Asympotic Capacity Bounds for Ad-hoc Networks Revisited: The Directional and Smart Antenna Cases

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Abstract— **Directional and smart antennas can be useful in increasing the capacity of wireless ad hoc networks. A number of media access and routing protocols have been recently proposed for use with such antennas, and have shown significant performance improvements over the omni-directional case. However, it is important to explore if and how different directional and smart antenna designs affect the asymptotic capacity bounds, derived by Kumar and Gupta [11]. These bounds are inherent to specific ad-hoc network characteristics, like the shared nature of the wireless media and multi-hop connectivity, and may pose major scalability limitations for such networks. In this paper, we look into how directional and smart antennas can affect the asymptotic behavior of an ad-hoc network's capacity. Specifically, we perform a capacity analysis for an ideal *flat-topped* antenna, a linear *phased-array* antenna, and a *fully adaptive array* antenna model. Finally, we explain how an ad-hoc network designer can manipulate different antenna parameters to mitigate the scalability problem of ad-hoc networks.**

Keywords-component; — capacity; ad-hoc; directional antennas; smart antennas;

I. INTRODUCTION

Wireless ad-hoc networks are multi-hop networks where all nodes cooperatively maintain network connectivity. The ability to be set up fast and operate without the need of any wired infrastructure (e.g. base stations, routers, etc.) makes them a promising candidate for military, disaster relief, and law enforcement applications. Furthermore, the growing interest in sensor network applications has created a need for protocols and algorithms for large-scale self-organizing ad-hoc networks, consisting of hundreds or thousands of nodes.

An important characteristic of a wireless ad-hoc network (or any other network) is its capacity. A number of recent papers have explored several issues related to the asymptotic capacity of ad-hoc networks using omni-directional [11] [13] [14] [7] and directional or smart antennas [2] [12] [16] [17]. Additionally, many protocols have been proposed for use with nodes equipped with directional or smart antennas, so as to increase network throughput [2] [3] [8] [9] [10] [20]. In many of these papers specific assumptions are being made, in terms of the technologies and protocols used by nodes, in order to model current practice in ad-hoc networks. Therefore, the respective capacity analysis and throughput results may be technology-dependent, and hold mainly for the specific scenarios modeled. In a more recent work, Xie and Kumar take a more information-theoretic approach, in order to derive scaling laws for the capacity of wireless networks, that hold regardless of specific technologies and protocols used [23].

Probably the most well-known scaling law for the capacity of ad-hoc networks is given by Kumar and Gupta [11]. In this work, the authors prove that in a multi-hop wireless network, where nodes are randomly placed on a planar disk and each node chooses a destination node at random, the capacity available to each node, for its own traffic, decreases as a function of $O(1/\sqrt{n \log n})$, n being the total number of nodes in the network. It is a scaling law that stems from two fundamental characteristics of ad-hoc networks, namely their multi-hop nature and the need for nodes to compete for the shared wireless media. The former implies that for each packet generated in the network, a growing number of intermediate nodes need to be involved in forwarding that packet from sender to destination, with increasing n, creating a higher per node overhead. The latter means that there is a restriction on the number of simultaneous transmissions at any time. The important and somewhat discouraging implication of this result, as the authors themselves note, is that there is an inherent scalability limitation for ad-hoc networks that should make designers focus on designing only small networks.

In order to overcome this apparent shortcoming, and increase the throughput available to each node as n grows large, the ideal case being a per node throughput of O(1), researchers have tried to mitigate the effect of each of the two aforementioned characteristics of ad-hoc networks. Specifically, in [7] [14], the authors are assuming that a fraction of the nodes are mobile, and use those nodes as relays, in order to reduce the number of hops from source to destination to a constant. An O(1) throughput is achieved in [14], but without any delay guarantees, while in [7] delay guarantees are provided for only a poly-logarithmic degradation in throughput. On the other hand, in order to reduce the interference caused by each transmission and, consequently, increase the number of simultaneous transmissions, the use of directional antennas and/or multiple packet reception (MPR) have been explored [16] [17].

In this paper we present how the use of directional and smart antennas can affect the asymptotic behavior of ad-hoc network capacity and improve their scalability. Our approach is similar to the one taken in [16], in that our analysis tries to incorporate appropriate directional antenna model parameters

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into the basic analysis of [11]. However, in addition to the ideal directional antenna models used in [16], we perform our asymptotic capacity analysis for a practical antenna model (*linear phased array*), as well as a *fully adaptive array* antenna model that has much greater capabilities in dealing with interference. Finally, we have incorporated both protocol-based and interference-based models in our capacity analysis, in order to decide on the success of a transmission, making the results more general.

In Section II, we discuss the directional and smart antenna models that we've used for our analysis. In Section III, we derive relations between the order of capacity growth and specific antenna parameters, like number of antenna elements, beam-width and normalized side-lobe gain. We use these relations in Section IV, in order to explore how simple antenna parameter manipulation can allow the ad-hoc network designer to improve the scaling law order of ad-hoc networks. Finally, we conclude our paper in Section V.

II. ANTENNA MODELS

A. Directional Antennas

Directional antennas have the ability to *steer* their main beam towards an arbitrary direction, in either or both azimuth and elevation plane, mechanically or electronically. The latter case is the one of most interest for the designer of ad-hoc networks, since mechanical movement consumes unacceptably large energy amounts to be applicable to the battery-constrained ad-hoc nodes. An example of an electronically steerable antenna is the *phased array antenna*, consisting of N antenna elements (e.g. linear or circular array) and whose antenna pattern can be directed by changing the relevant phases of its antenna elements.

Directional antennas are often modeled in the ad-hoc networks literature using an ideal, 2-D or 3-D, model, usually referred to as the *flat-topped* antenna model, as shown in Fig.1. This model, albeit quite simplistic, can provide valuable insight on how the directional antenna characteristics affect the capacity of an ad-hoc network. In addition to the *flat-topped* antenna, we will use in our analysis a simple phased array antenna model [18], in order to more accurately model real-world antenna systems. Its antenna pattern is also depicted in Fig. 1.

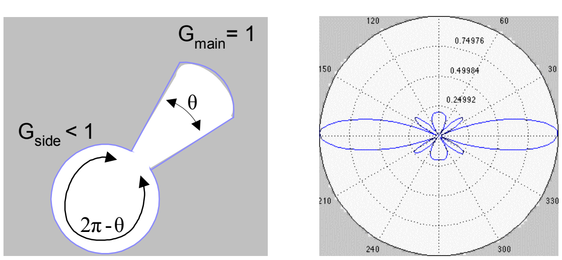

Figure 1. Normalized flat-topped antenna pattern (left) and linear array antenna pattern (right)

B. Smart Antenna Models

In this work, we are also interested in *fully adaptive array* antennas. Such antennas have the ability to automatically respond to an unknown interference environment, in real time, by steering nulls and reducing side lobe levels in the direction of the interference, while retaining some desired signal beam characteristics. This process is sometimes called *beamforming*. A large number of alternative beamforming designs (e.g. digital, microwave, aerial beamforming) and algorithms (e.g. Least Mean Square, Constant Modulus Algorithm, etc.) have been proposed in literature, a detailed tutorial of which can be found in [1].

In the past, adaptive array antennas had only been considered for the use on base station in cellular systems, due to their large size, high cost, considerable power consumption, and complexity of design. However, recently there have been proposed simple, analog, smart antenna designs [4] [5] that are low cost and energy-efficient enough to be used on wireless terminals. They're based on the concept of *aerial beamforming* and prototype antennas have been built and tested [6].

An adaptive array antenna consisting of N elements is said to have *N-1 degrees of freedom*. Without further details on how this is achieved, this roughly implies that such an antenna can independently track one node of interest and cancel $N-2$ non-coherent interferers. In our subsequent analysis we assume that a smart antenna of N elements can turn its main beam of gain $G_{max} = 1$ to an arbitrary direction while creating nulls of gain $G_{null} << 1$ towards at most $N-2$ different directions[1].

C. Protocol and Inteference Model

As mentioned earlier, the need for all nodes to share the common wireless media implies that there is a limit on the number of simultaneous transmissions that can successfully occur at any time. This limit could be dictated by some media access control (MAC) protocol. Such a protocol spaces concurrent transmissions far enough from each other, so as to guarantee avoidance of most or all collisions (e.g. CSMA/CA). Alternatively, this limit may be imposed by the physical properties of the media. Specifically, one could assume that any set of simultaneous transmissions is permissible, as long as the SINR (Signal to Interference and Noise Ratio) at each receiver is above a specific threshold β. The above two models, were first introduced in [11] for analyzing the capacity of wireless networks, when omni-directional antennas are used. We adapt these models for the cases of directional and smart antennas. We also assume a two-ray ground propagation model [22] and let P be the common transmitting power of all nodes, P_{th} the receiving power threshold and h the antenna height. We present asymptotic capacity results for three representative cases:

Case 1) Directional Antennas & Protocol Model:

We assume that all nodes are equipped with an ideal *flat-topped* directional antenna and implement a directional version of the 802.11 protocol. This protocol acquires the *floor* for a transmission by sending RTS and CTS packets directionally,

[1] In order to account for inherent inaccuracy in the algorithm, random noise and other propagation phenomena, we assume that the gain G_{null} at the direction of nulls is not zero, but instead has a finite, albeit much lower than G_{max}, value.

while also performing *directional virtual carrier* sensing [8] [9]. It establishes a *silence region* around any receiving node as follows [12]: If a node X_j is receiving a transmission from some angle φ (relevant to a reference angle), then:

1.a) No other node within a range R_1 and within an angle [$\varphi - \vartheta/2$, $\varphi + \vartheta/2$] from X_j can be receiving at the same time from any direction, where R_1 is given by

$$R_1 = \left((P/P_{th})h^2 G_{side}\right)^{\frac{1}{4}} \quad (1)$$

1.b) No other node within a range R_2 and within an angle [$\varphi + \theta/2$, $\varphi + 2\pi - \theta$] from X_j can be receiving at the same time from any direction, where R_2 is given by

$$R_2 = \left((P/P_{th})h^2 G_{side}^2\right)^{\frac{1}{4}} \quad (2)$$

Case 2) Directional Antennas & Interference Model:

All nodes are assumed to be equipped with a linear phased array antenna consisting of N elements, and choose a common transmitting power P. Let $\{X_k: k \in T\}$ be the subset of nodes simultaneously transmitting at some time instant. A transmission from a node X_i, $i \in T$, is successfully received by node X_j if

$$SINR_j = \frac{\dfrac{P}{\left|X_i - X_j\right|^4}}{N + \sum\limits_{\substack{k \in T \\ k \neq i}} \dfrac{P * \overline{G}_I}{\left|X_k - X_j\right|^4}} \geq \beta \quad (3)$$

$\overline{G}_I$ is the average receiving antenna gain for a random interferer. In the case of the (*edge-fire*) linear array antenna is given by

$$\overline{G}_I = \frac{1}{\pi}\int_0^{\pi}\left|\frac{\sin(0.25N\pi(\cos\theta - 1))}{N\sin(0.25\pi(\cos\theta - 1))}\right| d\theta \quad (4)$$

The above integral cannot be solved in a closed form. Table 1 contains its value for different numbers of elements N.

TABLE I. AVERAGE INTERFERENCE GAIN AS A FUNCTION OF N

N	3	4	5	6	7	8	9	10
G_I	0.536	0.433	0.406	0.355	0.340	0.308	0.298	0.275

Case 3) Smart Antenna & Protocol Model:

All nodes are assumed to be equipped with a fully adaptive array antenna of N elements. A media access protocol resolves simultaneous transmission request, such that within a range R from any receiving node, at most N-2 other nodes may be receiving at the same time. R is given by

$$R = \left((P/P_{th})h^2\right)^{\frac{1}{4}} \quad (5)$$

III. ASYMPTOTIC CAPACITY ANALYSIS

In this section we present asymptotic capacity laws for the three scenarios outlined in section 2. All proofs are based on the original capacity analysis by Kumar and Gupta [11], which we modify to incorporate appropriate antenna parameters into the equations. Due to space limitations, we only present the proof for case 1 in the Appendix. Furthermore, we are mainly concerned with the asymptotic behavior of the capacity equations. Therefore, all linear scaling factors, besides antenna parameters of interest, are captured in appropriate constants c_1, c_2, and c_3. We summarize here our assumptions:

- There are n nodes randomly distributed on a planar disk of unit area. If the size of the disk is A m^2, instead, then all results need to be scaled by $\sqrt{A}$, as explained in [11].
- Each node randomly picks a destination node for its traffic. The average distance $\overline{L}$ between sender-destination pairs is O(1).
- The network transports λnT bits over a period of T seconds, where λ denotes the average transmission rate for each node to its destination over a period T.
- For simplicity, we assume that there is only a single wireless channel of capacity W bits/sec, available to all nodes. All results hold also for the case of multiple channels, whose aggregate capacity is equal to W.

Case 1) Directional Antennas & Protocol Model:

In this case, the average rate sustainable by the network is bounded by two factors. First, each packet generated by a node will have to be carried over at least $\overline{L}/R$ hops, on the average. This imposes an aggregate load of $\lambda(n)n\overline{L}/R$ packets/sec on the network. Second, each receiving node establishes a *silence region* within which no other node can be active. The aggregate area of all such *disjoint* silence regions cannot exceed the total area of the planar disk. Based on these two conditions and assuming a *flat-topped* antenna model, we derive an upper bound for the (end-to-end) average sustainable transmission rate λ for each node, as follows:

$$\lambda(n) \leq \frac{c_1 W}{\sqrt{n\log n}} \frac{1}{\theta G_{side} + (2\pi - \theta)G_{side}^2} \quad (6)$$

Case 2) Directional Antennas & Interference Model:

When the interference model is used instead, λ is limited by the need for each receiving node to be able to decode the intended signal from incoming noise and interference from multiple nodes. As shown in section 2, a successfully received transmission implies an SINR that is higher than the receiver's threshold β. Assuming all nodes are using an N-element linear phased array antenna, the average sustainable transmission rate λ for each node, is bounded above as follows:

$$\lambda(n) \leq \frac{c_2 W}{\sqrt{n}}\left(\frac{1}{\beta \overline{G}_I}\right)^{\frac{1}{4}} \quad (7)$$

Case 3) Smart Antennas & Protocol Model:

When smart antennas are used on each node, the analysis is the same as the original one for the omni-directional antenna case [11], with only the following difference: Each receiving node creates a silence region of disk shape around it. However, up to *N-2* additional nodes in that disk may be receiving simultaneously. Hence, the resulting bound for λ is scaled by a factor proportional to *N-2* as follows:

$$\lambda(\mathrm{n}) \leq \frac{\mathrm{c}_3 \mathrm{W}}{\sqrt{\mathrm{nlogn}}}(\mathrm{N} - 2) \tag{8}$$

IV. IMRPOVING THE SCALING LAWS

As we can see by equations (6), (7), and (8), we have expressed the asymptotic capacity bounds for all three cases, as functions of different antenna parameters, like the number of elements, antenna gain and beamwidth. The importance of those results is easier seen from an ad-hoc network designer's perspective. Let us view all relevant antenna parameters as different functions of *n,* namely $N(n)$, $\overline{G}_I$ (n), $G_{side}(n)$ and $\theta(n)$, where *n* is the number of nodes. This does not necessarily mean that we assume antennas can dynamically modify their parameters. It merely implies that the designer can make its choice of directional or smart antenna parameters to be used on nodes, based on the expected scale of the ad-hoc network. For example, if a designer chooses to scale the number of elements *N* in a smart antenna, as a function of $\Theta\left(\sqrt{logn}\right)$ it would improve the scaling order of $\lambda(n)$ (see Equation 8) from $O\left(1/\sqrt{nlogn}\right)$ to $O\left(1/\sqrt{n}\right)$. This allows an asymptotically increased number of nodes in the network to sustain a specific per node transmission rate.

Of course, one should be aware that antenna parameters like number of elements, gain, and beam-width cannot be increased at will. This could require technologies or designs that would be conflicting with the constraint for simple, inexpensive, low-energy antennas for wireless terminals. Therefore, it is quite interesting to see how feasible different scaling requirements are, for the different antenna models we've assumed. We will do so, through two examples.

Let us consider the previous example of scaling the number of elements *N* in a smart antenna, as a function of $\Theta\left(\sqrt{logn}\right)$. We already saw how this approach affects the scalability of the network. Now we examine what this requirement implies in practice, for *N*. Let's assume that the scale of the ad-hoc network changes from n_1 to n_2 nodes. The relative increase in the number of antenna elements is given by $N_r = \sqrt{logn_2/logn_1)}$ and its value is shown in Table.2 for different values of n_1 and n_2. As we can see, the relative increase is small enough to be feasible for practical smart antennas. Finally, note that the relative increase in *N* per order of magnitude growth in network size becomes smaller for larger networks.

TABLE II. RELATIVE INCREASE IN NUMBER OF ELEMENTS (EXAMPLE 1)

$\mathbf{n_1}$	10	100	1000
$\mathbf{n_2}$	100	1000	10000
$\mathbf{N_r}$	1.414	1.228	1.155

As a second example, let's consider the case of the linear phased-array antenna (i.e. case 2), whose asymptotic capacity is given by (7). If we would like to improve the scaling order of $\lambda(n)$ by a factor of $\sqrt{logn}$, as in the previous example, then it follows from (7) that we need $\left(\overline{G}_I(n)\right)^{-\frac{1}{4}} \cong \Theta\left(\sqrt{logn}\right)$. Based on equation (4), we show in Table 3, how this requirement translates into necessary increase of the number of elements *N* in the array. We assume that we start with n_o nodes in the network, and that each node is using a 3-elemement linear phased-array antenna. We calculate the approximate new number of elements N_{new} necessary when the size of the network grows by one order of magnitude (x10), so as to obey the $\left(\overline{G}_I(n)\right)^{-\frac{1}{4}} \cong \Theta\left(\sqrt{logn}\right)$ requirement.

TABLE III. NUMBER OF NECESSARY ELEMENTS (EXAMPLE 2)

$\mathbf{n_o}$	10	100	1000
$\mathbf{N_{new}}$	N.F.	13	9

The N.F. in Table.3 stands for "Not Feasible", that is, one cannot achieve the required increase in $\left(\overline{G}_I(n)\right)^{-\frac{1}{4}}$ by just using a higher number of elements in a linear phased array, when going from 10 nodes to 100 nodes in an ad-hoc network. This confirms our previous argument, that it is not always possible to arbitrarily improve the asymptotic capacity bounds by manipulating the antenna parameters.

V. CONCLUSIONS

In this paper we have analyzed how the use of directional and smart antennas affects the asymptotic capacity behavior of wireless ad-hoc networks. We performed our analysis for an ideal *flat-topped* antenna model, as well as two realistic antenna models, namely a phased-array antenna, and a fully adaptive array antenna. We used two different models for the access to the wireless media, namely the Physical and Interference model, and combined them with the above three antenna models to derive asymptotic capacity equations that incorporate appropriate antenna parameters. Finally, we have shown how the use of directional and smart antennas can alleviate the intrinsic scalability limitations of wireless ad-hoc networks.

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APPENDIX

We present the proof for equation (6), namely the upper bound for the average sustainable transmission rate λ, when the *protocol model* is assumed, for nodes using flat-topped directional antennas as defined in Section 2. Our proof follows steps similar to the proof in [11].

Let X_j be a node that is successfully receiving a transmission from some other node X_i. This implies that a silence region defined by (1) and (2) has been established by the directional MAC protocol, around X_j. The area of this region is given by

$$A_{SR} = \frac{\theta R_1^2(G_{side},\theta) + (2\pi-\theta)R_2^2(G_{side},\theta)}{2} \tag{A.1}$$

Silence regions are disjoint up to a factor of c_{11}, where $0<c_{11}<1$. Hence, at least an area of size $c_{11}A_{SR}$ per receiving node does not overlap with any other such area. Consequently, the number of simultaneous transmissions on the single wireless channel, at any time is no more than

$$\frac{1}{c_{11}A_{SR}} = \frac{c_{12}}{\theta R_1^2(G_{side},\theta) + (2\pi-\theta)R_2^2(G_{side},\theta)} \tag{A.2}$$

where $c_{12} = 2/c_{11}$.

Now, assume $\overline{L}$ is the average distance between randomly chosen sender destination pairs and let $R(n)$ be the maximum range up to which a node can successfully receive a transmission, assuming there is neither noise nor interference. Then, each packet will have to traverse at least $\overline{L}/R(n)$ hops. This results in an aggregate traffic load of

$$\frac{\lambda(n) n\overline{L}}{R(n)} \tag{A.3}$$

To ensure that all the offered traffic load can be carried by the network, it is necessary that

$$\frac{\lambda(n) n\overline{L}}{R(n)} \le \frac{c_{12}W}{\theta R_1^2(G_{side},\theta) + (2\pi-\theta)R_2^2(G_{side},\theta)} \tag{A.4}$$

From (A.4) we can derive the upper bound for the average sustainable transmission rate $\lambda(n)$ per node as

$$\lambda(n) \le \frac{c_{12}WR(n)}{n\overline{L}\left[\theta R_1^2(G_{side},\theta) + (2\pi-\theta)R_2^2(G_{side},\theta)\right]} \tag{A.5}$$

For the normalized antenna pattern of the flat-topped antenna model, and the two-ray ground propagation model, $R(n)$ is given by (5). Additionally, it has been shown in [21] that, in order to guarantee connectivity in the ad-hoc network the transmission power P has to be high enough such that $R(n) > \sqrt{logn/(\pi n)}$. Combining these with (A.5) gives us equation (6)

$$\lambda(n) \le \frac{c_1 W}{\sqrt{nlogn}} \frac{1}{\theta G_{side} + (2\pi-\theta)G_{side}^2}$$

where $c_1 = c_{12}/\overline{L}$, since $\overline{L} = O(1)$ ■