes. As a matter of fact, the measured isotopic ratio turns out to be ≈ 0.02 whereas the normal one is ≈ 0.9. This is the same to say that ^{149}Sm was depleted due to neutron captures of thermal neutrons when the natural nuclear reactor was active. A possible explanation for the anomalous isotopic ratio is a different value of the capture resonance energy at the time of the reaction which, of course, depends on α. This can be qualitatively understood from very simple arguments. The mass of a given nucleus (A, Z) has electromagnetic and strong contributions. According to the Bethe–Weizäcker formula the electromagnetic contribution amounts to:

$$E_{\mathrm{EM}} = 98.25 \frac{Z(Z-1)}{A^{1/3}} \alpha \text{ MeV} \quad (37)$$

and, hence, any nuclear reaction will depend on α. From this, the following bound (Damour and Dyson 1996) on the variation of α can be derived:

$$-0.9 \times 10^{-7} < \frac{\Delta\alpha}{\alpha} < 1.2 \times 10^{-7} \quad (38)$$

A reanalysis of the existing data using the isotopic ratios of the Gadolinium isotope pairs ^{155}Gd/^{156}Gd and ^{157}Gd/^{158}Gd separately as well as those of Sm previously discussed (Fujii et al. 2000) yielded a tighter upper bound on the variation of the fine structure constant

$$\frac{\Delta\alpha}{\alpha} \leq (-0.8 \pm 1.0) \times 10^{-8}. \quad (39)$$

These estimates, however, involve a number of assumptions about, for instance, the temperature of neutrons, the original abundances of the different isotopes and about the dependence of nuclear energy on α that are difficult to control.

3.2 Solar system constraints

Similar values are obtained from the so-called "rhenium constraint" which relies on the fact that the β-decay energy is sensitive to the variation of α. Qualitatively the origin of this dependence is easy to understand. Since this process is the decay of a neutron into a proton, an electron and an electron antineutrino, $n \rightarrow p + e + \bar{\nu}_e$, it is highly sensitive to the neutron–proton mass difference which is described phenomenologically by an electromagnetic contribution and a weak interaction contribution determined by the difference between the d- and u-quark masses. It is the first contribution through which the dependence of the β-decay on the fine structure constant comes in. Thus, for the purpose of measuring a possible variation of α in the past, the most interesting targets are those isotopes which are both long-lived and have small differences in the binding energies. Of these isotopes ^{187}Re is the leading example. For these long-lived heavy isotopes the decay rate, λ, can be well approximated by its nonrelativistic expression $\lambda = \Lambda(\Delta E)^p$, where ΔE is the decay energy, Λ is a function which is almost independent of α, and

$$p \simeq 2 + \sqrt{1 - \alpha^2 Z^2} \tag{40}$$

is the degree of forbiddeness of the transition. It can be shown that the dependence of the decay rate on α is given by

$$\frac{\mathrm{d}\ln\lambda}{\mathrm{d}\ln\alpha} = 4\pi Z\alpha \frac{c}{v}\left[\left(\frac{0.6\ \mathrm{MeV}}{\Delta E}\right)\left(\frac{2Z+1}{A^{1/3}}\right) - 1\right] \tag{41}$$

v being the escape velocity of the electron. The analysis of ^{187}Re decay rate via the ^{187}Re/^{187}Os ratio of iron-rich meteorites (Olive et al. 2002) formed 4.6 Gyear ago (at $z \approx 0.45$) gives:

$$\left|\frac{\Delta\alpha}{\alpha}\right| < 3 \times 10^{-7}. \tag{42}$$

This result has been nevertheless recently disputed in Fujii and Iwamdo (2003, 2005), where it was argued that the central argument used in Olive et al. (2002)—namely that the limit of time variation of the decay rate is of the same order as the accuracy by which the decay rate is itself determined—was not correct. The corresponding reanalysis yielded:

$$\frac{\Delta\alpha}{\alpha} \sim 1.7 \times 10^{-4}. \tag{43}$$

3.3 Constraints from high-redshift quasars

Perhaps most of the recent interest in the variation of fundamental constants is partially motivated by the results of the observations of Drinkwater et al. (1998), Dzuba et al. (1999), Webb et al. (1999, 2001) and Murphy et al. (2001a,b, 2003), who claim to have detected a temporal variation of the fine structure constant over cosmological look-back times. In principle, there exist two methods to estimate the value of α in the early Universe. The first of this methods is the so-called alkali doublet method, first proposed in Bahcall et al. (1967). The second method is the many-multiplet method (Webb et al. 1999). Both methods rely on precise laboratory measurements of atomic transitions—for which nowadays we have unprecedented precisions, with relative errors smaller than 10^{-9}—and atomic masses—with accuracies of the order of parts per billion and even less. The idea of both methods is the following. All atomic transitions depend on α. However the dependence on α can be quite complex since it involves several terms, corresponding to the fine and hyperfine structure of the spectrum as well as several other interactions. In the leading approximation the splitting $\Delta\lambda = \lambda_1 - \lambda_2$ of two spectral lines of wavelength λ_1 and λ_2 is given by

$$\frac{\Delta\lambda}{\lambda} \propto (Z\alpha)^2 + \mathcal{O}(\alpha)^4, \tag{44}$$

where $\lambda = (\lambda_1 + \lambda_2)/2$ is the mean wavelength of the doublet and Z is the atomic number. This expression incorporates the effects of spin–orbit interactions, relativistic corrections, the hyperfine structure and electron–electron interactions and, consequently, is quite general. The remarkable feature is that the proportionality constant does not involve other fundamental parameters, like the Planck constant or the nucleon and electron masses. Additionally, a variation of α over cosmological timescales affects all the transitions in same way, that is, all wavelengths will be shifted by the same factor. Consequently, it turns out that in principle the spectral shifts are indistinguishable from the Doppler shift of the source. However, by comparing different transitions the dependence on α can be disentangled from the redshift of the source. Consequently, since laboratory measurements provide the frequencies of the lines according to the present value of the fine structure constant, α_0, with a large degree of accuracy, and highly precise observations of distant quasars can be used to derive the value of α at a given redshift, it follows that the time variation of α can be derived. Both the alkali doublet and the many-multiplet method use spectroscopic observations of gas clouds seen in absorption against background quasars. The only difference between them is that the alkali doublet method uses the relativistic fine-structure splitting of alkali-type doublets, whereas the many-multiplet method relies on the simultaneous analysis of spectroscopic data of *different* chemical species. It is worth mentioning that although the alkali doublet method is much simpler, it has been shown that using the many-multiple method results in a significant sensitivity gain. Quite generally (Dzuba et al. 1999) the frequencies of the atomic transitions are given by

$$\omega = \omega_0 + q_1 Z^2 \left[\left(\frac{\alpha}{\alpha_0} \right)^2 - 1 \right] + q_2 Z^4 \left[\left(\frac{\alpha}{\alpha_0} \right)^4 - 1 \right], \tag{45}$$

where ω_0 is the value of the frequency measured in the laboratory and the coefficients q_1 and q_2 can be determined numerically. Note that the dependence of the frequencies on Z suggests that large Z species should be preferentially used. This is the reason why in most of the analysis Mg, Ca, Si, Fe, ... are used. After some algebraic manipulations it can be shown that the splitting ratio, $(\Delta\lambda/\lambda)$, of the absorption system at a given z and the same ratio measured at the laboratory, $(\Delta\lambda/\lambda)_0$, directly yield the relative difference of α in the leading approximation:

$$\frac{\Delta\alpha}{\alpha} \simeq \frac{1}{2}\left[\frac{(\Delta\lambda/\lambda)}{(\Delta\lambda/\lambda)_0} - 1\right]. \tag{46}$$

Although both the alkali doublet method and the many-multiplet method provide an elegant way of measuring a hypothetical variation of α, the reader should be aware that these methods are possibly hampered by a series of problems and systematic effects. In particular, the laboratory measurements of the transition wavelengths should be highly accurate given the very small effects we are trying to detect. This, in turn, restricts the number of lines that can be used. Secondly, when the many-multiplet method is used the assumption that the different atomic transitions are produced in the same region of the absorbing cloud is questionable. Other problems involve the blending of the spectral lines, the spatial inhomogeneity of the absorbing species, atmospheric dispersion, the presence of magnetic fields (inducing Zeeman splitting) and instrumental effects. All these effects have been carefully studied in Murphy et al. (2001a) and, under reasonable assumptions, their influence on the measurement of $\Delta\alpha/\alpha$ is either negligible or, at least, can be controlled. However, it has been recently suggested (Ashenfelter et al. 2004) that the isotopic abundances of some of the species used in the analysis (particularly Mg) may have varied over cosmological timescales, thus producing a sizeable systematic effect for the absorption systems located at $z \leq 1.6$.

The alkali doublet method has been used for almost five decades (Savedoff 1956), whereas the many-multiplet method has been most widely used recently. Here we will only summarize the most recent analyses. For instance, in Potekhin et al. (1994) the absorption doublets of C IV, N V, O VI, Mg II, Al III and Si IV of a sample of quasars with redshifts ranging from 0.2 to 3.7 were used to obtain

$$\frac{\Delta\alpha}{\alpha} = (2.1 \pm 2.3) \times 10^{-3} \tag{47}$$

at $z \simeq 3.2$ at the 2σ level. This analysis was later improved in Cowie and Songaila (1995) where the following estimate was obtained:

$$\frac{\Delta\alpha}{\alpha} = (-0.3 \pm 1.9) \times 10^{-4} \tag{48}$$

Later on, using the Si IV absorption feature—which was demonstrated to be the most sensitive one—an improved constraint (Varshalovich et al. 1996a) was obtained:

$$\frac{\Delta\alpha}{\alpha} = (2 \pm 7) \times 10^{-5} \tag{49}$$

More recently, the same group of authors (Ivanchik et al. 1999) used the doublets of Si IV, C IV and Ng II of 20 absorption systems with z between 2.0 and 3.5 to obtain

$$\frac{\Delta\alpha}{\alpha} = (-3.3 \pm 14.5) \times 10^{-5} \tag{50}$$

at the 95% confidence level.

The many-multiplet method has been applied to several different sets of data so far. For instance, a sample of 30 relatively low-z quasar absorption Mg/Fe systems—spanning redshifts from 0.5 to 1.6—already gave indications (Webb et al. 1999) of a nonzero time variation of α:

$$\frac{\Delta\alpha}{\alpha} = (-1.9 \pm 0.5) \times 10^{-5} \tag{51}$$

Another sample of 49 additional absorption systems was analyzed by the same authors (Murphy et al. 2001b). This sample allowed to measure a nonzero $\Delta\alpha/\alpha$ at the 4.1σ confidence level for redshifts $0.5 < z < 3.5$:

$$\frac{\Delta\alpha}{\alpha} = (-0.72 \pm 0.18) \times 10^{-5} \tag{52}$$

Finally, a large sample of 128 new absorption systems was also analyzed. This sample gave the most precise determination

$$\frac{\Delta\alpha}{\alpha} = (-0.57 \pm 0.10) \times 10^{-5} \tag{53}$$

at the 5.7σ confidence level, averaged over $0.2 < z < 3.7$. Fig. 1 shows the sample of quasars of Murphy et al. (2001b)—grey symbols—along with their corresponding error bars and the binned results—black symbols—with their standard deviations.

Thus, these results seemed to provide evidence for the hypothesis that α was *smaller* in the past. It is worth emphasizing that all these last three measurements have error bars which are smaller than the value of $\Delta\alpha/\alpha$. Thus, in view of the existing data we are not facing just an upper bound but, instead, a direct measurement of the rate of change of α. As already mentioned, possible systematic effects are difficult to estimate (Ashenfelter et al. 2004). However, a careful analysis of systematic errors (Murphy et al. 2001a) has confirmed that most likely there are no systematic effects which can easily mimic a negative value of $\Delta\alpha/\alpha$ comparable to that found, and that the known systematic effects could lead only to more significant deviations of $\Delta\alpha/\alpha$ from zero. However, whether this detection is genuine or not is still the matter of an ongoing (and active) debate (Barrow 2003). This, in turn, has motivated other investigations. At the time the present review was written none of them support the hypothesis

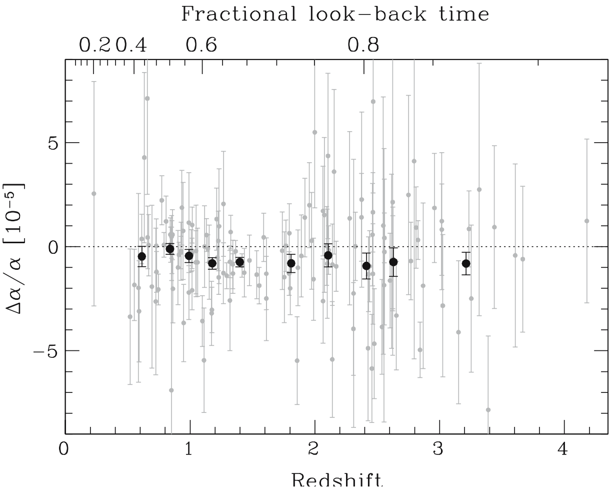

Fig. 1 $\Delta\alpha/\alpha$ for 128 absorption systems with 1σ errors (*grey symbols*) and binned results (*black symbols*) as a function of redshift, adapted from Murphy et al. (2001b). The error bars of the binned data correspond to the standard deviation of the ensemble average

of a varying α. For instance, in Srianand et al. (2004) a sample of quasars with redshifts between 0.4 and 2.3 was analyzed yielding

$$\frac{\Delta\alpha}{\alpha} = (-0.06 \pm 0.06) \times 10^{-5} \tag{54}$$

at the 3σ level. Making use of the O III emission lines of several quasars and galaxies from the *Sloan Digital Sky Survey* (York et al. 2000) led to

$$\frac{\Delta\alpha}{\alpha} = (0.7 \pm 1.4) \times 10^{-4} \tag{55}$$

in the redshift range $0.16 < z < 0.8$ (Bahcall et al. 2004). Later on, the many-multiplet method was slightly modified (Levshakov et al. 2005) to take into account the different biases. These authors applied this new technique—the so-called Single Ion Differential α Measurement (SIDAM)—to the Fe II lines of the quasar Q 1101−264 which is located at $z = 1.839$ and a null result was obtained:

$$\frac{\Delta\alpha}{\alpha} = (2.4 \pm 3.8) \times 10^{-6} \tag{56}$$

At the same time, and using observations of distant quasars obtained with the ESO–UVES spectrograph, 15 Si IV doublets were analyzed in (Chand et al. 2005) finding again a null result

$$\frac{\Delta\alpha}{\alpha} = (0.15 \pm 0.43) \times 10^{-5} \tag{57}$$

Using the OH microwave transition, first proposed in Darling (2003), the analysis of several sources led to

$$\frac{\Delta\alpha}{\alpha} = (-0.38 \pm 2.2) \times 10^{-3} \quad (58)$$

at $z \simeq 0.68$ (Chengalur and Kanekar 2003; Kanekar and Chengalur 2004). A negative result was also obtained in Kanekar et al. (2005) using 18 cm OH lines from the $z \simeq 0.765$ gravitational lens toward PMN J0134−0931:

$$\frac{\Delta\alpha}{\alpha} \leq 6.7 \times 10^{-6} \quad (59)$$

The most precise (and recent) single redshift bound to $\Delta\alpha/\alpha$ has been obtained (Levshakov et al. 2006) using the ESO Very Large Telescope and the SIDAM procedure to a damped Lyα system at $z = 1.15$ obtaining again a null result:

$$\frac{\Delta\alpha}{\alpha} = (-0.07 \pm 0.84) \times 10^{-6} \quad (60)$$

at the 1σ confidence level. Even more recently in Chand et al. (2006) the quasar HE 0514–4414, located at $z_{\rm abs} = 1.508$, has been used to obtain a somehow less restrictive bound:

$$\frac{\Delta\alpha}{\alpha} = (0.05 \pm 0.24) \times 10^{-5} \quad (61)$$

To summarize, although the most recent measurements seem not to confirm the results of Drinkwater et al. (1998), Dzuba et al. (1999), Webb et al. (1999, 2001), and Murphy et al. (2001b, 2003), the topic is still the subject of an active ongoing debate.

3.4 Cosmic microwave background constraints

Additional high-z constraints come also from observations of the Cosmic Microwave Background radiation (CMB). The CMB is composed of photons emitted at the time of recombination of H and He when the Universe became transparent for the radiation at $z \sim 10^3$ or, equivalently, at $t \approx 300$ Myear. A non-standard value of the fine structure constant modifies the CMB angular power spectrum by changing the epoch of recombination. This, in turn, translates into a modification of the height and of the position of the first and subsequent acoustic peaks of the CMB anisotropy spectrum. The underlying argument is, again, rather simple. Before recombination the photons and the electrons are coupled. However, after recombination this is no longer true. Hence, since the interaction of photons and electrons crucially depends on the precise value of α the characteristics of the CMB also depend on it. Since we now have very precise measurements of the angular power spectrum this can be used to set tight constraints on the precise value of α at very large redshifts.

To be specific, the temperature fluctuations for a given direction (θ, φ) is most commonly expressed as

$$\frac{\delta T}{T} = \sum_{\ell} \sum_{m=-\ell}^{m=+\ell} a_{\ell m} Y_{\ell m}(\theta, \varphi), \tag{62}$$

where $Y_{\ell m}(\theta, \varphi)$ is the spherical harmonics. The angular power spectrum multipole is then given by $C_{\ell m} = \langle |a_{\ell m}| \rangle$. These coefficients depend on the adopted cosmological model and on the values of the couplings. However, the dependence is complex and must be computed with the appropriate numerical codes. In particular, it has been shown that the recombination process is well approximated by the evolution of the proton fraction, x_{p}, the singly ionized helium fraction, x_{HeII} and the matter temperature, T_{M}.

The dependence of the proton fraction $x_{\mathrm{p}} = n_{\mathrm{p}}/n_{\mathrm{H}}$ on redshift is given by

$$\frac{\mathrm{d}x_{\mathrm{p}}}{\mathrm{d}z} = \frac{C_{\mathrm{H}}}{H(z)(1+z)} \left[x_{\mathrm{e}} x_{\mathrm{p}} n_{\mathrm{H}} R_{\mathrm{H}} - \beta_{\mathrm{H}}(1 - x_{\mathrm{p}}) e^{-\frac{h\nu_{\mathrm{H}}}{kT_{\mathrm{M}}}} \right] \tag{63}$$

where $x_{\mathrm{e}} = n_{\mathrm{e}}/n_{\mathrm{H}}$ is the electron fraction, ν_{H} is the Lyα frequency and R_{H} is the recombination coefficient for hydrogen. This coefficient is obtained from detailed numerical simulations and can be well fitted by the following expression:

$$R_{\mathrm{H}} = 10^{-19} F \frac{a T_4^b}{1 + c T_4^d} \ \mathrm{m^3\,s^{-1}}, \tag{64}$$

where $F = 1.14$, $a = 4.309$, $b = -0.6166$, $c = 0.6703$, $d = 0.5300$ (Péquingot et al. 1991) and T_4 is the matter temperature in units of 10^4 K. Additionally, β_{H} is the photoionization coefficient:

$$\beta_{\mathrm{H}} = R_{\mathrm{H}} \left(\frac{2\pi m_{\mathrm{e}} k T_{\mathrm{M}}}{h} \right)^{3/2} \exp\left(-\frac{B_{\mathrm{H2s}}}{kT_{\mathrm{M}}} \right), \tag{65}$$

where $B_{\mathrm{H2s}} = 3.4\,\mathrm{eV}$ is the binding energy of the 2s energy level. Finally, C_{H} is the Peebles reduction factor:

$$C_{\mathrm{H}} = \frac{1 + K_{\mathrm{H}} \Lambda_{\mathrm{H}} n_{\mathrm{H}} (1 - x_{\mathrm{p}})}{1 + K_{\mathrm{H}} (\Lambda_{\mathrm{H}} + \beta_{\mathrm{H}}) n_{\mathrm{H}} (1 - x_{\mathrm{p}})}, \tag{66}$$

where $\Lambda_{\mathrm{H}} = 8.22458\,\mathrm{s}^{-1}$ is the two-photon decay rate and $K_{\mathrm{H}} = c^3/(8\pi \nu_{\mathrm{H}}^3 H)$. A similar expression holds for the evolution of He I with redshift. Finally, the dependence of the matter temperature on redshift is given by:

$$\frac{\mathrm{d}T_{\mathrm{M}}}{\mathrm{d}z} = \frac{8\sigma_{\mathrm{T}} a_{\mathrm{R}} T_{\mathrm{R}}^4}{3H(z)(1+z) m_{\mathrm{e}}} \frac{x_{\mathrm{e}}}{1 + f_{\mathrm{He}} + x_{\mathrm{e}}} (T_{\mathrm{M}} - T_{\mathrm{R}}) + \frac{2T_{\mathrm{M}}}{1+z}. \tag{67}$$

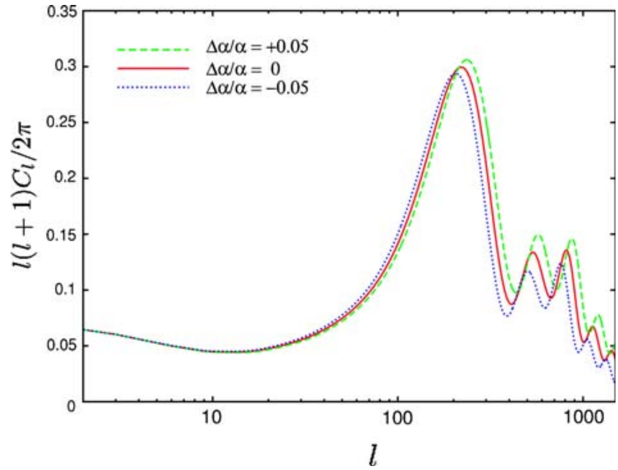

Fig. 2 The spectrum of the CMB fluctuations for several values of the $\Delta\alpha/\alpha$, from Ichikawa et al. (2006)

In this expression T_R is the radiation temperature, $\sigma_T = 2\alpha^2 h^2/(3\pi m_e c^2)$ is the Thomson cross section (which explicitly depends on α) and a_R is the Stefan–Boltzmann constant. The Compton scattering makes the radiation temperature and the matter temperature the same at sufficiently high redshifts. However, at lower redshifts they are different. As can be seen, the temperature of recombination largely depends on the fine structure constant. Nevertheless, this is not the only term in which a dependence on α can be found. For instance, the two-photon decay rates depend on α^8, the binding energies scale as α^2 and so do the Lyα frequency and the corresponding frequency for neutral He. Also, K_H—and the corresponding factor for He I also does—scales as $\sim \nu_H^{-3} \sim \alpha^{-6}$. Finally, the recombination coefficient R_H also depends on α. All in all, there are several subtle dependences of the various relevant quantities on α. In fact, it can be shown that an increase in α directly translates into a larger z recombination epoch. This is easy to understand, since the binding energies scale as α^2 and, consequently, photons should have larger energies to ionize hydrogen atoms. This, in turn, induces a shift in the spectrum to higher multipoles and an increase of the strength of the acoustic peaks of the spectrum, see Fig. 2.

On these grounds, a first attempt to bound $\Delta\alpha/\alpha$ based on reliable observational data was done in Avelino et al. (2000) by using the first release of data by the BOOMERANG (de Bernardis et al. 2000) and MAXIMA (Balbi et al. 2000) experiments. The authors concluded that these data preferred a value of α that was smaller in the past by a few percent. However, due to the intrinsic degeneracies on other cosmological parameters a definite bound was not obtained. It is also true that there were also earlier attempts (Hannestad 1999; Kaplinghat et al. 1999) to constrain the hypothetical variation of α—based on less competitive observational data—but these studies turned out to be inconclusive as well and can be only considered as projections of future work to be done using more reliable observational material.

A more conclusive study—based on the same data—was done in Battye et al. (2001) where the following (tentative) upper bound

$$\frac{\Delta\alpha}{\alpha} = -0.01 \tag{68}$$

was obtained. Later on, in Avelino et al. (2001) the authors updated their previous study and the following bound was obtained:

$$\frac{\Delta\alpha}{\alpha} = -0.005^{+0.07}_{-0.04} \tag{69}$$

at the 68% confidence level. In the same vein, in Landau et al. (2001) using not only BOOMERANG and MAXIMA data but also data from the COBE Differential Microwave Radiometer—see Boggess et al. (1992), and references therein—the following bounds to the variation of the fine structure constant were obtained:

$$0.02 < \frac{\Delta\alpha}{\alpha} < -0.14. \tag{70}$$

In this paper, a semianalytic approach was developed to deal with the uncertainties associated with the other cosmological parameters, such as the baryonic matter density, the total matter density and the value of the Hubble constant. All these parameters play a key role in computing the resulting power spectrum of the CMB, and their associated uncertainties considerably limited the results obtained by all the previously mentioned authors. It is thus important to realize that a careful analysis of these uncertainties would naturally explain the diversity of the results obtained using the same set of data.

Later on, using a combined analysis of the BOOMERANG, MAXIMA and DASI (Halverson et al. 1998) data sets and combining these data sets with constraints coming from large-scale structure observations—particularly, the 2dF survey (Smith et al. 1997)— it was shown (Martins et al. 2002) that the existing data were consistent with no variation of α from the epoch of recombination to the present day. Moreover, these authors restricted any such variation to be less than about 4%.

With the launch in 2001 of the WMAP satellite (Bennett et al. 2003) things have improved much. The instruments on board WMAP have measured the temperature of the Cosmic Microwave Background with unprecedented accuracy. Consequently, this kind of studies has benefited from this enhanced accuracy in the determination of the angular power spectrum of the CMB, and better sensitivities are now being obtained. For instance, using WMAP first-year data, in Martins et al. (2004) the bound

$$0.02 < \frac{\Delta\alpha}{\alpha} < -0.05 \tag{71}$$

was obtained at the 95% confidence level. Also, using WMAP first-year data, but carrying out a more detailed analysis, the same set of authors has recently updated (Rocha et al. 2003, 2004) their constraint to obtain

$$0.01 < \frac{\Delta\alpha}{\alpha} < -0.06 \tag{72}$$

at the 95% confidence level. More recently, this result has been confirmed (Ichikawa et al. 2006) using WMAP data only, but combining it with the constraint on the Hubble parameter from HST Hubble Key Project:

$$0.026 < \frac{\Delta\alpha}{\alpha} < -0.042. \tag{73}$$

In conclusion, the constraints on the variation of α obtained using the angular power spectrum of the CMB are not particularly strong. Moreover, it can be shown that strong constraints can only be obtained if the rest of the cosmological parameters are independently known. Consequently, using this method competitive bounds will not be obtained unless robust determinations of the rest of cosmological parameters are eventually obtained. Hopefully, the PLANCK mission will help in changing this situation—see (Maino et al. 1999), and references therein.

3.5 Big Bang Nucleosynthesis constraints

Big Bang Nucleosynthesis (BBN) also provides bounds on the variation of the fine structure constant, but unlike the methods studied up to now the redshift at which these bounds are significant are very large, $z \sim 10^{10}$. The high degree of sophistication and predictivity of BBN, which directly predicts abundances for ^{2}H, ^{4}He, ^{7}Li as a function of the baryon fraction and the value of the Hubble constant only, makes the comparison of BBN predictions with observational data a crucial test for the theoretical models. Particularly, the amount of ^{4}He produced during the nucleosynthesis is basically obtained from the neutron-to-proton ratio at the freeze-out of the weak interactions that convert neutrons and protons to each other. Consequently, we expect the result of BBN calculations to sensitively depend on the particular value of α.

Additionally, and from the observational point of view, there are several measurements of the deuterium and helium primordial abundances. Particularly, the measurements of the deuterium Lyα features in several quasar absorption systems at high redshift yield a relative deuterium abundance D/H $= (3.0 \pm 0.4) \times 10^{-5}$ (O'Meara et al. 2001). For the ^{4}He mass fraction, $Y_{\rm p}$, the most reliable data is obtained from the study of HII regions in blue compact galaxies. Specifically, in Olive et al. (1997) a value of $Y_{\rm p} = (0.244 \pm 0.002)$ was found, although the precise value is still the subject of some controversy. Measuring the primordial abundance of ^{7}Li is not an easy task, since it is strongly depleted in stars. Even though very careful analyses have been done so far—see, for instance, Ryan et al. (2000) and references therein—the results of standard BBN do not fully agree yet with the observational data and, hence, the primordial ^{7}Li abundance cannot yet be safely used to constrain nonstandard BBN. Thus, most of the constraints come from either the ^{2}H or the ^{4}He primordial abundances.

The effects of a varying α in the results of BBN are essentially twofold. On the one hand a nonstandard value of α affects the mass difference, Δm, of protons and neutrons whereas, on the other, it also affects the energy release and the Coulomb barrier of the relevant nuclear reactions. Both effects are important and cannot be neglected. In particular, the mass difference between neutrons and protons, which for a standard value of α amounts to $Q = \Delta mc^2 = 1.29$ MeV, affects the primordial abundances because it fixes the neutron to proton ratio at the freeze-out of the weak interactions and, thus, provides the initial conditions at the beginning of BBN. On its hand, the dependence on α of the most important nuclear reactions involved in BBN also has clear consequences on the final primordial abundances—we recall the discussion about nuclear reactions in Sect. 3.1—and must also be taken into account. Under these conditions—and assuming statistical equilibrium of the weak interactions before the freeze-out—the primordial ^{4}He mass fraction can be well approximated by

$$Y_{\rm p} \simeq 2\frac{1}{1+\left(\frac{n_{\rm p}}{n_{\rm n}}\right)_{\rm f}} = \frac{2}{1+e^{\Delta m/T_{\rm f}}}, \tag{74}$$

where $T_{\rm f}$ is the freeze-out temperature and the rest of the symbols have been previously defined or have their usual meanings. As it can be seen, the dependence of Q on α is, thus, one of the crucial elements. All the constraints on $\Delta\alpha/\alpha$ have to assume a dependence of Δm (or, equivalently, of Q) on α and, additionally, must take into account the effects of the nuclear reaction rates. Once these two inputs are fixed, standard BBN calculations are performed and compared with the existing observational data.

The pioneering works of Campbell and Olive (1995) and Kolb et al. (1986) showed that the abundance of primordial ^{4}He was the most sensitive to relatively small changes of Q. This is because ^{4}He has the larger binding energy and, thus, the effects of a varying α on its abundance are larger. They proceeded as follows. They first assumed a functional form for mass difference of protons and neutrons $Q = \alpha Q_1 + \beta Q_2$, where Q_1 stands for the electromagnetic contributions and Q_2 for the rest of the contributions. They furthermore assumed that Q_1 was the most important contribution. With all these inputs they carried out the calculations and concluded that

$$\left|\frac{\Delta\alpha}{\alpha}\right| < 0.01. \tag{75}$$

The work of Kolb et al. (1986) was revised and extended in Bergström et al. (1999) assuming a different dependence of Q on α:

$$Q = \left(1.29 - 0.76\frac{\Delta\alpha}{\alpha}\right) \text{ MeV}, \tag{76}$$

which was derived taking into account both the electromagnetic and the strong contributions (Gasser and Leutwyler 1982). They also evaluated the effects of a nonstandard value of α on the most important nuclear reactions and obtained:

$$\left|\frac{\Delta\alpha}{\alpha}\right| < 0.02. \tag{77}$$

Finally, using the same expression for Q the bound

$$-5.0 \times 10^{-2} < \frac{\Delta\alpha}{\alpha} < 1.0 \times 10^{-2} \tag{78}$$

has been recently derived (Ichikawa and Kawasaki 2002). Even more recently in Müller et al. (2004) the following upper limit was obtained:

$$\left|\frac{\Delta\alpha}{\alpha}\right| \lesssim 10^{-3} \tag{79}$$

whereas in Cyburt et al. (2005) the following bound was derived:

$$\left|\frac{\Delta\alpha}{\alpha}\right| < 0.06. \tag{80}$$

Very recently, in Landau et al. (2006) a semianalytic approach was adopted, and the following bound was obtained:

$$\frac{\Delta\alpha}{\alpha} < -0.136 \pm 0.041 \tag{81}$$

when all the isotopes were included in the analysis, whereas when the abundance of ^{7}Li was not taken into account the following bound resulted:

$$\frac{\Delta\alpha}{\alpha} < -0.054 \pm 0.097. \tag{82}$$

The most recent works (Coc et al. 2006a,b) have carefully analyzed all the dependences of the nuclear reaction rates on α and the Yukawa couplings, $h_{\rm Y}$—which characterize the interaction between quarks and the scalar Higgs boson—and using the deuterium abundance have provided bounds on the joint variation of the Yukawa couplings and the fine structure constant. In the scenario of a varying dilaton, which predicts that the coupling variations are related by $\Delta h_Y/h_Y = (1/2)\Delta\alpha/\alpha$ (**?**), the following tight bounds were derived:

$$-3.6 \times 10^{-5} < \frac{\Delta\alpha}{\alpha} < 4.2 \times 10^{-5}. \tag{83}$$

In summary, these results do not provide any evidence for a variation of α. Nevertheless, a final comment is in order. As it occurs with the bounds coming from the angular power spectrum of the CMB, the results of BBN also show a marked dependence on the choice of the cosmological model. Thus, although they probe very large look-back times, the results are model dependent and, consequently, should be treated with some caution.

4 Observational constraints on the variation of *G*

As already mentioned in Sect. 2 the constancy of the gravitational constant has been debated for a long time (Milne 1935, 1937; Jordan 1937, 1939) and it is still the subject of several theoretical investigations. Moreover, it is important to realize that alternative theories of gravitation in which G does not remain constant have attracted more theoretical efforts than other (more recent) theoretical settings in which all the couplings vary with time—see Sect. 2 and Danielsson (2001) for a recent review on the most recent theoretical framework. Consequently, there have also been more attempts to measure a time variation of the gravitational constant and several different methods have been proposed and used so far, in contrast with the situation for α and μ. However, most of these bounds come either from local measurements (the Sun, our Solar System or the solar neighborhood) or from very early times (basically Big–Bang nucleosynthesis), whereas at intermediate look-back times there are not many measurements, of which those obtained from the Hubble diagram of Type Ia supernovae are the most relevant ones. We discuss them in detail below. There is, however, another interesting point that should be stressed. Namely, that the *experimental* value of G is poorly determined, in sharp contrast with the situation for the case of α and μ. In fact the uncertainty in the determination of G is still today as large as 0.15% (Gundlach and Merkowitz 2000).

4.1 Terrestrial constraints

Our first constraint comes from paleontological arguments. In the early work of Teller (1948)—a piece of work of paramount importance—it was shown that the hypothesis of an evolving G would be in conflict with some paleontological evidence. In particular, using the virial theorem it can be shown that the temperature of the Sun scales linearly with the gravitational constant, $T_\odot \propto GM_\odot/R_\odot$. Consequently, since the luminosity of the Sun strongly depends on its temperature ($L_\odot \propto T_\odot^7 M_\odot^5$), the total flux received on the Earth very much depends on the exact value of G. Assuming further a keplerian orbit and conservation of orbital angular momentum it can be shown that the temperature on the surface of the Earth scales as $T \propto G^{2.25} M_{\odot}^{1.75}$. Assuming, moreover, that the mass of the Sun has remained constant, this allows to set bounds on the variation of G by considering that, under certain conditions, some bacteria and other organisms would not have developed 4.0×10^8 years ago, for which we have direct evidence they did. Later on, this argument was refined (Gamow 1967a) and it was shown that the temperature on Earth could have been in the acceptable range only if

$$\left| \frac{\Delta G}{G} \right| < 0.1. \tag{84}$$

Another approach to the problem consists of studying the primitive radius of the Earth. Obviously, if the strength of the gravitational force has changed significantly during geological timescales the radius of the Earth should have changed accordingly (Dicke 1962). Taking into account the equation of hydrostatic equilibrium and assuming an appropriate equation of state for the Earth (or any other planet) it can be shown (McElhinny et al. 1978) that

$$\frac{\Delta G}{G} = -\eta \frac{\Delta R_{\oplus}}{R_{\oplus}}, \tag{85}$$

where the proportionality constant η carries all the information about the equation of state. For the case of the Earth η amounts to $\simeq 11.7647$, and taking into account that the Earth has not changed significantly in radius (0.8% at most during the last 4.0×10^8 years) an upper bound can be obtained. If, additionally, one takes into consideration the existing data for the Moon and Mars the following upper bound is obtained:

$$\left| \frac{\dot{G}}{G} \right| \lesssim 8 \times 10^{-12} \text{ year}^{-1} \tag{86}$$

4.2 Solar system constraints

4.2.1 Earth–Moon system

We first start reviewing the constraints coming from the Earth–Moon system. For the case of a varying G the orbit of the Moon (and, of course, its period) would have change on the same timescale. Hence, in a series of pioneering works several authors tried to constrain a hypothetical variation of the gravitational constant by studying the motion of the Moon around the Earth. In particular, it is expected that if G varies the orbital period, P, should vary as

$$\frac{\dot{P}}{P} = -2\frac{\dot{G}}{G} \tag{87}$$

However, a practical problem arises when using this method. The problem consists in quantifying $\dot{P}/P$ for the Moon because there is a contribution from tidal interactions with the Earth that has first to be removed before the real effect of a varying G can be evaluated. Also, the determination of the period of the Moon in ancient epochs is not free of problems either. This can be done, for instance, by studying the historical records of solar eclipses which occurred long ago, or by studying astrometric data from lunar occultations. In all the cases most probably reliable data are only available for the most recent centuries and there is always the suspicion that early data are probably not accurate. This method was first used in Morrison (1973), which determined

$$\left|\frac{\dot{G}}{G}\right| < 2 \times 10^{-11}\ \text{year}^{-1}. \tag{88}$$

A couple of years later in van Flandern (1975) a positive detection of a varying G was reported:

$$\frac{\dot{G}}{G} = (-8 \pm 5) \times 10^{-11}\ \text{year}^{-1}. \tag{89}$$

Later on, the analysis of Muller (1978) gave no positive detection:

$$\frac{\dot{G}}{G} = (-2.6 \pm 15) \times 10^{-11}\ \text{year}^{-1}. \tag{90}$$

A more accurate evaluation of the effects of tidal friction (van Flandern 1981) led again to a positive detection but with a value somewhat smaller than the previous one:

$$\frac{\dot{G}}{G} = (-6.4 \pm 2.2) \times 10^{-11}\ \text{year}^{-1} \tag{91}$$

There is another method of constraining the rate of variation of G which very much resembles the previous one. It consists of estimating the number of sidereal days in a sidereal year and in a sidereal month in past epochs. If we denote by $\nu_\oplus$ the frequency of the motion of the Earth around its own axis, by ν_1 the frequency of the orbital motion of the Moon around the Earth, and by ν_2 that of the Earth around the Sun, we have that the number of sidereal days in a year and in a month are given by, respectively, $Y = \nu_\oplus/\nu_2$ and $M = \nu_\oplus/\nu_1$. It can be shown (Blake 1977) that the variation of G can be related to the variation of Y and M and to the variation of the moment of inertia of the Earth, I, by

$$2\frac{\Delta G}{G} = \left(\frac{\gamma}{3} - 1\right)\frac{\Delta Y}{Y} - \frac{\gamma}{3}\frac{\Delta M}{M} - \frac{\Delta I}{I}, \tag{92}$$

where $\gamma = 1.9856$. As previously explained in Sect. 4.1, a variation of G induces a variation of its radius and of its moment of inertia. Taking into account the available data at that time for the variation of the radius of the Earth and using the paleontological data of Pannella (1972)—obtained from the analysis of calcified structures of fossil organisms—the following bound was obtained:

$$\frac{\dot{G}}{G} = (-0.5 \pm 2.0) \times 10^{-11}\ \text{year}^{-1} \tag{93}$$

In summary, all these methods appear to be rather fragile, mainly due to the intrinsic uncertainties in determining the past observational properties of the orbits of the Moon around the Earth or of the Earth itself around the Sun. Hence, it can be concluded that the constraints on the variation of G derived from these methods are not reliable and, thus, more precise methods are needed.

4.2.2 Radar and laser ranging

Better constraints on the secular rate of change of G can be obtained using radar ranging to accurately obtain the separation of either the Moon, other planets or even interplanetary probes. The separation of these orbiting bodies can be monitored with high accuracy and can be afterwards compared with theoretical predictions to obtain upper bounds on $\dot{G}/G$. When using this method we implicitly assume that the clocks we use on Earth are not affected by the variation of any other fundamental constant. This comment is in order since, as discussed in Sect. 2, there are some theoretical models which predict a correlated variation of α and G. Obviously, a variation of α would directly translate into a variation of the atomic clocks used to derive the variation of G—see Sect. 3.1. Consequently, a negligible variation of the properties of the atomic clock signals for the time interval of the observations is implicitly assumed.

The pioneering work of Shapiro et al. (1971) used Venus and Mercury as targets and compared the time delays between those two planets with a Cesium atomic clock. The time interval analyzed was of almost 5 years. The perturbations of the Moon and other planets were taken into account. The observational data were then compared with the theoretical expectations and the conclusion was that a loose upper bound could be placed:

$$\left|\frac{\dot{G}}{G}\right| < 4 \times 10^{-10}\ \text{year}^{-1} \tag{94}$$

This method has since then been used quite frequently, employing several targets. These targets include mainly, but not only, several planetary probes, for which we have excellent data. We list below the most relevant works.

The first of these works is a reanalysis of the data presented in Shapiro et al. (1971). The radar ranging data of Venus remained unused in this analysis because of the large observational uncertainties. This bound was updated in Reasenberg and Shapiro (1976, 1978) by including radar ranging data of the

Mariner 9 and Mars Orbiter spacecrafts to yield

$$\left|\frac{\dot{G}}{G}\right| < 1.5 \times 10^{-10} \text{ year}^{-1} \tag{95}$$

Later on, in Shapiro (1990), the analysis was further improved and it was shown that the observational data were consistent with

$$\frac{\dot{G}}{G} = (-2 \pm 10) \times 10^{-12} \text{ year}^{-1}. \tag{96}$$

A couple of years later, in Anderson et al. (1992) it was shown that the available data of the Mariner 10 probe and of Mercury and Venus could be used to provide the bound

$$\frac{\dot{G}}{G} = (0 \pm 2) \times 10^{-12} \text{ year}^{-1} \tag{97}$$

The data from the Viking landers has also been used to constrain the variation of G. In this case, however, the modeling of the asteroid belt contributes significantly to the nominal error bars. Using this set of data in Reasenberg et al. (1979) the following upper limit was obtained:

$$\left|\frac{\dot{G}}{G}\right| \lesssim 10^{-12} \text{ year}^{-1}. \tag{98}$$

Later on, and also using data from the Viking landers, in Hellings et al. (1983) the conclusion that

$$\left|\frac{\dot{G}}{G}\right| = (2 \pm 4) \times 10^{-12} \text{ year}^{-1} \tag{99}$$

was reached. However, around the same time, and using the same observational data, a different result (Reasenberg 1983) was obtained:

$$\left|\frac{\dot{G}}{G}\right| \lesssim 3 \times 10^{-11} \text{ year}^{-1}. \tag{100}$$

The data from the Viking landers has been recently reanalyzed (Chandler et al. 1993) to yield

$$\left|\frac{\dot{G}}{G}\right| \lesssim 10^{-11} \text{ year}^{-1}. \tag{101}$$

Finally, the measurement of the frequency shift of the radio signal sent and received from the Cassini spacecraft has recently led (Bertotti et al. 2003) to the following stringent (although model-dependent) bound:

$$\left|\frac{\dot{G}}{G}\right| \lesssim 10^{-14}\ \text{year}^{-1} \tag{102}$$

The Lunar Laser Ranging experiment has also provided valuable results. The Apollo 11, 14 and 15 manned missions and the Lunakhod 1 and 4 Soviet missions placed reflectors on the surface of the Moon. These mirrors reflect laser beams from the Earth and provide us with accurate measurements of the Earth–Moon distance–see, for instance, Dickey et al. (1994) and references therein. The first constraint using this method was presented in Williams et al. (1976):

$$\left|\frac{\dot{G}}{G}\right| \lesssim 3 \times 10^{-11}\ \text{year}^{-1}. \tag{103}$$

This constraint was later improved in Müller et al. (1991) by extending the time baseline to 20 years, yielding

$$\left|\frac{\dot{G}}{G}\right| \lesssim 1 \times 10^{-11}\ \text{year}^{-1}. \tag{104}$$

Later on, the previous analysis was further improved in Dickey et al. (1994) to obtain

$$\left|\frac{\dot{G}}{G}\right| \lesssim 6 \times 10^{-12}\ \text{year}^{-1}. \tag{105}$$

By extending even further the time baseline to 24 years, in Williams et al. (1996) it was shown that the available observational data were consistent with

$$\left|\frac{\dot{G}}{G}\right| \lesssim 8 \times 10^{-12}\ \text{year}^{-1}. \tag{106}$$

The differences between the last two bounds were due to the evaluation of tidal acceleration of the Moon. Finally, the most recent tests (Williams et al. 2004) have yielded

$$\frac{\dot{G}}{G} = (4 \pm 9) \times 10^{-13}\ \text{year}^{-1} \tag{107}$$

4.3 Stellar constraints

In this subsection we review the main stellar constraints on the rate of variation of G. These constraints come mainly (but not only) from white dwarfs and neutron stars, that is, from compact objects in which gravity plays a very significant role, or from globular clusters in which the considerations about the nuclear lifetimes are crucial. As we will discuss below there are two types of methods: those in which we can directly probe the variation of G by measuring a physical characteristic of a star, and those in which only a statistical measure of the rate of variation of G is possible. Obviously, those methods involving a direct measure of $\dot{G}$ are more reliable and robust. However, the main drawback of these direct methods is that they usually rely on astronomical measurements in which very precise determinations of the physical characteristics of the stellar object are needed. These include, for instance, the luminosity of the star, the effective temperature and the radius. Consequently, accurate distance determinations are, in general, needed. This in turn has two direct effects. The first one is that the results rely on a limited number of objects for which these physical characteristics can be determined unambiguously. The second (undesired) effect is that most of these objects are, consequently, nearby stars, thus resulting in short look-back times.

4.3.1 Ages of globular clusters

The theory of stellar evolution has nowadays reached such a predictive power that allows one to use it to set constraints on the rate of variation of G. The early calculations—see Pochoda et al. (1963), Roeder and Demarque (1966), Gamow (1967b), Shaviv and Bahcall (1969), Chin and Stothers (1975, 1976) and Ezer and Cameron (1966)—of stellar models with a varying G were primarily concerned with the evolution of our Sun. However, most of these calculations turned out to be rather inconclusive except for the most extreme assumptions about $\dot{G}/G$, for which it was found that such very large variation rates would eventually lead to the exhaustion of nuclear resources nowadays. But these very large rates of change of G were in most of the cases already in conflict with other constraints. The reason for this was twofold. On the one hand, the lack of good observational datasets hampered a reliable comparison with the existing observations. On the other, the theoretical uncertainties in the modeling of the properties of our Sun were at that time also very large. Particularly, the main drawback of this method is that the presolar helium abundance *and* the mixing length parameter must be fine tuned to reproduce the present-day luminosity and radius of the Sun.

A different approach can be adopted. Any possible secular variation of G leads to a modification of the strength of the gravitational force. Consequently, the pressure profile inside a star changes accordingly to fulfill the conditions of the equation of hydrostatic equilibrium. For nondegenerate stars the only control parameter is the temperature, since increasing (decreasing) the local density directly translates an increase (decrease) of gravity. Since the nuclear reaction rates are very sensitive to the temperature it turns out that any nuclear evolutionary timescale becomes affected. Hence, by comparing the ages of the globular clusters using, for instance, the luminosity of the main-sequence turn-off, and by adopting a maximum and a minimum acceptable age of the Galaxy, one readily obtains an upper bound to the rate of variation of the gravitational constant. This method was carefully analyzed in Degl'Innocenti et al. (1996)

and the bound

$$-3.5\times 10^{-11}\ \mathrm{year}^{-1} \lesssim \frac{\dot{G}}{G} \lesssim 7\times 10^{-12}\ \mathrm{year}^{-1} \tag{108}$$

was obtained.

4.3.2 Asteroseismological constraints

White dwarfs represent the final state of the evolution of objects with masses smaller than $\sim 10\, M_{\odot}$. Consequently, most of the stars will end up their lives as a white dwarf. The general properties of white dwarfs were firmly established long ago. The most relevant one is the fact that their structure is largely supported against gravitational collapse by the pressure of degenerate electrons. It is, however, interesting to note that the outer layers are mildly degenerate. In the simplest picture, most of the energy release of white dwarfs results from the residual gravothermal energy, while nuclear energy release usually represents a minor contribution and it is nonnegligible only for the hottest white dwarfs. Moreover, the outermost nondegenerate layers effectively control the energy leaks and, hence, are crucial in our understanding of their rate of cooling. Some white dwarfs are nonradial pulsators. This has provided us with the interesting possibility of using seismological techniques to investigate the internal constitution of white dwarfs. Asteroseismology of white dwarfs has proved to be a powerful tool in searching for the internal properties of these objects which would be otherwise inaccessible. Moreover, all white dwarfs should undergo non-radial pulsations as they cool down across the relevant range of effective temperatures. For our purposes it is enough to note here that the properties of the nonradial oscillations of white dwarfs are basically determined by the so-called Brunt–Vaïsälä frequency N, which is defined as

$$N = g\left(\frac{1}{\Gamma_1}\frac{\mathrm{d}\ln p}{\mathrm{d}\ln r} - \frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r}\right), \tag{109}$$

where g is the gravitational acceleration, Γ_1 is the first adiabatic exponent, p is the pressure and ρ is the density. If G has a secular variation, it will have a direct impact on the structure of white dwarfs. For instance, if G increases, the star will tend to shrink, the reverse being also true. In other words, a variation of G will have a direct impact on the degree of degeneracy of the white dwarf interior and on the local acceleration of gravity, consequently modifying the value of N. This, in turn, has two noticeable effects (Benvenuto et al. 2004); on the one hand, it modifies the period, P, of the pulsations and, on the other, it modifies the rate of cooling thus modifying as well the period derivative, $\dot{P}$. Particularly, it can be shown that the rate of change of the period is given, to a good approximation, by

$$\frac{\dot{P}}{P} = -a\frac{\dot{T}}{T} + b\frac{\dot{R}}{R}, \tag{110}$$

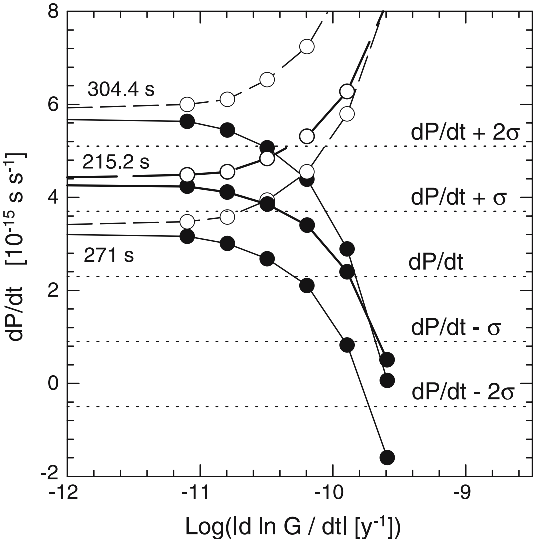

Fig. 3 The derivatives of the periods of oscillation for the white dwarf models G117–B15A as a function of $|\mathrm{d}\ln G/\mathrm{d}t|$. For the cases of $\dot{G} > 0$ ($\dot{G} < 0$) *dashed* (*solid*) *lines* have been used. In order to emphasize the case of the main period (215.2 s) *thick lines* have been used. *Dots* represent the values for which models have been computed. The observed secular variation of the main period of G117–B15A together with one and two standard deviations are depicted with *dotted lines*. Reprinted from Benvenuto et al. (2004)

where T is the temperature of the nearly isothermal core, R is the radius and a and b are constants of the order of unity which depend on the detailed structure and chemical composition of the white dwarf. Consequently, any change in the strength of the gravitational interaction is immediately translated into an anomalous rate of change of the period.

Among variable white dwarfs the most suitable star is G117–B15A because it has been monitored for more than 20 years. The time baseline for this particular star is so large that has allowed us to measure the secular rate of change of its main period (at ~ 215 s) with unprecedented accuracy $\dot{P} = (2.3 \pm 1.4) \times 10^{-15}$ s s^{-1}, making this object the most stable optical clock found so far (Kepler et al. 2000). Using this particular white dwarf in Benvenuto et al. (2004), a careful analysis was done—see Fig. 3—resulting in the following bounds at the 2σ confidence level:

$$-2.5 \times 10^{-10}\ \mathrm{year}^{-1} < \frac{\dot{G}}{G} < +4.0 \times 10^{-11}\ \mathrm{year}^{-1} \tag{111}$$

This bound was later revised (Biesiada and Malek 2004), obtaining the following value:

$$\left|\frac{\dot{G}}{G}\right| \lesssim 4.1 \times 10^{-11}\ \mathrm{year}^{-1} \tag{112}$$

Helioseismology also provides us with an excellent tool to perform tests on the constancy of G. In this case the underlying idea is to probe how our Sun would eventually evolve in the case of an evolving G. The idea of using stellar evolution to constrain possible variations in G comes originally from Teller (1948) who showed—see also appendix in Degl'Innocenti et al. (1996)—that the luminosity L of a star depends on G according to $L \propto G^7$. If G were to vary on a nuclear timescale (billions of years), then the rates of nuclear burning of hydrogen into helium on the main-sequence would also vary. This, in turn, would affect the current central abundances of hydrogen and helium of the Sun. An additional effect of a varying G is that the inner edge of the outer convective region also changes its location, thus affecting the vibrational modes. Because helioseismology enables us to probe the structure of the solar interior, we can use the observed p-mode oscillation frequencies to constrain the rate of variation of G. Specifically, from helioseismology we can determine the run of the sound speed in the core of the Sun, which, with the aid of an accurate equation of state, can be used to determine the central densities and the abundances of hydrogen and helium. Using this technique in Demarque et al. (1994) the following constraint was obtained:

$$\left|\frac{\dot{G}}{G}\right| \leq 10 \times 10^{-10}\ \mathrm{year}^{-1} \tag{113}$$

At that time they used the best available solar p-mode oscillation data (Libbrecht et al. 1990). In a subsequent work the data from the GONG instrument—Global Oscillation Network Group (Harvey et al. 1996; Christensen-Dalsgaard et al. 1996)—and the BiSON experiment—Birmingham Solar Oscillation Network (Chaplin et al. 1996)—which provided much more accurate p-mode frequencies for low ℓ modes, were analyzed yielding the constraint

$$\left|\frac{\dot{G}}{G}\right| \leq 1.6 \times 10^{-12}\ \mathrm{year}^{-1} \tag{114}$$

It has also been shown (Guenther et al. 1995) that the g-modes of the Sun could provide even tighter constraints. However, these modes have most probably not been observed so far, although there are some claims that they have been detected (Hill and Gu 1990). If this indeed were the case then the constraint

$$\left|\frac{\dot{G}}{G}\right| \leq 4.5 \times 10^{-12}\ \mathrm{year}^{-1} \tag{115}$$

would be obtained.

4.3.3 Late stages of stellar evolution

In this subsection we review the constraints coming from white dwarf cooling and from the light curves of thermonuclear supernovae. Both types of objects are gravitationally supported against the pressure of degenerate electrons, their densities can be rather high and, consequently, their properties are very sensitive to the precise value of the gravitational constant.

White dwarf stars provide an independent method for measuring the rate of change of G. There are two reasons for this. First, when they are cool enough their energy is entirely of gravitational and thermal origin, and any change in the value of G modifies the energy balance which, in turn, translates into a change of luminosity. Secondly, since they are long-lived objects, ~ 10 Gyr, even extremely small values of the rate of change of G can become prominent. The first attempts to obtain constraints on $\dot{G}$ from the cooling of white dwarfs (Vila 1976) were unsuccessful due to the lack of reliable observational data and the uncertainties in the cooling theory of white dwarfs. Since then both the observational data (Liebert et al. 1988) and the cooling theory itself have seen an impressive advance—see, for instance Salaris et al. (2000) and references therein. It can be shown (García-Berro et al. 1995) that for the case of a secularly varying G, the luminosity of a cool enough white dwarf is given by

$$L = -\frac{\mathrm{d}B}{\mathrm{d}t} + \frac{\dot{G}}{G}\Omega, \tag{116}$$

where $B = U + \Omega$ is the binding energy, U is the thermal energy and Ω is the gravitational energy. As the white dwarf cools down the thermal content decreases and the second term in the previous equation dominates. Note as well that the cooling process is accelerated if $\dot{G}/G < 0$. On the other hand, the white dwarf luminosity function directly measures the rate of cooling of white dwarfs and shows a sharp cut-off at $\log(L/L_\odot) \simeq -4.5$, a consequence of the finite age of the Galactic disk. By comparing the results of the previous equation with the observational data and, more specifically, with the position of the observed drop-off of the white dwarf luminosity function, the following tight bound was obtained:

$$-(1 \pm 1) \times 10^{-11}\ \mathrm{year}^{-1} < \frac{\dot{G}}{G} < 0 \tag{117}$$

at the 1σ confidence level (García-Berro et al. 1995). This result was challenged in Benvenuto et al. (1999), where a much tighter bound was obtained using the same method, but their analysis turned out to be flawed.

Type Ia supernovae (SNIa) are supposed to be one of the best examples of standard candles. This is because, although the nature of their progenitors and the detailed mechanism of explosion are still the subject of a strong debate, their observational light curves are relatively well understood and their individual intrinsic differences can be accounted for. Under these assumptions, thermonuclear supernovae are objects well suited to study the Universe at large, especially at high redshifts ($z \sim 1.0$), where the rest of standard candles

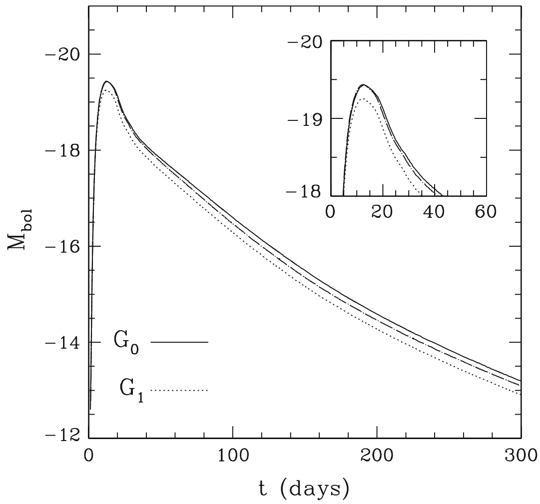

Fig. 4 Bolometric light curves of SNIa for the local value of the gravitational constant, G_0 (*solid line*), for $G_1 = 1.1G_0$ (*dotted line*) and for G_1 shifted upwards by 0.18 magnitudes, from Gaztañaga et al. (2002)

fail in deriving reliable distances, thus providing an unique tool for determining cosmological parameters or discriminating among different alternative cosmological theories. The observations carried out by the High-z Supernova Search Team (Riess et al. 1998) and the Supernova Cosmology Project (Perlmutter et al. 1999) indicate that distant SNIa appear to be *dimmer* than local ones.

Simple analytical models of the light curve—see, for instance, Arnett (1982)—predict that the peak luminosity is proportional to the mass of nickel synthesized, which in turn, to a good approximation, is a fixed fraction of the Chandrasekhar mass ($M_{\rm Ni} \propto M_{\rm Ch}$), which depends on the value of gravitational constant

$$M_{\rm Ch} = \frac{(\hbar c)^{3/2}}{m_{\rm p} G^{3/2}}. \tag{118}$$

The actual fraction varies when different specific SNIa scenarios are considered (Khokhlov et al. 1993; Gómez-Gomar et al. 1998), but the physical mechanisms relevant for type Ia supernovae naturally relates the energy yield to the Chandrasekhar mass. Figure 4 shows the results of detailed calculations (Gaztañaga et al. 2002) which confirm this assumption. The apparent magnitude of a thermonuclear supernova is then given by

$$m(z) = M_0 + \frac{15}{4} \log \frac{G}{G_0} + 5 \ln d_L(z; \Omega_{\rm M}, \Omega_\Lambda; G) + 25, \tag{119}$$

where M_0 is the (intrinsic) absolute magnitude of SNIa, d_L is the luminosity distance and G_0 is the present day value of the gravitational constant. A variation of G affects both the luminosity distance and the Chandrasekhar mass. In Gaztañaga et al. (2002) and Riazuelo and Uzan (2002) it was argued that the

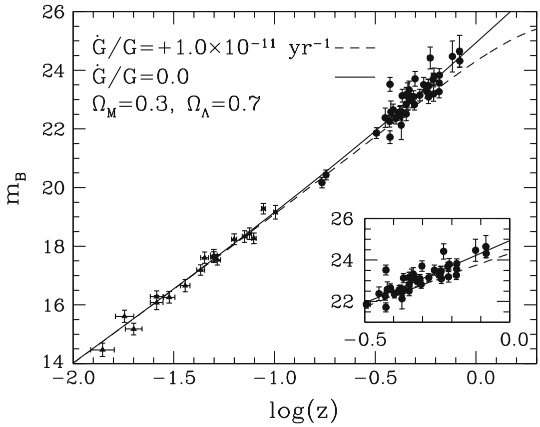

Fig. 5 The Hubble diagram of distant supernovae, assuming the preferred cosmological scenario of the Supernova Cosmology Project, for $\dot{G}/G = 0$ (*solid line*), $\dot{G}/G = +10^{-11}$ yr^{-1} (*dashed line*). For $\dot{G}/G = +10^{-15}$ year^{-1} the results are indistinguishable from those of the case with constant G. The observational data are taken from Perlmutter et al. (1999). The inset shows an enlarged view of the region around $z \sim 0.5$. From Lorén-Aguilar et al. (2003)

former dependence is small and can be ommitted. Figure 5 shows the resulting Hubble diagram. Moreover, using the 2σ confidence contours for $z = 0.5$ obtained from the fit to the Hubble diagram of SNIa obtained in Gaztañaga et al. (2002) bounds on $\dot{G}/G$ have been computed. The bounds depend on the cosmological scenario. For example, for the currently favored cosmological scenario $(\Omega_M, \Omega_\Lambda) = (0.3, 0.7)$ and for the flat matter-dominated Universe with $(\Omega_M, \Omega_\Lambda) = (1.0, 0.0)$ the following estimates:

$$-1.4 \times 10^{-11}\ \text{year}^{-1} < \frac{\dot{G}}{G} < +2.6 \times 10^{-11}\ \text{year}^{-1} \tag{120}$$

and

$$-2.9 \times 10^{-11}\ \text{year}^{-1} < \frac{\dot{G}}{G} < -0.3 \times 10^{-11}\ \text{year}^{-1} \tag{121}$$

were respectively obtained (Lorén-Aguilar et al. 2003). This issue has been recently reanalyzed in García-Berro et al. (2006), where no explicit bound was obtained but, instead, the dependence of G on redshift as deduced from the Hubble flow of distant supernovae was obtained.

Gravitational collapse supernovae (Type II supernovae) can also be used to set upper bounds on a varying G. This is so because the Chandrasekhar mass limit also sets the scale for the late evolutionary stages of massive stars, including the formation of neutron stars in core collapse supernovae. As the Chandraskhar mass depends on G, the masses of these newly born neutron stars retain a record of past values of G. To be specific, the line of reasoning is the following: if G were to vary significantly, old neutron stars should have on average masses considerably different from young ones. In Thorsett (1996) it was shown that the measurements of the masses of five young and old neutron stars in pulsar binaries limit the variation of G. In fact, at the 95% confidence level the following upper bound was obtained:

$$\frac{\dot{G}}{G} < (-0.6 \pm 4.2) \times 10^{-12}\ \text{year}^{-1}. \tag{122}$$

4.3.4 Pulsar constraints

Observations of the period of pulsars—whether they belong to binary systems or whether they are single—yield strong bounds on the rate of variation of G. However, when pulsars are considered the upper limits to the variation of G are, for most of the cases, model-dependent. The reason is quite simple. We have already argued in Sect. 4.2 that the rate of period change for orbiting bodies provides a good method to measure the rate of change of G, provided that all the other effects that contribute to $\dot{P}$ are known and, moreover, accurate measurements are feasible. For the case of binary pulsars the latter is certainly true but the theoretical modeling is not as easy as in the case of Solar System bodies. Specifically, for the case of pulsars the gravitational binding energy cannot be discarded and its effects must be accounted for (Eardley 1975). This, in turn, complicates very much the task of retrieving useful constraints from the observational data and, consequently, in some of the works discussed below a phenomenological approach is instead adopted.

For the case of binary pulsars it can be shown that besides the emission of gravitational radiation a varying G also induces a variation of the orbital period which can be measured with timing data (Nordtvedt 1990). If the masses of the components of binary system are similar (which is not always the case) the period drift is given by

$$\frac{\dot{P}}{P} \simeq (2 + 5c)\,\frac{\dot{G}}{G}, \tag{123}$$

where c is a coefficient which is model-dependent and must be computed theoretically. Obviously, for the case of Solar System bodies c is rather small, and the expression used in Sect. 4.2 is recovered. Using this technique and the available observational data for the pulsar PSR 1913+16 in Damour et al. (1988) the following bound was obtained:

$$\frac{\dot{G}}{G} < (1.0 \pm 2.3) \times 10^{-11}\ \text{year}^{-1}. \tag{124}$$

Later on, this constraint was re-evaluated Damour and Taylor (1991) and the following upper bound

$$\frac{\dot{G}}{G} < (1.10 \pm 1.07) \times 10^{-11}\ \text{year}^{-1} \tag{125}$$

was derived. More recently, in Kaspi et al. (1994), the following value was reported:

$$\frac{\dot{G}}{G} < (4 \pm 5) \times 10^{-12} \text{ year}^{-1} \tag{126}$$

for the same pulsar, whereas examining the data for the pulsar PSR 1855+09 they obtained

$$\frac{\dot{G}}{G} < (-9 \pm 18) \times 10^{-12} \text{ year}^{-1}. \tag{127}$$

Yet there is another technique that can be used to set meaningful constraints on $\dot{G}$ using pulsars. The underlying idea is to use the spin down rate of single pulsars. The spin down rate of pulsars can be obtained with good accuracy for a handful of pulsars. The problem here comes from estimating the theoretical value, which includes contributions from electromagnetic losses, emission of gravitational radiation and the contribution of a varying G (Heintzmann and Hillebrandt 1975). Note that by considering isolated pulsars we are explicitly discarding any possible spin up due to accretion from a secondary. Consequently, the spin down rate can be expressed as

$$\frac{\dot{P}}{P} = \left(\frac{\dot{P}}{P}\right)_G + \left(\frac{\dot{P}}{P}\right)_{\text{EM}} + \left(\frac{\dot{P}}{P}\right)_{\text{GW}}. \tag{128}$$

The contributions to the period derivative of the emission of gravitational waves and of the electromagnetic losses are both positive. The contribution of a secularly varying G can be computed assuming that angular momentum remains constant, and so

$$\left(\frac{\dot{P}}{P}\right)_G = \left(\frac{\mathrm{d}\ln I}{\mathrm{d}\ln G}\right)\frac{\dot{G}}{G}, \tag{129}$$

where I is the moment of inertia. Clearly, the measured rate of spin down places an upper limit to the rate of variation of the gravitational constant. Using this technique and the measured spin down rate of the pulsar JP 1953 the following upper bound

$$\left|\frac{\dot{G}}{G}\right| \lesssim 10^{-10} \text{ year}^{-1} \tag{130}$$

was obtained (Heintzmann and Hillebrandt 1975), which was later updated to:

$$0 < -\frac{\dot{G}}{G} \lesssim 6.8 \times 10^{-11} \text{ year}^{-1} \tag{131}$$

in Mansfield (1976). Later on, using data from the pulsar PSR 0655+64 a tighter bound was obtained:

$$0 \leq -\frac{\dot{G}}{G} \lesssim 5.5 \times 10^{-11}\ \text{year}^{-1} \tag{132}$$

in Goldman (1990).

The most recent constrain on the hypothetical variation of G using neutron stars comes from the nearest millisecond pulsar, PSR J0437–4715. The basic reasoning is the following (Jofré et al. 2006). A time variation of G would cause neutron star matter to depart from β equilibrium, due to the changing hydrostatic equilibrium. This induces non-equilibrium β processes, which release energy that is invested partly in neutrino emission and partly in internal heating. Eventually, the star arrives at a stationary state in which the temperature remains nearly constant, as the forcing through the change of G is balanced by the ongoing reactions. Using the surface temperature of this pulsar, inferred from ultraviolet observations, the following upper limit was obtained:

$$\left|\frac{\dot{G}}{G}\right| < 4 \times 10^{-12}\ \text{year}^{-1} \tag{133}$$

if only modified Urca reactions are considered, whereas a somewhat lower bound is obtained if direct Urca reactions are allowed:

$$\left|\frac{\dot{G}}{G}\right| < 2 \times 10^{-10}\ \text{year}^{-1}. \tag{134}$$

4.4 Miscellaneous constraints

Another method of constraining a possible secular evolution of G comes from the analysis of cluster of galaxies (Dearborn and Schramm 1974). It is rather natural to think that a smaller value of the gravitational constant would eventually allow member galaxies to escape from the potential well of the cluster since in this case the gravitational binding energy of the cluster is also smaller. This hypothesis was further analyzed in Dearborn and Schramm (1974), where it was deduced that if G were to decrease the very existence of cluster of galaxies would imply the following bound:

$$0 \lesssim \frac{\dot{G}}{G} \lesssim 4 \times 10^{-11}\ \text{year}^{-1}. \tag{135}$$

4.5 Cosmic microwave background constraints

The analysis of the Cosmic Microwave Background for the case of a varying G should also provide a powerful method to constraint its rate of variation ar very large look-back times ($z \sim 10^3$). As previously discussed in Sect. 3.4, the Cosmic Microwave Background angular power spectrum is sensitive to the epoch of recombination. Thus, a different value of G should have left noticeable imprints in it. Particularly, since varying the value of G varies the strength of the gravitational interaction, the Friedmann equations are affected and, consequently, the structure of the angular power spectrum should be changed, very much in an analogous way as it occurs for the case of a varying α. This has been investigated in Liddle et al. (1998) for the particular case of the Jordan–Brans–Dicke theory. They were able to set constraints only on $w_{\rm BD}$. To the best of our knowledge and at the time of writing this review, only two specific bounds on the rate of change of G have been derived from the analysis of the CMB anisotropy spectrum using WMAP data. The first of these works (Nagata et al. 2004) yielded the following constraint:

$$\left|\frac{\Delta G}{G}\right| < 0.05 \tag{136}$$

at the 2σ confidence level. We stress, however, that this bound is model-dependent. In the second one (Chan and Chu 2006), 3-year WMAP data and two different phenomenological models for the variation of G were used, and the following constraints were obtained:

$$\left|\frac{\Delta G}{G}\right| < 0.05 \tag{137}$$

when a step function was adopted, whereas if a linear function was used for the variation of G, the bound turned out to be less stringent:

$$-0.11 < \frac{\Delta G}{G} < 0.13. \tag{138}$$

4.6 Big Bang nucleosynthesis constraints

As previously discussed in Sect. 3.5 the amount of ^{4}He, ^{7}Li, depends on the freeze-out temperature, which, in turn is a function of several fundamental constants. In particular, the freeze-out temperature depends on the expansion rate, which is governed by the precise value of G (Steigmann 1976; Schramm and Wagoner 1977). This procedure, thus, also allows to set constraints on the rate of variation of G. Much of the work done using BBN to constraint $\dot{G}$ has been done for the Jordan–Brans–Dicke model, as it has been previously discussed for the case of the CMB constraints, and most of them place constraints on $w_{\rm BD}$ (Casas et al. 1992a,b; Serna et al. 1992; Clifton et al. 2005). A few others place constraints on $\dot{G}$ assuming a certain functional dependence for the scalar field, which translates into a power-law dependence for $G(t)$. Apparently, the first constraint on $\dot{G}$ obtained using this method is that of Barrow (1978), where the following bound

$$\left|\frac{\dot{G}}{G}\right| = (1.5 \pm 0.7) \times 10^{-12}\ \text{year}^{-1} \tag{139}$$

was obtained. In this preliminary analysis only the abundance of ^{4}He was taken into account. Later on, in Yang et al. (1979) it was shown that using the abundances of the rest of primordial elements would constraint considerably better the rate of variation of G, and they obtained the following (optimistic) bound:

$$\frac{\dot{G}}{G} \lesssim -5 \times 10^{-13}\ \text{year}^{-1}. \tag{140}$$

The analysis of (Stecker 1980) provided the bound

$$\frac{\dot{G}}{G} \lesssim -1.1 \times 10^{-11}\ \text{year}^{-1}. \tag{141}$$

This analysis was further improved (Rothman and Matzner 1982) to yield

$$\left|\frac{\dot{G}}{G}\right| = 1.7 \times 10^{-13}\ \text{year}^{-1}. \tag{142}$$

In the thorough analysis of Accetta et al. (1990) the most recent measurements (for that time) of the neutron half-life and of the uncertainties in the reaction rates were taken into account, arriving at the conclusion that

$$\left|\frac{\dot{G}}{G}\right| < 9 \times 10^{-13}\ \text{year}^{-1}. \tag{143}$$

Later on, in Damour and Gundlach (1991), the range of allowed values was constrained to

$$-1.1 \times 10^{-12}\ \text{year}^{-1} \lesssim \frac{\dot{G}}{G} \lesssim 4 \times 10^{-12}\ \text{year}^{-1} \tag{144}$$

In the analysis of Kim and Lee (1995) the role of the electron chemical potential was assessed and the following bound

$$\frac{\dot{G}}{G} \lesssim -2.7 \times 10^{-12}\ \text{year}^{-1} \tag{145}$$

was obtained. Later on, almost the same set of authors Kim et al. (1998) included neutrino degeneracy arriving at the same approximate upper limit. Finally, the recent analysis of Copi et al. (2004) yielded

$$\frac{\Delta G}{G} = 0.01^{+0.20}_{-0.16}, \tag{146}$$

whereas the even more recent analysis of Cyburt et al. (2005) has provided the following constraint:

$$-0.10 < \frac{\Delta G}{G} < 0.13. \tag{147}$$

5 Observational constraints on the variation of μ

Another fundamental constant whose cosmological variation has been the subject of numerous studies, both experimental and observational, is the proton-to-electron mass ratio, $\mu \equiv m_{\rm p}/m_{\rm e}$. The present value of the proton-to-electron mass ratio, $\mu = 1836.15267261(85)$, is now known with a relative accuracy of 2×10^{-9} (Mohr and Taylor 2002, 2005). On the other hand, several authors—see the introductory remarks in Srianand et al. (2004) and Quast et al. (2004)—have argued that the quantum chromodynamic scale, $\Lambda_{\rm QCD}$, should vary considerably faster than that of quantum electrodynamics, $\Lambda_{\rm QED}$. As a consequence, the secular change in the proton-to-electron mass ratio (if any) should be larger than that of the fine structure constant. Therefore, the search for a secular change of μ should be considerably easier than the search for a varying α and, hence, this makes μ a very interesting target to search for possible cosmological variations of the fundamental constants.

Before reviewing the observational results on $\dot{\mu}$ it is important to realize that many of the most recent (and restrictive) bounds on the possible variation of μ come from spectroscopic observations of distant sources, such as quasars. Note that this was also one of the most successful methods for imposing limits on the variation of α, as previously discussed in Sect. 3.3. In fact, atomic transitions depend on *both* the fine structure constant and on the proton-to-electron mass ratio. It can be shown that the non-relativistic part of the spectrum depends mostly on μ, and that the fine and hyperfine structures depend primarily on α^2. Consequently, depending on which atomic transitions are used to obtain bounds on the variation of the fundamental constants and *to which* spectroscopic transitions these are compared a combination of μ and α is usually probed.

More specifically, if fine structure doublets are studied the variation of α is obtained, whereas if the H fine structure is compared to the hyperfine structure the hypothetical variation of the quantity $\mu\alpha^2$ is probed. We will see below that in order to obtain the variation of μ the best method is to compare rotational versus vibrational modes of molecules, given that it is only sensitive to a variation of μ, and for these reasons the H_2 molecule is usually employed.

5.1 Geochemical constraints

The first constraint on the variation of the proton-to-electron mass ratio is based on geochemical arguments (Yahil 1975). It is based on the comparison of the geochemical ages of K–Ar and Rb–Sr. It was obtained about 20 years ago and gives

$$\left|\frac{\Delta\mu}{\mu}\right| \lesssim 1.2 \tag{148}$$

over the past 10^{10} year.

5.2 Quasar spectroscopy constraints

The first suggestion that the variation of the electron-to-proton mass ratio could be determined using observations of molecular absorption lines of distant quasars was given in Thompson (1975). The method was developed and implemented much later (almost twenty years later) in Varshalovich and Levshakov (1993). The method is based on the fact that the wavelengths of vibrorotational lines of molecules depend on the reduced mass, M, of the molecule. To be precise, the energy difference between to consecutive levels of the rotational spectrum of a diatomic molecule scale as M, whereas, to a first approximation, the energy difference between to adjacent levels of the vibrational spectrum is proportional to $\sqrt{M}$. For the case of H_2 we have that $M = m_{\rm p}$ and, consequently, by studying the vibrorotational spectrum of molecular hydrogen we may obtain very useful information about μ. More precisely, in the Born–Oppenheimer approximation the frequency of a vibrorotational transition can be expressed as

$$\nu \propto c_{\rm e} + \frac{c_{\rm v}}{\sqrt{\mu}} + \frac{c_{\rm r}}{\mu}, \tag{149}$$

where $c_{\rm e}$, $c_{\rm v}$ and $c_{\rm r}$ stand, respectively, for the electronic, the vibrational and the rotational contributions. As a result, the observed wavelength λ of any given line in an absorption system at the redshift z differs from the local rest-frame wavelength λ_0 of the same line in the laboratory according to the relation

$$\lambda(z) \simeq \lambda_0(1+z)\left(1 + K\frac{\Delta\mu}{\mu}\right), \tag{150}$$

where K is a sensitivity coefficient which can be computed theoretically for some cases, such as, for example, for the Lyman and Werner bands of the H_2 molecule (Varshalovich and Potekhin 1995). Using this expression, the cosmological redshift of a line can be distinguished from the shift due to a variation of μ. This method has been used in a number of papers to obtain upper bounds on the secular variation of the electron-to-proton mass ratio from observations of distant absorption systems in the spectra of quasars at several redshifts. Generally speaking, the various constraints obtained so far have limited the variation of μ to roughly

$$\left|\frac{\Delta\mu}{\mu}\right| \lesssim 0.5 \times 10^{-4} \tag{151}$$

although most of them have shown no hint of a variation of μ. We review them in detail below. We nevertheless would like to mention before going into details that the observations from which these bounds are obtained are, in general, very demanding and require long exposure times in large telescopes. Hence, most of the bounds come from different absorption systems at different redshifts and using increasingly accurate laboratory data.

In Varshalovich and Levshakov (1993) the damped Lyα system associated to the quasar PKS 0528–250 (at $z = 2.811$) was used to obtain

$$\left|\frac{\Delta\mu}{\mu}\right| < 4.0 \times 10^{-3}. \tag{152}$$

Later on, the same quasar absorption system was studied in Cowie and Songaila (1995), and the authors reached the conclusion that

$$\left|\frac{\Delta\mu}{\mu}\right| < (0.75 \pm 6.25) \times 10^{-4}. \tag{153}$$

Again, the same quasar was used with more accurate theoretical data in Varshalovich and Potekhin (1995) and the following upper bound to the rate of variation of μ was obtained:

$$\left|\frac{\Delta\mu}{\mu}\right| < 2.0 \times 10^{-4} \tag{154}$$

at the 95% confidence level. Another study of the same quasar absorption system was performed in Varshalovich et al. (1996b). In this study 59 rotational transitions of molecular hydrogen were used, yielding the following upper bound:

$$\left|\frac{\Delta\mu}{\mu}\right| < (-1.0 \pm 1.2) \times 10^{-4} \tag{155}$$

at the 2σ level. The analysis of this quasar absorption system was repeated in Potekhin et al. (1998), using in this case 83 absorption lines and more accurate theoretical data, to yield

$$\left|\frac{\Delta\mu}{\mu}\right| < (-7.5 \pm 9.5) \times 10^{-5}. \tag{156}$$

Two other quasar absorption systems, Q 1232+082 and Q 0347$-$382, respectively located at $z = 2.3377$ and $z = 3.0249$, have also been used (Ivanchik et al. 2000) to obtain

$$\left|\frac{\Delta\mu}{\mu}\right| < (-5.7 \pm 3.8) \times 10^{-5} \tag{157}$$

at the 1.5σ level. Note that in this case a positive detection for $\dot{\mu}$ was obtained. The authors, however, pointed out that more measurements were needed in order to secure this result, since using another laboratory dataset the results obtained using the same set of observational data were significantly different. The quasar absorption system towards Q 0347−382 was further studied using high-resolution spectra obtained with the very large telescope/ultraviolet-visual echelle spectrograph (VLT/UVES) in Levshakov et al. (2002). The result was

$$\left|\frac{\Delta\mu}{\mu}\right| < 5.5 \times 10^{-5} \tag{158}$$

More recently, in Ivanchik et al. (2003), the same authors of Ivanchik et al. (2000) studied again the quasar absorption system towards Q 0347−382, updating their results with a refined analysis and a nonzero variation of μ was reported:

$$\frac{\Delta\mu}{\mu} = (3.0 \pm 2.4) \times 10^{-5}. \tag{159}$$

Note, however, that there exists some discrepancy in the recent literature in this regard, since using the same quasar and very similar physical inputs the analysis of observational data performed in Ubachs and Reinhold (2004) yields

$$\frac{\Delta\mu}{\mu} = (-0.5 \pm 3.6) \times 10^{-5}. \tag{160}$$

at the 2σ level. However, in Ivanchik et al. (2005), the previous controversial result was revised and confirmed using the combined analysis of the Lyman and Werner bands of the quasar absorption systems of Q 1232+082 and Q 0347−382. The new analysis used high-resolution spectra, updated laboratory data of the energy levels (Abgrall et al. 1993) and of the rest–frame wavelengths of the H_2 molecule (Philip et al. 2004). These two sets of laboratory data yielded, respectively:

$$\frac{\Delta\mu}{\mu} = (3.05 \pm 0.75) \times 10^{-5}, \tag{161}$$

and

$$\frac{\Delta\mu}{\mu} = (1.65 \pm 0.74) \times 10^{-5}. \tag{162}$$

The most recent observational constraint using vibrorotational transitions of the H_2 molecule comes from the absorption systems of the quasars Q 0347−383 and Q 0405−443. In this analysis accurate ab initio calculations of the relevant molecular data was used (Hinnen et al. 1994). Another significant step beyond the Bohr–Oppeheimer approximation was also made by including several other corrections (Dressler and Wolniewicz 1986; Wolniewicz 1994). Also accurate laboratory data were used (Ubachs and Reinhold 2004). Moreover, a total of 76 lines were used in a very thorough analysis. The results were shown to be compatible with a nonnegligible variation of the proton-to-electron mass ratio

$$\frac{\Delta\mu}{\mu} = (2.4 \pm 0.6) \times 10^{-5} \tag{163}$$

at the 3.5σ level.

Up to now we have discussed the bounds on the variation of μ obtained by using the vibrorotational transitions of molecular hydrogen. Its use is fully justified by the fact that H_2 is very abundant. However, there exist other molecules of interest. Unfortunately, very few studies have used other molecules. The reason is twofold: first, the lack of reliable laboratory datasets and, second, and most importantly, the inherent difficulty of detecting and measuring accurately such molecules at large redshifts. Perhaps the only exception is the bound obtained observing the quasar PKS 1413+135 (Wiklind and Combes 1997), with redshif $z = 0.247$. In this work the lines of HCO^+, HCN and CO were used and

$$\frac{\Delta\mu}{\mu} \sim 10^{-5} \tag{164}$$

was obtained at the 3σ level. However, the authors admittedly recognize that the precision of the measurements was poor.

Finally, in the most recent work (Tzanavaris et al. 2006) the following bound was obtained:

$$\frac{\Delta\mu}{\mu} = (0.58 \pm 1.95) \times 10^{-5} \tag{165}$$

using nine quasar absorption spectra at 21-cm and UV rest-wavelengths. The redshift range is $0.23 < z < 2.35$. It is important to stress that this technique is completely independent of the molecular hydrogen observations.

As previously mentioned, the variation of the proton-to-electron mass ratio is correlated with the variation of the fine structure constant in some theoretical models. For example, in the framework of Supersymmetric theories of Grand Unification the cosmological variation $\Delta\mu/\mu$ is related to the variation of the fine structure constant by

$$\frac{\Delta\mu}{\mu} \sim P\frac{\Delta\alpha}{\alpha}, \tag{166}$$

where P is a constant factor which depends on the model. According to Banks et al. (2002) and Dine et al. (2003) the theoretical expectation is $|P| < 50$, but the exact value of P is poorly determined from theoretical considerations only. Since we already have studied the secular variations of α and μ it is then possible to obtain an observational determination of P using only data from absorption systems of distant quasar. Accordingly, in Ivanchik et al. (2005) estimates of the value of P were derived by combining the above expression with constraints on the variation of the fine structure constant discussed previously in Sect. 3.3 above. Thus, if the constraint from Murphy et al. (2003) for the variation of α is adopted, then the results of Ivanchik et al. (2005) imply $-9.5 < P < -0.2$, whereas the result from Chand et al. (2004) implies $|P| > 1$.

5.3 Other spectroscopical methods

Another method to constrain the secular variation of the proton-to-electron mass ratio is based on the spectral lines of heavy elements. For this type of atoms, the nuclei are so heavy that the mass of the nucleus can be considered, to a very good approximation, to be virtually infinite. This is in sharp contrast with the case of the hydrogen molecule. Consequently, any hypothetical variation of μ will directly translate into a discrepancy with the redshift determined from hydrogen. In Pagel (1977) this additional redshift was quantified:

$$\Delta z = z_{\rm H} - z = (1+z)\frac{\mu - \mu_0}{1+\mu_0}, \tag{167}$$

where μ_0 is the local value of the proton-to-electron mass ratio. The method was subsequently applied to PKS 0237−23, PHL 957 and a number of other quasars with redshifts ranging from 2.1 to 2.7, leading to

$$\left|\frac{\Delta\mu}{\mu}\right| = 4 \times 10^{-1} \tag{168}$$

at the 3σ level.

6 Some final caveats: confronting models with observations

It has been shown in Sect. 2 that within the framework of several theoretical models the variability of the scale factor necessarily implies the *correlated variations* of several fundamental constants and, particularly, of α and G. Let us compare them. Specifically, let us use the claimed variation of the fine structure constant (Murphy et al. 2001b, 2003) discussed above to set some constraint on the variation of G. The multidimensional models analyzed above predict that the gravitational constant G was *smaller* in the past and, therefore, that $\dot{G}/G$ should be *positive*. Assuming, for the sake of simplicity, a constant rate of variation for both α and G, and using a typical age of the Universe of $\sim$ 14 Gyr, the estimate $\dot{G}/G \sim +10^{-15}$ year^{-1} can be obtained for both the Kaluza–Klein and the Einstein–Yang–Mills models, whereas it is a factor of 10^2 larger for the Randall–Sundrum model. Let us note now that the existing bounds on the variation of G (see Sect. 4) are negative—independently of whether they are local or they are obtained at moderately high redshifts. Consequently, positive values of $\dot{G}/G$ seem to be not allowed by the present astrophysical data. To be more precise, if we adopt the preferred cosmological scenario, namely $\Omega_{\rm M} = 0.3$, $\Omega_\Lambda = 0.7$, a *positive* value of $\dot{G}/G$ would make distant supernovae to appear *brighter*—see the discussion in Sect. 4.3.3. This effect is opposite to what it is found observationally. That is, distant thermonuclear supernovae appear to be *dimmer* than local ones. Thus, the available observational data sets seem to be in contradiction with each other. However, it is rather obvious that the current observational bounds are not yet very precise and we cannot yet totally discard these models. Thus, observations of distant supernovae at even larger redshifts are needed since they would cast some light on this issue.

The arguments presented in Banks et al. (2002) also add some elements against those claims of positive detections. In Banks et al. (2002) it was assumed that the variation of α is modeled by coupling a dynamical scalar field ϕ with a potential $V(\phi)$ to $F^2_{\mu\nu}$. A variation $\delta\phi$ around a ground state value, ϕ_0, generates not only the variation of the fine structure constant, but also a corresponding change of the potential

$$\delta V(\phi) = V_0'\delta\phi + \frac{1}{2}V_0''(\delta\phi)^2 + \ldots, \tag{169}$$

where it is supposed that the cosmological constant $\Lambda_C = V(\phi_0) = 0$. The variation of the potential is thus interpreted as the variation of the cosmological constant. By simple and rather general arguments it can be shown that this variation is related to the variation of α as $\delta\Lambda_C = \delta V = C\delta\alpha\Lambda^4$, where C is a constant and Λ is a typical physical scale, for example $\Lambda = \Lambda_{QCD} \sim 100$ MeV. The variation of the potential cannot exceed the energy density ρ_{m*} in the Universe at the time of galaxy formation, when the light of distant quasars was emitted. This argument yields the condition

$$\delta V = \left|C\frac{\delta\alpha}{\alpha}\right| \times 10^{29}\ \mathrm{eV}^4 < \rho_{m*} \sim 10^{-8}\ \mathrm{eV}^4 \tag{170}$$

or

$$\left|C\frac{\delta\alpha}{\alpha}\right| \lesssim 10^{-37}. \tag{171}$$

To describe the variation $|\delta\alpha/\alpha| \sim 10^{-5}$ an extreme fine tuning is needed. The problem is essentially the huge back-reaction produced by varying couplings on the vacuum energy. This difficulty could be intimately related to the long–standing cosmological constant problem. Hence, its satisfactory solution could also provide a mechanism to suppress the enormous variation of the vacuum energy due to the time variation of ϕ. However, consistent and "natural" quantitative descriptions of this sort of new physics are still missing and represent a big challenge for theoretical physicists.

7 Conclusions

There are a number of generalizations of the Standard Model which contain built-in mechanisms that allow for a variation of the fundamental constants. This can be viewed as another outcome of the rich interface between cosmology, astrophysics and particle physics. Either a positive or a negative result of the observational efforts to detect such variations would provide us with very valuable additional information about these mechanisms, and would eventually help to confirm or discard alternative theories beyond the Standard Model. These models have been reviewed in Sect. 2. In particular, we have elaborated several models, starting with the classical Jordan–Brans–Dicke theory and continuing with the Bekenstein–Sandvik–Barrow–Magueijo model, the string-inspired models, quintessence theories and some models with extra dimensions. These last models predict correlated variations of the fundamental couplings. Hence, correlated variations of α, G...Consequently, the quest for secularly evolving fundamental couplings is of the maximum theoretical interest.

The current state of the art of the observational methods used to constrain a hypothetical variation of the fundamental constants has been reviewed in the second part of the paper. Particularly, we have focused on the three best studied cases. We have assembled most of the relevant observational work done so far in setting astronomical constraints on the variation of the fine structure constant (Sect. 3), of the gravitational constant (Sect. 4) and of the proton-to-electron mass ratio (Sect. 5). Although we have collected information of the early work in this field we have paid more attention to the most recent and accurate astronomical measurements. Also, we have elaborated on some terrestrial constraints which provide the most accurate (although local) upper limits. We have shown that for all three couplings in which we have focused there are several independent constraints to their rate of change. These limits come either from local environments or from cosmological scales. We have argued that although the local constraints turn out to be very tight, the bounds at intermediate look-back times and at cosmological distances are rather loose. We have also discussed that although there are some claims for direct detections of a variation of the fundamental couplings (namely for α and μ) this issue is still far from being closed and, moreover, such claims are still controversial. It is, nevertheless, important to realize that although this specific topic is still the subject of strong debate an eventual confirmation of the reported variations of α and μ would provide us with clear indications of the need of new physics. Additionally, in Sect. 6 we have provided evidence that the current positive detections of a secularly varying α are in conflict with the upper limits derived from the Hubble diagram of distant Type Ia supernovae. Since the current searches for distant thermonuclear supernovae are providing us with a powerful tool to probe very large look-back times we expect that in the near future this method will be able to provide reliable constraints on these types of theoretical models.

Finally, one should bear in mind that the interpretation of the observational data on the variation of the fundamental constants depends on the underlying theoretical framework and, generally speaking, it is based on a number of assumptions which are very different from each other depending on the adopted model. Much work has been done so far for the case of the Jordan–Brans–Dicke model but, as shown in Sect. 2, this is not the only model which predicts a variation of the fundamental couplings. Since some of the theoretical calculations needed to obtain those constraints on the variation of the couplings from the observations are model-dependent, much theoretical work still remains to be done. Another concern comes from the lack of reliable observational data.

With the advent of the new generation of very large telescopes the situation has clearly improved, and during the last few years some of the most tight bounds (or even claims for positive detections) have been set. Clearly, the last word on this subject has not been told yet and, therefore, more observational and theoretical efforts are needed. Surely, in the following years more interesting results—both theoretical and observational—will show up.

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