

## ASYMPTOTIC EXPANSIONS AND COMPARISON OF BIVARIATE PARAMETER MEANS

NEVEN ELEZOVIĆ AND LENKA VUKŠIĆ

*Abstract.* The subject of this paper is the analysis of bivariate parameter means: general power mean, generalized logarithmic mean, Gini mean and Stolarsky mean. Asymptotical analysis of these means are made and series of corresponding coefficients are calculated. Using these information, a necessary conditions for the comparison of these means are derived. This approach enables better understanding of relations between these means.

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