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# Asymptotic Approaches in Nonlinear Dynamics

New Trends and Applications

With 58 Figures



Springer

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