A Letters Journal Exploring the Frontiers of Physics

September 2009

EPL, **87** (2009) 60004 www.epljournal.org
doi: 10.1209/0295-5075/87/60004

Asymmetric cost in snowdrift game on scale-free networks

W.-B. Du[1], X.-B. Cao[1,2], M.-B. Hu[3] and W.-X. Wang[4]

[1] *Department of Computer Science and Technology, University of Science and Technology of China Hefei 230026, China*
[2] *Anhui Province Key Laboratory of Software in Computing and Communication - Hefei, Anhui, 230026, China*
[3] *School of Engineering Science, University of Science and Technology of China - Hefei 230026, China*
[4] *Department of Electronic Engineering, Arizona State University - Tempe, AZ 85287-5706, USA*

received 1 April 2009; accepted in final form 9 September 2009
published online 6 October 2009

PACS 02.50.Le – Decision theory and game theory
PACS 89.75.Hc – Networks and genealogical trees
PACS 89.65.-s – Social and economic systems

Abstract – We study the effects of asymmetric cost on the cooperative behavior in the snowdrift game on scale-free networks. The asymmetric cost reflects the inequality in mutual cooperation and the diversity of cooperators. We focus on the evolution of cooperation and the inequality in wealth distribution influenced by the degree of asymmetry in cost, related with cooperators' connections. Interestingly, we find that when cooperators with more neighbors have the advantage, cooperative behavior is highly promoted and the rich exploits the poor to get richer; while if cooperators with less neighbors are favored, cooperation is highly restricted and the rich are forced to offer some payoff to the poor so that the wealth is more homogeneously distributed. The wealth distribution in population is investigated by using the Gini coefficient and the Pareto exponent. Analytical results and discussions are provided to better explain our findings. The asymmetric cost enhances the leader effects in the decision making process by heterogeneous wealth distribution, leading not only to very high cooperator density but also to very stable cooperative behavior.

Understanding cooperative behavior ranging from natural to society by means of game theory has been an active topic of interdisciplinary research [1–4]. A central and challenging issue is how cooperative behavior emerges among selfish individuals, which apparently contradicts Darwinian selection. So far, game theory and simple models have provided a powerful framework to address this issue, in particular, incorporated with the approach of complex networks [5–9]. Two simple games, the Prisoner's Dilemma game (PDG) and the snowdrift game (SG) as a paradigm to explore the pairwise interactions among selfish individuals, have drawn growing attention [10]. The proposal of the SG as an alternative to the PDG is, on the one hand, because of the difficulties in assessing proper payoffs in the PDG, and more importantly the biological interest[1], on the other hand. In these games, both players can choose to cooperate or defect to maximize their payoffs which are determined by both their strategies and the payoff matrix. The main difference between the two games is that in the PDG a cooperator will have the lowest sucker's payoff if he/she encounters a defector; while in the SG, mutual defection leads to the lowest payoffs of both players. Apparently, the SG favors the cooperation compared to the PDG. However, defectors are always offered the highest payoff if they encounter cooperators, which results in the instability of cooperation in both games. The difference between the two-person games and real observations thus leads to the important consideration of interaction pattern among individuals, in other words, the combination of games and complex networks, to better mimic the game dynamics and explain the emergence of cooperation.

Since the original finding by Nowak and May that a simple spatial structure can induce the emergence and persistence of cooperation [11], the interplay between games and network structures, associated with new decision making strategies has been extensively studied, such as games on regular networks [12–21], complex networks [22–32] and adaptive networks with alternative interactions [33–37]. There are a variety of surprising findings. For instance, cooperation is inhibited by the spatial structure in the SG [15]; adaptive networks [33,35] and connection diversity in scale-free networks can considerably enhance cooperation [26]; phase transition and hysteresis behavior [17]; optimal connectivity density [29]; resonance-type phenomenon [30] etc. Besides, there are

[1]The snowdrift game is equivalent to the hawk-dove game.

some natural mechanisms in the real world, such as reward and costly punishment [38], variation in strategy transfer capability [39], noise [30,40–43] and memory effects [16,44], have been explored to explain cooperative behaviors.

In most previously studied games, symmetric payoff with sharing the same cost is assumed to be an intrinsic property in mutual cooperation. In other words, the diversity in cooperators is overlooked. In reality, there are no absolutely fair negotiations in social and economical systems due to the difference in status, strength and influence between two parties. For example, In commercial activities, some powerful international corporations often use their strength or power to force their weak co-players to make an unfair deal. On the contrary, many welfare policies are established to protect disadvantaged groups. We thus argue that asymmetric cost distribution in cooperation would be ubiquitous, ranging from nature to society. To be general, we investigate the SG by taking the asymmetric cost into account to better characterize the evolution of cooperation in the real world. The asymmetric cost is related with the connectivity of individuals and governed by a parameter. Our model can be simply reduced to the original SG by setting the parameter value to be zero. We find that the asymmetric cost can nearly induce a complete cooperation state in the SG on scale-free networks, regardless of the total cost in cooperation. The striking finding is explained by some illustration of strategy evolution in a small network example. We investigate the wealth distribution in terms of the Gini coefficient and Pareto exponent in the SG with asymmetric cost. We provide some theoretical analysis based on the mean-field approximation to give a firm support for simulation results. We also study the effect of asymmetric cost on cooperation by restraining connectivity diversity. Cooperation is still remarkably enhanced, sufficiently supporting the strong positive effect of asymmetric cost on cooperation. Our work may shed some new light in the study of cooperation in games on networks.

In this paper, we adopt the Barabási-Albert (BA) scale-free network model [45] to represent the population structure. In this model, starting from m_0 fully connected nodes, a new node with m ($m \leqslant m_0$) edges is added to the system at every step. The new node links to m different nodes by a "preferential attachment" mechanism: the probability of connecting to an existing node i is proportional to its degree, *i.e.*, $p_i = k_i / \sum_j k_j$, where j runs over all existing nodes and k_i is the degree of node i. In this paper, we set $m_0 = m = 4$. Initially, cooperators and defectors are randomly distributed on nodes. At each time step, all players involved in the SG play with their direct neighbors and get payoffs according to the payoff matrix:

$$\begin{array}{ccc} & C & D \\ C & 1-r/2 & 1-r \\ D & 1 & 0 \end{array} \tag{1}$$

For the original SG, the elements in the payoff matrix are $T = b$, $R = b - r/2$, $S = b - r$, and $P = 0$, with the constraint $b \geqslant r \geqslant 0$. It is natural to set $b = 1$ and $b \geqslant r \geqslant 0$ (see 1). Thus we can investigate evolutionary behavior with a single parameter while the essentials of the SG are preserved. One can see that, when both players cooperate, they get the same payoff, ignoring their differences, which may not reflect the real scenarios. So we introduce an unbalanced payoff distribution to the SG game model:

$$\begin{array}{ccc} & C & D \\ C & 1-r\cdot\Lambda & 1-r \\ D & 1 & 0 \end{array} \tag{2}$$

where Λ is the asymmetric coefficient and r is the cost. If one cooperates (C) and the other defects (D), the cooperator bears the whole cost r and his/her payoff is $1-r$ while the defector costs nothing and gains payoff 1. For mutual cooperation, the costs of two cooperators i and j are $r\Lambda_i$ and $r\Lambda_j$, where Λ_i and Λ_j are defined as

$$\Lambda_i = \frac{k_j^\alpha}{k_i^\alpha + k_j^\alpha} \quad \text{and} \quad \Lambda_j = \frac{k_i^\alpha}{k_i^\alpha + k_j^\alpha}, \tag{3}$$

where α is an adjustable parameter. From the definition, we have $\Lambda_i + \Lambda_j = 1$ and the total cost of cooperators i and j is fixed to be $r(\Lambda_i + \Lambda_j) = r$. Thus, the total payoff for mutual cooperation is fixed to be $2 - r$, but it can be unequally shared by two cooperators because of the unequal costs of $r\Lambda_i$ and $r\Lambda_j$ between nodes i and j. Who will gain higher or lower payoffs is determined by the parameter α associated with the node degrees of two cooperators. If $\alpha > 0$, high-degree nodes exploit low-degree nodes to get richer. If $\alpha < 0$, low-degree nodes get more payoff from high-degree nodes. Irrespective of the value of α, it can be easily proved that the payoff rank $T > R > S > P$ is strictly satisfied. For $\alpha = 0$, the model is reduced to the standard SG. This mechanism reflects the diversity of individuals in the real world as well, because the asymmetric cost distribution is induced by the diversity of individuals which is quantified by heterogeneous node degrees.

In each step, all pairs of directly linked nodes are engaged in a single round of the SG. The total payoff of player i is stored as P_i. The accumulative payoff (wealth) of player i since the beginning of simulation is stored as W_i. The learning strategy of nodes is as follows: for each node i, a neighbor j is randomly selected; then node i adopts j's strategy with a probability [12,46]:

$$H_{i\to j} = \frac{1}{1 + \exp[(P_i - P_j)/\kappa]}. \tag{4}$$

Here $0 < \kappa < \infty$ characterizes the environmental noise, including bounded rationality, individual trials, errors in decision, etc. The effect of noise κ on the stationary

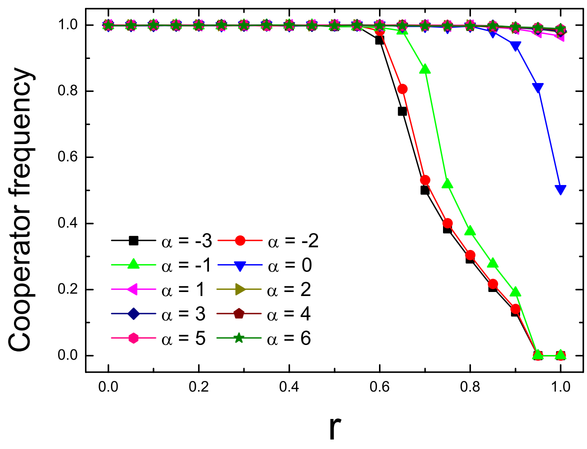

Fig. 1: (Color online) Frequency of cooperators as a function of the total cost r for different values of the asymmetric parameter α on scale-free networks.

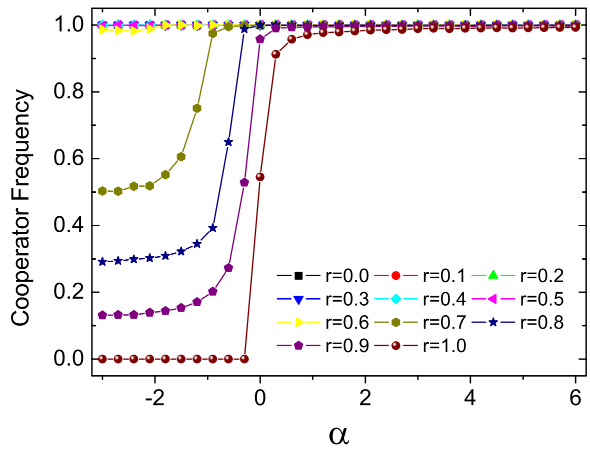

Fig. 2: (Color online) Frequency of cooperators as a function of the asymmetric parameter α for different values of the total cost r on scale-free networks.

density of cooperation (cooperator frequency) has been well studied in ref. [40]. In this paper, we set $\kappa = 0.1$ as in other studies.

Simulations are carried out on BA scale-free networks with size $N = 1000$ and $m_0 = m = 4$. Initially, strategies (C and D) are randomly distributed among population. Equilibrium frequencies of cooperators are obtained by averaging over 3000 generations after a transient time of 10^4 generations. Each data is averaged by 100 runs on 100 different networks.

Figure 1 shows the variation of cooperator frequency as a function of r for different values of α. One can see that the cooperator frequency is highly affected by α. When $\alpha = 0$, high cooperator frequency is maintained for a large range of r and only falls below 0.8 when $r > 0.9$. For $\alpha = [-3, -2, -1]$, the cooperator frequency is remarkably depressed and it decreases monotonously with the decrement of α. For $\alpha = [1, 2, 3]$, the cooperator frequency is promoted: the cooperator frequency keeps 1.0 for almost the entire range of r.

Figure 2 shows the variation of cooperator frequency with α. One can see that the cooperator frequency keeps 1.0 when r is small. But if r is larger than 0.6, where the temptation to defect is significantly magnified and players should be more favorable of defect, the asymmetric payoff distribution has a profound influence on the cooperator frequency. There is a sharp jump around $\alpha = 0$. For large values of r, the cooperator frequency is greatly depressed when $\alpha < 0$ and rapidly increases to 1.0 when $\alpha > 0$. This surprising phenomenon may contradict our expectation at the first glance. When $\alpha > 0$, the interest of majority in the population will be hurt, but the cooperative behavior is promoted. On the contrary, when $\alpha < 0$, the mechanism restrains cooperation.

Figure 3 provides some explanation for the above findings. It reveals how the asymmetric payoff mechanism affects the evolution of cooperative behavior. Figure 3(a) is a typical hub structure in the scale-free networks. Assume that the hub node is a cooperator and its neighbors are randomly assigned strategies. If $\alpha = 0$, the hub cooperator's total payoff is 1.0, equalling to its two defector neighbors (see fig. 3(a)). Thus, cooperation cannot outperform defection. If $\alpha = 1$, the hub cooperator can exploit its two cooperative neighbors to collect a larger payoff 1.6 (fig. 3(b)). According to the payoff difference, two non-hub defectors are more likely to be invaded by C strategy in

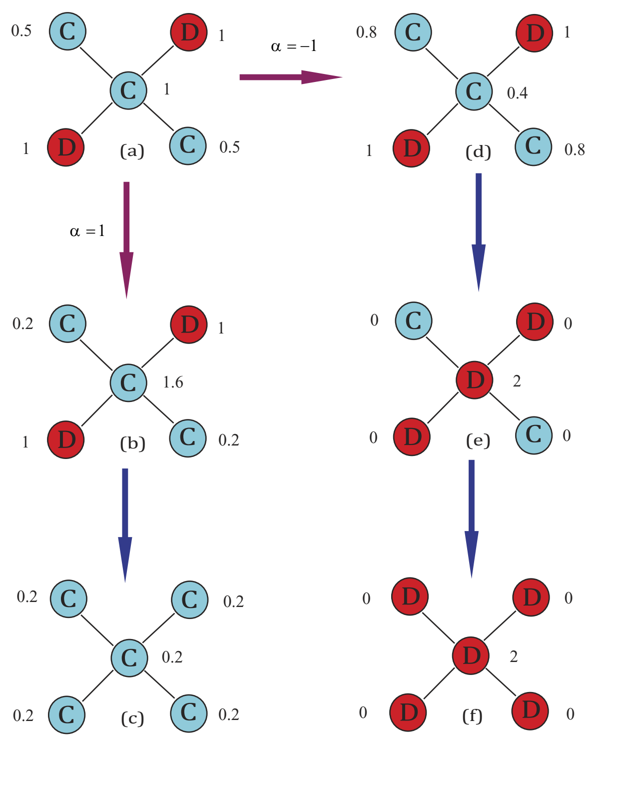

Fig. 3: (Color online) Illustration of the evolution for $r = 1$.

the next generation (fig. 3(c)). In the pattern of fig. 3(c), the steady cooperative behavior is easily maintained: the hub cooperator surrounded by non-hub cooperators has great influence on its neighbors and can hardly reverse to D strategy due to its payoff advantage. If $\alpha = -1$, the hub cooperator can only get 0.2 from each cooperative neighbor and its total payoff is only 0.4, which is much smaller than those of its defector neighbors (their payoffs are both 1.0, see fig. 3(d)). Thus in the next step, the hub node is probably invaded by D strategy (fig. 3(e)). In the pattern of fig. 3(e), the hub node chooses to defect and its collective payoff is 2.0, while all its neighbors get nothing. So in the next generation, all players learn to choose the D strategy and the system falls to a pure D state (fig. 3(f)).

Besides the cooperative behavior, the game dynamics on networks are also suitable to characterize the wealth accumulating behavior in the society [46–49]. Since the asymmetric cost can evidently affect the collection of wealth, next we will investigate the wealth distribution in terms of two important economic parameters: the Gini coefficient and the Pareto exponent. They both measure the inequity of wealth distribution and have important implications for the economics and the sociology.

The Gini coefficient, which was developed by the Italian statistician Corrado Gini in 1912, varies from 0 (when the players have the same wealth) to 1 (when the whole wealth is possessed by only one player). Thus, a low Gini coefficient indicates more equal income or wealth distribution, while a high Gini coefficient indicates more unequal distribution. 0 corresponds to perfect equality and 1 corresponds to perfect inequality. Worldwide, Gini coefficients range from approximately 0.249 in Japan to 0.707 in Namibia.

It is well known that even in the developed countries, it is common that 40% of the total wealth is owned by only 10% of the population. The wealth distribution is often described as "Pareto tail" that decays as a power law of wealth:

$$P(W) \propto W^{-(1+v)}, \tag{5}$$

where $P(W)$ is the probability of finding a person with wealth larger than W, and the value of v is usually called the Pareto exponent. Empirical studies show that $v = 1.6$ for USA, $v = 2.6$ for Italy, and $v = 0.8$ for India [46–48].

Figure 4 shows the variation of Gini coefficient and Pareto exponent with α for different values of r. One can see that the Gini coefficient monotonously increases and the Pareto exponent monotonously decreases with the increment of α, which means that the wealth becomes more unevenly distributed. Moreover, the values of the Gini coefficient (0.2–0.7) and the Pareto exponent (0.8–3.0) above are in accordance with empirical data of real world. Next, we will give a simple analysis of nodes' payoff.

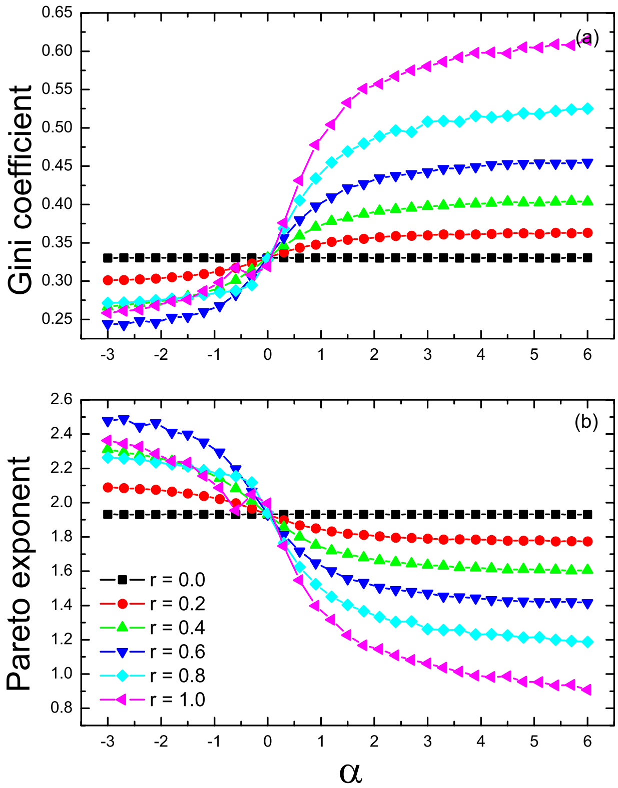

Fig. 4: (Color online) Gini coefficient (a) and Pareto exponent (b) *vs.* α for different values of r.

For individual i in the SG, its payoff could be estimated as

$$\begin{aligned} P_i &= k_i\rho_c \sum_{k=k_{min}}^{k_{max}} p(k|k_i)\rho_c\left(1-\frac{rk^\alpha}{k_i^\alpha+k^\alpha}\right) \\ &\quad + k_i\rho_c \sum_{k=k_{min}}^{k_{max}} p(k|k_i)(1-\rho_c)(1-r) \\ &\quad + k_i(1-\rho_c) \sum_{k=k_{min}}^{k_{max}} p(k|k_i)\rho_c \\ &= k_i\rho_c^2 \sum_{k=k_{min}}^{k_{max}} \frac{kp(k)}{\langle k\rangle} - k_i\rho_c^2 r \sum_{k=k_{min}}^{k_{max}} \frac{kp(k)}{\langle k\rangle}\frac{k^\alpha}{k_i^\alpha+k^\alpha} \\ &\quad + k_i\rho_c(1-\rho_c)(1-r) \sum_{k=k_{min}}^{k_{max}} \frac{kp(k)}{\langle k\rangle} \\ &\quad + k_i\rho_c(1-\rho_c) \sum_{k=k_{min}}^{k_{max}} \frac{kp(k)}{\langle k\rangle} \\ &= k_i\rho_c + k_i\rho_c(1-\rho_c)(1-r) \\ &\quad - k_i\rho_c^2 r\frac{1}{\langle k\rangle}\left\langle \frac{k}{1+(\frac{k_i}{k})^\alpha}\right\rangle, \end{aligned} \tag{6}$$

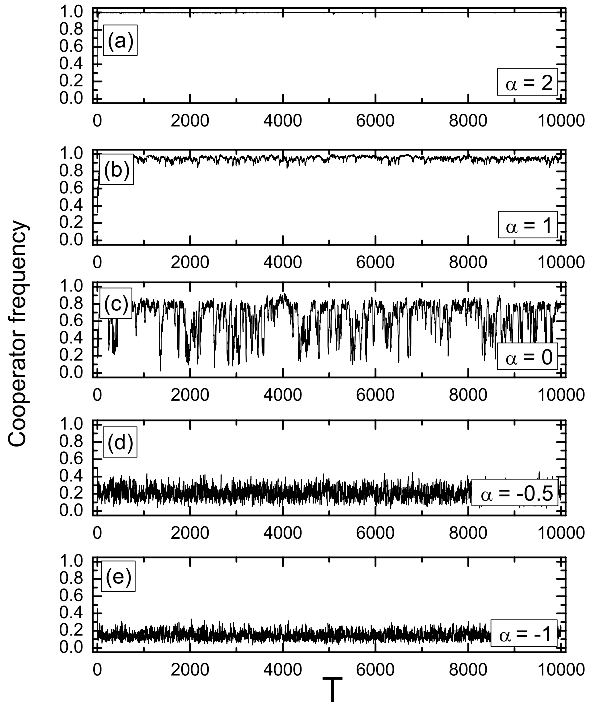

Fig. 5: Time series of frequency of cooperators for different values of α.

where $\langle . \rangle$ denotes the ensemble average over all nodes and $\sum_{k=k_{min}}^{k_{max}} kp(k) = \langle k \rangle = 2m$. Assume there is a hub node i and a non-hub node $j(k_i \gg k \geqslant k_j)$, one can easily obtain that P_i increases with the increment of α and P_j decreases with the increment of α. Thus the gap between the rich and the poor becomes wider as α grows.

For $\alpha = 0$, the model is reduced to the standard snowdrift game. We thus have

$$P_i = k_i\rho_c + k_i\rho_c(1-\rho_c)(1-r) - \frac{1}{2}k_i\rho_c^2 r. \qquad (7)$$

Since P_i is obtained, the wealth distribution can be obtained from

$$p(k)\mathrm{d}(k) = p(P)\mathrm{d}P, \qquad (8)$$

where $p(k)$ is the degree distribution of BA scale-free network: $p(k) = 2m^2k^{-\gamma}$. For finite network size, γ is a little bit smaller than 3. The wealth distribution thus is

$$p(P) = p(k(P))\frac{\mathrm{d}k(P)}{\mathrm{d}P}, \qquad (9)$$

where $\mathrm{d}k(P)/\mathrm{d}P$ can be obtained from eq. (6). In particular, for large positive values of α and in the tail of the wealth distribution, we can approximately write

$$P_i \approx k_i\rho_c + k_i\rho_c(1-\rho_c)(1-r) - k_i^{1-\alpha}\rho_c^2 r\langle k^\alpha \rangle. \qquad (10)$$

For very small negative values of α,

$$P_i \approx k_i\rho_c + k_i\rho_c(1-\rho_c)(1-r) - k_i\rho_c^2 r. \qquad (11)$$

Next, we investigate the effect of α on the time series of cooperator frequency. We find that α can also affect the evolution behaviors of cooperation. As shown in fig. 5, the evolution process are more stable when the asymmetric payoff distribution mechanism is applied, especially when $\alpha > 0$. When $\alpha = 0$, the cooperator frequency is driven by the hub nodes. If some hub nodes reverse their strategies (C to D, or D to C), there will be a large fluctuation in the system's cooperator frequency. When $\alpha > 0$, the hub nodes become richer and the non-hub nodes become poorer. The gap between the rich and the poor is more evident. According to the learning strategy (4), a high-degree node can easily influence its neighbors' strategy but their own strategies are extremely difficult to be changed. So the evolution is steady. When $\alpha < 0$, the influence of hub nodes is weakened and the wealth distribution in population becomes homogeneous (see figs. 3 and 4). Since the evolution is no longer driven by a few hub nodes, there will not be large fluctuations in the time series. We have examined the time series of different system sizes from $N = 1000$ to $N = 5000$ and the situation is alike.

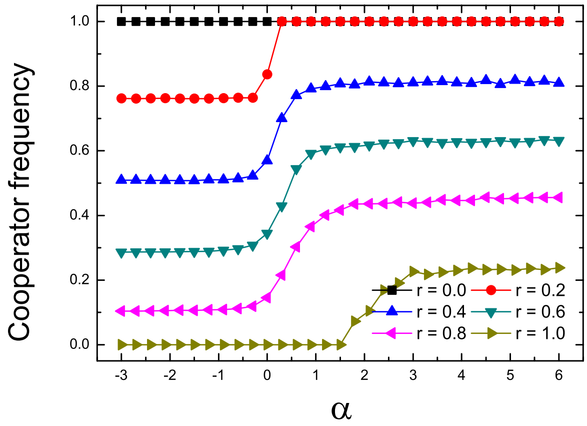

Fig. 6: (Color online) Frequency of cooperators as a function of the asymmetric parameter α for different values of the total cost r, for the normalized updating.

Moreover, previous researches have proposed a normalized learning strategy to avoid additional bias caused by the heterogeneity of degrees:

$$H_{i\to j} = \frac{1}{1+\exp[(P_i/k_i - P_j/k_j)/\kappa]}, \qquad (12)$$

where the ratio of the total payoff of a player and its degree P_i/k_i is defined as the normalized total payoff. Figure 6 shows the cooperator frequency as a function of α with the normalized learning rule. One can see that the asymmetric cost can still remarkably enhance cooperator frequency without the affection of diversity in node degrees.

In summary, we have introduced an asymmetric cost in cooperation into the evolutionary snowdrift game on scale-free networks and then investigated its influence on cooperation behavior and wealth distribution in the population. In our model, the asymmetric cost distribution can be adjusted by only one parameter α. If $\alpha > 0$, the rich get richer and the poor get poorer. Simulation results show that the cooperator frequency and the non-identity of population both monotonously increase with α. The effect of α on the systems wealth distribution haven been studied by using the Pareto exponent and the Gini coefficient as well. It is found that with the increment of α, the Pareto exponent decreases monotonously, while the Gini coefficient increases monotonously. The study of time series of cooperator frequency shows that the asymmetric cost results in very stable cooperative behavior, in particular for $\alpha > 0$. All these results demonstrate that the asymmetric cost mechanism considerably promotes the cooperative behavior in evolutionary games but with an inequality wealth distribution. Since the asymmetric cost and payoff distribution are common in nature and human society, we expect our findings to be more relevant to understanding the emergence of cooperation in the real world.

* * *

We thank G. Szabó, G. Yan, H.-X. Yang and B.-H. Wang for useful discussions. This work is partly supported by the Major State Basic Research Development Program of China (No. 2004CB318109), Program for New Century Excellent Talents in University (NCET-07-0787) and the National Nature Science Foundation of China (NNSFC Nos. 70601026, 10635040).

REFERENCES

[1] von Neumann J. and Morgenstern O., *Theory of Games and Economic Behavior* (Princeton University Press, Princeton) 1953.

[2] Smith J. M., *Evolution and the Theory of Games* (Cambridge University Press, Cambridge) 1982.

[3] Axelrod R., *The Evolution of Cooperation* (Basic Books, New York) 1984.

[4] Hofbauer J. and Sigmund K., *Evolutionary Games and Population Dynamics* (Cambridge University Press, Cambridge) 1998.

[5] Albert R. and Barabási A.-L., *Rev. Mod. Phys.*, **74** (2002) 47.

[6] Dorogovtsev S. N. and Mendes J. F. F., *Adv. Phys.*, **51** (2002) 1079.

[7] Newman M. E. J., *SIAM Rev.*, **45** (2003) 167.

[8] Boccaletti S., Latora V., Moreno Y., Chavez M. and Hwang D. U., *Phys. Rep.*, **424** (2006) 175.

[9] Dorogovtsev S. N., Goltsev A. V. and Mendes J. F. F., *Rev. Mod. Phys.*, **80** (2008) 1275.

[10] Szabó G. and Fáth G., *Phys. Rep.*, **446** (2007) 97.

[11] Nowak M. and May R. M., *Nature*, **359** (1992) 826.

[12] Szabó G. and Töke C., *Phys. Rev. E*, **58** (1998) 69.

[13] Doebeli M. and Knowlton N., *Proc. Natl. Acad. Sci. U.S.A.*, **95** (1998) 8676.

[14] Szabó G. and Hauert C., *Phys. Rev. Lett.*, **89** (2002) 118101.

[15] Hauert C. and Doebeli M., *Nature*, **428** (2004) 643.

[16] Wang W.-X., Ren J., Chen G. and Wang B.-H., *Phys. Rev. E*, **74** (2006) 056113.

[17] Wang W.-X., Lv J.-H., Chen G. and Hui P. M., *Phys. Rev. E*, **77** (2008) 046109.

[18] Wu Z.-X., Xu X.-J., Huang Z.-G., Wang S.-J. and Wang Y.-H., *Phys. Rev. E*, **74** (2006) 021107.

[19] Perc M., *New J. Phys.*, **8** (2006) 22.

[20] Wu Z.-X. and Wang Y.-H., *Phys. Rev. E*, **75** (2007) 041114.

[21] Jiang R., Deng H., Hu M.-B., Wu Y.-H. and Wu Q.-S., *Phys. Rev. E*, **78** (2008) 041101.

[22] Abramson G. and Kuperman M., *Phys. Rev. E*, **63** (2001) 030901.

[23] Kim B. J., Trusina A., Holme P., Minnhagen P., Chung J. S. and Choi M. Y., *Phys. Rev. E*, **66** (2002) 021907.

[24] Wu Z.-X., Xu X.-J., Chen Y. and Wang Y.-H., *Phys. Rev. E*, **71** (2005) 037103.

[25] Hauert C. and Szabó G., *Am. J. Phys.*, **73** (2005) 405.

[26] Santos F. C. and Pacheco J. M., *Phys. Rev. Lett.*, **95** (2005) 098104.

[27] Santos F. C., Rodrigues J. F. and Pacheco J. M., *Phys. Rev. E*, **72** (2005) 056128.

[28] Santos F. C. and Pacheco J. M., *J. Evol. Biol.*, **19** (2006) 726.

[29] Tang C.-L., Wang W.-X., Wu X. and Wang B.-H., *Eur. Phys. J. B*, **53** (2006) 411.

[30] Ren J., Wang W.-X and Qi F., *Phys. Rev. E*, **75** (2007) 045101.

[31] Fu F., Liu L.-H. and Wang L., *Eur. Phys. J. B*, **56** (2007) 367.

[32] Fu F., Chen X.-J., Liu L.-H. and Wang L., *Phys. Lett. A*, **371** (2007) 58.

[33] Zimmermann M. G., Eguíluz V. M. and Miguel M. S., *Phys. Rev. E*, **69** (2004) 065102R; Zimmermann M. G. and Eguíluz V. M., *Phys. Rev. E*, **72** (2005) 056118.

[34] Pacheco J. M., Traulsen A. and Nowak M. A., *Phys. Rev. E*, **97** (2006) 258103.

[35] Wu Z.-X., Xu X.-J., Huang Z.-G., Wang S.-J. and Wang Y.-H., *Phys. Rev. E*, **74** (2006) 021107.

[36] Li W., Zhang X. and Hu G., *Phys. Rev. E*, **76** (2007) 045102.

[37] Fu F., Hauert C. and Nowak M. A., *Phys. Rev. E*, **78** (2008) 026117.

[38] Hauert C., Traulsen A., Brandt H., Nowak M. A. and Sigmund K., *Science*, **316** (2007) 1905.

[39] Szolnoki A. and Perc M., *New J. Phys.*, **10** (2008) 043036.

[40] Szabó G., Vukov J. and Szolnoki A., *Phys. Rev. E*, **72** (2005) 047107.

[41] Vukov J., Szabó G. and Szolnoki A., *Phys. Rev. E*, **73** (2006) 067103.

[42] Perc M. and Marhl M., *New J. Phys.*, **8** (2006) 142.

[43] Perc M., *Phys. Rev. E*, **75** (2007) 022101.

[44] Qin S.-M., Chen Y., Zhao X.-Y. and Shi J., *Phys. Rev. E*, **78** (2008) 041129.

[45] Barabási A.-L. and Albert R., *Science*, **286** (1999) 509.

[46] Hu M.-B., Wang W.-X., Jiang R., Wu Q.-S., Wang B.-H. and Wu Y.-H., *Eur. Phys. J. B*, **53** (2006) 273.

[47] Hu M.-B., Jiang R., Wu Q.-S. and Wu Y.-H., *Physica A*, **381** (2007) 467.

[48] Du W.-B., Zheng H.-R. and Hu M.-B., *Physica A*, **387** (2008) 3796.

[49] Hu M.-B., Jiang R., Wu Y.-H., Wang R.-L. and Wu Q.-S., *Physica A*, **387** (2008) 5862.