

# ASYMMETRIES IN THE CAPACITY-INFLATION TRADE-OFF

PEDRO PABLO ÁLVAREZ LOIS

**ABSTRACT.** This paper analyzes the joint dynamics of two key macroeconomic variables for the conduct of monetary policy: inflation and the aggregate capacity utilization rate. An econometric procedure useful for estimating dynamic rational expectation models with unobserved components is developed and applied in this context. The method combines the flexibility of the unobserved components approach, based on the Kalman recursion, with the power of the general method of moments estimation procedure. A ‘hybrid’ Phillips curve relating inflation to the capacity utilization gap and incorporating forward and backward looking components is estimated. The results show that such a relationship is non-linear: the slope of the Phillips curve depends significantly on the magnitude of the capacity gap. These findings provide support for studying the implications of asymmetric monetary policy rules.

## 1. INTRODUCTION

As a first step towards a truly understanding of the effects stemming from monetary policy, one needs to explore the precise nature of the relationship between prices and real economic activity, namely the Phillips curve. This is so because the appropriate course of monetary policy depends crucially on the short-run inflation dynamics, as a recent strand of research has stressed.<sup>1</sup> This article explores the precise nature of such a relationship, providing evidence on its asymmetric shape. The argument behind this finding involves the existence of capacity utilization constraints in the real side of the economy. When analyzing the activity-inflation trade off, the standard approach in the literature is to consider some sort of linear (or linearized) Phillips curve. Specifically, the change in inflation relative to expected inflation is assumed to be proportional to some measure of overall real activity, with such relationship being constant over time. However, there are strong arguments, both theoretical and empirical, that can be put forward in support of an asymmetric activity-inflation trade-off. One of these arguments is known as the capacity constraint hypothesis.<sup>2</sup> The idea is that some firms find it difficult to increase their capacity to produce in the short run, creating production bottlenecks and supply

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<sup>1</sup>See, *inter alia*, the work of Clarida, Galí and Gertler (1999) and Goodfriend and King (1997).

<sup>2</sup>Other arguments are based on “menu costs” and nominal wage rigidities. Dupasquier and Ricketts (1997) briefly survey some of the different sources of asymmetries, performing an empirical investigation in this regard. See also Weise (1999).

shortages. Thus, when the economy experiences strong aggregate demand, the impact on inflation will be greater when more firms are restricted in their ability to raise output in the short run. In this framework, the short-run aggregate supply equation or Phillips curve has a convex shape, which has relevant consequences for the performance of a monetary policy aimed at controlling inflation. If the economy is initially weak, easing monetary conditions will primarily affect output, but if the economy is initially strong, a monetary expansion will mainly affect prices, like Friedmans's *pushing a string* argument. Another implication of a convex Phillips curve is that the more stable output is, the higher the level of output will be in the economy, on average.<sup>3</sup> Again, the issue of asymmetry has important implications for the design and conduct of monetary policy<sup>4</sup>

To carry out the analysis, an econometric procedure useful for estimating dynamic rational expectation models with unobserved components is developed. The method combines the flexibility of the unobserved components approach based on the Kalman recursion, with the power of the general method of moments (GMM) estimation procedure. The method is applied to the estimation of a 'hybrid' Phillips curve that relates inflation to the capacity utilization gap. The term 'hybrid' means that in the specification of the Phillips curve, both forward and backward looking components are considered. The empirical results that are obtained with the application of this method provide clear evidence on the asymmetric relationship existing in the joint dynamics of prices and economic activity, the source of such an asymmetry being the existence of capacity constraints in the real side of the economy. The Phillips curve considered here is based on the capacity gap, which is the difference between the non accelerating inflationary capacity utilization rate (NAICU) and the current aggregate capacity utilization rate.<sup>5</sup> As a by product of the econometric procedure, time-varying estimates of the NAICU are obtained.

An additional relevant implication over the role of capacity utilization in the conduct of monetary policy is the possibility of developing asymmetric monetary policy rules. In light of the arguments and results given here, such kinds of rules can be easily justified. For example, to the extent that the effect on inflation becomes disproportionately larger as the capacity gap increases, the policy response should become more aggressive with each incremental increase in the gap. From either a theoretical and an empirical point of view, it would be desirable to explore these issues in depth but, unfortunately, this is beyond the scope of the present analysis and has to be left for future research.

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<sup>3</sup>The thinking behind this result is that given the lags in the effects of monetary policy, there is an incentive for pre-emptive tighten responses to inflationary pressure. If central bankers act in this way, it will prevent the economy from moving too far up the level where inflation begins to rise more rapidly, thereby avoiding the need for a larger negative output gap in the future to reverse this large rise in inflation. See MacKlem (1997) for a full discussion in this respect.

<sup>4</sup>In this sense, Nobay and Peel (2000) have shown that the analysis of optimal discretionary monetary policy under a non-linear Phillips Curve yields results that are in marked contrast with those obtained under conventional linear paradigm. Shaling, (1999) extends Svensson's inflation targeting model with a convex Phillips curve and derives optimal monetary policy rules.

<sup>5</sup>Conventionally, Phillips curves are based on the employment gap or on the output gap. The latter is defined as the difference between potential output and current output, where the former is typically viewed as the trend component of the latter, with the trend estimated in various ways. See Gerlach and Smets (1999) and Kichian (1999) with the references there in.

The outline of the paper is as follows. Section 2 introduces the Phillips curve relationship. Section 3 presents the econometric methodology used in the estimation of the empirical model, with the results being reported in Section 4. Some conclusions and possible lines for further research are offered in Section 5.

## 2. AN EMPIRICAL ‘HYBRID’ SHORT-RUN PHILLIPS CURVE

There is evidence that capacity is a leading indicator of future inflationary pressures, despite the apparently spurious dynamics existing between the joint behavior of these two magnitudes.<sup>6</sup> Figure 1 shows the dynamics of the capacity utilization and the level of the inflation rate for the U.S. economy over the last three decades. The associated cross correlogram is shown in Figure 2. The simple correlation between the contemporaneous level of these two variables is negative and very low (-0.2) a fact that could suggest that the specification of the Phillips curve below could be at odds with the data. However, this correlation increases (0.18) when the variables under consideration are inflation and lagged, at least four periods, capacity utilization rate. This result reinforces, but not sufficiently, the idea of capacity as a leading indicator of inflation pressures. Indeed, from Figures 3 and 4, where it is shown the historical series and the corresponding scattered graph, it is not clear the existence of a positive relationship in the dynamics of prices and capacity. Consequently, these dynamics are not well captured simply by taking into account the level of the inflation rate and some lag in the capacity utilization rate. Better results are found, however, when inflationary influences are subsumed in last period’s inflation rate. A measure of acceleration in the inflation rate is considered in this case. The cross correlations are shown in Figure 5. It is noticeably the positive correlation (0.7) reached when acceleration and the two-quarter lagged capacity are considered.<sup>7</sup> In this case, the historical series, shown in Figure 6, seem to follow a close pattern over time and the corresponding scattered graph, shown in Figure 7, points toward a positive relationship. Hence, given that one of the overriding objectives of most central banks around the world is to achieve and maintain a low and stable rate of inflation, the information provided in capacity utilization rate series is expected to be an important and useful input in the design of monetary policy rules, a fact that policy makers cannot disregard.

In light of the discussion above, in this section I analyze the short run trade-off between inflation and economic activity, this latter variable being measured by the capacity utilization rate of the economy. More specifically, I specify and estimate a capacity-based Phillips curve, stressing the (non-linearity) asymmetry in such a relationship. As a by-product of the estimation procedure, time-varying NAICU rate estimates are obtained. This is of special interest given the ongoing debate on this concept and the scepticism on the reliability of existing NAICU estimates. I start by specifying an expectations-augmented Phillips curve as follows

$$(2.1) \quad \pi_t = \pi_t^e + \gamma_t (C_t - C_t^*) + v_t ,$$

<sup>6</sup>de Kock and Nada-Vicens (1996) revealed that capacity pressures provide a signal about future inflation at the 5 per cent level for Canada, the U.S., Japan and Germany. See also Staiger, Stock and Watson (1997).

<sup>7</sup>Stock and Watson (1999) using univariate time series models conclude that Phillips curves based on capacity utilization outperform, in a forecasting dimension, those based on unemployment. In multivariate time series models, capacity utilization also tends to be among the most important indicators of inflation as Cecchetti (1995) shows. See Corrado and Matthey (1997) for additional arguments.

where  $\pi_t$ ,  $\pi_t^e$ ,  $C_t^*$  and  $C$  are, respectively, the inflation rate, the expected inflation, the NAICU and the observed capacity utilization rate. The error term,  $v_t$  is assumed to be normally distributed with mean zero. Notice that the NAICU is implicitly defined as the capacity rate that makes inflation expectations consistent with observed inflation. In equation (2.1) there are two unobserved variables,  $\pi_t^e$  and  $C_t^*$ . The manner in which these variables are dealt with is now analyzed.

Regarding expected inflation, several approaches have been followed in the literature. In the ‘traditional’ formulation of the Phillips curve, expectations are assumed to be rational and backward-looking, that is,  $\pi_t^e \equiv E_{t-1}\pi_t$ . A simple forecasting structure is considered which implies that

$$(2.2) \quad \pi_t^e = \pi_{t-1}$$

However, it must be pointed out that expectations formation is sensitive to monetary policy inflation and expectations based only on past inflation rates might thus be inappropriate. This argument is at the heart of the New Keynesian paradigm, where the use of forward looking expectations is advocated. Reconciling the new Phillips curve with the data has not been always successful, however. One of the possible explanations is the loss of inertia in inflation when only future inflation is considered. Hence, a plausible correct specification for (2.2) is to use a combination of backward- and forward-looking components, that is,

$$(2.3) \quad \pi_t^e = \lambda E_t \pi_{t+1} + (1 - \lambda) E_{t-1} \pi_t .$$

where the parameter  $\lambda \in (0, 1)$  measures the importance of future and lag inflation in the component of inflation expectations.<sup>8</sup> Taking into account (2.2), the previous equation can be rewritten as

$$(2.4) \quad \pi_t^e = \lambda E_t \pi_{t+1} + (1 - \lambda) \pi_{t-1}$$

The expression for the ‘hybrid’ Phillips curve is obtained after introducing (2.4) into (2.1),

$$(2.5) \quad \pi_t = \lambda E_t \pi_{t+1} + (1 - \lambda) \pi_{t-1} + \gamma_t (C_t - C_t^*) + v_t$$

Notice that this formulation nests the ‘traditional’ specification of the Phillips curve as a special case, when  $\lambda = 0$ . Notice also that both the trade-off parameter  $\gamma_t$  and the NAICU are potentially time varying. A fairly general formulation for this non-observable variable is specified as a random walk<sup>9</sup>

$$(2.6) \quad C_t^* = C_{t-1}^* + \eta_t$$

while the slope coefficient  $\gamma_t$  is assumed to be a linear function of the NAICU gap

$$(2.7) \quad \gamma_t = k_0 + 100k_1 (C_{t-1} - C_{t-1}^*)$$

Under the capacity constraint hypothesis, therefore, the value of  $k_1$  is expected to be positive and significant: when the gap is positive and the economy is booming, prices should go up very rapidly. This argument is illustrated in Chart 1, which represents a (non-linear) asymmetric Phillips curve. When the economy experiences

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<sup>8</sup>This specification of expected inflation is the basis for what Galí and Gertler (1999) and some others authors call a ‘hybrid New Phillips’ curve. The motivation for this approach is largely empirical. Some plausible justification for it can be achieved making reference to the existence of adaptive expectations on part of a subset of price setters.

<sup>9</sup>This assumption is also considered, among others, in Gordon (1997) and Stock (1999).

an excess of supply, that is, when the capacity gap is negative, the response in prices will be moderate. The opposite is likely to happen in the case of excess demand.

**2.1. Econometric Methodology.** Next, I proceed to describe the general econometric methodology that will be used in the estimation of the model consisting of equations (2.5), (2.6) and (2.7). Such a methodology uses the general method of moments (GMM) to estimate the parameters of the model and the Kalman recursion to obtain optimal forecasts of the unobserved components, in this case the NAICU.<sup>10</sup> The Kalman-GMM algorithm is a recursive procedure where in each iteration, given a set of structural parameters, a series of the unobserved variables is generated using the Kalman filter. With this data, a set of orthogonality conditions is constructed and a new set of parameter estimates are obtained applying an appropriate GMM technique consisting of the minimization of a loss function. In what follows, a more detailed description of the econometric approach is presented.

**2.1.1. State-Space Representation.** The model to be estimated consists of the ‘hybrid’ Phillips curve

$$\pi_t = \lambda E_t \pi_{t+1} + (1 - \lambda) \pi_{t-1} + \gamma_t (\mathcal{C}_t - \mathcal{C}_t^*) + v_t$$

with the unobservable NAICU given by the equation

$$\mathcal{C}_t^* = \mathcal{C}_{t-1}^* + \eta_t$$

and where the slope or trade-off parameter is assumed to evolve according to the following equation

$$\gamma_t = k_0 + 100k_1 (\mathcal{C}_{t-1} - \mathcal{C}_{t-1}^*)$$

In order to make the model estimable, a few manipulations have to be made with respect to the forward looking component in the Phillips curve. In particular, the unobserved forecast variable  $E_t \pi_{t+1}$  is replaced by its realized value plus a forecast error term. This yields an equation of the form

$$(2.8) \quad \pi_t = \lambda \pi_{t+1} + (1 - \lambda) \pi_{t-1} + \gamma_t (\mathcal{C}_t - \mathcal{C}_t^*) + \varepsilon_t$$

where the error term,  $\varepsilon_t$ , is a linear combination of the error in the forecast of  $\pi_{t+1}$  and the exogenous disturbance term,  $v_t$ , that is

$$(2.9) \quad \varepsilon_t \equiv v_t + \lambda (E_t \pi_{t+1} - \pi_{t+1})$$

Notice that the terms  $(E_t \pi_{t+1} - \pi_{t+1})$  can be thought of as an inflation surprise. The error term has the following distribution

$$(2.10) \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

It is important to notice the normality assumption made in the distribution function of the disturbance term  $\varepsilon_t$ . The basic derivation of the Kalman algorithm works under this condition. As shown in Hansen and Hodrick (1980), the disturbance term in the class of models considered here has a moving average representation of order  $\kappa - 1$ ,  $\kappa$  being the forecasting horizon. In this particular case,  $\kappa = 0$ , so that the model is effectively normal. A generalization to higher order disturbances

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<sup>10</sup>For a good exposition of state space models and the Kalman algorithm see Chapter 3 of Harvey (1989) and Chapter 14 in Hamilton (1994). This latter reference also covers the GMM methodology.

could be achieved without much difficulty. A more complex task is to consider non-Gaussian disturbances.<sup>11</sup>

The transformed model consists of equations (2.6), (2.7) and (2.8). Since it contains unobserved components, as well as time-varying parameters, it is useful to adopt a state-space representation for it. In particular, the ‘hybrid’ Phillips curve in (2.5) is taken as the *measurement* equation which can be written, in general form, as

$$(2.11) \quad \mathbf{y}_t = \mathbf{Z}_t \alpha_t + \mathbf{d}_t + \varepsilon_t, \quad t = 1, \dots, T$$

where  $\mathbf{y}_t$  is a time series containing  $N$  elements, that in the specific case at hand is  $\mathbf{y}_t = \pi_t$ , while  $\mathbf{Z}_t$  is an  $N \times m$  matrix containing observed explanatory variables,

$$\mathbf{Z}_t = [\pi_{t+1}, \pi_{t-1}, -\gamma_t]$$

$\mathbf{d}_t$  is an  $N \times 1$  vector also of observed variables,  $\varepsilon_t$  is an  $N \times 1$  vector of serially uncorrelated disturbances with mean zero with  $\mathbf{H}$  being its covariance matrix. In this model these elements become

$$\mathbf{d}_t = \gamma_t \mathcal{C}_t$$

The  $N \times 1$  vector  $\alpha_t$  contains elements that are not observable, but are known to be generated by a first-order Markov process,

$$(2.12) \quad \alpha_t = \mathbf{T}_t \alpha_{t-1} + \eta_t, \quad t = 1, \dots, T$$

where  $\mathbf{T}_t$  is an  $m \times m$  matrix and  $\eta_t$  is a  $m \times 1$  vector of serially uncorrelated disturbances with means zero and covariance matrix  $\mathbf{Q}_t$ . The covariance matrix  $\mathbf{Q}_t$  and the transition matrix  $\mathbf{T}_t$  are assumed to be time invariant, so that  $\mathbf{Q}_t = \mathbf{Q}$  and  $\mathbf{T}_t = \mathbf{T}$  for all  $t$ . The error term  $\eta_t$  is assumed to be normally distributed and uncorrelated, in all time periods, with the error in the measurement equation,  $\varepsilon_t$ . Equation (2.12) is the *transition* equation of the state-space system. For the particular model presented above,

$$\alpha_t = [a_{1t}, a_{2t}, \mathcal{C}_t^*]'$$

in which,  $a_{1t} = \lambda$  and  $a_{2t} = 1 - \lambda$  implying that  $\mathbf{Q}$  has all elements zero except the last one in the diagonal, that will be denoted as  $\sigma_\eta^2$ . Hence, the only parameter that really varies with time is the NAICU,  $\mathcal{C}_t^*$ . The transition matrix is assumed to have the following form,

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 \end{bmatrix}$$

from where it follows that the NAICU is modeled, in a parsimonious manner, as a random walk<sup>12</sup>

$$(2.13) \quad \mathcal{C}_t^* = \mathcal{C}_{t-1}^* + \eta_t$$

<sup>11</sup>A more general specification of the error term in this context is considered, for instance, in Kichian (1999). This author analyses the possibility of having ARCH errors in a state-space framework for measuring potential output.

<sup>12</sup>This assumption is also considered, among others, in Staiger et al. (1997), Gordon (1997) and Stock (1999)

The specification of the state-space model is completed with two further assumptions. First, the initial state vector  $\alpha_0$  has a mean of  $\mathbf{a}_0$  and a covariance matrix  $\mathbf{P}_0$ ; and second, the disturbances  $\varepsilon_t$  and  $\eta_t$  are uncorrelated with the initial state. Notice that the system matrices  $\mathbf{Z}_t$ ,  $\mathbf{H}$  and  $\mathbf{Q}$  depend on a set of unknown parameters that have to be estimated. These elements are usually known as hyperparameters and will be denoted by an  $n \times 1$  vector  $\psi_0$ . In the present context, the vector of hyperparameters is

$$(2.14) \quad \psi_0 = \{\lambda, \gamma(k_0, k_1); \sigma_\varepsilon^2, \sigma_\eta^2\}'$$

and will be estimated by GMM. Once the model has been put in state space form, it is possible to apply the Kalman filter, which is a recursive procedure for computing the state vector at time  $t$ , based on the information available at time  $t$ .

**2.1.2. The Kalman-GMM Algorithm.** In order to estimate the model by GMM, it is necessary to obtain the values of the unobserved components described in the transition equation. The Kalman filter takes the hyperparameters  $\psi_0$  as given and produces time-series estimates of the state variables and the error term in the measurement equation. This error term will be used to construct the orthogonality conditions that will be the basis for the GMM estimation.

More specifically, let  $\mathbf{a}_{t-1|t-1}$  denote the optimal estimator of  $\alpha_{t-1}$ , based on the observations up to and including  $\mathbf{y}_{t-1}$ . Let  $\mathbf{P}_{t-1|t-1}$  denote the covariance matrix of the estimated error. Now, given  $\mathbf{a}_{t-1|t-1}$  and  $\mathbf{P}_{t-1|t-1}$ , the optimal estimator of  $\alpha_t$  is given by

$$\mathbf{a}_{t|t-1} = \mathbf{c}_t + \mathbf{T}_t \mathbf{a}_{t-1|t-1}$$

while the covariance matrix of the estimated error is

$$\mathbf{P}_{t|t-1} = \mathbf{T}_t \mathbf{P}_{t-1|t-1} \mathbf{T}_t' + \mathbf{Q}_t, \quad t = 1, \dots, T$$

These two equations are known as the *prediction* equations. Once the new observation,  $\mathbf{y}_t$ , becomes available, the estimator of  $\alpha_t$ ,  $\mathbf{a}_{t|t-1}$ , can be updated. The *updating* equations are:

$$\mathbf{a}_{t|t} = \mathbf{a}_{t|t-1} + \mathbf{P}_{t|t-1} \mathbf{Z}_t' \mathbf{F}_t^{-1} \varepsilon_{t|t-1}$$

and

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1} \mathbf{Z}_t' \mathbf{F}_t^{-1} \mathbf{Z}_t \mathbf{P}_{t|t-1}$$

where  $\varepsilon_{t|t-1}$  can be interpreted as a vector of prediction errors,

$$\varepsilon_{t|t-1} = \mathbf{y}_t - \mathbf{Z}_t \mathbf{a}_{t|t-1} - \mathbf{d}_t$$

having a conditional variance of the following form

$$\mathbf{F}_t = \mathbf{Z}_t \mathbf{P}_{t|t-1} \mathbf{Z}_t' + \mathbf{H}_t, \quad t = 1, \dots, T$$

Next, giving starting values for the conditional mean of the state vector  $\mathbf{a}_0$  and its conditional variance  $\mathbf{P}_0$ , the Kalman filter proceeds iteratively for  $t = 1$  to  $t = T$ , delivering optimal estimators of the state vector as new observations become available. An important aspect of this procedure is the initialization of the algorithm. In this respect, if prior information is available on all the elements of the state vector,  $\alpha_0$ , then it has a proper prior distribution with known mean,  $\mathbf{a}_0$ , and bounded covariance matrix,  $\mathbf{P}_0$ . Once the filter estimates of the state vector  $\alpha_t$  have been computed and the hyperparameters been estimated, it is possible to improve their

*quality* by the procedure of smoothing.<sup>13</sup> The idea of smoothing is to estimate  $\alpha_t$  taking into account the information available after time  $t$ . The smoothed estimator, denoted by  $\mathbf{a}_{t|T}$ , can be obtained by applying the *fixed-interval* algorithm. This procedure consists of a set of recursions which start with the final quantities  $\mathbf{a}_T$ , and  $\mathbf{P}_T$ , given by the Kalman filter, and works backwards. The equations are

$$\mathbf{a}_{t|T} = \mathbf{a}_t + \mathbf{P}_t^* (\mathbf{a}_{t+1|T} - \mathbf{T}_{t+1} \mathbf{a}_t)$$

and

$$\mathbf{P}_{t|T} = \mathbf{P}_t + \mathbf{P}_t^* (\mathbf{P}_{t+1|T} - \mathbf{P}_{t+1|t}) \mathbf{P}_t^{*'}$$

where

$$\mathbf{P}_t^* = \mathbf{P}_t \mathbf{T}_{t+1}' \mathbf{P}_{t+1|t}^{-1}, \quad t = T-1, \dots, 1$$

with  $\mathbf{a}_{T|T} = \mathbf{a}_T$  and  $\mathbf{P}_{T|T} = \mathbf{P}_T$ .

The Kalman algorithm procedure above takes as given the vector of hyperparameters, but these elements have to be estimated. Given the particular nature of the model, an appropriate estimation procedure is the Generalized Method of Moments (GMM). This method assumes that the statistical model implies a set of  $r$  orthogonality conditions of the form

$$(2.15) \quad E \{ \mathbf{h} (\psi_0, \mathbf{w}_{t|t-1}) \} = \mathbf{0}$$

where  $\mathbf{w}_{t|t-1}$  is an  $(h \times 1)$  vector of variables observed and predicted at date  $t$ , the vector of true parameters is  $\psi_0$ , and  $\mathbf{h}(\cdot)$  is a differentiable  $r$ - dimensional vector value function. In the specific case at hand, the orthogonality condition is given by the combination of the disturbance term in (2.8),

$$(2.16) \quad \varepsilon_{t|t-1} = \pi_t - \lambda \pi_{t+1} - (1 - \lambda) \pi_{t-1} - \gamma_t (\mathcal{C}_t - \mathcal{C}_{t|t-1}^*)$$

with an  $r$ - dimensional vector  $\mathbf{u}_t$  of variables (instruments) dated at time  $t$  or earlier that are orthogonal to  $\varepsilon_{t|t-1}$ , the vector of prediction errors and where  $\mathcal{C}_{t|t-1}^*$  is the Kalman-estimate of the NAICU. The orthogonality condition has the following expression

$$E (\varepsilon_{t|t-1} | \mathbf{u}_t) = 0$$

so that the function  $\mathbf{h} (\psi_0, \mathbf{w}_{t|t-1}) = \varepsilon_{t|t-1} \mathbf{u}_t$ , which by equation (2.16) and (2.15) results in

$$(2.17) \quad E \left\{ \left( \pi_t - \lambda \pi_{t+1} - (1 - \lambda) \pi_{t-1} - \gamma_t (\mathcal{C}_t - \mathcal{C}_{t|t-1}^*) \right) \mathbf{u}_t \right\} = 0$$

The next step involves the choice of the instrument set  $\mathbf{u}_t$ . In this case, one could include those variables that help predict changes in the inflation rate. Bivariate Granger-causality tests are performed as a manner to check the forecasting capability of each instrument. Given that the set of instruments will possibly exceed the number of parameters, the model is said to be overidentified. In such a case, the GMM method consists in a two-step nonlinear two-stage least squares procedure that yields consistent estimates of the true vector of parameters  $\psi_0 = \{ \lambda, \gamma (k_0, k_1); \sigma_\varepsilon^2, \sigma_\eta^2 \}'$ . Thus, the GMM estimate  $\hat{\psi}$  is the value of the vector  $\psi$  that minimizes

$$Q (\psi, \mathcal{Y}_{T|T-1}) \equiv [\mathbf{g} (\psi, \mathcal{Y}_{T|T-1})]' \hat{S}_T^{-1} [\mathbf{g} (\psi, \mathcal{Y}_{T|T-1})]$$

<sup>13</sup>The term *quality* in this context refers to obtaining estimates of the state variables with a lower mean squared error.



where  $\mathcal{Y}_{T|T-1} \equiv (\mathbf{w}_{T|T-1}, \mathbf{w}_{T-1|T-2}, \dots, \mathbf{w}_{1|0})'$  is a  $(Th \times 1)$  vector and  $\mathbf{g}(\cdot)$  is the sample mean of  $\mathbf{h}(\cdot)$ , that is,

$$\mathbf{g}(\psi, \mathcal{Y}_{T|T-1}) = \frac{1}{T} \sum_{t=1}^T \mathbf{h}(\psi, \mathbf{w}_{t|t-1})$$

and  $\hat{S}$  is the asymptotic covariance matrix of  $\sqrt{T}\mathbf{g}(\cdot)$ . In the first step,  $\mathbf{S}_T$  is set equal to the identity matrix  $\mathbf{I}_N$ . The resulting estimate  $\hat{\psi}$  is used to construct a better estimate of  $\mathbf{S}_T$ . For the case that is heteroskedastic and serially correlated, one could use the following White's estimate of  $\mathbf{S}_T$

$$\hat{S} = \hat{\Gamma}_{0,T} + \sum_{v=1}^q \{1 - [v/(q+1)]\} (\hat{\Gamma}_{v,T} + \hat{\Gamma}'_{v,T})$$

with

$$\hat{\Gamma}_{v,T} = (1/T) \sum_{t=v+1}^T [\mathbf{h}(\psi, \mathbf{w}_{t|t-1}) \mathbf{h}(\psi, \mathbf{w}_{t-v|t-1-v})]'$$

In order to analyze the overall specification of the model, the use of a simple procedure has been suggested in the GMM literature which consists of checking that the orthogonality conditions are effectively close to zero when evaluated at the estimated parameters  $\hat{\psi}$ . This test is known as test for overidentifying restrictions and the statistic associated with it has asymptotically a  $\chi^2$  distribution with  $(r - a)$  degrees of freedom. The test statistic has the following functional form

$$\left[ \sqrt{T}\mathbf{g}(\hat{\psi}, \mathcal{Y}_{T|T-1}) \right]' \hat{S}_T^{-1} \left[ \sqrt{T}\mathbf{g}(\hat{\psi}, \mathcal{Y}_{T|T-1}) \right] \xrightarrow{L} \chi^2(r - a)$$

The Kalman-GMM is a sequential procedure that starts by making an initial guess as to the numerical values of the unknown parameters,  $\psi_0^{(1)}$ . For these initial numerical values for the population parameters, the matrices  $\mathbf{T}$ ,  $\mathbf{Q}$  and  $\mathbf{H}$  can be constructed from the expressions just given and iterate on the Kalman recursion. The sequences  $\{\mathbf{a}_{t|t-1}\}_{t=1}^T$  and  $\{\mathbf{P}_{t|t-1}\}_{t=1}^T$  resulting from these iterations could then be used to compute the error terms  $\{\varepsilon_{t|t-1}\}_{t=1}^T$  which together with the set of instruments  $\mathbf{u}_t$  allows to define the orthogonality conditions  $\mathbf{h}(\psi_0^{(1)}, \mathbf{w}_{t|t-1})$  and then to obtain the loss function  $Q(\psi_0^{(1)}, \mathcal{Y}_{T|T-1})$ . The procedure continues until this function  $Q(\psi_0^{(j)}, \mathcal{Y}_{T|T-1})$  is minimized and estimates of the hyperparameters of the model,  $\psi_0^{(j)}$ , are thereby found. This completes the description of econometric methodology used in the estimation of the empirical model described in equations (2.5)-(2.7) above.

### 3. ESTIMATION RESULTS AND DISCUSSION

The model consisting of equations (2.5), (2.6) and (2.7) is estimated by means of a procedure that uses the general method of moments (GMM) to estimate the parameters of the model and the Kalman recursion to obtain optimal forecasts of the unobserved components, in this case the NAICU. The estimation uses quarterly data for the economy of the United States. The sample period extends from 1960:Q1 to 2000:Q1. Quarterly inflation at an annual rate was computed from data on the U.S. GDP Implicit Price Deflator according to the following formula:

$400 \cdot \log(p_t/p_{t-1})$ , where  $p_t$  is the price deflator with 1996 as base year, seasonally adjusted and provided by the U.S. Department of Commerce, Bureau of Economic Analysis. Data on the capacity utilization rate, seasonally adjusted, was obtained from the series provided by the Federal Reserve Board. This data refers to the percentage of capacity in the manufacturing sector.

The analysis has several steps. Firstly, a preliminary analysis of the data is performed. The series for inflation and capacity utilization rate are pictured in Figure 1. At first glance, capacity seems to be stationary, whereas inflation displays a more erratic behavior. To confirm this intuition, a stationary analysis is performed for each of the series. Results are presented in the first two rows of Table 1. The null hypothesis of non-stationarity is more clearly rejected for capacity utilization than for inflation, which seems to be weakly stationary. In light of these results, the Phillips curve in equation (2.5) is a relationship between two stationary variables. Nevertheless, the characterization of inflation as a stationary process must be regarded with caution.

Before moving into the estimation of the Phillips Curve, it is worthwhile investigating the behavior of the capacity utilization rate alone it contains valuable information that could be used in the estimation of the Phillips curve. In particular, the presence of non-linearities in the capacity utilization rate is explored using a two-regime threshold autoregressive (TAR) model.<sup>14</sup> Specifically, capacity is assumed to follow a process of the form

$$C_t = (\alpha_0 + \alpha_1 C_{t-1} + \dots + \alpha_p C_{t-p}) 1(q_t \leq \gamma) + (\beta_0 + \beta_1 C_{t-1} + \dots + \beta_p C_{t-p}) 1(q_t > \gamma) + \varepsilon_t$$

where  $1(\cdot)$  denotes the indicator function and  $q_t$  is a known function of the data. The error  $\varepsilon_t$  is a martingale difference sequence with respect to the past history of  $C_t$ . The autoregressive order is  $p \geq 1$  and  $\gamma$  is the threshold parameter. The parameters of interest are estimated by least squares. Importantly, the threshold function that better fits the data is  $q_t = (C_t + C_{t-1})/2$ . The TAR model is tested against a linear specification. The results of these tests are illustrated in Figures 8 and 9. The estimate  $\hat{\gamma} \simeq 0.83$  implies that the TAR model splits the regression function into two regimes. Hence, depending on the past value of the capacity utilization rate, the economy moves from a low capacity regime to a high capacity one. This is illustrated in Figure 10.

Once the basic structure of the empirical model has been vindicated, the strategy of the rest of this section is the following. I first estimate a benchmark model in which, both, the NAICU and the trade-off parameter,  $\gamma_t$ , are constant. These are constraints that will be relaxed in due course. In the benchmark set-up, therefore, any kind of time variation in the parameters is allowed. The equation under consideration is

$$(3.1) \quad E\{(\pi_t - \lambda\pi_{t+1} - (1-\lambda)\pi_{t-1} - \gamma C_t + \gamma C^*) \mathbf{u}_t\} = 0$$

where the vector of hyperparameters to be estimated is  $\psi_0 = \{\lambda, \gamma, C^*\}'$ . The instruments will include all variables that are potentially useful for forecasting inflation. Among these variables are lagged values of inflation, capacity, the long-short term interest rate spread, wage inflation and a measure of productivity. The interest rate spread is computed as the difference between the 10-Year Treasury Rate, at

<sup>14</sup>See Hansen (1997) for a detailed description of TAR models.

constant maturity rates, and the 3-Month Treasury Bill Rate, at auction averages, both series provided by the Federal Reserve Board of Governors. Wage inflation is computed as the annualized rate of growth of quarterly unit labor costs in the nonfarm business sector. The series is seasonally adjusted and provided by the U.S. Department of Labor, Bureau of Labor Statistics. The same source provides a series of seasonally adjusted output per hour in the nonfarm business sector that is used to construct a measure for overall productivity. In order to check the forecasting performance of each instrument, bivariate Granger-causality tests are performed. The results are displayed in Table 2. The capacity utilization rate and wage inflation contain significant information for forecasting inflation. The interest rate spread and the productivity measure also have this capability but to a lesser extent than the other two variables. On the other hand, inflation has a poor forecasting capability with respect to these variables, the exception being wage inflation so that a problem of endogeneity could arise. However, this is not expected to be the case since the order of the magnitude of the causality of inflation with respect to wages is not very high. Equation (??) was estimated using a two step non-linear two stage procedure with four lags of the instruments described above. The results are shown in Table 3. The overall specification of the model is well supported by the results of the test for overidentifying restrictions. The  $\chi^2$  statistic is 15.6 and the 99% critical value with 18 degrees of freedom is 34.8. It is remarkable, furthermore, that estimates for the equilibrium capacity rate and the weighting parameter  $\lambda$  are very significant. The estimate of the NAICU is approximately equal to the average capacity utilization rate of 82% which is the value the Board of Governors assign to the non inflationary rate of capacity utilization. The value of the weighting parameter  $\lambda$  is slightly smaller than the widely used value of 0.5 which is the basis of the ‘sticky’ inflation model of Fuhrer and Moore (1995).

The important point is that the slope coefficient is not very significant, probably because the model does not capture the dynamics of the data in a right way. Hence, the next step is to relax the assumption of constancy in the slope of the ‘hybrid’ Phillips curve (??). To that end, the parameter  $\gamma$  is now considered to be a time-invariant linear function of the lagged NAICU gap.

$$\gamma_t = k_0 + 100k_1 (\mathcal{C}_{t-1} - \mathcal{C}^*)$$

At this stage, the non accelerating inflationary capacity utilization rate is kept constant.

$$E \{ (\pi_t - \lambda\pi_{t+1} - (1 - \lambda)\pi_{t-1} - \gamma_t (\mathcal{C}_t - \mathcal{C}^*)) \mathbf{u}_t \} = 0$$

where the vector of hyperparameters to estimate is  $\psi_0 = \{\lambda, \mathcal{C}^*, \gamma(k_0, k_1)\}'$ . The set of instruments include the second to the fifth lagged value of inflation, capacity, the long-short term interest rate spread, wage inflation and a measure of productivity. The results of the GMM estimation yield a value for the  $\chi^2$  statistic in the overidentifying test of 11.9 which is smaller than 26.2, the 99% critical value with 12 degrees of freedom. Thus, the model is well specified. The estimated coefficients are shown in Table 3 from where it can be seen that the weighting parameter  $\lambda$  and the NAICU are slightly higher than in the previous model, but they are again very significant. The important finding refers to the estimated value of the coefficient  $k_1$  which is positive and significant at the 5 percent level. This suggest that the capacity constraint hypothesis cannot be disregarded as a source of (nonlinearity) asymmetry in the short-run Phillips curve.

The final step is to estimate the model with both the slope and the NAICU being time variant, which is the model given by equations (2.5)-(2.7). Results are shown in Table 5. The Kalman-GMM estimates of the parameter vector  $\psi_0 = \{\lambda, \gamma(k_0, k_1); \sigma_\varepsilon^2, \sigma_\eta^2\}'$  are significant with a  $\chi^2$  statistic in the overidentification test of 12.9 and since the 99% critical value with 13 degrees of freedom is equal to 27.7, the model seems to be well specified. It must be pointed out that the variances of the transition and measurement equation,  $\sigma_\varepsilon^2$  and  $\sigma_\eta^2$  respectively, are not estimated directly, but with an iterative procedure. In Figure 11, the capacity utilization rate and the time-varying NAICU, obtained from the estimation of the Phillips curve, are displayed. It can be inferred that the NAICU has exhibited a low variability in the last two decades, with values ranging from 82 to 83 per cent. This result could complement those obtained by McConnell and Pérez-Quirós (2000), who document a decline in the volatility of real GDP growth in the first quarter of 1984.

The results in this section have two main contributions: first, it responds to the need for a reliable measure of the NAICU put forward by some authors, since the only available measure is the *ad hoc* Federal Reserve Board estimate of 82%; and second, it provides empirical support for the consideration of asymmetries in the design of monetary policy rules by central bankers.

#### 4. CONCLUDING REMARKS AND EXTENSIONS

The goal of this paper has been to shed light into the nature of the monetary transmission mechanism. To that end, the joint dynamics of two key macroeconomic variables, prices and the capacity utilization rate, the latter being a measure of real economic activity, have been studied. To carry out the analysis, an econometric procedure useful for estimating dynamic rational expectation models with unobserved components is developed. The method combines the flexibility of the unobserved components approach based on the Kalman recursion, with the power of the GMM estimation procedure. The method is applied to the estimation of a 'hybrid' Phillips curve that relates inflation to the capacity utilization gap, where the term 'hybrid' means that in the specification of the Phillips curve, both forward and backward looking components are considered. The results have shown that such a relationship is non-linear: the slope of the Phillips curve depends significantly on the magnitude of the capacity gap. This finding has important implications for the conduct of monetary policy and particularly for the design of monetary policy rules. These rules could be evaluated within an appropriate framework and its performance compared to simple feedback rules, such as Taylor-type rules which have been the object of increased attention among monetary policy practitioners and monetary theorists. Taylor rules are characterized by an aggressive response of the interest rate to high inflation and a high output gap. As an alternative to the output gap, one could use the NAICU gap. Indeed, recent studies that analyze the implications of measurement errors for the design of monetary policy, such as Orphanides et al. (1999), show that the results using the capacity utilization rate are more encouraging than those based on the output gap, despite the high correlation between both series. Hence, an interesting extension will be the consideration of a reduced form model in the spirit of Rudebusch and Svensson (1999), where the particularities described here will be considered.

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**TABLE 1: One-sided test of  $H_0$ : Unit root vs.  $H_1$ : Stationary**

Series	Model	Lags	ADF (t - statistic)
Inflation, $\pi_t$	Constant - No Trend	0	-3.00*
Capacity, $C_t$	Constant - No Trend	3	-4.07***
Wage Inflation	Constant - No Trend	1	-4.72***
Spread	Constant - No Trend	2	-3.51*
Productivity	Constant - No Trend	1	-8.08***

Note: Asterisks denote the rejection of the null hypothesis at the (\*) 10%, (\*\*) 5% and (\*\*\*) 1% significance levels. Critical values are taken from Hamilton (1994) Table B.6 page 763, these are -2.57 at 10%, -2.88 at the 5% and -3.46 at the 1% level. The model Constant-No Trend is  $y_t = \xi_1 \Delta y_{t-1} + \xi_2 \Delta y_{t-2} + \dots + \xi_p \Delta y_{t-p} + \alpha + \rho y_{t-1} + u_t$  refers to the Case 2 in Hamilton (1994).

**TABLE 2: Bivariate Granger-Causality Tests:  
 $H_0$ :  $y$  does not Granger-cause  $x$** 

Causality Sense	F-Statistic	Causality Sense	F-Statistic
Capacity-Inflation	7.21***	Inflation-Capacity	1.95
Wages-Inflation	8.23***	Inflation-Wages	2.89**
Productivity-Inflation	3.26**	Inflation-Productivity	1.12
Spread-Inflation	3.75***	Inflation-Spread	1.11

Note: Asterisks denote rejection of the null hypothesis at the (\*\*) 5% and (\*\*\*) 10% significance levels. Critical values are taken from Hamilton (1994) Table B.4 page 760, these are 2.44 at the 5% and 3.47 at the 1% level.. The F statistic has p degrees of freedom in the nominator and (t-2p-1) in the denominator, where p (=4) is the number of lags and t (=156) is the sample size.

**TABLE 3:** ‘Hybrid’ Phillips Curve Constant Slope and Constant NAICU

$$E\{(\pi_t - \lambda\pi_{t+1} - (1 - \lambda)\pi_{t-1} - \gamma\mathcal{C}_t + \gamma\mathcal{C}^*)\mathbf{u}_t\} = 0$$

Variable	Coefficient	Standar Error	T-statistic	P-value
$\lambda$	0.43295	0.0644	6.7238	0.00
$\gamma$	1.4155	1.0515	1.3462	0.08
$\mathcal{C}^*$	0.8283	0.0248	33.333	0.00

**TABLE 4:** Estimates ‘Hybrid’ Phillips Curve Time Varying Slope and Constant NAICU

$$E\{(\pi_t - \lambda\pi_{t+1} - (1 - \lambda)\pi_{t-1} - \gamma_t(\mathcal{C}_t - \mathcal{C}^*))\mathbf{u}_t\} = 0$$

$$\gamma_t = k_0 + 100k_1(\mathcal{C}_{t-1} - \mathcal{C}^*)$$

Variable	Coefficient	Standar Error	T-statistic	P-value
$\lambda$	0.44003	0.007791	5.63294	0.00
$k_0$	8.22689	2.769037	2.99514	0.05
$k_1$	0.92696	0.349778	2.65446	0.05
$\mathcal{C}^*$	0.84971	0.006993	121.8313	0.00

**TABLE 5:** Estimates ‘Hybrid’ Phillips Curve Time Varying Slope and NAICU

$$E\left\{\left(\pi_t - \lambda\pi_{t+1} - (1 - \lambda)\pi_{t-1} - \gamma_t(\mathcal{C}_t - \mathcal{C}_{t|T}^*)\right)\mathbf{u}_t\right\} = 0$$

$$\gamma_t = k_0 + 100k_1(\mathcal{C}_{t-1} - \mathcal{C}_{t-1|T}^*)$$

Variable	Coefficient	Standar Error	T-statistic	P-value
$\lambda$	0.43443	0.761940	5.70172	0.00
$k_0$	7.05163	0.905789	7.78507	0.00
$k_1$	1.16649	0.490279	2.37924	0.00
$\sigma_\varepsilon^2$	0.93472	-	-	-
$\sigma_\eta^2$	0.00025	-	-	-

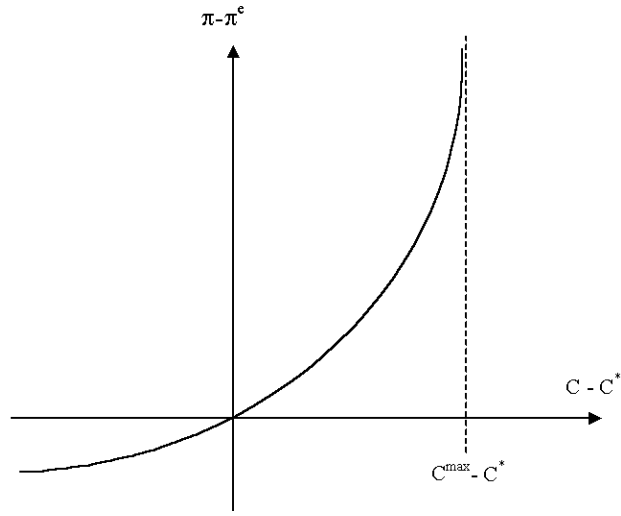
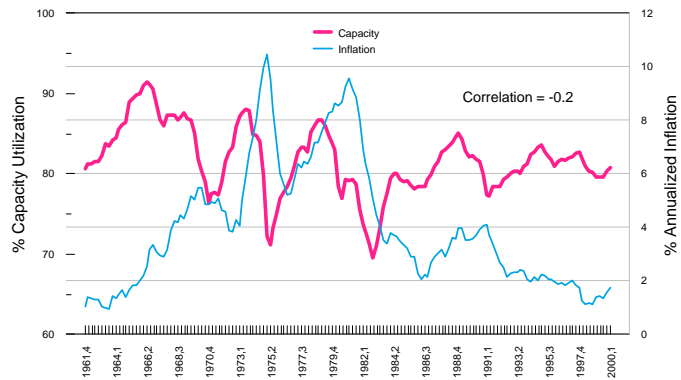


Chart 1: Phillips Curve under Capacity Constraints



**Note:** Inflation is the four-quarter moving average of the annualized quarterly GDP-Deflator  
 Capacity Utilization is the quarter average percent of capacity in manufacturing sector  
**Source:** Board of Governors Federal Reserve System and the U.S. Department of Commerce

Figure 1: Capacity and Inflation



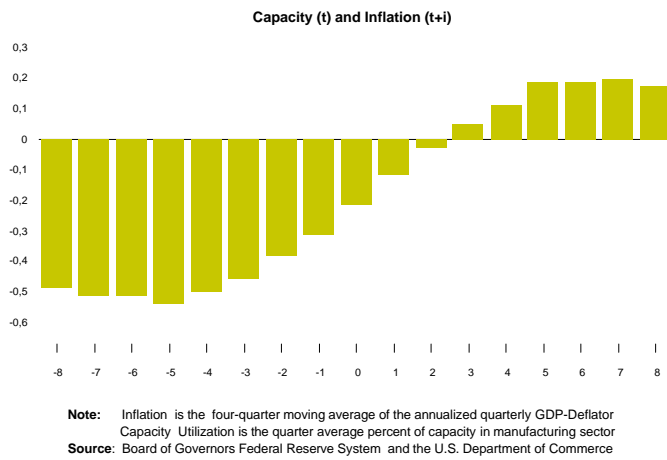


Figure 2: Cross Correlogram Capacity and Inflation

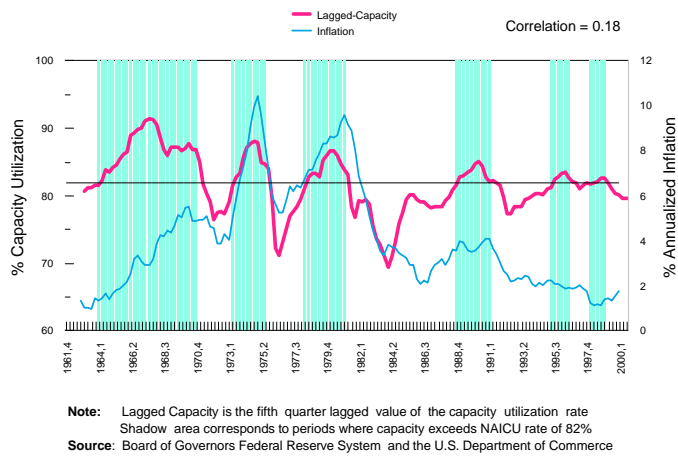


Figure 3: Capacity and Inflation (Historical)

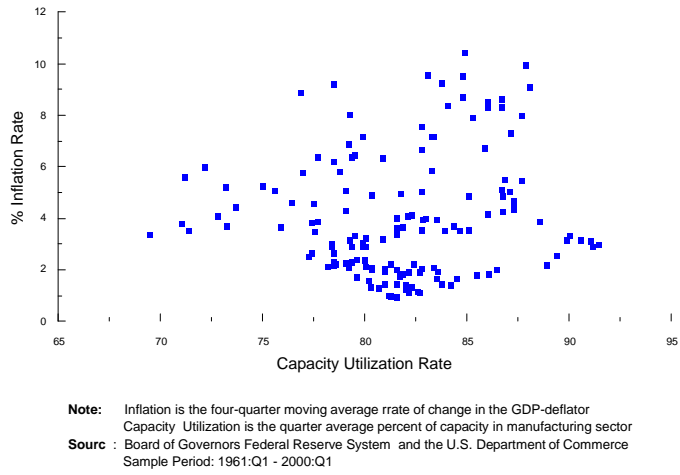


Figure 4: Capacity and Inflation (Scattered)

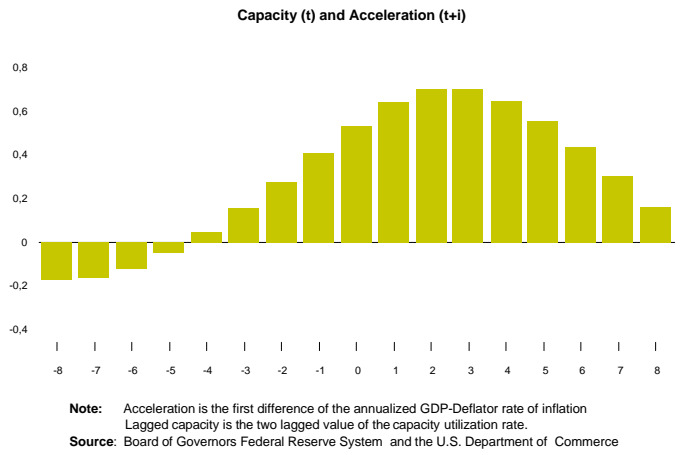


Figure 5: Cross Correlogram Capacity and Acceleration

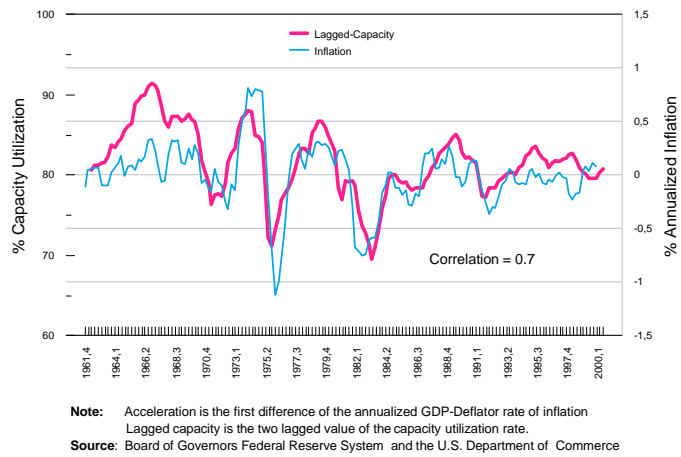


Figure 6: Capacity and Acceleration (Historical)

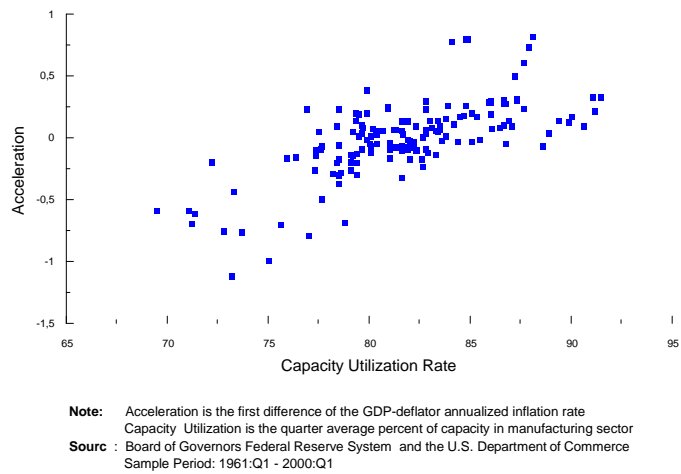


Figure 7: Capacity and Acceleration (Scattered)

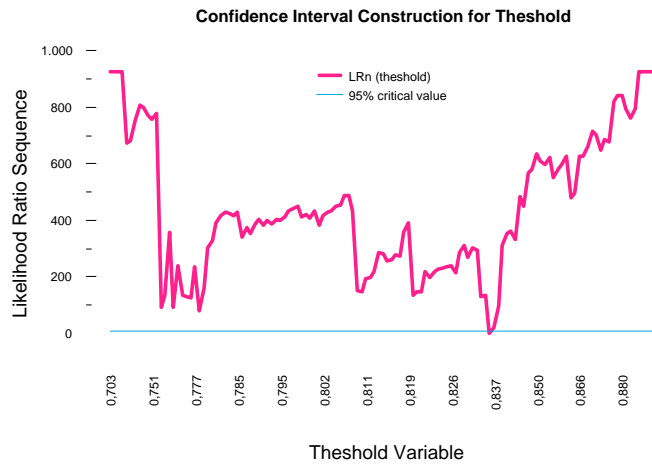
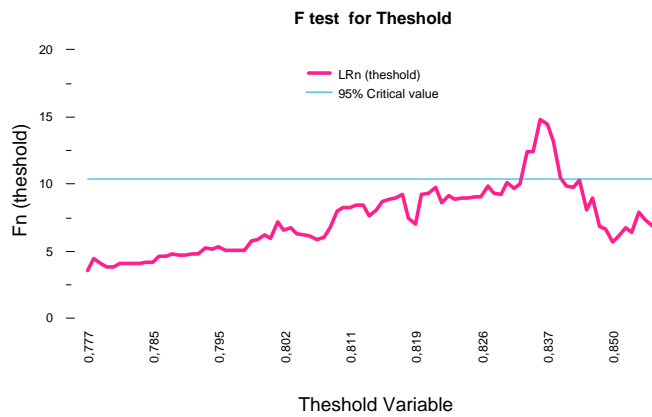


Figure 8: Confidence Interval Construction for Threshold



**Note:** Reject the null H0: Linearity if F sequence exceeds Critical Value

Figure 9: F test for Threshold

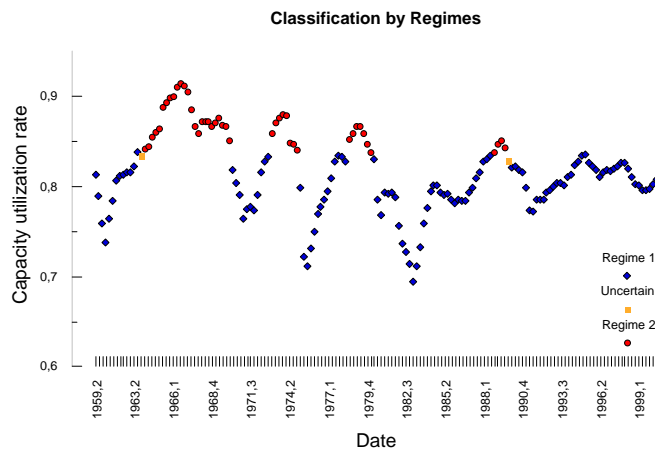


Figure 10: Classification By Regimes

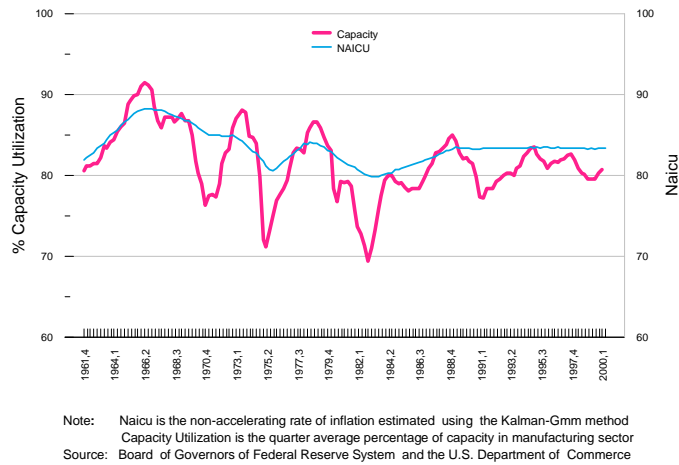


Figure 11: Capacity and Time Varying Naicu

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*E-mail address:* [palvarez@idea.uab.es](mailto:palvarez@idea.uab.es)

*URL:* <http://idea.uab.es/~palvarez/>