Asymptotic safety of gravity-matter systems

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IMPRS

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Outline

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Motivation

Why gravity-matter systems?

- Pure quantum gravity does not exist in nature
- Matter seems to matter Percacci, Perini (2003)

  Donà, Eichhorn, Percacci (2013)

Does the non-trivial UV fixed point persist within inclusion of matter?

Motivation

Why a vertex expansion?

Features:

- Systematic approximation scheme
- Disentangling background and fluctuation fields
- Newton's coupling from graviton three-point function

Challenges:

- Long algebraic computations due to large tensor structures
- Physical choice of tensor projection
- Technical issues for non-flat backgrounds (so far)

Vertex expansion

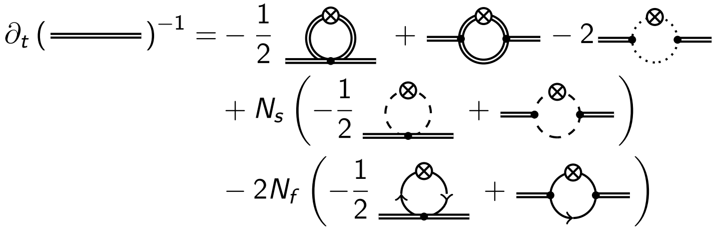
$$\Gamma_k[\bar{g},\phi] = \sum_{n=0}^{\infty} \frac{1}{n!}\Gamma_k^{(\phi_1\ldots\phi_n)}[\bar{g},0]\phi_1\cdots\phi_n \qquad \text{with} \qquad \phi = (h, c, \bar{c}, \psi, \bar{\psi}, \varphi)$$

$$= \Gamma_k[\bar{g},0] + \Gamma_k^{(h)}[\bar{g},0]h + \frac{1}{2}\Gamma_k^{(2h)}[\bar{g},0]h^2 + \frac{1}{3!}\Gamma_k^{(3h)}[\bar{g},0]h^3 + \ldots$$

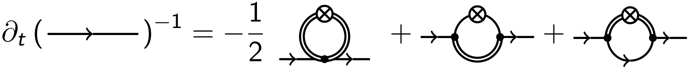
$$+ \frac{1}{2}\Gamma_k^{(\bar{c}c)}[\bar{g},0]\bar{c}\,c + \frac{1}{2}\Gamma_k^{(\bar{\psi}\psi)}[\bar{g},0]\bar{\psi}\,\psi + \frac{1}{2}\Gamma_k^{(\varphi\varphi)}[\bar{g},0]\varphi^2 + \ldots$$

- Computation of all *n*-point correlation functions up to third order
- Flow equation for each correlation function from Wetterich equation

Graviton two-point function:



Fermionic two-point function:



Generation of vertex dressing

Generating action:

$$S = S_{\text{EH}} + S_{\text{gh}} + S_{\text{gf}} + \int \mathrm{d}^4x \sqrt{g}\,\bar{\psi}_i \not{\nabla} \psi_i + \frac{1}{2} \int \mathrm{d}^4x \sqrt{g}\, g_{\mu\nu} \partial^\mu \varphi_j \partial^\nu \varphi_j$$

with $i = 1, \ldots, N_f$ and $j = 1, \ldots, N_s$

Tensor structures for vertices:

$$\mathcal{T}^{(\phi_1 \ldots \phi_n)}(p_1, \ldots, p_n; \Lambda_n) = S^{(\phi_1 \ldots \phi_n)}(p_1, \ldots, p_n; \Lambda \to \Lambda_n, G_N \to 1) \qquad (1)$$

Spin-connection via spin-base invariance formalism Weldon (2000); Gies, Lippoldt (2013)

Vertex construction

RG-invariant ansatz for n-point function Christiansen, Knorr, Pawlowski, Rodigast (2014)

$$\Gamma^{(\phi_1\ldots\phi_n)} = \prod_{i=1}^{n} \sqrt{Z_{\phi_i}(p_i^2)}\, G_n^{\frac{n}{2}-1} \mathcal{T}^{(\phi_1\ldots\phi_n)}(p_1,\ldots,p_n;\Lambda_n) \qquad (2)$$

Graviton two-point function:

$$\Gamma^{(hh)} = Z_h(p^2)\mathcal{T}^{(hh)} \sim Z_h(p^2)(p^2 - 2\Lambda_2)\Pi_{\mathrm{TT}} + \text{other tensor structures}$$

Fermionic two-point function:

$$\Gamma^{(\overline{\psi}\psi)} = Z_\psi(p^2) i\gamma_\mu p^\mu$$

Vertex construction

RG-invariant ansatz for n-point function Christiansen, Knorr, Pawlowski, Rodigast (2014)

$$\Gamma^{(\phi_1 \ldots \phi_n)} = \prod_{i=1}^{n} \sqrt{Z_{\phi_i}(p_i^2)}\, G_n^{\frac{n}{2}-1} \mathcal{T}^{(\phi_1 \ldots \phi_n)}(p_1, \ldots, p_n; \Lambda_n) \qquad (2)$$

Graviton two-point function:

$$\Gamma^{(hh)} = Z_h(p^2)\mathcal{T}^{(hh)} \sim Z_h(p^2)(p^2 + M^2)\Pi_{\text{TT}} + \text{other tensor structures}$$

$$M^2 = -2\Lambda_2 = \text{graviton mass parameter}$$

Projection methods

- Transverse-traceless part of graviton *n*-point functions
- $\Gamma^{(3h)}$ at symmetric momentum configuration
- Bilocal momentum projection for $\Gamma^{(3h)}$ Christiansen, Knorr, Meibohm, Pawlowski, MR
- Momentum dependence of $\Gamma^{(\phi\phi)}$ encoded in anomalous dimensions

Truncation

- Flow of *n*-point functions up to order three
- Flat Euclidean background $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$
- Closure of flow equations (gravity sector)

$$G_n(p_1, \ldots, p_n) \equiv G_3 =: G$$

$$\Lambda_5 \equiv \Lambda_4 \equiv \Lambda_3$$

- Minimally coupled scalars and fermions
- No matter-matter-interactions

Parameters of the truncation

$$G, \Lambda_3, M^2, \eta_h(p^2), \eta_c(p^2), \eta_\psi(p^2), \eta_\varphi(p^2), N_s, N_f$$

Outline

Fully momentum dependent anomalous dimensions

The anomalous dimensions obey coupled Fredholm integral equations, e.g.

$$\eta_h(p^2) \sim \int \mathrm{d}\Omega \mathrm{d}q \; q^3 \left(\dot{r}(q^2) - \eta_i(q^2) r(q^2)\right) f_i(p, q, G, \Lambda_3, M^2)$$

In our setup: Four coupled Fredholm integral equations

$\rightarrow$ Requires large numerical effort for solution (Liouville-Neumann series)

Approximation

Evaluate anomalous dimensions at the peak of the integral $q = k$

Quality of approximated momentum dependence ($N_s = N_f = 0$)

Full momentum dependence Christiansen, Knorr, Meibohm, Pawlowski, MR (2015)

$$
\begin{aligned}
(g^*, \mu^*, \lambda_3^*) &= (0.66, -0.59, 0.11) \\
(\theta_{1/2}, \theta_3) &= (-1.4 \pm 4.1\, i, 14)
\end{aligned} \tag{3}
$$

Approximated momentum dependence

$$
\begin{aligned}
(g^*, \mu^*, \lambda_3^*) &= (0.62, -0.57, 0.095) \\
(\theta_{1/2}, \theta_3) &= (-1.3 \pm 4.1\, i, 12)
\end{aligned} \tag{4}
$$

Error of 6% (15%) on fixed point values (critical exponents)

Key message

Momentum dependence of the anomalous dimensions is important

Bounds on the anomalous dimensions - from the regulator

The regulator is proportional to $Z_\phi \sim k^{-\eta_\phi}$

$$R_k^\phi(p^2) = \Gamma_k^{(\phi\phi)}(p^2) r_k^\phi(p^2)\Big|_{M^2=0} \tag{5}$$

But the regulator has to satisfy (with $i = 1$ for fermions and $i = 2$ else)

$$\lim_{k\to\infty} R_k^\phi(p) \sim \lim_{k\to\infty} Z_\phi k^i \sim \lim_{k\to\infty} k^{i-\eta_\phi} \overset{!}{\to} \infty \tag{6}$$

Bounds on the anomalous dimensions - from the regulator

$$\eta_h < 2\,, \qquad \eta_c < 2\,, \qquad \eta_\varphi < 2\,, \qquad \eta_\psi < 1$$

Bounds on the anomalous dimensions - from the flow

Contributions to the flow have the typical shape

$$\text{diagram} \sim (\# - \eta_\phi)$$

with $\# = \{4, 5, 6, \dots\}$ for analytic approximation

The sign of the diagram is regulator dependent for $\eta_\phi > \#$

Bounds on the anomalous dimensions - from the flow

$$\eta_h \lessapprox 4\,, \qquad \eta_c \lessapprox 10\,, \qquad \eta_\varphi \lessapprox 8\,, \qquad \eta_\psi \lessapprox 5$$

We take the most conservative bound!

Outline

Questions so far?

Flow equations in analytic approximation

Flow of dynamical couplings (matter contributions)

$$\partial_t g \sim -\frac{43}{570\pi} g^2 N_s - \frac{3599}{11400\pi} g^2 N_f \,,$$
$$\partial_t \lambda_2 \sim -\frac{1}{24\pi} g N_s + \frac{4}{9\pi} g N_f \,, \tag{7}$$

compared to background couplings in background-field approximation

$$\partial_t \bar{g} \sim +\frac{1}{6\pi} \bar{g}^2 N_s + \frac{1}{3\pi} \bar{g}^2 N_f \,,$$
$$\partial_t \bar{\lambda} \sim +\frac{1}{12\pi} (3 + 2\bar{\lambda}) \bar{g} N_s - \frac{1}{3\pi} (3 - \bar{\lambda}) \bar{g} N_f \,. \tag{8}$$

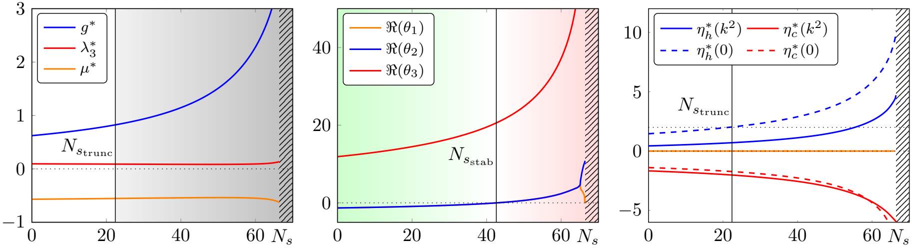
Do the different signs indicate an error?

No, it's just a computation of different couplings!

Reminder: Truncation

- g from $\left(\Gamma^{(3h)}(p=k) - \Gamma^{(3h)}(p=0)\right)$; numerical integration necessary
  $Z_h(p^2)$ models remaining p-dependence Christiansen, Knorr, Meibohm, Pawlowski, MR
- λ_3 from $\Gamma^{(3h)}(p=0)$; analytic equation
- μ from $\Gamma^{(2h)}(p=0)$; analytic equation
- $\eta_\phi(p^2)$ from $\Gamma^{(\phi\phi)}(p^2)$ with approximated p-dependence
- Remark: $\eta_\varphi(p^2) = 0$ due to choice of graviton gauge

Inclusion of scalars

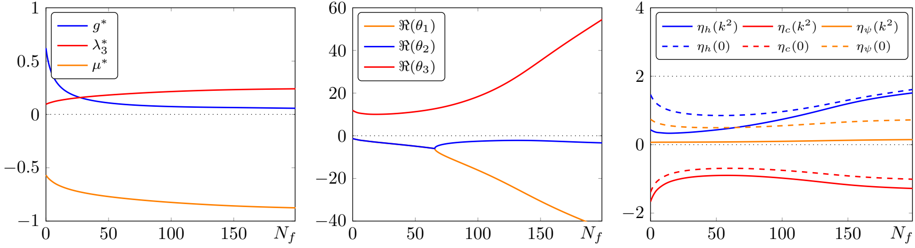


Reliability bound on truncation ($\eta_h^* > 2$) $N_{s_{\rm trunc}} \approx 21.5$

Loss of an attractive UV fixed point $N_{s_{\rm stab}} \approx 42.6$

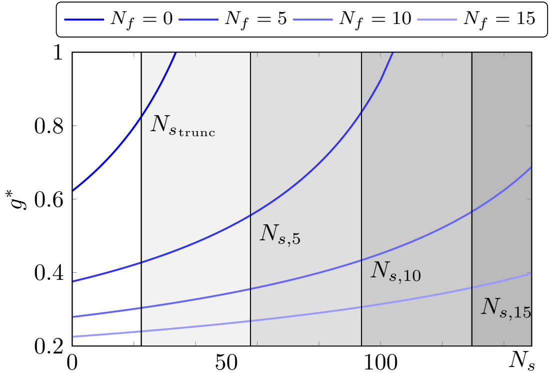
Complete loss of the UV fixed point $N_{s_{\rm max}} \approx 66.4$

Inclusion of fermions



Reliable, attractive UV fixed point for all N_f

Mixed scalar-fermion systems



One fermion compensates for ≈ 7.1 scalars

Computation of background couplings

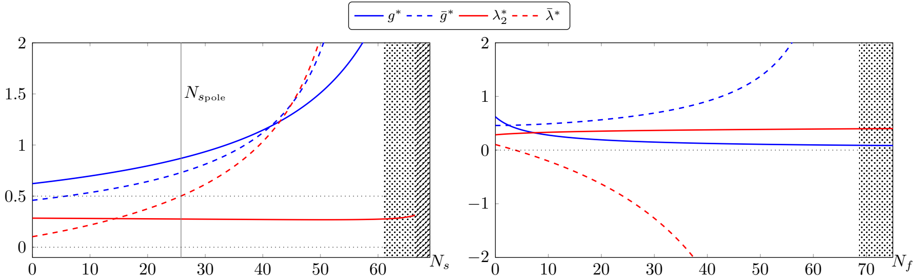
- The background couplings $\bar{g}$ and $\bar{\lambda}$ are observables for $k \to 0$
- Their flow depends only on dynamical quantities

$$\partial_t \left(\frac{k^2}{\bar{g}} \right) = F_{R^1}(g, \lambda; N_s, N_f)$$
$$\partial_t \left(\frac{\bar{\lambda} k^4}{\bar{g}} \right) = F_{R^0}(g, \lambda; N_s, N_f) \tag{9}$$

- Same equations as earlier computations if $(g, \lambda) \to (\bar{g}, \bar{\lambda})$

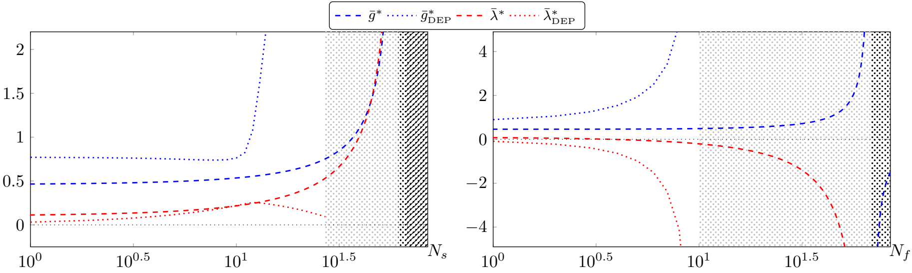
Codello, Percacci, Rahmede (2008); Donà, Eichhorn, Percacci (2013)

Background couplings vs dynamical couplings



Qualitative agreement only for small N_s and N_f

Comparison with previous results



DEP: Donà, Eichhorn, Percacci (2013)

Good qualitative agreement for small N_s and N_f

Summary

Setup:

- Vertex expansion up to third order
- Genuine Newton's coupling from graviton three-point function
- Minimally coupled scalars and fermions

Results:

- All numbers of fermions are compatible with asymptotic safety
- Need to improve truncation for intermediate and large N_s
- Background and dynamical couplings deviate for large N_s and N_f

Outlook - Road towards better truncations

Improvement in the matter sector (Matter-matter-interactions)

- Yukawa-coupling Held, Eichhorn, Pawlowski; in preparation
- φ^4-coupling Fischer, Pawlowski; in preparation

Improvement in the gravity/mixed sector

- Graviton four-point function Denz, Pawlowski, MR; in preparation
- $h\varphi\varphi$-coupling Donà, Eichhorn, Labus, Percacci; arXiv:1512.01589
  Eichhorn, Labus, MR; in preparation
- Expansion about non-flat backgrounds