Progress of Theoretical Physics, Vol. 59, No. 2, February 1978

Asymptotic Completeness and Confinement in the Massive Schwinger Model

Noboru NAKANISHI

Research Institute for Mathematical Sciences
Kyoto University, Kyoto 606

(Received October 18, 1977)

Qualitative discussions are made on the operator solution of the massive Schwinger model in covariant gauges under the postulate of asymptotic completeness. It is shown that the confinement of the spinor field is compatible with asymptotic completeness; this is possible because gauge invariance is spontaneously broken and the longitudinal photon becomes the Goldstone boson.

It is pointed out that the equivalent boson theory of the massive Schwinger model does not involve the new parameter θ introduced by Coleman, Jackiw and Susskind.

§ 1. Introduction and summary

Asymptotic completeness is one of the fundamental principles in quantum field theory. It is well known that asymptotic completeness does not follow from other basic axioms. This postulate is very important because of the following reasons.

1. Generally, field equations and equal-time commutation relations do not uniquely specify the solution of quantum field theory; they admit various representations of field operators. The postulate of asymptotic completeness implies that the representation is usually described by the Fock space generated by asymptotic fields and hence it is irreducible.

2. Asymptotic completeness provides the basis of particle interpretation of quantum field theory. It excludes pathological theories such as generalized free fields.

3. In the Heisenberg picture, the unitarity of the (physical) S-matrix can be established on the basis of asymptotic completeness. This point is particularly important in the manifestly covariant formulation of gauge theory.

Asymptotic completeness is stated in two different ways:

(a) The totality of asymptotic states is complete.

(b) Any well-defined operator is expressible in terms of asymptotic fields.

Both statements are equivalent to each other if all commutators between asymptotic fields are c-numbers.*) In this case, any operator has the Glaser-Lehmann-Zimmermann (GLZ) expansion,[2] which is a power-series expansion in asymptotic fields written in the normal-product form.

*) This property can be proved under certain general assumptions.[1]

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As we have pointed out recently,[3] in various solvable two-dimensional models, the widely accepted solutions do not meet the requirement of asymptotic completeness, because the spinor field ψ is *confined*, that is, it does not have the corresponding asymptotic field. We have, therefore, proposed new solutions expressed in terms of asymptotic fields alone in the Thirring model,[3),4)] in the Schwinger model,[3),4)] in the Schroer model,[5] and in the pre-Schwinger model.[6] In all those models, we have found that gauge invariance (of the first kind) is *spontaneously broken*. The confinement of ψ and spontaneous breakdown of gauge invariance are intimately related under the postulate of asymptotic completeness. Indeed, since the spinor field ψ, which is the only carrier of the charge Q, has no corresponding asymptotic field, if there were no Goldstone boson, Q could not be expressed in terms of asymptotic fields in contradiction with asymptotic completeness. But confinement is compatible with asymptotic completeness if gauge invariance is spontaneously broken. Such phenomena cannot take place neither in the Federbush model nor in the massive Thirring model because they involve no massless particles.

Now, the purpose of the present paper is to analyze the massive Schwinger model, i.e., the two-dimensional quantum electrodynamics, under the postulate of asymptotic completeness. Of course, this model is not exactly solvable, but it is particularly interesting because Coleman, Jackiw and Susskind (CJS)[7] showed in an indirect way that the spinor field ψ is confined. In § 2, we make some qualitative considerations on the expressions for Heisenberg fields in terms of asymptotic fields. It is found there that gauge invariance is spontaneously broken as in the other models in which ψ is confined and that the Goldstone boson is nothing but the longitudinal photon. Furthermore, though ψ is certainly confined, we point out that there must instead exist a *neutral* spinor asymptotic field.

In § 3, we derive the massive sine-Gordon theory from the massive Schwinger model on the basis of our asymptotic-field solution to the (massless) Schwinger model.[3] Though the massive sine-Gordon theory derived by CJS contains a new phase parameter, θ, independent of the mass and the coupling constant, our result contains no such extra parameter. This discrepancy originates from the fact that CJS's analysis is based on the Lowenstein-Swieca (LS) solution[8] to the (massless) Schwinger model. As has been clarified in detail,[9] compared with the asymptotic-field solution, the LS one contains some extra degrees of freedom introduced artificially, from which CJS's parameter θ arises when the mass term is added. We thus argue that the parameter θ is not intrinsic to the massive Schwinger model itself.

In the Appendix, we calculate the second-order self-energy part of ψ and find that it is badly infrared divergent on the mass shell. If the massive Schwinger model is a sensible theory, this fact also supports the confinement of ψ.

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§ 2. Qualitative form of the solution

The Lagrangian density of the massive Schwinger model in covariant gauges is given by

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + B\partial^\mu A_\mu + \tfrac{1}{2}\alpha B^2 - \bar{\psi}(-i\gamma^\mu\partial_\mu + M_0)\psi$$
$$-e\bar{\psi}\gamma^\mu\psi A_\mu\,, \tag{2·1}$$

where ψ is a two-component spinor field with a bare mass $M_0>0$ ($\bar{\psi}=\psi^\dagger\gamma^0$), A_μ a two-dimensional massless vector (or electromagnetic) field ($F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$), B an auxiliary scalar field, e (>0) a coupling constant and α a gauge parameter.

The field equations derived from (2·1) are

$$(-i\gamma^\mu\partial_\mu + M_0 + e\gamma^\mu A_\mu)\psi = 0\,, \tag{2·2}$$

$$\partial^\nu F_{\nu\mu} - \partial_\mu B = j_\mu\,, \tag{2·3}$$

$$\partial^\mu A_\mu + \alpha B = 0\,, \tag{2·4}$$

where j_μ denotes the current which is formally defined by $e\bar{\psi}\gamma_\mu\psi$. Since $\partial^\mu j_\mu = 0$, (2·3) yields

$$\Box B = 0\,. \tag{2·5}$$

Canonical quantization (The canonical conjugates of A_0 and A_1 are B and F_{01}, respectively.) can be carried out consistently.[10] Since B satisfies a free field equation, we can easily show that

$$[A_\mu(x), B(y)] = -i\partial_\mu{}^x D(x-y)\,, \tag{2·6}$$

$$[B(x), B(y)] = 0\,, \tag{2·7}$$

where

$$D(\xi) \equiv -\tfrac{1}{2}\epsilon(\xi^0)\theta(\xi^2)\,. \tag{2·8}$$

As before,[11),3)] the positive-frequency part of B is defined by

$$B^{(+)}(x) \equiv -i\int dz^1 D^{(+)}(x-z)\overleftrightarrow{\partial}_0{}^z B(z)\,, \tag{2·9}$$

where $f\overleftrightarrow{\partial}g \equiv (\partial f)g - f\partial g$ and

$$D^{(+)}(\xi) \equiv -(4\pi)^{-1}\log(-\mu^2\xi^2 + i0\xi^0)\,, \tag{2·10}$$

μ being the infrared cutoff introduced by Klaiber.[12] The physical states $|\text{phys}\rangle$ are defined by the constraint

$$B^{(+)}(x)|\text{phys}\rangle = 0\,. \tag{2·11}$$

Because of (2·5) B is an asymptotic field, and consequently the true vacuum $|0\rangle$ is a physical state as it should be.

The corrected version[13] of Johnson's theorem[14] states that an abelian gauge

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field A_μ remains massless if it interacts with other fields minimally and if there are no massless physical states when a small mass term $\frac{1}{2}m_0{}^2A^\mu A_\mu$ is fictitiously added to the Lagrangian density. Accordingly, A_μ should have a massless asymptotic field in the massive Schwinger model. That A_μ remains massless was demonstrated also by CJS by showing the presence of the two-dimensional Coulomb potential.

Since A_μ is a *two-dimensional* vector field, it can be expressed as

$$A_\mu = \partial_\mu W - \epsilon_{\mu\nu}\partial^\nu V\,, \qquad (2\cdot 12)$$

where W and V are Lorentz invariant. On substituting (2·12) in (2·4), we find

$$\Box W = -\alpha B\,. \qquad (2\cdot 13)$$

Hence we can write

$$W = (\sqrt{\pi}/e)X + aB\,, \qquad (2\cdot 14)$$

where X is a massless scalar field satisfying

$$\Box X = -\alpha(e/\sqrt{\pi})B\,, \qquad (2\cdot 15)$$

$$[X(x), B(y)] = -i(e/\sqrt{\pi})D(x-y)\,, \qquad (2\cdot 16)$$

a being an undetermined real constant. Here (2·16) follows from (2·6) and (2·12); a possible additional constant term is dropped because of the local commutativity between asymptotic fields. Because of (2·16) and (2·11), X produces unphysical quanta, which are nothing but the longitudinal photons. As in the (massless) Schwinger model, X is a dipole ghost field for $\alpha \neq 0$ and satisfies

$$[X(x), X(y)] = ibD(x-y) + i\alpha(e^2/\pi)E(x-y) \qquad (2\cdot 17)$$

because of (2·16) and (2·15), where

$$E(\xi) \equiv -(1/8)\epsilon(\xi^0)\xi^2\theta(\xi^2)\,,\ \ \Box E = D\,, \qquad (2\cdot 18)$$

b being an undetermined real constant. There are no massless asymptotic fields other than B and X at least for generic values of M_0 and e. Accordingly, *all asymptotic fields other than X commute with B.* According to (2·6) with (2·12), V, and therefore each order of its GLZ expansion, must commute with B. Hence V does not involve X, where "a quantity Φ involves X" means that Φ *cannot* be expressed in terms of asymptotic fields without using X.

Now, since our space-time is of two-dimensions, $F_{\mu\nu}$ is essentially spinless. More precisely, we can write

$$F_{\mu\nu} = \epsilon_{\mu\nu}F_{01} \qquad (2\cdot 19)$$

and F_{01} is a spinless field, which is seen to be

$$F_{01} = \Box V \qquad (2\cdot 20)$$

from (2·12). Hence it does not involve X. Furthermore, F_{01} has an odd parity, while B has an even one. We can therefore conclude that F_{01} has no massless

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spectrum (though it may depend on B), that is, $\langle 0|F_{01}(x)|p\rangle=0$ for any $p^2=0$ (but $p_\mu\neq0$). Of course, this fact corresponds to the nonexistence of the transverse photons. Accordingly,*)

$$\lim_{x^1\to\pm\infty} F_{01}(x)=0\,. \tag{2·21}$$

From the zeroth component of (2·3), therefore, we find that the charge operator Q is expressed as**)

$$Q\equiv\int dx^1 j_0(x)=-\int dx^1\partial_0 B(x)\,. \tag{2·22}$$

Thus, from (2·16) we have

$$[Q, X(x)]=-ie/\sqrt{\pi}\neq 0\,. \tag{2·23}$$

The vacuum expectation value of (2·23) shows that *gauge invariance is spontaneously broken* and that X is the Goldstone-boson field. Unlike the Schwinger model, however, we remark that the Higgs phenomenon does not take place because A_μ remains massless. This interesting situation happens because the longitudinal photon is a Lorentz scalar in the two-dimensional space-time.

Since the dependence on X of A_μ is explicitly known, we can determine the dependence on X of ψ from (2·2). We can write

$$\psi=:\exp(-i\sqrt{\pi}X):\Psi\,, \tag{2·24}$$

where Ψ does not involve X. An important consequence of (2·24) is that Ψ is gauge invariant, that is,

$$[\Psi(x), B(y)]=0\,. \tag{2·25}$$

Thus Ψ is a functional of physical asymptotic fields only.

CJS proved that there are no integrally-charged physical particles in the massive Schwinger model. From this result, they claimed that there would be no fermions, but we emphasize that the latter statement is not a logical consequence of the former. Their result cannot exclude the possible existence of *neutral* fermions. Indeed, (2·24) shows that *this possibility must really happen.* This can be shown in the following way.

Since (2·2) is a field equation containing a non-zero mass term,***) that is,

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*) Here if $\langle 0|F_{01}(x)|0\rangle$ happens to be a non-zero constant c, then F_{01} in (2·21) should be replaced by $F_{01}-c$.

**) Note that in n-dimensions the formula $Q=-\int d\boldsymbol{x}\,\partial_0 B(x)$, or equivalently $F_{\mu\nu}(x)=o(|\boldsymbol{x}|^{-n+2})$ as $|\boldsymbol{x}|\to\infty$ (Remember that the Coulomb force behaves like $|\boldsymbol{x}|^{-n+2}$.) implies the absence of charged particles, because all massive asymptotic fields then commute with B and any electrically charged particle should be massive. We believe that as long as the Higgs phenomenon does not happen the above situation is not realized for $n\geqq3$, contrary to the reasoning of Maison and Zwanziger.[15]

***) For the massless ψ, special q-number transformations are consistent with the Lorentz covariance of the field equation of ψ.[16] Note that Lorentz transformation properties given in Refs. 3) and 4) are wrong.

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it is not γ^5-invariant, it can be Lorentz covariant only when ψ is transformed in the standard way.[16] That is, for

$$x'^{\mu}=(\Lambda^{-1})^{\mu}{}_{\nu}x^{\nu},\quad \Lambda^{\mu}{}_{\nu}=\begin{pmatrix}\cosh\chi & \sinh\chi\\ \sinh\chi & \cosh\chi\end{pmatrix}, \tag{2·26}$$

we have

$$\psi'(x')=\exp(-\chi\gamma^5/2)\psi(x). \tag{2·27}$$

Since (2·27) is a c-number multiplicative transformation, by comparing the GLZ expansions of both $\psi(\mathrm{x})$ and $\psi'(x')$, we find that *each order* of the GLZ expansion of $\psi(x)$ must be transformed in the same way as in (2·27). If, however, there were no spinor asymptotic field, any homogeneous *polynomial* P in asymptotic fields could not be transformed like $P\rightarrow\exp(\pm\chi/2)P$; hence we encounter a contradiction. Thus there must exist a spinor asymptotic field, $\psi^{(0)}$ say.

In order to reproduce (2·27), Ψ should be proportional to $\psi^{(0)}$ in the sense of the spinor index. As shown above, $\psi^{(0)}$ commutes with B, that is, it is neutral and physical.

In summary, ψ is confined and correspondingly gauge invariance is indeed spontaneously broken, but a neutral spinor asymptotic field is present as an observable entity.

§ 3. Massive sine-Gordon theory

An interesting feature of the two-dimensional models is that any spinor field theory has an equivalent boson one. Of course, the latter is logically independent of the solution of the former in terms of asymptotic fields. We can therefore derive the equivalent boson theory of the massive Schwinger model on the basis of the results obtained in the (massless) Schwinger model, quite independently of the analysis made in § 2. Before doing this, however, we review CJS's derivation of the massive sine-Gordon theory involving a new parameter θ.

The reasoning of CJS is based on the solution of the Schwinger model in a covariant gauge ($\alpha=0$) given by LS. The total Hamiltonian density $\mathcal{H}$ of the massive Schwinger model is obtained as a sum of the Hamiltonian density, $\mathcal{H}_0$, of the Schwinger model and the mass term, $\mathcal{H}_I$, of the spinor field. CJS work out the computation in the gauge-invariant algebra. Then, according to LS, the Schwinger model is described by a free massive scalar field φ and a constant unitary operator U, which commutes with φ. Though $\mathcal{H}_0$ is expressed in terms of φ alone, $\mathcal{H}_I$ is explicitly dependent on U. In this way, CJS obtain

$$\begin{aligned}\mathcal{H}=\tfrac{1}{2}:[(\partial_0\varphi)^2+(\partial_1\varphi)^2+(e^2/\pi)\varphi^2]:\\ +\tfrac{1}{2}ceM_0[:\exp(2i\sqrt{\pi}\varphi):U+:\exp(-2i\sqrt{\pi}\varphi):U^{\dagger}],\end{aligned} \tag{3·1}$$

c being a numerical constant. Let $|\theta\rangle$ be an eigenstate of U, that is,

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$$U|\theta\rangle = e^{i\theta}|\theta\rangle. \tag{3·2}$$

Then in the subspace containing a particular $|\theta\rangle$ alone, (3·1) reduces to

$$\mathcal{H}_\theta = \tfrac{1}{2}:[(\partial_0\varphi)^2 + (\partial_1\varphi)^2 + (e^2/\pi)\varphi^2]: + ceM_0 :\cos(2\sqrt{\pi}\varphi + \theta):. \tag{3·3}$$

It is important to note that θ cannot be eliminated from (3·3) by redefining φ.

From the above derivation, it is evident that θ originates from the "spurion" operator U. Since U commutes with every operator in the algebra, the representation is *reducible* so that multiple vacua $\{|\theta\rangle\}$ are encountered. Thus the appearance of θ can be traced back to the representation implied by the LS solution. As has been analyzed very recently,[9] the LS solution can be reconstructed from our asymptotic-field one by additionally introducing three constant operators, called σ_1, σ_2 and Q^5, which are independent of asymptotic fields. That is, the LS solution contains some extra degrees of freedom introduced artificially. This fact is not surprising because the LS solution was not directly found in the operator-level analysis but constructed so as to reproduce the known $2n$-point Wightman functions correctly. LS tacitly made a presumption that all Wightman functions containing unequal numbers of ψ and $\bar{\psi}$ should vanish identically. We have shown, however, that such a requirement is inconsistent with asymptotic completeness and hence with the irreducibility of the representation.

The operator U of CJS arises from $\sigma_1^{-1}\sigma_2$ of the LS solution, that is, it is the freedom introduced artificially. Consequently, CJS tacitly consider an indefinite-metric Hilbert space containing some redundant state vectors. When the mass term is added as the interaction Hamiltonian, those extra state vectors couple with physical states and acquire physical relevance in their formalism. This is the origin of CJS's parameter θ; we therefore believe that the appearance of θ is not intrinsic to the genuine massive Schwinger model.

Now, we derive the equivalent boson Hamiltonian of the massive Schwinger model from our operator solution of the Schwinger model expressed in terms of asymptotic fields. In the Schwinger model, we have three spinless asymptotic fields B, X and φ (φ stands for $m^{-1}\tilde{U}$ of Ref. 3)). As shown previously,[4] the Hamiltonian density is expressed as

$$\begin{aligned}\mathcal{H}_0 = {} & \tfrac{1}{2}:[(\partial_0\varphi)^2 + (\partial_1\varphi)^2 + (e^2/\pi)\varphi^2]: \\ & - (e/\sqrt{\pi})^{-1}:[(\partial_0 X)(\partial_0 B) + (\partial_1 X)(\partial_1 B)]: \\ & - \tfrac{1}{2}(e^2/\pi)^{-1}:[(\partial_0 B)^2 + (\partial_1 B)^2]: - \tfrac{1}{2}\alpha :B^2:.\end{aligned} \tag{3·4}$$

The total Hamiltonian density $\mathcal{H}$ of the massive Schwinger model is obtained by adding a mass term

$$\mathcal{H}_I = M_0\bar{\psi}\psi = M_0(\psi_1^\dagger\psi_2 + \psi_2^\dagger\psi_1). \tag{3·5}$$

In the interaction picture, ψ is the ψ of the (massless) Schwinger model, that is,

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we have

$$\psi = :\exp\{-i\sqrt{\pi}[X+\gamma^5(\sqrt{\pi}e^{-1}\widetilde{B}+\varphi)]\}:u\,, \tag{3·6}$$

where $\widetilde{B}$ is the conjugate field of B, defined by

$$\widetilde{B}(x) \equiv \int_{-\infty}^{x^1} dz^1 \partial_0 B(x^0, z^1)\,, \tag{3·7}$$

and u is a constant, c-number, two-component quantity such that

$$|u_r|^2 = (\mu\lambda)^{1/2}/2\pi\,, \quad (r=1, 2) \tag{3·8}$$

with

$$\lambda = \tfrac{1}{2}e^{\gamma}(e/\sqrt{\pi})\,, \tag{3·9}$$

γ being Euler's constant. Since the relative phase between u_1 and u_2 is undetermined, we set*)

$$u_2/u_1 = ie^{-i\eta}\,. \quad (0 \leqq \eta < 2\pi) \tag{3·10}$$

From (3·6), with the aid of the commutation relations between asymptotic fields,[3] we have

$$\psi_1{}^\dagger\psi_2 = u_1{}^*u_2 \lim_{\varepsilon\to 0}[\exp M_{12}^{(+)}(\varepsilon)]:\exp[-2i\sqrt{\pi}(\sqrt{\pi}e^{-1}\widetilde{B}+\varphi)]: \tag{3·11}$$

with

$$M_{12}^{(+)}(\xi) \equiv \pi[D^{(+)}(\xi) - \Delta^{(+)}(\xi; e^2/\pi) + \alpha(e^2/\pi)E^{(+)}(\xi) - \tfrac{1}{2}i]\,. \tag{3·12}$$

Since

$$\lim_{\varepsilon\to 0} \exp M_{12}^{(+)}(\varepsilon) = -i(\lambda/\mu)^{1/2}\,, \tag{3·13}$$

we obtain

$$\psi_1{}^\dagger\psi_2 = e^{-i\eta}(\lambda/2\pi):\exp[-2i\sqrt{\pi}(\sqrt{\pi}e^{-1}\widetilde{B}+\varphi)]:\,. \tag{3·14}$$

Hence

$$\bar{\psi}\psi = (\lambda/\pi):\cos[2\sqrt{\pi}(\sqrt{\pi}e^{-1}\widetilde{B}+\varphi)+\eta]:\,. \tag{3·15}$$

The parity operator commutes with both free and interaction Hamiltonians, and therefore it does with the interaction Hamiltonian in the interaction picture. Since under the parity operation $\bar{\psi}\psi$ is even but $\widetilde{B}$ and φ are odd, we must have $\eta=0$ or π. Thus

$$\mathcal{H}_I = \pm(M_0\lambda/\pi):\cos[2\sqrt{\pi}(\sqrt{\pi}e^{-1}\widetilde{B}+\varphi)]:\,. \tag{3·16}$$

If we confine ourselves to the gauge-invariant algebra, X and therefore B should be set equal to zero. Then the total Hamiltonian density $\mathcal{H}=\mathcal{H}_0+\mathcal{H}_I$ is effectively given by

*) One should not confuse η with θ. The phase factor $e^{-i\eta}$ is merely a multiplicative c-number factor which is not related to any extra degree of freedom at all.

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$$\mathcal{H} \approx \tfrac{1}{2}:[(\partial_0{}^2\varphi)^2+(\partial_1\varphi)^2+(e^2/\pi)\varphi^2]:$$
$$\pm ceM_0: \cos(2\sqrt{\pi}\varphi): \qquad (3\cdot 17)$$

with $c=e^{\gamma}/2\pi^{3/2}$.

Unlike the massless sine-Gordon theory, the sign of the last term is physically non-trivial, but we do not know a criterion for determining the right sign. The positive-sign case of (3·17) exactly reproduces the massive sine-Gordon equation derived by Yoneya,[17),*)] who has shown that it has classical soliton solutions if e is small compared with M_0. The negative-sign case of (3·17) does not have such a property. Since, as emphasized in § 2, there must exist a spinor asymptotic field in the massive Schwinger model, the above consideration suggests that the right choice is the positive-sign case.**)

From the analysis made in this section, we conclude that CJS's parameter θ is spurious. There is, however, a strong objection to this conclusion.

Shortly after CJS's work, Coleman[18)] analyzed the massive Schwinger model in a non-covariant gauge ($A_1\equiv 0$). The equation for F_{01} can be easily integrated and θ is essentially introduced as an integration constant.***) After eliminating A_0, he finds that the Hamiltonian for $Q=0$ is explicitly dependent on θ. Coleman's result may be regarded as a support to the relevance of θ.

For a generic value of θ, Coleman's Hamiltonian is explicitly parity non-conserving. He regards this fact as spontaneous breakdown of parity invariance, but such interpretation is not acceptable because the commutativity between the parity operator and the Hamiltonian must hold regardless to whether or not parity is spontaneously broken. Recently, the real reason for this point has been clarified by Pak:[19)] Coleman's Hamiltonian, whose density is essentially the 00 component of the *symmetric* energy-momentum tensor, is different from the *canonical* Hamiltonian by a surface term which is non-vanishing for $\theta\neq 0$.

Pak's comment has been further criticized by Halpern and Senjanović (HS) very recently. Unless both $Q=0$ and $\theta=0$, the surface-term trouble is present in the non-covariant gauge. It invalidates the variational principle itself and yields inconsistency. Thus the controversy between Coleman and Pak is groundless. HS explicitly add a surface term to the Lagrangian by introducing new freedom, denoted by G and λ, and show that everything can then be formulated consistently. Their final result is favorable to Coleman's conclusion that the parameter θ is relevant.

The present author does not believe that HS have established the relevance

*) He sets $\theta=0$ without discussion.

**) We expect that because of quantum fluctuation quantum solitons may continue to exist even for e large.

***) Unfortunately, there might arise some confusion about the definition of θ because Coleman identified his parameter with $\pi-\theta$. This discrepancy is owing to the sign ambiguity of the cosine term in the sine-Gordon Hamiltonian equivalent to the free massive Dirac theory.

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of the parameter θ *in the massive Schwinger model.* The point is their very starting point. Since to change the Lagrangian generally implies to change the model, it is not clear whether or not the HS formalism really deals with the genuine massive Schwinger model.*)

As long as one admits the freedom corresponding to the parameter θ, the HS formalism seems to be quite satisfactory, but we deny its very existence. In this case, we encounter no surface-term trouble from the beginning if $Q=0$. One might feel that it is unsatisfactory that one must restrict oneself to the $Q=0$ case only. From our standpoint, however, the restriction to $Q=0$ is quite natural. As shown in (2·22), Q is expressed in terms of the self-commuting field $B(x)$, whence $Q|\text{phys}\rangle$ always has *zero norm.* Since Q is gauge invariant, this fact must remain true in any gauge. On the other hand, in the $A_1=0$ gauge, any state vector has positive norm. Hence we encounter no inconsistency if and only if $Q=0$.

Our conclusion is that the parameter θ is not relevant in the *genuine* massive Schwinger model; its appearance is merely owing to the freedom introduced artificially.

The author would like to thank Professor S. Coleman for correspondence on CJS's paper, though his opinion on the parameter θ is different from the author's.

Appendix

——*Self-energy Feynman integral*——

The second-order photon self-energy part of the massive Schwinger model can be calculated easily,[21] but the Feynman integrals other than it contain photon propagators which are inherently infrared divergent. Since our formalism presented in Ref. 11) has resolved this infrared difficulty characteristic to the two-dimensional theories, we are now able, at least in principle, to calculate any Feynman integral involving massless boson propagators. In the following, we explicitly demonstrate how to calculate the second-order self-energy Feynman integral $\Sigma^{(2)}(p)$ of ψ.

Since $D^{(+)}(\xi)$ is given by (2·10), the photon propagator in the Feynman gauge ($\alpha=1$) is given by $-g^{\mu\nu}D_F(\xi)$ with

$$D_F(\xi)\equiv-(4\pi)^{-1}\log(-\mu^2\xi^2+i0). \qquad (\text{A}\cdot 1)$$

Since $D_F(\xi)$ is a tempered distribution, its two-dimensional Fourier transform must exist, but it seems that it cannot be expressed in terms of well-known distributions. Hence we first consider a massive propagator

$$\Delta_F(\xi;m^2)=(2\pi)^{-1}K_0[m(-\xi^2+i0)^{1/2}], \qquad (\text{A}\cdot 2)$$

*) In the covariant quantization of gauge theory, one usually introduces a gauge-fixing term to the Lagrangian, but in that case one also introduces a subsidiary condition to compensate the extra freedom. If a subsidiary condition $G|\text{phys}\rangle=0$ is introduced in the HS formalism, θ disappears for $Q=0$.

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where K_0 stands for a modified Bessel function. Comparing (A·2) with (A·1), we find

$$D_F(\xi)=\lim_{\varepsilon\to 0}[\Delta_F(\xi;\varepsilon^2)+(2\pi)^{-1}\log(|\varepsilon|/2\kappa)] \tag{A·3}$$

with $\kappa=\mu e^{-\gamma}$, that is,

$$D_F(\xi)=\frac{1}{(2\pi)^2}\lim_{\varepsilon\to 0}\int d^2p\exp(-ip_\mu\xi^\mu)\Big[\frac{-i}{\varepsilon^2-p^2-i0}$$

$$+2\pi\log(|\varepsilon|/2\kappa)\delta^2(p)\Big]. \tag{A·4}$$

Therefore,

$$\Sigma^{(2)}(p)=\lim_{\varepsilon\to 0}\frac{-e^2}{(2\pi)^2 i}\int d^2k\frac{\gamma^\mu[M+\gamma(p-k)]\gamma_\mu}{M^2-(p-k)^2-i0}$$

$$\cdot\Big[\frac{1}{\varepsilon^2-k^2-i0}+2\pi i\log\frac{|\varepsilon|}{2\kappa}\cdot\delta^2(k)\Big]$$

$$=\lim_{\varepsilon\to 0}\frac{ie^2}{(2\pi)^2}\cdot 2M\Big[I_\varepsilon(p)+2\pi i\frac{\log(|\varepsilon|/2\kappa)}{M^2-p^2-i0}\Big], \tag{A·5}$$

where M stands for the "mass" of ψ and

$$I_\varepsilon(p)\equiv\int d^2k[M^2-(p-k)^2-i0]^{-1}(\varepsilon^2-k^2-i0)^{-1}. \tag{A·6}$$

The explicit computation of (A·6) is straightforward. The final expression is

$$\Sigma^{(2)}(p)=\frac{e^2}{\pi}\cdot\frac{M}{M^2-p^2-i0}\log\frac{2\kappa M}{M^2-p^2-i0}. \tag{A·7}$$

Thus $\Sigma^{(2)}(p)$ is completely well-defined off the mass shell. On the mass shell, however, it is strongly divergent as expected. Because of the celebrated cancellation of infrared divergences[22] in quantum electrodynamics, we believe that the leading-order divergences may disappear in any observable quantity, but there is no reason for expecting that the *non-leading* divergences may also cancel. Thus (A·7) suggests that if the massive Schwinger model is a sensible theory, ψ should be confined.

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