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**Citation for published version (APA):**

Kamp, van de, M. M. J. L. (1998). Asymmetric signal level distribution due to tropospheric scintillation. *Electronics Letters*, 34(11), 1145-1145. <https://doi.org/10.1049/el:19980805>

**DOI:**

[10.1049/el:19980805](https://doi.org/10.1049/el:19980805)

**Document status and date:**

Published: 01/01/1998

**Document Version:**

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

**Please check the document version of this publication:**

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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# Asymmetric signal level distribution due to tropospheric scintillation

M.M.J.L. van de Kamp

Short term distribution of signal fluctuations due to tropospheric scintillation (in decibels) is usually modelled as Gaussian. For long term distribution, this results in a symmetrical distribution function. However, various experiments have shown a significant asymmetry in long term distribution for strong scintillations. It is shown that the observed asymmetry follows directly from theory, if a different modelling approach is applied.

**Introduction:** Tropospheric scintillation is fast fluctuation of amplitude and phase in the mm-wave range, caused by atmospheric turbulence. In general, the impact of rain attenuation on communication signals is dominant in the design of the link. Scintillation, however, becomes important for low fade margin systems operating at high frequencies and low elevation angles. Prediction models of the effects of tropospheric scintillation have been developed semi-empirically by various researchers, by combining theoretical relations and experimentally observed dependencies.

**Short term signal fluctuations:** It is generally assumed that the signal level fluctuations due to turbulence on a short term basis (up to several minutes) are distributed in a Gaussian manner when expressed in decibels. This is why they are often characterised only by the signal standard deviation. This standard deviation has been found to depend on frequency, the path length through the turbulent layer and the intensity of the turbulence. However, it has been found that for the largest scintillation intensities, the short term distribution significantly departs from a Gaussian distribution. The negative signal deviations, generally referred to as 'fade', are on average larger than the positive ones, generally referred to as 'enhancement', especially for large signal fluctuations.

The physical background for the Gaussian probability density function (PDF) of signal fluctuations in decibels lies in a certain modelling approach. Strohbehn, Wang and Speck [1] pointed this out for optical signals scattered by a random medium. If the receiver is located inside the turbulent medium, and the scattering volume is long and narrow, the received field is a result of multiple scattering. Since the received field  $E$  is then the product of the incident field multiplied by a number of independent factors, the application of the central-limit theorem to  $\log E$  leads to a Gaussian distribution for  $\log E$ , a log-normal distribution for  $|E|$ , and a Gaussian distribution for  $y \sim \text{Re}(\log E) = \log |E|$ .

Conversely, if the turbulent medium is a thin layer far from the receiver, the distortion component of the received field is the sum of a large number of waves scattered from different regions in the layer. Vilar and Haddon [2] stated that the thin layer model is probably more realistic for a slant path. Applying the central-limit theorem in this situation leads to the conclusion that the distortion is a zero mean Gaussian process. Resulting from this approach, the electric field amplitude  $X = |E|$  will not follow a log-normal, but a Rice-Nakagami distribution:

$$p(X) = \frac{X}{\sigma_x^2} e^{-(X^2 + X_0^2)/2\sigma_x^2} I_0\left(\frac{X X_0}{\sigma_x^2}\right) \quad (1)$$

where  $\sigma_x$  is the standard deviation of the distortion,  $X_0$  is the amplitude of the received main signal, and  $I_0(\cdot)$  is the modified Bessel function of zeroth-order. For very small variances, the difference between the log-normal and the Rice-Nakagami distributions is small, but as the variance becomes moderate or large, the difference becomes easily recognisable.

Using eqn. 1, the distribution function of signal level in decibels can be derived. It can be assumed that the interfering signal, originating from refraction on the turbulent eddies, is proportional to the main received signal  $X_0$ . Under this assumption,  $\sigma_x$  will be proportional to  $X_0$ :

$$\sigma_x = \xi X_0 \quad (2)$$

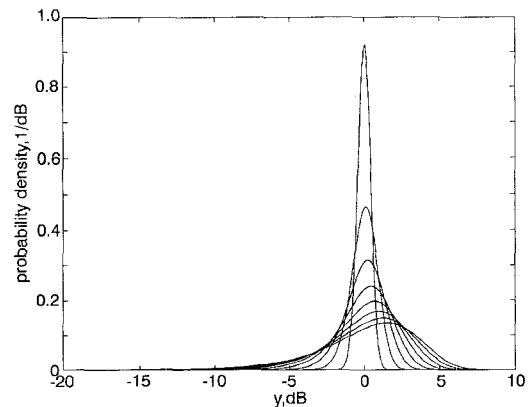
The signal deviation from the mean on a dB-scale can be calculated from

$$y = 20 \log(X/X_0) \quad (3)$$

The resulting distribution of  $y$  is then given by

$$p(y) = \frac{\ln 10}{20} \frac{10^{2y/20}}{\xi^2} e^{-(10^{2y/20} + 1)/2\xi^2} I_0\left(\frac{10^{y/20}}{\xi^2}\right) \quad (4)$$

Thus, the only parameter influencing this distribution is  $\xi$ , which is a measure of the intensity of the scintillation. Fig. 1 shows the distribution  $p(y)$  for various values of  $\xi$ . It is clearly seen that as  $\xi$  increases, not only does the spread increase, but also the skewness of the distribution. Even though this skewing effect has often been observed, the short-term signal level distribution  $p(y)$  is still mostly modelled as Gaussian.



**Fig. 1** Probability density distribution of signal level  $y$  (in decibels), for values of parameter  $\xi = 0.05$  (sharpest peak), 0.1, 0.15, ..., 0.4 (largest spread)

**Long-term signal level distribution:** An estimate of the long-term distribution of signal level deviation can be calculated as [3]

$$p(y) = \int_0^\infty p(\sigma_y) p(y|\sigma_y) d\sigma_y \quad (5)$$

where  $p(\sigma_y)$  is the distribution function of the short term standard deviations  $\sigma_y$ , and  $p(y|\sigma_y)$  is the short term distribution function of signal level  $y$  for a given standard deviation  $\sigma_y$ . For  $p(y|\sigma_y)$ , a Gaussian distribution may be assumed, or the alternative distribution of eqn. 4.

Karasawa *et al.* [4] present a prediction model for the long term cumulative distribution of amplitude deviation  $y$ , derived from eqn. 5 with a Gaussian distribution assumed for  $p(y|\sigma_y)$  and a Gamma distribution for  $p(\sigma_y)$ . The resulting amplitude deviation, exceeded for a time percentage of  $P$ , is given by

$$y = (-0.0597 \log^3 P - 0.0835 \log^2 P - 1.258 \log P + 2.672) \sigma_{it} \quad (6)$$

for  $0.01\% < P < 50\%$ , where  $\sigma_{it}$  is the long term signal standard deviation, which is given by another expression which depends on frequency, elevation angle, effective antenna diameter, and a meteorological parameter. In practice,  $\sigma_{it}$  is equal to the long term average of  $\sigma_y$ .

It appears that the long-term cumulative distribution, when expressed as  $y(P)$ , is proportional to  $\sigma_{it}$ . This results from the theoretical calculation using a Gamma distribution for  $p(\sigma_y)$ , under the assumption that the mean and the standard deviation of  $\sigma_y$  are proportional. If this condition is not satisfied, the dependence of  $y(P)$  on  $\sigma_y$  becomes more complicated.

Eqn. 6 agreed well with the measurements of Karasawa *et al.* for signal enhancement. However, signal fading they measured to be larger, especially in the low probability region. They fitted a curve to these measurement results, giving an expression of the same form as eqn. 6, and also proportional to  $\sigma_{it}$ . A similar difference between fade and enhancement in the long-term distribution was also observed by other experimenters [5-7].

**Model for asymmetry:** The observed difference between fading and enhancement in the long-term distribution may be modelled adequately by considering the asymmetry of short-term signal level fluctuations described previously. We did this by using the Rice-Nakagami distribution to derive a theoretical expression for  $p(y)$ . The distribution of eqn. 4 has been substituted for  $p(y|\sigma_y)$  in eqn. 5. The relation between  $\xi$  and  $\sigma_y$  was derived directly from eqn. 4.

Furthermore, we took the Gamma distribution for  $p(\sigma_y)$  as proposed by Karasawa *et al.* The result of this integration is shown in Fig. 2, for a normalised signal deviation  $y/\sigma_{it}$ , and for different values of  $\sigma_{it}$ .

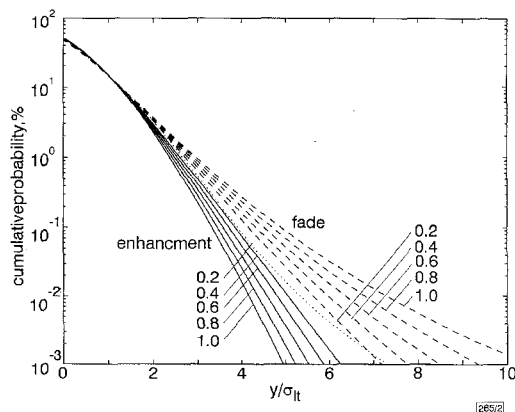


Fig. 2 Cumulative distribution of normalised signal enhancement and fade, calculated using distribution of eqn. 4, for indicated values of  $\sigma_{it}$  (dB), and model by Karasawa

..... Karasawa model

In this Figure, the difference between signal fade and enhancement is clear. Furthermore, it shows that this difference significantly increases with increasing long term standard deviation  $\sigma_{it}$ : the normalised enhancement decreases and the normalised fade increases with  $\sigma_{it}$  for fixed exceedance probabilities. This is also in agreement with various measurements; the difference between fade and enhancement in the long-term distribution of  $y/\sigma_{it}$  has been found to be large at sites with strong scintillation [7] and negligible at sites with weak scintillation [3]. Karasawa's empirical model, however, predicts the difference to be constant.

The dependencies observed in Fig. 2 can be modelled as follows. If the cumulative distribution of signal fade is  $y_f(P)$  and of signal enhancement  $y_e(P)$ , these distributions can be written as

$$y_f(P) = \gamma(P) + \delta(P) \quad y_e(P) = \gamma(P) - \delta(P) \quad (7)$$

where  $\gamma(P)$  can be approximated by Karasawa's expression (eqn. 6), and  $\delta(P)$  is approximated by

$$\delta(P) = (0.100 \log^2 P - 0.375 \log P + 0.297) \sigma_{it}^2 \quad (8)$$

for  $0.001\% < P < 50\%$ .

**Conclusions:** The modelling approach of scintillation due to turbulence which assumes a thin turbulent layer leads to the prediction that, for strong scintillation, the long term distribution of signal level deviation becomes asymmetrical. This is in contrast to the modelling approach currently used, but consistent with the results obtained by various experimenters. The short term PDF of signal level is therefore better described by eqn. 4 than by a Gaussian distribution. The long-term distribution of signal level is qualitatively well described by eqns. 6–8. The quantitative agreement of this new model with measurements may still be improved by tuning the relation between the mean and standard deviation of the Gamma distribution  $p(\sigma_y)$  in eqn. 5, from a comparison with measurement results.

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13 March 1998

Electronics Letters Online No: 19980805

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## Modelling diversity reception over narrowband fixed wireless channels

L.J. Greenstein, V. Erceg and D.G. Michelson

A complete first-order statistical characterisation of two-branch diversity reception over narrowband fixed wireless channels can be given in terms of just five channel parameters: the average path gains and Ricean  $K$ -factors for each branch and the complex envelope correlation coefficient between the time-varying parts of the two path gains. Propagation measurements collected in typical suburban environments have shown that each of these parameters, or its logarithm, is close to Gaussian. Accordingly, the set may be cast as a five-element vector of jointly random Gaussian processes which are completely specified by the means, standard deviations, and mutual correlation coefficients of the five parameters.

**Introduction:** Simulation studies of communications systems which use diversity reception over narrowband fixed wireless channels require a statistical characterisation of the relevant channel parameters. Extensive measurements of fixed wireless channels in suburban New Jersey (by AT&T Labs) and Illinois (by AT&T Wireless Services) have taught us that multipath fading on a fixed wireless channel is non-dispersive over a bandwidth of  $< \sim 100$  kHz. We can therefore characterise the complex signal path gain of such a channel by a frequency-flat (but possibly time-varying) response:

$$g(t) = V + v(t) \quad (1)$$

where  $V$  is a complex number and  $v(t)$  is a complex, zero-mean random time variation caused by wind-blown foliage, vehicular traffic, etc. This description applies to a particular frequency segment (eg. 100 kHz wide) and a particular time segment (eg. 15 min wide). Both the 'constant'  $V$  and the parameters of the random process  $v(t)$  may change from one such time-frequency segment to another.

**Model description:** The first step in characterising the diversity channel is to recast eqn. 1 so that it can be expressed in terms of quantities that can be derived from measurements of received power on each branch against time. (Assessment of the corresponding Doppler spectra has shown that sampling the received power at a rate of 10 Hz is generally sufficient to characterise the channel.) The corresponding power gain is  $|g(t)|^2$ , and its average over a finite time segment is

$$G_{ave} = \overline{|g(t)|^2} = |V|^2 + \overline{|v(t)|^2} \quad (2)$$

Experience has shown that  $v(t)$  can be reasonably modelled as a complex Gaussian process. In this case,  $|g(t)|^2$  has a Ricean distribution over time, with a total power equal to  $G_{ave}$  and a  $K$ -factor given by

$$K = |V|^2 / \overline{|v(t)|^2} \quad (3)$$