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Asymptotic structure of $\mathcal{N}=2$ supergravity in 3D: extended super-BMS$_3$ and nonlinear energy bounds

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Abstract: The asymptotically flat structure of $\mathcal{N}=(2,0)$ supergravity in three spacetime dimensions is explored. The asymptotic symmetries are found to be spanned by an extension of the super-BMS$_3$ algebra, endowed with two independent affine $\hat{u}(1)$ currents of electric and magnetic type. These currents are associated to U(1) fields being even and odd under parity, respectively. Remarkably, although the U(1) fields do not generate a backreaction on the metric, they provide nontrivial Sugawara-like contributions to the BMS$_3$ generators, and hence to the energy and the angular momentum. Consequently, the entropy of flat cosmological spacetimes endowed with U(1) fields acquires a nontrivial dependence on the zero modes of the $\hat{u}(1)$ charges. If the spin structure is odd, the ground state corresponds to Minkowski spacetime, and although the anticommutator of the canonical supercharges is linear in the energy and in the electric-like $\hat{u}(1)$ charge, the energy becomes bounded from below by the energy of the ground state shifted by the square of the electric-like $\hat{u}(1)$ charge. If the spin structure is even, the same bound for the energy generically holds, unless the absolute value of the electric-like charge is less than minus the mass of Minkowski spacetime in vacuum, so that the energy has to be nonnegative. The explicit form of the global and asymptotic Killing spinors is found for a wide class of configurations that fulfills our boundary conditions, and they exist precisely when the corresponding bounds are saturated. It is also shown that the spectra with periodic or antiperiodic boundary conditions for the fermionic fields are related by spectral flow, in a similar way as it occurs for the $\mathcal{N}=2$ super-Virasoro algebra. Indeed, our supersymmetric extension of BMS$_3$ can be recovered from the Inönü-Wigner contraction of the superconformal algebra with $\mathcal{N}=(2,2)$, once the fermionic generators of the right copy are truncated.

Keywords: Conformal and W Symmetry, Space-Time Symmetries, Gauge-gravity correspondence, Supergravity Models

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Contents

1 Introduction

The symmetries of asymptotically flat spacetimes at null infinity were proposed to be spanned by the BMS algebra since long ago [1, 2]. More recently, this analysis has been further developed and expanded in [3–10], and it has led to the proposal of [11, 12], which might be promising in order to resolve the information loss paradox [13]. Nonetheless, some open issues still remain to be suitably understood in the four-dimensional case (see e.g., [14]), which naturally motivates one to explore them in a simplified setup, as it is the case of General Relativity in three spacetime dimensions. As shown in [4, 15, 16], the three-dimensional version of the BMS algebra (BMS_3) describes the asymptotically flat symmetries. The BMS_3 algebra turns out to be isomorphic to the Galilean conformal algebra in two dimensions, and it has been shown to be relevant in the context of flat holography [17–20], as well as for the tensionless limit of string theory [21–23] (see also [24]). It is also worth pointing out that the generators of the BMS_3 algebra can be seen to emerge in a unique way through a twisted Sugawara-like construction made out from composite operators of affine currents describing the asymptotic symmetries of the "soft hairy" type of boundary conditions recently discussed in [25–27]. Similar results along these lines have also been found in the context of near horizon (twisted) warped conformal symmetry algebras in [28–30].

Besides, in the context of $\mathcal{N}=1$ supergravity in three spacetime dimensions [31–33], the minimal supersymmetric extension of BMS_3 has been shown to arise from a suitable set of asymptotically flat boundary conditions [34], which are not necessarily given at null infinity. The superalgebra turns out to be isomorphic to the supersymmetric extension of the two-dimensional Galilean conformal algebra in [35, 36] (see also [37]), which was found from a non-relativistic limit of the superconformal algebra, and hence their generators do not possess the same physical interpretation.

The extension to the case of $\mathcal{N}=(1,1)$ has also been recently explored in [38], where it was shown that two inequivalent possibilities can be recovered from different flat limiting processes of $\mathcal{N}=(1,1)$ AdS_3 supergravity [39]. The homogeneous or "democratic" possibility corresponds to the straightforward extension of the case with $\mathcal{N}=1$, which agrees with the results found in [22, 23, 40] in the context of Galilean superconformal algebras. In the "despotic" possibility, the algebra becomes isomorphic to the inhomogeneous Galilean superconformal algebra [23, 36].

In the next section we make a brief revision of the $\mathcal{N}=(2,0)$ Poincaré supergravity theory constructed out in [41]. We show that demanding the action to be parity-invariant implies that the U(1) field that is minimally coupled to the complexified gravitino is even under parity, while the remaining U(1) field has to be odd. In section 3 we propose a set of boundary conditions that includes a generic choice of Lagrange multipliers, which is strictly necessary in order to accommodate solutions of physical interest. The asymptotic symmetries are shown to be spanned by a supersymmetric extension of the BMS_3 algebra with $\mathcal{N}=(2,0)$, endowed with two independent affine $\hat{u}(1)$ currents of electric and magnetic type. Specifically, the nonvanishing anticommutator of the complexified fermionic generators acquires a central extension and depends on the supertranslations as well as on the electric-like affine $\hat{u}(1)$ current. It is also shown that our supersymmetric extension of BMS_3 can be recovered from the Inönü-Wigner contraction of the superconformal algebra with $\mathcal{N}=(2,2)$, once the fermionic generators of the right copy are truncated. In section 4 we show that for fermionic fields that fulfill antiperiodic boundary conditions, the ground state is given by Minkowski spacetime, possibly endowed with $\hat{u}(1)$ charges of electric type. Remarkably, although the anticommutator of the supercharges is linear in the energy and

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in the electric-like $\hat{u}(1)$ charge, the energy becomes bounded from below by the energy of Minkowski spacetime in vacuum, shifted by the square of the electric-like $\hat{u}(1)$ charge. If the spin structure is even, the same bound for the energy generically holds, unless the absolute value of the electric-like charge is less than minus the mass of Minkowski spacetime in vacuum, so that the energy has to be nonnegative.

Bosonic configurations that fulfill our boundary conditions are revisited in section 5, where we pay special attention to the conditions that ensure their regularity in terms of gauge fields. We also discuss them in the metric formalism, and carry out a thorough analysis of the thermodynamic properties of cosmological spacetimes endowed with U(1) fields. The presence of U(1) fields of electric and magnetic type also unveils some remarkable properties of Minkowski spacetime, as well as locally flat configurations with conical defects or surpluses. In section 6 we focus in the analysis of bosonic solutions possessing unbroken supersymmetries that saturate the corresponding energy bounds, and we also find the explicit form of the (asymptotic) Killing spinors. Section 7 is devoted to show that the spectra with periodic or antiperiodic boundary conditions for the fermionic fields are related by spectral flow, in a similar way as it occurs for the super-Virasoro algebra $\mathcal{N} = 2$ [42]. We conclude with some comments about the extension of our results in section 8. Our conventions are discussed in appendix A.

Note added: while this manuscript was in the process of typesetting, ref. [43] was posted in the arxiv, which possesses some overlap with particular cases of our results.

2 $\mathcal{N} = (2, 0)$ Poincaré supergravity in three spacetime dimensions

As shown in [41], $\mathcal{N} = (2, 0)$ Poincaré supergravity in three spacetime dimensions can be formulated as a Chern-Simons theory for a suitable extension of the super-Poincaré group. The algebra has to be endowed with two additional bosonic U(1) generators that respectively correspond to an automorphism and a central charge, so that it admits a non-degenerate invariant bilinear form. The nonvanishing (anti-)commutators read

$$\begin{aligned} &[J_a, J_b] = \epsilon_{abc} J^c \,, && [J_a, P_b] = \epsilon_{abc} P^c \,, \\ &[J_a, Q^I_\alpha] = \frac{1}{2} \left(\Gamma_a\right)^{\beta}{}_{\alpha} Q^I_\beta \,, && [Q^I_\alpha, T] = \epsilon^{IJ} Q^J_\alpha \,, \\ &\{Q^I_\alpha, Q^J_\beta\} = -\frac{1}{2} \delta^{IJ} \left(C\Gamma^a\right)_{\alpha\beta} P_a + C_{\alpha\beta} \epsilon^{IJ} Z \,, \end{aligned} \tag{2.1}$$

where $C_{\alpha\beta}$ and $(\Gamma_a)^{\alpha}{}_{\beta}$ stand for the charge conjugation and Dirac matrices, respectively (for our conventions see appendix A).

The existence of a nontrivial Casimir operator, given by

$$I = 2J^a P_a - Q^I_\alpha C^{\alpha\beta} Q^I_\beta - 2TZ \,, \tag{2.2}$$

allows to define an invariant bilinear form whose nonvanishing components read

$$\langle J_a, P_b \rangle = \eta_{ab} \,, \qquad \langle Q^I_\alpha, Q^J_\beta \rangle = C_{\alpha\beta} \delta^{IJ} \,, \qquad \langle T, Z \rangle = -1 \,. \tag{2.3}$$

It can be seen that this nondegenerate metric turns out to be the unique one that leads to a parity even action, once suitable parity properties of the fields are taken into account (see below).

The entire field content can be arranged within a single connection for the gauge supergroup

$$A = e^a P_a + \omega^a J_a + \psi_I^\alpha Q_\alpha^I + B\,T + C\,Z\,, \tag{2.4}$$

so that apart from the dreibein e^a, the (dualized) spin connection ω^a, and the $\mathcal{N}=2$ gravitini ψ_I^α, the theory possesses two additional U(1) fields, given by B and C, respectively being even and odd under parity.

The supergravity theory can then be described by a Chern-Simons action

$$I = \frac{k}{4\pi}\int \langle AdA + \frac{2}{3}A^3\rangle\,, \tag{2.5}$$

which by virtue of (2.3) and (2.4), reduces to

$$I = \frac{k}{4\pi}\int \left(2e^a R_a + i\bar{\psi}_I \nabla \psi_I - 2BdC\right)\,, \tag{2.6}$$

up to a boundary term.

The level and the Newton constant are related as $k=\frac{1}{4G}$, while $R^a = d\omega^a + \frac{1}{2}\epsilon^{abc}\omega_b\omega_c$ stands for the dualized curvature two-form. The covariant derivative acting on spinors reads

$$\nabla\psi_I = d\psi_I + \frac{1}{2}\omega^a\Gamma_a\psi_I + B\epsilon^{IJ}\psi_J\,. \tag{2.7}$$

The field equations then imply that the field strength

$$F = \tilde{T}^a P_a + R^a J_a + \nabla\psi_I^\alpha Q_\alpha^I + dB\,T + \tilde{F}_C\,Z \tag{2.8}$$

vanishes, where $\tilde{T}^a$ and $\tilde{F}_C$ stand for the supercovariant torsion and the supercovariant U(1) curvature along Z, respectively. They are given by

$$\tilde{T}^a = T^a - \frac{1}{4}i\bar{\psi}_I\Gamma^a\psi_I\,, \tag{2.9}$$

$$\tilde{F}_C = dC + \frac{1}{2}i\epsilon^{IJ}\bar{\psi}_I\psi_J\,, \tag{2.10}$$

with $T^a = de^a + \epsilon^{abc}\omega_b e_c$.

By construction, the action (2.6) is invariant under local supersymmetries that correspond to gauge transformations, $\delta A = d\lambda + [A,\lambda]$, spanned by a Lie-algebra-valued fermionic parameter $\lambda = \epsilon_I^\alpha Q_\alpha^I$. The nontrivial supersymmetry transformations of the fields are then given by

$$\delta e^a = \frac{1}{2}i\bar{\epsilon}_I\Gamma^a\psi_I\,, \qquad \delta\psi_I = \nabla\epsilon_I\,, \qquad \delta C = -i\epsilon^{IJ}\bar{\epsilon}_I\psi_J\,, \tag{2.11}$$

and therefore, along the lines of [44], the algebra of the local supersymmetries spanned in (2.11) can be seen to close off-shell according to super-Poincaré with $\mathcal{N}=(2,0)$ in (2.1), without the need of introducing auxiliary fields.

In the next section we perform an exhaustive analysis of the asymptotic structure of the theory.

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3 Asymptotic structure: supersymmetric extension of BMS_3 with $\mathcal{N} = (2,0)$

The analysis of the asymptotic structure of $\mathcal{N}=1$ Poincaré supergravity was carried out in [34]. Following the lines of [45] and [46], it was extended in [47] so as to incorporate a generic choice of Lagrange multipliers at infinity (for the case of fermionic fields of spin $s=\frac{3}{2}$).

Here we extend these results to the case of Poincaré supergravity with $\mathcal{N}=(2,0)$. It is then useful to change the basis of the extended super-Poincaré algebra in (2.1) according to

$$\begin{aligned} L_{-1} &= -\sqrt{2}J_0\,, & L_1 &= \sqrt{2}J_1\,, & L_0 &= J_2\,,\\ M_{-1} &= -\sqrt{2}P_0\,, & M_1 &= \sqrt{2}P_1\,, & M_0 &= P_2\,,\\ G^I_{-\frac{1}{2}} &= \sqrt{2}Q^I_+\,, & G^I_{\frac{1}{2}} &= \sqrt{2}Q^I_-\,, \end{aligned} \tag{3.1}$$

so that the superalgebra now reads

$$\begin{aligned} [L_m, L_n] &= (m-n)\,L_{m+n}\,, & [L_m, M_n] &= (m-n)\,M_{m+n}\,,\\ [L_m, G^I_p] &= \left(\frac{m}{2}-p\right) G^I_{m+p}\,, & [G^I_p, T] &= \epsilon^{IJ} G^J_p\,,\\ \{G^I_p, G^J_q\} &= \delta^{IJ} M_{p+q} - 2\,(p-q)\,\epsilon^{IJ} Z\,, \end{aligned} \tag{3.2}$$

where $m,n=\pm1,0$, and $p,q=\pm\frac{1}{2}$. Thus, the nonvanishing components of the invariant bilinear form in (2.3) reduce to

$$\begin{aligned} \langle L_1, M_{-1}\rangle = \langle L_{-1}, M_1\rangle &= -2\,, & \langle L_0, M_0\rangle &= 1\,,\\ \langle G^I_{-\frac{1}{2}}, G^J_{\frac{1}{2}}\rangle &= 2\delta^{IJ}\,, & \langle T, Z\rangle &= -1\,. \end{aligned} \tag{3.3}$$

In order to propose a suitable set of asymptotic conditions, it is useful to follow some general criteria as the ones spelled out in [48–52]. Once adapted to the theory under discussion, they are:

(*i*) The asymptotic symmetries of the set must include BMS_3 as well as the two fermionic ones.

(*ii*) The fall-off of the fields has to be relaxed enough so as to incorporate the bosonic solutions of interest.

(*iii*) The decay must simultaneously be sufficiently fast in order to ensure finiteness of the variation of the global charges.

(*iv*) The boundary conditions have to guarantee that the variation of the charges fulfills suitable functional integrability conditions.

Taking into account these four requirements, the asymptotic behaviour is proposed to be of the form

$$A = h^{-1}ah + h^{-1}dh\,, \tag{3.4}$$

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where as in [53] the group element h entirely captures the dependence on radial coordinate r, so that the auxiliary connection a depends only on the remaining ones u, ϕ.

In concrete, we choose $h = e^{\frac{r}{2}M_{-1}}$, and the spacelike component of a to be given by

$$a_\phi = L_1 - \frac{\pi}{k}\left[\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right)L_{-1} + \left(\mathcal{J} + \frac{2\pi}{k}\mathcal{T}\mathcal{Z}\right)M_{-1} + \psi G^1_{-\frac{1}{2}} + \mathcal{S}G^2_{-\frac{1}{2}} + 2\mathcal{Z}T + 2\mathcal{T}Z\right], \tag{3.5}$$

so that deviations with respect to the background configuration, which we assume to be given by the null orbifold [54], are described by arbitrary functions of u, ϕ that go along the highest weight generators. According to [45, 46], the bosonic functions $\mathcal{P}$, $\mathcal{J}$, $\mathcal{Z}$, $\mathcal{T}$, and the fermionic ones ψ, $\mathcal{S}$, correspond to the dynamical fields.

Moreover, as required by criterion (*ii*), in order to accommodate the widest possible class of bosonic solutions, the Lagrange multipliers associated to the dynamical fields have to be explicitly incorporated in the asymptotic behaviour. They turn out to be defined through the lowest weight components of

$$a_u = \Lambda[\mu_\mathcal{J}, \mu_\mathcal{P}, \mu_\psi, \mu_\mathcal{S}, \mu_\mathcal{T}, \mu_\mathcal{Z}]\,, \tag{3.6}$$

given by

$$\begin{aligned}\Lambda = {} & \mu_\mathcal{J} L_1 + \mu_\mathcal{P} M_1 + \mu_\psi G^1_{\frac{1}{2}} + \mu_\mathcal{S} G^2_{\frac{1}{2}} + \left(\mu_\mathcal{T} - \frac{2\pi}{k}\mu_\mathcal{J}\mathcal{Z}\right)T + \left(\mu_\mathcal{Z} - \frac{2\pi}{k}\mu_\mathcal{J}\mathcal{T} + \frac{8\pi}{k}\mu_\mathcal{P}\mathcal{Z}\right)Z \\ & - \mu_\mathcal{J}{}' L_0 - \mu_\mathcal{P}{}' M_0 + \left[\frac{1}{2}\mu_\mathcal{J}{}'' - \frac{\pi}{k}\mu_\mathcal{J}\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right)\right]L_{-1} \\ & + \left[\frac{1}{2}\mu_\mathcal{P}{}'' - \frac{\pi}{k}\mu_\mathcal{J}\left(\mathcal{J} + \frac{2\pi}{k}\mathcal{T}\mathcal{Z}\right) - \frac{\pi}{k}\mu_\mathcal{P}\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right) + \frac{\pi}{2k}i\psi\mu_\psi + \frac{\pi}{2k}i\mathcal{S}\mu_\mathcal{S}\right]M_{-1} \\ & - \left(\mu_\psi{}' + \frac{\pi}{k}\mu_\mathcal{J}\psi - \frac{2\pi}{k}\mathcal{Z}\mu_\mathcal{S}\right)G^1_{-\frac{1}{2}} - \left(\mu_\mathcal{S}{}' + \frac{\pi}{k}\mu_\mathcal{J}\mathcal{S} + \frac{2\pi}{k}\mathcal{Z}\mu_\psi\right)G^2_{-\frac{1}{2}}\,,\end{aligned} \tag{3.7}$$

where prime denotes ∂_ϕ.

The bosonic Lagrange multipliers $\mu_\mathcal{J}$, $\mu_\mathcal{P}$, $\mu_\mathcal{T}$, $\mu_\mathcal{Z}$ as well as the fermionic ones μ_ψ, $\mu_\mathcal{S}$, can be assumed to be given by arbitrary independent functions of u, ϕ, that are held fixed at the boundary without variation.

The asymptotic symmetries are then described by the subset of gauge transformations $\delta a = d\lambda + [a, \lambda]$ that preserve the asymptotic form of the auxiliary connection in (3.5), (3.6).

The spacelike component of a in (3.5) is maintained for Lie-algebra-valued parameters of the form

$$\lambda = \Lambda[\epsilon_\mathcal{J}, \epsilon_\mathcal{P}, \epsilon_\psi, \epsilon_\mathcal{S}, \epsilon_\mathcal{T}, \epsilon_\mathcal{Z}]\,, \tag{3.8}$$

provided that the transformation law of the dynamical fields is given by

$$
\begin{aligned}
\delta\mathcal{P} &= 2\mathcal{P}\epsilon_{\mathcal{J}}{}' + \mathcal{P}'\epsilon_{\mathcal{J}} - \frac{k}{2\pi}\epsilon_{\mathcal{J}}{}''' - 4\mathcal{Z}\epsilon_{\mathcal{T}}{}' \,, \\
\delta\mathcal{J} &= 2\mathcal{J}\epsilon_{\mathcal{J}}{}' + \mathcal{J}'\epsilon_{\mathcal{J}} + 2\mathcal{P}\epsilon_{\mathcal{P}}{}' + \mathcal{P}'\epsilon_{\mathcal{P}} - \frac{k}{2\pi}\epsilon_{\mathcal{P}}{}''' \\
&\quad + \mathcal{Z}\epsilon_{\mathcal{Z}}{}' + \mathcal{T}\epsilon_{\mathcal{T}}{}' - \frac{3}{2}i\psi\epsilon_{\psi}{}' - \frac{1}{2}i\psi'\epsilon_{\psi} - \frac{3}{2}i\mathcal{S}\epsilon_{\mathcal{S}}{}' - \frac{1}{2}i\mathcal{S}'\epsilon_{\mathcal{S}} \\
\delta\psi &= \frac{3}{2}\psi\epsilon_{\mathcal{J}}{}' + \psi'\epsilon_{\mathcal{J}} - \mathcal{S}\epsilon_{\mathcal{T}} - \mathcal{P}\epsilon_{\psi} + \frac{k}{\pi}\epsilon_{\psi}{}'' - 2\mathcal{Z}'\epsilon_{\mathcal{S}} - 4\mathcal{Z}\epsilon_{\mathcal{S}}{}' \,, \\
\delta\mathcal{S} &= \frac{3}{2}\mathcal{S}\epsilon_{\mathcal{J}}{}' + \mathcal{S}'\epsilon_{\mathcal{J}} + \psi\epsilon_{\mathcal{T}} - \mathcal{P}\epsilon_{\mathcal{S}} + \frac{k}{\pi}\epsilon_{\mathcal{S}}{}'' + 2\mathcal{Z}'\epsilon_{\psi} + 4\mathcal{Z}\epsilon_{\psi}{}' \,, \\
\delta\mathcal{T} &= -i\mathcal{S}\epsilon_{\psi} + i\psi\epsilon_{\mathcal{S}} - \frac{k}{2\pi}\epsilon_{\mathcal{Z}}{}' + \epsilon_{\mathcal{J}}{}'\mathcal{T} + \epsilon_{\mathcal{J}}\mathcal{T}' - 4\epsilon_{\mathcal{P}}{}'\mathcal{Z} - 4\epsilon_{\mathcal{P}}\mathcal{Z}' \,, \\
\delta\mathcal{Z} &= -\frac{k}{2\pi}\epsilon_{\mathcal{T}}{}' + \epsilon_{\mathcal{J}}{}'\mathcal{Z} + \epsilon_{\mathcal{J}}\mathcal{Z}' \,.
\end{aligned}
\tag{3.9}
$$

Note that Λ in (3.8) is precisely the same as in (3.7), but now depends on arbitrary bosonic and fermionic functions of u, ϕ, given by $\epsilon_{\mathcal{J}}$, $\epsilon_{\mathcal{P}}$, $\epsilon_{\mathcal{T}}$, $\epsilon_{\mathcal{Z}}$, and ϵ_{ψ}, $\epsilon_{\mathcal{S}}$, respectively.

Preserving the form of a_u then implies that the field equations have to hold at the asymptotic region, and also provides additional suitable conditions for the parameters that span the asymptotic symmetries. In the reduce phase space, the field equations then read

$$
\begin{aligned}
\dot{\mathcal{P}} &= 2\mathcal{P}\mu_{\mathcal{J}}{}' + \mathcal{P}'\mu_{\mathcal{J}} - \frac{k}{2\pi}\mu_{\mathcal{J}}{}''' - 4\mathcal{Z}\mu_{\mathcal{T}}{}' \,, \\
\dot{\mathcal{J}} &= 2\mathcal{J}\mu_{\mathcal{J}}{}' + \mathcal{J}'\mu_{\mathcal{J}} + 2\mathcal{P}\mu_{\mathcal{P}}{}' + \mathcal{P}'\mu_{\mathcal{P}} - \frac{k}{2\pi}\mu_{\mathcal{P}}{}''' \\
&\quad + \mathcal{Z}\mu_{\mathcal{Z}}{}' + \mathcal{T}\mu_{\mathcal{T}}{}' - \frac{3}{2}i\psi\mu_{\psi}{}' - \frac{1}{2}i\psi'\mu_{\psi} - \frac{3}{2}i\mathcal{S}\mu_{\mathcal{S}}{}' - \frac{1}{2}i\mathcal{S}'\mu_{\mathcal{S}} \\
\dot{\psi} &= \frac{3}{2}\psi\mu_{\mathcal{J}}{}' + \psi'\mu_{\mathcal{J}} - \mathcal{S}\mu_{\mathcal{T}} - \mathcal{P}\mu_{\psi} + \frac{k}{\pi}\mu_{\psi}{}'' - 2\mathcal{Z}'\mu_{\mathcal{S}} - 4\mathcal{Z}\mu_{\mathcal{S}}{}' \,, \\
\dot{\mathcal{S}} &= \frac{3}{2}\mathcal{S}\mu_{\mathcal{J}}{}' + \mathcal{S}'\mu_{\mathcal{J}} + \psi\mu_{\mathcal{T}} - \mathcal{P}\mu_{\mathcal{S}} + \frac{k}{\pi}\mu_{\mathcal{S}}{}'' + 2\mathcal{Z}'\mu_{\psi} + 4\mathcal{Z}\mu_{\psi}{}' \,, \\
\dot{\mathcal{T}} &= -i\mathcal{S}\mu_{\psi} + i\psi\mu_{\mathcal{S}} - \frac{k}{2\pi}\mu_{\mathcal{Z}}{}' + \mu_{\mathcal{J}}{}'\mathcal{T} + \mu_{\mathcal{J}}\mathcal{T}' - 4\mu_{\mathcal{P}}{}'\mathcal{Z} - 4\mu_{\mathcal{P}}\mathcal{Z}' \,, \\
\dot{\mathcal{Z}} &= -\frac{k}{2\pi}\mu_{\mathcal{T}}{}' + \mu_{\mathcal{J}}{}'\mathcal{Z} + \mu_{\mathcal{J}}\mathcal{Z}' \,,
\end{aligned}
\tag{3.10}
$$

which can be readily obtained from (3.9) by taking into account that time evolution is generated by gauge transformations whose parameters correspond to the Lagrange multipliers. The conditions for the parameters are explicitly given by

$$
\begin{aligned}
\dot{\epsilon}_{\mathcal{J}} &= \mu_{\mathcal{J}}\epsilon_{\mathcal{J}}{}' - \mu_{\mathcal{J}}{}'\epsilon_{\mathcal{J}} \,, \\
\dot{\epsilon}_{\mathcal{P}} &= \mu_{\mathcal{P}}\epsilon_{\mathcal{J}}{}' + \mu_{\mathcal{J}}\epsilon_{\mathcal{P}}{}' - \mu_{\mathcal{J}}{}'\epsilon_{\mathcal{P}} - \mu_{\mathcal{P}}{}'\epsilon_{\mathcal{J}} - i\mu_{\mathcal{S}}\epsilon_{\mathcal{S}} - i\mu_{\psi}\epsilon_{\psi} \,, \\
\dot{\epsilon}_{\psi} &= \mu_{\mathcal{J}}\epsilon_{\psi}{}' + \frac{1}{2}\mu_{\psi}\epsilon_{\mathcal{J}}{}' - \frac{1}{2}\mu_{\mathcal{J}}{}'\epsilon_{\psi} - \mu_{\psi}{}'\epsilon_{\mathcal{J}} + \mu_{\mathcal{S}}\epsilon_{\mathcal{T}} - \mu_{\mathcal{T}}\epsilon_{\mathcal{S}} \,, \\
\dot{\epsilon}_{\mathcal{S}} &= \mu_{\mathcal{J}}\epsilon_{\mathcal{S}}{}' + \frac{1}{2}\mu_{\mathcal{S}}\epsilon_{\mathcal{J}}{}' - \frac{1}{2}\mu_{\mathcal{J}}{}'\epsilon_{\mathcal{S}} - \mu_{\mathcal{S}}{}'\epsilon_{\mathcal{J}} + \mu_{\mathcal{T}}\epsilon_{\psi} - \mu_{\psi}\epsilon_{\mathcal{T}} \,, \\
\dot{\epsilon}_{\mathcal{Z}} &= \mu_{\mathcal{J}}\epsilon_{\mathcal{Z}}{}' - \mu_{\mathcal{Z}}{}'\epsilon_{\mathcal{J}} - 4\mu_{\mathcal{P}}\epsilon_{\mathcal{T}}{}' + 4\mu_{\mathcal{T}}{}'\epsilon_{\mathcal{P}} - 2i\mu_{\psi}\epsilon_{\mathcal{S}}{}' + 2i\mu_{\mathcal{S}}\epsilon_{\psi}{}' - 2i\mu_{\mathcal{S}}{}'\epsilon_{\psi} + 2i\mu_{\psi}{}'\epsilon_{\mathcal{S}} \,, \\
\dot{\epsilon}_{\mathcal{T}} &= \mu_{\mathcal{J}}\epsilon_{\mathcal{T}}{}' - \mu_{\mathcal{T}}{}'\epsilon_{\mathcal{J}} \,,
\end{aligned}
\tag{3.11}
$$

and they can be seen to ensure that the variation of the canonical generators is conserved. This is discussed next.

3.1 Canonical generators and their algebra

In the canonical approach [55], the variation of the surface integrals that define the global charges is given by

$$\delta Q[\lambda] = -\frac{k}{2\pi}\int \langle \lambda \delta a_\phi \rangle d\phi \,, \tag{3.12}$$

which by virtue of (3.3), (3.5), (3.8), readily integrates as

$$Q[\epsilon_{\mathcal{J}}, \epsilon_{\mathcal{P}}, \epsilon_{\psi}, \epsilon_{S}, \epsilon_{\mathcal{T}}, \epsilon_{\mathcal{Z}}] = -\int \left(\epsilon_{\mathcal{J}}\mathcal{J} + \epsilon_{\mathcal{P}}\mathcal{P} + i\epsilon_{\psi}\psi + i\epsilon_{\mathcal{S}}\mathcal{S} + \epsilon_{\mathcal{T}}\mathcal{T} + \epsilon_{\mathcal{Z}}\mathcal{Z}\right) d\phi \,. \tag{3.13}$$

The asymptotic symmetry algebra can then be obtained from the direct evaluation of the Poisson brackets of the global charges in (3.13). As a shortcut, if one takes into account the variation of the dynamical fields in (3.9), the asymptotic symmetry algebra can be directly read from $\delta_{\lambda_2} Q[\lambda_1\} = \{Q[\lambda_1], Q[\lambda_2]\}$, so that once expanding in Fourier modes according to $X_n = \int X e^{-in\phi} d\phi$, it is given by

$$\begin{aligned}
i\{\mathcal{J}_m, \mathcal{J}_n\} &= (m-n)\,\mathcal{J}_{m+n}\,,\\
i\{\mathcal{J}_m, \mathcal{P}_n\} &= (m-n)\,\mathcal{P}_{m+n} + km^3\delta_{m+n,0}\,,\\
i\{\mathcal{J}_m, \mathcal{G}^I_p\} &= \left(\frac{m}{2} - p\right)\mathcal{G}^I_{m+p}\,,\\
i\{\mathcal{J}_m, \mathcal{Z}_n\} &= -n\mathcal{Z}_{m+n}\,,\\
i\{\mathcal{J}_m, \mathcal{T}_n\} &= -n\mathcal{T}_{m+n}\,,\\
i\{\mathcal{P}_m, \mathcal{T}_n\} &= 4n\mathcal{Z}_{m+n}\,,\\
i\{\mathcal{T}_m, \mathcal{Z}_n\} &= -km\delta_{m+n,0}\,,\\
i\{\mathcal{G}^I_p, \mathcal{T}_m\} &= i\epsilon^{IJ}\mathcal{G}^J_{m+p}\,,\\
i\{\mathcal{G}^I_p, \mathcal{G}^J_q\} &= \delta^{IJ}\left(\mathcal{P}_{p+q} + 2kp^2\delta_{p+q,0}\right) + 2i\epsilon^{IJ}\left(p-q\right)\mathcal{Z}_{p+q}\,,
\end{aligned} \tag{3.14}$$

where $\mathcal{G}^1 = \psi$ and $\mathcal{G}^2 = \mathcal{S}$.

Here, m, n stand for integers, and p, q are given by (half-)integers when fermions fulfill (anti)periodic boundary conditions, while the reality conditions of the modes are given by $(\mathcal{J}_m)^* = \mathcal{J}_{-m}$, $(\mathcal{P}_m)^* = \mathcal{P}_{-m}$, $(\mathcal{T}_m)^* = \mathcal{T}_{-m}$, $(\mathcal{Z}_m)^* = \mathcal{Z}_{-m}$, $(\mathcal{G}^I_m)^* = \mathcal{G}^I_{-m}$.

The asymptotic symmetry algebra (3.14) then manifestly contains BMS_3 with the central extension found in [16] as the subalgebra spanned by $\mathcal{J}_m$ and $\mathcal{P}_m$. The remaining part of the bosonic subalgebra[1] consists on two commuting independent affine algebras generated by the spin-one currents $J^{\pm}_m = \frac{1}{\sqrt{2}}\left(\mathcal{Z}_m \pm \mathcal{T}_m\right)$, whose level is determined by k.

The super-Poincaré algebra in (3.2) tuns out to be a subalgebra of (3.14) in the antiperiodic case, being spanned by $\left\{\mathcal{J}_m, \mathcal{P}_n, \mathcal{G}^{(i)}_p, \mathcal{Z}_0, \mathcal{T}_0\right\}$ provided the labels are restricted according to $m, n = \pm 1, 0$ and $p, q = \pm\frac{1}{2}$, and the supertranslation generator $\mathcal{P}_0$ is shifted

[1] As an interesting remark, it is worth pointing that the bosonic subalgebra of (3.14) coincides with the one obtained in [56] for pure gravity with a nonstandard set of boundary conditions, which also corresponds to the vanishing cosmological constant limit of the asymptotic symmetry algebra found in [57].

as $\mathcal{P}_0 \to \mathcal{P}_0 + \frac{k}{2}$. The explicit matching is recovered provided that $L_m = \mathcal{J}_m$, $M_m = \mathcal{P}_m$, $G_p^{(i)} = \mathcal{G}_p^{(i)}$, $T = -\mathcal{T}_0$ and $Z = -\mathcal{Z}_0$.

3.1.1 Asymptotic symmetry algebra from a truncation of the homogeneous contraction of the superconformal algebra with $\mathcal{N} = (2, 2)$

It is simple to verify that the supersymmetric extension of the BMS_3 algebra with $\mathcal{N} = (2,0)$ in (3.14), can be recovered from a homogeneous (democratic) or ultra-relativistic Inönü-Wigner contraction of the superconformal algebra with $\mathcal{N} = (2,2)$, provided that the fermionic generators of the right copy are truncated (see e.g., [40]). Indeed, this is so regardless the truncation is performed before or after the contraction process. This can be seen as follows.

The superconformal algebra with $\mathcal{N} = (2,2)$ is given by two independent (left and right) copies of the super-Virasoro algebra with $\mathcal{N} = 2$, which reads

$$
\begin{aligned}
i\{\mathcal{L}_m, \mathcal{L}_n\} &= (m-n)\,\mathcal{L}_{m+n} + \frac{\kappa}{2} m^3 \delta_{m+n,0}\,, && (3.15)\\
i\{\mathcal{L}_m, \mathcal{Q}_p^I\} &= \left(\frac{m}{2} - p\right) \mathcal{Q}_{m+p}^I\,, && (3.16)\\
i\{\mathcal{L}_m, \mathcal{R}_n\} &= -n\mathcal{R}_{m+n}\,, && (3.17)\\
i\{\mathcal{R}_m, \mathcal{R}_n\} &= 2\kappa m \delta_{m+n,0}\,, && (3.18)\\
i\{\mathcal{Q}_p^I, \mathcal{R}_m\} &= -i\epsilon^{IJ} \mathcal{Q}_{m+p}^J\,, && (3.19)\\
i\{\mathcal{Q}_p^I, \mathcal{Q}_q^J\} &= \delta^{IJ}\left(\mathcal{L}_{p+q} + \kappa p^2 \delta_{p+q,0}\right) + \frac{1}{2} i\epsilon^{IJ}(p-q)\mathcal{R}_{p+q}\,. && (3.20)
\end{aligned}
$$

It is then useful to change the basis according to

$$
\begin{aligned}
\mathcal{J}_m &= \mathcal{L}_m^+ - \mathcal{L}_{-m}^-\,, & \mathcal{P}_m &= \frac{1}{\ell}\left(\mathcal{L}_m^+ + \mathcal{L}_{-m}^-\right)\,, && (3.21)\\
\mathcal{T}_m &= -\left(\mathcal{R}_m^+ - \mathcal{R}_{-m}^-\right)\,, & \mathcal{Z}_m &= \frac{1}{4\ell}\left(\mathcal{R}_m^+ + \mathcal{R}_{-m}^-\right)\,, && (3.22)\\
\mathcal{G}_p^{+I} &= \sqrt{\frac{2}{\ell}}\mathcal{Q}_r^{+I}\,, & \mathcal{G}_p^{-I} &= \sqrt{\frac{2}{\ell}}\mathcal{Q}_{-r}^{-I}\,, && (3.23)
\end{aligned}
$$

so that the contraction process is performed through rescaling the level as $\kappa = k\ell$, and then taking the limit $\ell \to \infty$. It is then clear that if one truncates the fermionic generators of the right copy, the super-BMS_3 algebra with $\mathcal{N} = (2,0)$ in (3.14) is recovered with $\mathcal{G}_p^I = \mathcal{G}_p^{+I}$.

As a final remark of this section, it is also worth pointing out that there is a different, so-called inhomogeneous, or "despotic", Inönü-Wigner contraction of the superconformal algebra with $\mathcal{N} = (2,2)$, which leads to an inequivalent supersymmetric extension of the super-BMS_3 algebra with $\mathcal{N} = (2,0)$. This alternative possibility has been simultaneously analyzed in the context of the asymptotic structure of a different theory in [58].

4 Nonlinear energy bounds from super-BMS_3 with $\mathcal{N} = (2, 0)$

Supersymmetric bounds for different definitions of the global charges in the context of the supergravity theory under consideration, have been previously discussed in [41, 59]. In this section we show that the Poisson brackets of the fermionic generators of the asymptotic supersymmetries in (3.14), in spite of being linear in the bosonic generators, yield an infinite number of nonlinear bounds for the energy. Only a finite number of them are able to saturate, precisely corresponding to the same number of unbroken supersymmetries. In order to carry out this task, as explained in [46], generic bosonic configurations can be brought to the "rest frame" by acting on them with a suitable combination of the asymptotic symmetries. Therefore, it is enough to focus in the case of bosonic configurations endowed with zero mode charges, given by $\mathcal{P}_0 = 2\pi\mathcal{P}$ and $\mathcal{Z}_0 = 2\pi\mathcal{Z}$, regardless the value of the remaining ones ($\mathcal{J}$ and $\mathcal{T}$). The supersymmetric bounds we look for can then be obtained along the semi-classical reasoning in [60–66]. In particular, we follow a similar strategy as the one in [67]. The fermionic Poisson brackets in (3.14) are then promoted to anticommutators. In the case of $p = -q = r$ they become

$$(2\pi)^{-1}\left(\hat{\mathcal{G}}_r^I\hat{\mathcal{G}}_{-r}^J + \hat{\mathcal{G}}_{-r}^J\hat{\mathcal{G}}_r^I\right) = \delta^{IJ}\left(\hat{\mathcal{P}} + \frac{k}{\pi}r^2\right) + 4ir\epsilon^{IJ}\hat{\mathcal{Z}} = B_r^{IJ}\,, \tag{4.1}$$

so that $(B_r^{IJ})^\dagger = B_r^{JI}$.

Therefore, when $I = J$, the left hand side of (4.1) is a positive definite hermitian operator for any value of r and I, which implies that in the classical limit, the bosonic charges have to fulfill the following bounds

$$B_r^{II} = \frac{k}{\pi}r^2 + \mathcal{P} \geq 0\,. \tag{4.2}$$

In the case of periodic boundary conditions for the fermionic fields, the strongest bound in (4.2) corresponds to $r = 0$, which implies that the energy is nonnegative ($\mathcal{P} \geq 0$). Analogously, for antiperiodic boundary conditions, the strongest bound is given by $r = \pm\frac{1}{2}$, so that the energy becomes bounded from below according to $\mathcal{P} \geq -\frac{k}{4\pi}$.

Additional bounds also arise in the case of $I \neq J$. In order to obtain them, it is useful to define the following complex fields

$$\hat{\mathcal{G}}_r^{\pm} := \frac{1}{\sqrt{2}}\left(\hat{\mathcal{G}}_r^1 \pm i\hat{\mathcal{G}}_r^2\right)\,, \tag{4.3}$$

which fulfill $\hat{\mathcal{G}}_r^{\pm}\left(\hat{\mathcal{G}}_r^{\pm}\right)^\dagger + \left(\hat{\mathcal{G}}_r^{\pm}\right)^\dagger\hat{\mathcal{G}}_r^{\pm} \geq 0$. The latter bounds then imply that $B_r^{11} \geq \pm iB_r^{12}$, which by virtue of (4.1) yields

$$r^2 \pm \frac{4\pi}{k}r\mathcal{Z} + \frac{\pi}{k}\mathcal{P} \geq 0\,. \tag{4.4}$$

It is convenient to factorize the bounds in (4.4) according to

$$\left(r \pm \lambda_{[+]}\right)\left(r \pm \lambda_{[-]}\right) \geq 0\,, \tag{4.5}$$

with

$$\lambda_{[\pm]} = -\frac{2\pi}{k}\mathcal{Z} \pm \sqrt{\frac{\pi}{k}\left(\frac{4\pi}{k}\mathcal{Z}^2 - \mathcal{P}\right)}\,, \tag{4.6}$$

where $\Lambda_{[\pm]} = i\lambda_{[\pm]}$ correspond to the eigenvalues of the spacelike components of the $sl(2,R) \oplus u(1)$ connection $\hat{\omega} = \omega + B$ that minimally couples to the fermionic fields (see section eq. (5.15) below).

It is then clear from (4.6) that for periodic or antiperiodic boundary conditions, in the case of $\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2 > 0$, the bounds in (4.5) are automatically fulfilled and never saturated.

In the remaining possibility, $\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2 \leq 0$, the bounds in (4.5) can be satisfied only provided that $\lambda_{[+]} - \lambda_{[-]} \leq 1$, which by virtue of (4.6) implies $\mathcal{P} \geq -\frac{k}{4\pi} + \frac{4\pi}{k}\mathcal{Z}^2$.

In sum, taking into account the infinite number of bounds in (4.2) and (4.5), one deduces that the stronger ones imply that energy has to be bounded from below as follows.

Antiperiodic boundary conditions: $\mathcal{P} \geq -\frac{k}{4\pi} + \frac{4\pi}{k}\mathcal{Z}^2$

Periodic boundary conditions: $\mathcal{P} \geq \begin{cases} 0 & , \quad |\mathcal{Z}| < \frac{k}{4\pi} \\ -\frac{k}{4\pi} + \frac{4\pi}{k}\mathcal{Z}^2 & , \quad |\mathcal{Z}| \geq \frac{k}{4\pi} \end{cases}$

As a closing remark of this section, it is worth emphasizing that, although the algebra of the asymptotic symmetries is a linear one, the lower bounds for energy turn out to be quadratic in the electric-like $\hat{u}(1)$ charge.

5 Bosonic configurations and some of their properties

Here we explore some properties of stationary spherically symmetric bosonic solutions that fit within the asymptotic fall off described in section 3. The solutions are endowed just with zero-mode bosonic charges, and hence they are described through dynamical gauge fields given by

$$a_\phi = L_1 - \frac{\pi}{k}\left[\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right)L_{-1} + \left(\mathcal{J} + \frac{2\pi}{k}\mathcal{T}\mathcal{Z}\right)M_{-1} + 2\mathcal{Z}T + 2\mathcal{T}Z\right], \tag{5.1}$$

with $\mathcal{J}$, $\mathcal{P}$, $\mathcal{T}$, $\mathcal{Z}$, constants.

In the absence of fermionic charges, it is consistent to switch off their corresponding Lagrange multipliers; i.e., the "chemical potentials" μ_ψ, μ_S, can be set to vanish. For the sake of simplicity, the remaining bosonic ones, given by $\mu_\mathcal{J}$, $\mu_\mathcal{P}$, $\mu_\mathcal{T}$, $\mu_\mathcal{Z}$, are assumed to be constants and held fixed without variation at the boundary. Therefore, the timelike component of the gauge field in (3.6) reduces to

$$\begin{aligned} a_u = {} & \mu_\mathcal{J} L_1 + \mu_\mathcal{P} M_1 + \left(\mu_\mathcal{T} - \frac{2\pi}{k}\mu_\mathcal{J}\mathcal{Z}\right)T + \left(\mu_\mathcal{Z} - \frac{2\pi}{k}\mu_\mathcal{J}\mathcal{T} + \frac{8\pi}{k}\mu_\mathcal{P}\mathcal{Z}\right)Z \\ & - \left[\frac{\pi}{k}\mu_\mathcal{J}\left(\mathcal{J} + \frac{2\pi}{k}\mathcal{T}\mathcal{Z}\right) + \frac{\pi}{k}\mu_\mathcal{P}\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right)\right]M_{-1} - \frac{\pi}{k}\mu_\mathcal{J}\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right)L_{-1}\,. \end{aligned} \tag{5.2}$$

5.1 Regularity conditions

In the case of regular solutions, their smoothness can be established through the fact that the holonomy of the gauge fields along a contractible cycle $\mathcal{C}$ has to be trivial, i.e.,

$$H_\mathcal{C} = Pe^{\int_\mathcal{C} a_\mu dx^\mu} = \Gamma^\pm\,, \tag{5.3}$$

where the sign of $\Gamma^{\pm}$ corresponds to different choices of spin structures (see, e.g, [68]). Indeed, Γ^{+} belongs to the center of the group, but this is not the case for Γ^{-}, since for antiperiodic boundary condition it must anticommute with the fermionic generators, i.e., $\{\Gamma^{-}, G_p^I\} = 0$.

In order to evaluate the regularity condition (5.3), it is worth pointing out that neither the Poincaré algebra nor the super-Poincaré algebra with $\mathcal{N} = (2, 0)$ in (2.1) possess a suitable standard matrix representation from which neither the invariant bilinear form in (2.3) nor the Casimir operator in (2.2) can be obtained from the trace of a product of two generators. Hence, regularity cannot be directly carried out through "diagonalizing" the holonomy in (5.3). Nonetheless, this task might be carried out by virtue of a nonstandard matrix representation [69] along the lines of [70, 71]. Hereafter, regularity conditions are implemented according to [72], which possesses the advantage of being independent of the existence of a suitable matrix representation for the entire gauge group. Once adapted to super-Poincaré with $\mathcal{N} = (2, 0)$, one proceeds as follows:

(i) One begins finding a group element of the form $g = e^{\lambda_n M_n + \lambda Z}$ that allows gauging away the components of the gauge field along M_n and Z projected over the contractible cycle, so that the corresponding components of the dreibein e and the U(1) field C can be consistently set to vanish. Consequently, the Lagrange multipliers of electric type become generically fixed in terms of the ones of magnetic type and the global charges.

(ii) Regularity conditions can then be straightforwardly evaluated through the diagonalization of the holonomy matrix along the contractible cycle for the fundamental representation of the remaining $sl(2, R) \oplus u(1)$ connection $\hat{\omega} = \omega + B$.

In order to continue with the analysis of the bosonic solutions under consideration, it is necessary to identify the suitable contractible cycles along which the regularity conditions have to be applied. Indeed, these cycles might correspond either to the circles along Euclidean time or the ones for the angular coordinate. These are the cases of cosmological spacetimes or solitonic-like solutions, respectively. This is discussed in what follows.

5.2 Metric formalism

The spacetime metric can be readily constructed from identifying the dreibein from the components of the full gauge field A in (3.4) along the generators P_a. The dreibein then reads

$$e^0 = \frac{\sqrt{2}\pi}{k}\left(\mathcal{J} + \frac{2\pi}{k}\mathcal{T}\mathcal{Z}\right)(d\phi + \mu_{\mathcal{J}} du) - \frac{1}{\sqrt{2}}dr + \frac{\sqrt{2}\pi}{k}\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right)\mu_{\mathcal{P}} du\,, \quad (5.4)$$

$$e^1 = \sqrt{2}\mu_{\mathcal{P}} du\,, \quad (5.5)$$

$$e^2 = r\left(d\phi + \mu_{\mathcal{J}} du\right)\,, \quad (5.6)$$

so that the line element is recovered in outgoing null coordinates, as (see e.g., [70, 72])

$$ds^2 = -\frac{4\pi}{k}\left(\frac{\pi\mathcal{N}^2}{kr^2} - \mathcal{M}\right)\mu_{\mathcal{P}}^2 du^2 - 2\mu_{\mathcal{P}} dr du + r^2\left[d\phi + \left(\mu_{\mathcal{J}} + \frac{2\pi\mu_{\mathcal{P}}\mathcal{N}}{kr^2}\right)du\right]^2\,, \quad (5.7)$$

where the integration constants $\mathcal{M}$, $\mathcal{N}$ are related to the global charges according to eqs. (5.10), (5.11). The entire configuration is then determined by the metric in (5.7) together with the U(1) fields of electric and magnetic type, given by

$$B = -\frac{2\pi}{k}\mathcal{Z}\left(d\phi + \mu_{\mathcal{J}}du\right) + \mu_{\mathcal{T}}du\,, \tag{5.8}$$

$$C = -\frac{2\pi}{k}\mathcal{T}\left(d\phi + \mu_{\mathcal{J}}du\right) + \left(\mu_{\mathcal{Z}} + \frac{8\pi}{k}\mu_{\mathcal{P}}\mathcal{Z}\right)du\,, \tag{5.9}$$

respectively.

It is worth pointing out that the spacetime metric does not acquire a back reaction due to the presence of the U(1) fields. Indeed, as it can be seen from the action in (2.6), this has to be so because, as they are described by a "BF"-type of Lagrangian, they do not couple to the metric and hence they cannot contribute to the stress-energy tensor. Nonetheless, their presence does not go unnoticed because they manifestly contribute to the energy and the angular momentum of the configuration, given by

$$\mathcal{P} = \mathcal{M} + \frac{4\pi}{k}\mathcal{Z}^2\,, \tag{5.10}$$

$$\mathcal{J} = \mathcal{N} - \frac{2\pi}{k}\mathcal{T}\mathcal{Z}\,, \tag{5.11}$$

where $\mathcal{Z}$ and $\mathcal{T}$ stand for the U(1) charges of electric and magnetic type, respectively.

It is also amusing to verify that static configurations, for which $\mathcal{N} = 0$, are able to carry a nontrivial angular momentum, because in the "dyonic" case the product of the electric and magnetic U(1) charges manifestly contribute to $\mathcal{J}$ in (5.11).

Regularity of the configurations can also be analyzed from demanding smoothness of the spacetime metric, so that $\mu_{\mathcal{P}}$ would be related to the inverse of the Hawking temperature ($\mu_{\mathcal{P}} = -\beta$), and $\mu_{\mathcal{J}}$ to the chemical potential associated to the angular momentum $\mathcal{J}$, when it corresponds.

5.3 Cosmological configurations and their thermodynamics

In this subsection we discuss the class of stationary spherically symmetric bosonic solutions for the case $\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2 \geq 0$, which clearly fulfills the energy bounds in section 4. The line element generically describes the class of locally flat cosmological spacetimes discussed in [73–75] endowed with a generic choice of chemical potentials as in [70, 72], and reduces to the null orbifold in [54] for $\mathcal{P} = \frac{4\pi}{k}\mathcal{Z}^2$. The thermodynamics of cosmological spacetimes in vacuum has been analyzed in [70, 72, 76, 77]. Here we extend the analysis to the case of cosmological spacetimes endowed with U(1) charges of electric and magnetic type.

As explained in [72] the Euclidean geometry possesses the topology of a solid torus, so that the circles spanned by the Euclidean time $\tau = -iu$, correspond to contractible cycles, and the orientation is reversed as compared with the one of a BTZ black hole [78, 79]. Following refs. [45] and [46], the solid torus can be chosen to be parametrized as a "straight" one, i.e., described by a fixed range of the coordinates, $0 \leq \tau < 1$ and $0 \leq \phi < 2\pi$. In this way, the Hawking temperature and the chemical potential associated to the angular momentum explicitly appear in the metric through $\mu_{\mathcal{P}}$ and $\mu_{\mathcal{J}}$, respectively. Thus, all

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of the chemical potentials, including the ones for the U(1) charges, given by $\mu_{\mathcal{Z}}$ and $\mu_{\mathcal{T}}$ become treated in the same footing. It is worth pointing out that the chemical potentials of the U(1) charges have to be switched on, otherwise an interesting class of physical solutions would fail to be regular.

Regularity of the configurations then implies that the holonomy of the connection along a thermal circle has to be trivial. As explained in section 5.1, according to criterion (i) we apply a suitable gauge transformation given by $g = e^{\lambda_0 M_0}$, with λ_0 constant, so that the timelike component of the connection (5.2) transforms as

$$\begin{aligned} a_u^g = {} & \mu_{\mathcal{J}} L_1 - \frac{\pi}{k}\mu_{\mathcal{J}}\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right) L_{-1} + \left(\mu_{\mathcal{P}} + \lambda_0 \mu_{\mathcal{J}}\right) M_1 \\ & + \left(\mu_{\mathcal{T}} - \frac{2\pi}{k}\mu_{\mathcal{J}}\mathcal{Z}\right) T + \left(\mu_{\mathcal{Z}} - \frac{2\pi}{k}\mu_{\mathcal{J}}\mathcal{T} + \frac{8\pi}{k}\mu_{\mathcal{P}}\mathcal{Z}\right) Z \\ & - \left[\frac{\pi}{k}\mu_{\mathcal{J}}\left(\mathcal{J} + \frac{2\pi}{k}\mathcal{T}\mathcal{Z}\right) + \frac{\pi}{k}\left(\mu_{\mathcal{P}} - \lambda_0\mu_{\mathcal{J}}\right)\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right)\right] M_{-1} . \end{aligned} \tag{5.12}$$

Therefore, the components of a_u^g along M_n and Z are gauged away for $\lambda_0 = -\frac{\mu_{\mathcal{P}}}{\mu_{\mathcal{J}}}$, provided that the chemical potentials of electric type, $\mu_{\mathcal{P}}$ and $\mu_{\mathcal{Z}}$, are fixed in terms of the magnetic one $\mu_{\mathcal{J}}$ and the global charges.

$$\mu_{\mathcal{P}} = -\frac{\mu_{\mathcal{J}}\left(\mathcal{J} + \frac{2\pi}{k}\mathcal{T}\mathcal{Z}\right)}{2\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right)} , \qquad \mu_{\mathcal{Z}} = \frac{2\pi}{k}\mu_{\mathcal{J}}\mathcal{T} - \frac{8\pi}{k}\mu_{\mathcal{P}}\mathcal{Z} . \tag{5.13}$$

Note that the gauge transformation spanned by $g = e^{-\frac{\mu_{\mathcal{P}}}{\mu_{\mathcal{J}}} M_0}$ depends only on chemical potentials that are kept fixed at the boundary, and hence it turns out to be a permissible one in the sense of [46]. The connection a_u^g then reduces to the following one for the $sl(2,R) \oplus u(1)$ algebra

$$a_u^g = \mu_{\mathcal{J}}\left[L_1 - \frac{\pi}{k}\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right) L_{-1}\right] + \left(\mu_{\mathcal{T}} - \frac{2\pi}{k}\mu_{\mathcal{J}}\mathcal{Z}\right) T . \tag{5.14}$$

The remaining regularity condition (ii) then corresponds to demanding that (5.14) possesses a trivial holonomy, i.e., $H_{\mathcal{C}} = \pm\mathbb{I}_{2\times 2}$, which can be readily performed in the fundamental representation of $sl(2,R) \oplus u(1)$ (see appendix A). Therefore, the eigenvalues of $a_\tau^g = i a_u^g$, given by

$$\Lambda_{[\pm]} = \frac{1}{2}\left(\mathrm{tr}\left[\hat{\omega}\right] \pm \sqrt{2\mathrm{tr}\left[\hat{\omega}^2\right] - \mathrm{tr}\left[\hat{\omega}\right]^2}\right) , \tag{5.15}$$

with $\hat{\omega} = a_\tau^g$, have to be given by $\Lambda_{[\pm]} = \pm i\pi m$, with m an arbitrary integer (even for $\mathbb{I}_{2\times 2}$, and odd for $-\mathbb{I}_{2\times 2}$). The regularity condition then reduces to

$$\pm\, i\pi m = -\left(\mu_{\mathcal{T}} - \frac{2\pi}{k}\mu_{\mathcal{J}}\mathcal{Z}\right) \pm i\sqrt{\frac{\pi}{k}}|\mu_{\mathcal{J}}|\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right)^{1/2} , \tag{5.16}$$

which implies that

$$\mu_{\mathcal{T}} = \frac{2\pi}{k}\mu_{\mathcal{J}}\mathcal{Z} , \qquad |\mu_{\mathcal{J}}| = \frac{m\sqrt{\pi k}}{\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right)^{1/2}} . \tag{5.17}$$

In sum, regularity implies that the chemical potentials become fixed in terms of the global charges according to

$$\begin{aligned}
\mu_{\mathcal{P}} &= -\frac{m\sqrt{\pi k}|\mathcal{J}+\frac{2\pi}{k}\mathcal{T}\mathcal{Z}|}{2\left(\mathcal{P}-\frac{4\pi}{k}\mathcal{Z}^2\right)^{3/2}}\,,\\
\mu_{\mathcal{J}} &= \operatorname{sgn}\left(\mathcal{J}+\frac{2\pi}{k}\mathcal{T}\mathcal{Z}\right)\frac{m\sqrt{\pi k}}{\left(\mathcal{P}-\frac{4\pi}{k}\mathcal{Z}^2\right)^{1/2}}\,,\\
\mu_{\mathcal{T}} &= \operatorname{sgn}\left(\mathcal{J}+\frac{2\pi}{k}\mathcal{T}\mathcal{Z}\right)\frac{2m\pi^{3/2}\mathcal{Z}}{\sqrt{k}\left(\mathcal{P}-\frac{4\pi}{k}\mathcal{Z}^2\right)^{1/2}}\,,\\
\mu_{\mathcal{Z}} &= \operatorname{sgn}\left(\mathcal{J}+\frac{2\pi}{k}\mathcal{T}\mathcal{Z}\right)\frac{2m\pi^{3/2}\mathcal{T}}{\sqrt{k}\left(\mathcal{P}-\frac{4\pi}{k}\mathcal{Z}^2\right)^{1/2}}+\frac{4m\pi^{3/2}|\mathcal{J}+\frac{2\pi}{k}\mathcal{T}\mathcal{Z}|\mathcal{Z}}{\sqrt{k}\left(\mathcal{P}-\frac{4\pi}{k}\mathcal{Z}^2\right)^{3/2}}\,.
\end{aligned}\tag{5.18}$$

It is worth noting that the branch that is connected to the standard cosmological spacetime, so that the Hawking temperature is given by $\mu_{\mathcal{P}}=-\beta$, corresponds to $m=1$, which agrees with what is found from requiring smoothness of the Euclidean metric.

Since we are dealing with a Chern-Simons theory, the entropy associated to the cosmological horizon can be directly found from the following formula [46, 80–82]

$$S=\frac{k}{2\pi}\int\langle a_\tau a_\phi\rangle d\phi\,,\tag{5.19}$$

which by virtue of (5.1) and (5.2), evaluates as

$$S=2\pi\left(2\mu_{\mathcal{J}}\mathcal{J}+2\mu_{\mathcal{P}}\mathcal{P}+\mu_{\mathcal{T}}\mathcal{T}+\mu_{\mathcal{Z}}\mathcal{Z}\right)\,.\tag{5.20}$$

Therefore, as required by the action principle, the entropy of regular configurations, by virtue of (5.18) reduces to

$$S=2\pi\left[m\sqrt{\frac{\pi k}{\mathcal{P}-\frac{4\pi}{k}\mathcal{Z}^2}}|\mathcal{J}+\frac{2\pi}{k}\mathcal{T}\mathcal{Z}|\right]\,.\tag{5.21}$$

Note that for $m=1$, the entropy

$$S=2\pi\sqrt{\frac{\pi k}{\mathcal{P}-\frac{4\pi}{k}\mathcal{Z}^2}}|\mathcal{J}+\frac{2\pi}{k}\mathcal{T}\mathcal{Z}|\,,\tag{5.22}$$

precisely gives a quarter of the cosmological horizon area over $4G$.

It is amusing to verify that, unlike the case of pure gravity, configurations without angular momentum ($\mathcal{J}=0$) can still be stationary and carry a nonvanishing entropy.

It is also worth pointing out that the entropy expressed in terms of the extensive variables, manifestly depends not only on the energy and the angular momentum, but also on the electric and magnetic U(1) charges. Indeed, it is simple to verify that the first law holds in the grand canonical ensemble, since

$$\delta S=2\pi\left(\mu_{\mathcal{P}}\delta\mathcal{P}+\mu_{\mathcal{J}}\delta\mathcal{J}+\mu_{\mathcal{T}}\delta\mathcal{T}+\mu_{\mathcal{Z}}\delta\mathcal{Z}\right)\,,\tag{5.23}$$

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provided that the chemical potentials are given by (5.18). In other words, an attempt of fixing the chemical potentials in a different way as in eq. (5.18), might generate a severe clash with the first law of thermodynamics.

As an ending remark of this section, it is worth mentioning that the entropy of the class of cosmological spacetimes with U(1) fields in (5.22), has also been obtained simultaneously and in an independent way through a different approach in [58], which is certainly reassuring.

5.4 Conical defects with U(1) fluxes

In the case of $-\frac{k}{4\pi} < \mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2 < 0$, the configurations are described by spacetime metrics (5.7) that generically describe rotating conical defects [83, 84] with nontrivial lapse and shift functions, endowed with U(1) fields of electric and magnetic type that do not generate a back reaction. In the case of antiperiodic boundary conditions the energy bounds in section 4 are clearly satisfied, but this is not necessarily so for the case of periodic boundary conditions. Indeed, for periodic boundary conditions, if $|\mathcal{Z}| \geq \frac{k}{4\pi}$, the stronger energy bound is trivially satisfied and never saturates; while for $|\mathcal{Z}| < \frac{k}{4\pi}$, the bound is fulfilled only for configurations with nonnegative energy ($\mathcal{P} \geq 0$). Note that in vacuum ($\mathcal{Z} = \mathcal{T} = 0$), this class of configurations does not fulfill the energy bounds when the spin structure is even.

Although this class of configurations fails to be regular due to the presence of sources at the origin, it turns out to be very interesting. This is because they might admit Killing spinors provided that the U(1) fluxes are suitably tuned with the angular deficit [41, 59]. In our terms, these U(1) fluxes correspond to charges of electric type. In section 6, the explicit form of the Killing spinors is constructed, and we also show that two of our supersymmetry bounds in section 4 are saturated, so that the configurations correspond to half-BPS states.

5.5 Minkowski spacetime and conical surpluses endowed with U(1) fields

Configurations with $\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2 < -\frac{k}{4\pi}$ correspond to spacetime metrics with conical surpluses, so that they possess angular excess. Despite they do not fulfill the energy bounds, they might be interesting because it can be seen that they could also admit Killing spinors provided that suitable U(1) fluxes are switched on. Nonetheless, it should be emphasized that they cannot describe BPS states.

In the case of $\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2 = -\frac{k}{4\pi}$ the bosonic solution is described by the Minkowski spacetime endowed with electric and magnetic U(1) fields. Noteworthy, the energy bounds for the antiperiodic boundary conditions are clearly saturated, but this is not necessarily so for periodic boundary conditions.

Remarkably, in the absence of angular momentum and magnetic U(1) charges, these configurations can be regarded as regular ones provided that the energy and the electric U(1) charges take discrete values, according to

$$\mathcal{P} = \frac{k}{\pi} n_+ n_- \,, \qquad \mathcal{Z} = -\frac{k}{4\pi}\left(n_+ + n_-\right) \,, \tag{5.24}$$

where $n_\pm$ correspond to (half-)integers in the case of (anti)periodic boundary conditions. Note that for antiperiodic boundary conditions, the electric U(1) charge is then given by an integer multiple of the mass of Minkowski spacetime in vacuum ($\mathcal{P} = -\frac{k}{4\pi}$), while for periodic boundary conditions, it corresponds to an even multiple.

This can be seen as follows. In these cases, regularity of the configurations corresponds to requiring the holonomy of the connection for a contractible circle along the angular coordinate to be trivial. Thus, following the criterion (i) in section 5.1, one can show that there is no group element of the form $g = e^{\lambda_n M_n + \lambda Z}$ that helps in order to gauge away the components of a_ϕ in (5.1) along the generators M_n and Z, and hence regularity necessarily implies that the angular momentum and the magnetic U(1) charge vanish, i.e., $\mathcal{J} = \mathcal{T} = 0$.

The connection a_ϕ then reduces to

$$a_\phi = L_1 - \frac{\pi}{k}\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right) L_{-1} - \frac{2\pi}{k}\mathcal{Z}T\,, \tag{5.25}$$

that takes values on the $sl(2,R) \oplus u(1)$ subalgebra. Criterion (ii) then implies that the holonomy of a_ϕ in (5.25) along the angular circle is trivial ($H_{\mathcal{C}} = \pm\mathbb{I}_{2\times 2}$). Hence, the eigenvalues of $\hat{\omega} = a_\phi$, can be obtained from (5.15), and they have to be given by $\Lambda_{[\pm]} = i\lambda_{[\pm]} = in_\pm$, where $n_\pm$ stand for integers or half-integers in the cases of even or odd spin structures, respectively, and $\lambda_{[\pm]}$ are given in eq. (4.6). The regularity condition then reads

$$n_\pm = -\frac{2\pi}{k}\mathcal{Z} \pm \sqrt{\frac{\pi}{k}\left(\frac{4\pi}{k}\mathcal{Z}^2 - \mathcal{P}\right)}\,, \tag{5.26}$$

which implies that the energy and the electric U(1) charge take discrete values given by (5.24).

In particular, it is worth pointing out that the configurations that correspond to Minkowski spacetime endowed with electric U(1) charge turn out to be regular for $n_+ = n_- + 1$, which implies that

$$\mathcal{Z} = -\frac{k}{4\pi}\left(2n_- + 1\right)\,, \tag{5.27}$$

is given by even or odd multiples of the mass of Minkowski spacetime in vacuum for antiperiodic or periodic boundary conditions, respectively; while the total energy of the configuration is given by

$$\mathcal{P} = \frac{k}{\pi}n_-\left(n_- + 1\right)\,. \tag{5.28}$$

Remarkably, Minkowski spacetime endowed with an electric U(1) charge given by (5.27) fulfills all of the energy bounds and saturates four of them for periodic or antiperiodic boundary conditions for the fermions, so that the configurations turn out to be maximally supersymmetric, admitting four Killing spinors whose explicit form is given in the next section. This is in stark contrast with what occurs for Minkowski spacetime in vacuum, which does not fulfill the energy bounds in the case of periodic boundary conditions for the fermions. In other words, for an even spin structure, Minkowski spacetime can be brought back into the allowed spectrum provided that it is endowed with an electric-like U(1) charge given by an odd multiple of the mass of the Minkowski spacetime in vacuum.

As a closing remark of this subsection, it is worth pointing out that similar classes of solitonic-like objects with conical surpluses have also been disscused in the context of three-dimensional gravity coupled to higher spin fields in refs. [47, 68, 85–91].

6 Bosonic solutions with unbroken supersymmetries

6.1 Asymptotic Killing spinor equations

Here we look for the class of bosonic configurations that asymptotically behave as the stationary spherically symmetric ones described by the gauge fields in (5.1) and (5.2), that admit Killing spinors being well-defined in the asymptotic region. These unbroken supersymmetries are spanned by fermionic gauge transformations that leave the bosonic configuration invariant. Thus, they fulfill $\delta a = d\lambda + [a,\lambda] = 0$, where λ stands for the purely fermionic asymptotic symmetries that can be obtained from eqs. (3.7) and (3.8). The fermionic Lie-algebra-valued parameter then reads

$$\lambda\left[\epsilon_\psi,\epsilon_\mathcal{S}\right] = \epsilon_\psi G^1_{\frac{1}{2}} - \left(\epsilon_\psi{}' - \frac{2\pi}{k}\mathcal{Z}\epsilon_\mathcal{S}\right)G^1_{-\frac{1}{2}} + \epsilon_\mathcal{S} G^2_{\frac{1}{2}} - \left(\epsilon_\mathcal{S}{}' + \frac{2\pi}{k}\mathcal{Z}\epsilon_\psi\right)G^2_{-\frac{1}{2}} = \epsilon^\alpha_I Q^I_\alpha\,, \tag{6.1}$$

so that the explicit form of the spinors ϵ^α_I in terms of the Grasmann-valued functions $\epsilon_\psi, \epsilon_\mathcal{S}$, can then be obtained by virtue of the change of basis in (3.1). The spinors are then given by

$$\epsilon_1 = \sqrt{2}\begin{pmatrix} -\epsilon_\psi{}' + \frac{2\pi}{k}\mathcal{Z}\epsilon_\mathcal{S} \\ \epsilon_\psi \end{pmatrix}\,, \qquad \epsilon_2 = \sqrt{2}\begin{pmatrix} -\epsilon_\mathcal{S}{}' - \frac{2\pi}{k}\mathcal{Z}\epsilon_\psi \\ \epsilon_\mathcal{S} \end{pmatrix}\,. \tag{6.2}$$

The asymptotic Killing spinor equations can be partially read from the transformation law of the fields in (3.9) under the asymptotic supersymmetries. The nontrivial ones are given by

$$\begin{aligned}
\delta\psi &= -\mathcal{P}\epsilon_\psi + \frac{k}{\pi}\epsilon_\psi{}'' - 4\mathcal{Z}\epsilon_\mathcal{S}{}' = 0\,,\\
\delta\mathcal{S} &= -\mathcal{P}\epsilon_\mathcal{S} + \frac{k}{\pi}\epsilon_\mathcal{S}{}'' + 4\mathcal{Z}\epsilon_\psi{}' = 0\,.
\end{aligned} \tag{6.3}$$

Analogously, the conditions for the Grasmann-valued parameters can be obtained from (3.11), so that they read

$$\begin{aligned}
\dot\epsilon_\psi &= \mu_\mathcal{J}\epsilon_\psi{}' - \mu_\mathcal{T}\epsilon_\mathcal{S}\,,\\
\dot\epsilon_\mathcal{S} &= \mu_\mathcal{J}\epsilon_\mathcal{S}{}' + \mu_\mathcal{T}\epsilon_\psi\,.
\end{aligned} \tag{6.4}$$

In order to perform the analysis it is useful to define a single complex Grasmann-valued parameter defined as

$$\xi = \frac{1}{\sqrt{2}}\left(\epsilon_\psi + i\epsilon_\mathcal{S}\right)\,, \tag{6.5}$$

so that the asymptotic Killing spinor equations in (6.3) and (6.4) can be rewritten as

$$\xi'' + \frac{4\pi}{k}i\mathcal{Z}\xi' - \frac{\pi}{k}\mathcal{P}\xi = 0\,, \tag{6.6}$$

$$\dot\xi - \mu_\mathcal{J}\xi' - i\mu_\mathcal{T}\xi = 0\,. \tag{6.7}$$

The solution of the asymptotic Killing spinor equations in (6.6) and (6.7) can then readily found. In the generic case $\left(\mathcal{P} \neq \frac{4\pi}{k}\mathcal{Z}^2\right)$, the solution reads

$$\xi = \xi_1 e^{i\mu_\mathcal{T}u}e^{i\lambda_{[+]}\hat\phi} + \xi_2 e^{i\mu_\mathcal{T}u}e^{i\lambda_{[-]}\hat\phi}\,, \tag{6.8}$$

where $\lambda_{[\pm]}$ correspond to the eigenvalues of the spacelike components of the $sl(2,R) \oplus u(1)$ connection $\hat{\omega} = \omega + B$ in (4.6), and $\hat{\phi} := \phi + \mu_{\mathcal{J}} u$. Here ξ_1 and ξ_2 stand for arbitrary complex (Grasmann-valued) constants. In the special case of $\mathcal{P} = \frac{4\pi}{k}\mathcal{Z}^2$, which corresponds to energy of the null orbifold endowed with U(1) fields, the eigenvalues degenerate ($\lambda_{[+]} = \lambda_{[-]} = -\frac{2\pi}{k}\mathcal{Z}$), and hence the solution is given by

$$\xi = \xi_1 e^{i\mu_{\mathcal{T}} u} e^{i\lambda_{[+]}\hat{\phi}} + \xi_2 \phi e^{(\mu_{\mathcal{J}} + i\mu_{\mathcal{T}})u} e^{i\lambda_{[+]}\hat{\phi}} . \tag{6.9}$$

One is then ready to analyze whether they are well-defined for the different classes of bosonic solutions discussed in section 5.

6.1.1 Cosmological configurations

In this case, the energy fulfills $\mathcal{P} > \frac{4\pi}{k}\mathcal{Z}^2$, so that the eigenvalues $\lambda_{[\pm]}$ in (4.6) turn out to be complex. Therefore, the solution for the Killing spinor equations in (6.8) is not globally well-defined because it cannot fulfill neither periodic nor antiperiodic boundary conditions. Consequently, all of the supersymmetries are broken, which goes by hand with the fact the energy bounds in this case are always satisfied, but never saturated.

6.1.2 Null orbifold with U(1) fields

For this class of configurations, the energy is given by $\mathcal{P} = \frac{4\pi}{k}\mathcal{Z}^2$, so that the solution of the asymptotic Killing spinor equations is given by (6.9). It is then clear that it can only be globally defined provided that $\xi_2 = 0$, where the eigenvalue has to be given by $\lambda_{[+]} = n$, with n a (half-)integer for fermions that fulfill (anti)periodic boundary conditions. Note that the electric U(1) charge is then restricted to take the following values: $\mathcal{Z} = -\frac{kn}{2\pi}$. The solution of the asymptotic Killing spinor equation then acquires the form

$$\xi = \xi_1 e^{i\mu_{\mathcal{T}} u} e^{-in\hat{\phi}} , \tag{6.10}$$

so that it is clear that this class of configurations possesses two unbroken supersymmetries. This is precisely the number of bounds that saturate, which correspond to the ones in (4.5) for $r = \mp\lambda_{[+]} = \mp n = \pm\frac{2\pi}{k}\mathcal{Z}$.

6.1.3 Conical defects with U(1) fluxes

In this case the energy fulfills the condition $-\frac{k}{4\pi} < \mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2 < 0$, which in terms of the eigenvalues $\lambda_{[\pm]}$ in (4.6), reads

$$\lambda_{[+]} - \lambda_{[-]} < 1 . \tag{6.11}$$

Therefore, the solution of the asymptotic Killing spinor equation in (6.8) is globally well-defined provided that $\xi_2 = 0$ and $\lambda_{[+]} = n_+$ is given by a (half-)integer, or $\xi_1 = 0$ and $\lambda_{[-]} = n_-$ a (half-)integer, for (anti)periodic boundary conditions. Therefore, this class of configurations preserves half of the supersymmetries, which goes by hand with the fact that the energy bounds are fulfilled, and the number of them that saturate is just two. They correspond to the ones in (4.5) either for $r = \mp\lambda_{[+]}$, or $r = \mp\lambda_{[-]}$.

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It is worth then mentioning that configurations with conical defects become half-BPS for even or odd spin structures, provided that the electric U(1) charge and the energy are suitably tuned according to

$$\mathcal{P} = \frac{k}{\pi} n_+ n_- \,, \qquad \mathcal{Z} = -\frac{k}{4\pi}\left(n_+ + n_-\right) . \tag{6.12}$$

6.1.4 Minkowski spacetime with U(1) fields

In the case of Minkowski spacetime dressed with U(1) fields, the energy is given by $\mathcal{P} = -\frac{k}{4\pi} + \frac{4\pi}{k}\mathcal{Z}^2$, which implies that the eigenvalues $\lambda_{[\pm]}$ in (4.6) fulfill

$$\lambda_{[+]} - \lambda_{[-]} = 1\,. \tag{6.13}$$

Therefore, the asymptotic Killing spinors are given by (6.8), and they are globally well-defined provided that $\lambda_{[\pm]}$ are given by (half-)integers for (anti)periodic boundary conditions. This is the only maximally supersymmetric case that fulfill the bounds in section 4, which agrees with the fact that four of them in (4.5) saturate, corresponding to the cases of $r = \mp(\lambda_{[-]} + 1)$ and $r = \mp\lambda_{[-]}$.

Note that the energy and the electric U(1) charge are then given by (5.28) and (5.27), respectively, with $\lambda_{[-]} = n_-$.

It is worth emphasizing that this case includes the regular one described in section 5.5, which corresponds to configurations with $\mathcal{J} = \mathcal{T} = 0$.

6.1.5 Configurations with conical surpluses

For this class of configurations the energy lies in the range $\mathcal{P} < -\frac{k}{4\pi} + \frac{4\pi}{k}\mathcal{Z}^2$, which amounts to

$$\lambda_{[+]} - \lambda_{[-]} > 1\,. \tag{6.14}$$

As aforementioned, the energy bounds are not fulfilled, but nonetheless this class of solutions might admit unbroken supersymmetries. In the maximally supersymmetric case the asymptotic Killing spinors are given by (6.8) provided that $\lambda_{[\pm]}$ correspond to (half-)integers for (anti)periodic boundary conditions. Note that in the case of $\mathcal{J} = \mathcal{T} = 0$, this class of configurations turns out to be regular (see section 5.5). The global charges are then given by (5.24), with $\lambda_{[\pm]} = n_\pm$.

The remaining possibility consists on configurations with angular excess endowed with suitable electric U(1) charges, which are not regular. The asymptotic Killing spinors are then given by (6.8), and they are globally well-defined either for $\xi_2 = 0$ and a (half-)integer $\lambda_{[+]} = n_+$, or $\xi_1 = 0$ and $\lambda_{[-]} = n_-$ a (half-)integer, for (anti)periodic boundary conditions.

6.2 Global Killing spinor equation

For the class of exact solutions described in section 5, one can also proceed in the standard way in order to identify the configurations that admit globally well-defined Killing spinors, as well as finding the explicit form of them. It is reassuring to verify that the results in this section precisely agree with the ones in section 6.1. Indeed, the results within this section can be readily reconstructed from the ones in section 6.1 by virtue of eq. (6.2). Nonetheless,

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carrying out the analysis in the standard way turns out to be a healthy exercise. In order to perform this task it is useful to complexify the spinors according to

$$\chi = \frac{1}{\sqrt{2}} \left(\epsilon_1 + i \epsilon_2 \right) , \tag{6.15}$$

so that the Killing spinor equation can be obtained from the local supersymmetry transformations in (2.11) ($\delta \psi_I = 0$), and it turns out to be given by

$$\delta \chi = d\chi + \frac{1}{2} \omega^a \Gamma_a \chi - i B \chi = 0 . \tag{6.16}$$

The generic solution of (6.16) is given by

$$\chi = \mathcal{P}\mathrm{exp} \left[- \int_{\mathcal{C}} \left(\frac{1}{2} \omega^a \Gamma_a - i B \right) \right] \chi_0 , \tag{6.17}$$

with χ_0 a constant Dirac spinor.

As it can be read from section 5, the spin connection and the electric U(1) field read

$$\begin{aligned} \omega &= \sqrt{2} \left[J_1 + \frac{\pi}{k} \left(\mathcal{P} - \frac{4\pi}{k} \mathcal{Z}^2 \right) J_0 \right] \left(d\phi + \mu_{\mathcal{J}} du \right) , && (6.18) \\ B &= - \frac{2\pi}{k} \mathcal{Z} \left(d\phi + \mu_{\mathcal{J}} du \right) + \mu_{\mathcal{T}} du , && (6.19) \end{aligned}$$

and hence, the solution in (6.17) reduces to

$$\chi = e^{i \left(- \frac{2\pi}{k} \mathcal{Z} \hat{\phi} + \mu_{\mathcal{T}} u \right)} \mathrm{exp} \left\{ - \sqrt{2} \left[J_1 + \frac{\pi}{k} \left(\mathcal{P} - \frac{4\pi}{k} \mathcal{Z}^2 \right) J_0 \right] \hat{\phi} \right\} \chi_0 , \tag{6.20}$$

with $\hat{\phi} = \phi + \mu_{\mathcal{J}} u$.

In the generic case of ($\mathcal{P} \neq \frac{4\pi}{k} \mathcal{Z}^2$), the solution in (6.20) then acquires the form

$$\begin{aligned} \chi = e^{i \left(- \frac{2\pi}{k} \mathcal{Z} \hat{\phi} + \mu_{\mathcal{T}} u \right)} \Bigg\{ & \cosh \left[- \frac{i}{4} (\lambda_{[+]} - \lambda_{[-]}) \hat{\phi} \right] \mathbb{I}_{2\times 2} && (6.21) \\ & - \frac{4 i \sqrt{2}}{\lambda_{[+]} - \lambda_{[-]}} \left[J_1 - \frac{1}{16} (\lambda_{[+]} - \lambda_{[-]})^2 J_0 \right] \sinh \left[- \frac{i}{4} (\lambda_{[+]} - \lambda_{[-]}) \hat{\phi} \right] \Bigg\} \chi_0 , \end{aligned}$$

with $\lambda_{[\pm]}$ defined in eq. (4.6).

For $\mathcal{P} = \frac{4\pi}{k} \mathcal{Z}^2$, the solution is given by

$$\chi = \left(\mathbb{I}_{2\times 2} - \sqrt{2} J_1 \hat{\phi} \right) e^{i \left(- \frac{2\pi}{k} \mathcal{Z} \hat{\phi} + \mu_{\mathcal{T}} u \right)} \chi_0 . \tag{6.22}$$

Here we have made use of the fundamental matrix representation of $sl(2, R) \oplus u(1)$ that appears in appendix A.

The remaining analysis can then be directly performed as in the previous section.

In the case of cosmological configurations ($\mathcal{P} > \frac{4\pi}{k} \mathcal{Z}^2$) all of the supersymmetries are broken because the Killing spinors in (6.21) are clearly not globally well-defined.

The null orbifold with U(1) fields ($\mathcal{P} = \frac{4\pi}{k}\mathcal{Z}^2$) possesses half of the supersymmetries provided that $\mathcal{Z} = \frac{kn}{2\pi}$, so that the Killing spinors can be obtained from (6.22). They are explicitly given by

$$\chi = e^{i\left(-n\hat{\phi}+\mu_{\mathcal{T}}u\right)}\chi_0\,, \tag{6.23}$$

where the constant spinor has to fulfill the projection $J_1\chi_0 = 0$, and n is a (half-)integer for (anti)periodic boundary conditions.

In the case of conical defects with U(1) fluxes ($-\frac{k}{4\pi} < \mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2 < 0$), the Killing spinor in (6.21) becomes

$$\begin{aligned}\chi = e^{i\left(-\frac{2\pi}{k}\mathcal{Z}\hat{\phi}+\mu_{\mathcal{T}}u\right)}\Bigg\{&\cos\left[\frac{1}{4}(\lambda_{[+]}-\lambda_{[-]})\hat{\phi}\right]\mathbb{I}_{2\times 2}\\ &-\frac{4\sqrt{2}}{\lambda_{[+]}-\lambda_{[-]}}\left[J_1-\frac{1}{16}(\lambda_{[+]}-\lambda_{[-]})^2J_0\right]\sin\left[\frac{1}{4}(\lambda_{[+]}-\lambda_{[-]})\hat{\phi}\right]\Bigg\}\chi_0\,,\end{aligned} \tag{6.24}$$

which is globally well-defined provided that the spinor χ_0 satisfies the projection

$$-\frac{4\sqrt{2}}{\lambda_{[+]}-\lambda_{[-]}}\left[J_1-\frac{1}{16}(\lambda_{[+]}-\lambda_{[-]})^2J_0\right]\chi_0 = \pm i\chi_0\,. \tag{6.25}$$

The Killing spinor in (6.24) then reduces to

$$\chi = e^{i\mu_{\mathcal{T}}u}\exp\left\{i\lambda_{[\pm]}\hat{\phi}\right\}\chi_0\,. \tag{6.26}$$

with $\lambda_{[\pm]}$ given by (4.6), and the sign of the projection in (6.25) coincides with the choice of the label of $\lambda_{[\pm]}$ in (6.26). The spinors are then consistent with (anti)periodic boundary conditions provided that $\lambda_\pm$ is a (half-)integer. Note that the projection condition in (6.25), preserves half of the supersymmetries.

In the case of Minkowski spacetime dressed with U(1) fields ($\mathcal{P} = -\frac{k}{4\pi} + \frac{4\pi}{k}\mathcal{Z}^2$), equation (6.21) reduces to

$$\chi = \left[\cos\left(\frac{\hat{\phi}}{2}\right)\mathbb{I}_{2\times 2} - 2\sqrt{2}\left(J_1 - \frac{1}{4}J_0\right)\sin\left(\frac{\hat{\phi}}{2}\right)\right]e^{i\left(-\frac{2\pi}{k}\mathcal{Z}\hat{\phi}+\mu_{\mathcal{T}}u\right)}\chi_0\,, \tag{6.27}$$

so that the Killing spinors are globally well-defined provided the electric U(1) charge is fixed by even or odd multiples of $-\frac{k}{4\pi}$ for antiperiodic or periodic boundary conditions, respectively. Noteworthy, this case is maximally supersymmetric for even and odd spin structures.

For the class of conical surpluses endowed with U(1) fluxes ($\mathcal{P} < -\frac{k}{4\pi} + \frac{4\pi}{k}\mathcal{Z}^2$) there are two possibilities.

In the maximally supersymmetric case the Killing spinors are given by (6.24), where $\lambda_{[\pm]} = n_\pm$ stand for (half-)integers in the case of (anti)periodic boundary conditions.

In the remaining possibility the configurations possess two independent Killing spinors, now given by (6.26), so that χ_0 fulfills the projection in (6.25). The Killing spinors fulfill (anti)periodic boundary conditions when $\lambda_\pm$ is a (half-)integer.

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7 Spectral flow: from Ramond to Neveu-Schwarz boundary conditions

As shown in [42], the spectrum spanned by the super-Virasoro algebra with $\mathcal{N} = 2$ in the case of periodic boundary conditions for the fermionic fields relates to the one for antiperiodic boundary conditions through spectral flow. These results were generalized for $\mathcal{N} > 2$ in [92].

Here we show that this is also the case for the super-BMS$_3$ algebra with $\mathcal{N} = (2,0)$ in eq. (3.14). This occurs by virtue of a field redefinition that is induced by a suitable U(1) gauge transformation. The field redefinition then amounts to a precise change of basis in the canonical generators that turns out to be an automorphism of the super-BMS$_3$ algebra with $\mathcal{N} = (2,0)$. This can be seen as follows.

The Dirac spinors $\xi^{\pm} = \frac{1}{\sqrt{2}}(\psi_1 \pm i\psi_2)$ are such that ξ^+ is the hermitian conjugate of ξ^-, and vice versa. Therefore, for generic "anyonic" boundary conditions characterized by some parameter η that is held fixed at the boundary, the spinors fulfill

$$\xi^{\pm}(\phi + 2\pi) = e^{\pm 2i\pi\eta}\xi^{\pm}(\phi) \,, \tag{7.1}$$

where $\eta = 0, \frac{1}{2}$ corresponds to Ramond (periodic), and Neveu-Schwarz (antiperiodic) boundary conditions, respectively.

Besides, under U(1) gauge transformation spanned by $g = e^{fT}$, the connection transforms as $A^f = g^{-1}Ag + g^{-1}dg$, so that the nontrivial transformations in terms of the components of the gauge fields read

$$B^f = B + df \,, \qquad \xi^{\pm f} = e^{\pm if}\xi^{\pm} \,. \tag{7.2}$$

Therefore, a generic choice of boundary conditions labelled by η, can be obtained from the one of periodic boundary conditions ($\eta = 0$) if one chooses $f = \eta\phi$. For this choice, the dynamical fields that describe the asymptotic structure in (3.5) then transform as

$$\begin{aligned} \mathcal{J}^{\eta} &= \mathcal{J} + \eta\mathcal{T} \,, & \mathcal{P}^{\eta} &= \mathcal{P} - 4\eta\mathcal{Z} + \frac{k}{\pi}\eta^2 \,, \\ \mathcal{Z}^{\eta} &= \mathcal{Z} - \frac{k}{2\pi}\eta \,, & \mathcal{G}^{\pm\eta} &= e^{\pm i\eta\phi}\mathcal{G}^{\pm} \,, \end{aligned} \tag{7.3}$$

with $\mathcal{T}^{\eta} = \mathcal{T}$. In terms of Fourier modes, eq. (7.3) reads

$$\begin{aligned} \mathcal{J}^{\eta}_m &= \mathcal{J}_m + \eta\mathcal{T}_m \,, \\ \mathcal{P}^{\eta}_m &= \mathcal{P}_m - 4\eta\mathcal{Z}_m + 2\eta^2 k\delta_{m,0} \,, \\ \mathcal{Z}^{\eta}_m &= \mathcal{Z}_m - \eta k\delta_{m,0} \,, \\ \mathcal{T}^{\eta}_m &= \mathcal{T}_m \,, \\ \mathcal{G}^{\eta\pm}_{p\pm\eta} &= \mathcal{G}^{\pm}_p \,, \end{aligned} \tag{7.4}$$

and it is then simple to verify that the new "η" generators fulfill the super-BMS$_3$ algebra with $\mathcal{N} = (2,0)$ for an anyonic choice of boundary conditions. In this sense (7.4) can be regarded as an automorphism of the asymptotic symmetry algebra (3.14) for generic boundary conditions for the fermions as in (7.1). Therefore, in particular, the algebras with

Ramond and Neveu-Schwarz boundary conditions become related by spectral flow. One can then say that a generic choice of boundary conditions for the fermions can be "gauged away" by virtue of the U(1) automorphism of the super-BMS$_3$ algebra with $\mathcal{N} = (2,0)$. Thus, different theories characterized by inequivalent choices of boundary conditions, actually coincide after a suitable field redefinition that is induced through an appropriate U(1) gauge transformation.

8 Extension of the results

One of the advantages of formulating supergravity in terms of a Chern-Simons action as in [39, 41, 93], is that the theory can be extended to include parity odd terms in the Lagrangian in a straightforward way along the lines of [94] (see also [34, 47, 95]). The procedure amounts to introduce an additional coupling through a simple modification of the invariant bilinear form of the gauge group, as well as further couplings through shifting the magnetic-like gauge fields by the corresponding ones of electric type. Therefore, as explained in [34, 47, 96], the global charges become suitably modified, so that the asymptotic symmetry algebra is able to acquire additional central extensions. It would then be interesting to explicitly construct this kind of extension for the supergravity theory with $\mathcal{N} = (2,0)$ in [41]. It is also worth pointing out that this procedure has recently been applied in [58] for the case of the inhomogeneous (despotic) theory with $\mathcal{N} = (2,0)$, so that an additional central extension manifestly shows up in the asymptotic symmetry algebra, as well as through an additional contribution to the entropy of cosmological spacetimes.

Another possibility that deserves to be explored is to consider the extension of our analysis for $\mathcal{N} > 2$, since it is natural to expect that the extended super-BMS$_3$ algebra has to be nonlinear, as it can be directly seen from the flat limit of the superconformal algebra in two spacetime dimensions. Indeed, nonlinear extensions of the BMS$_3$ algebra have been shown to arise in the context of hypergravity [47, 96] as well as for higher spin gravity without cosmological constant in [70, 72, 97, 98].

As a closing remark, it is worth pointing out that induced representations of BMS$_3$, as well as its extensions that include higher spin generators have been discussed in [99–102]. In the case of (higher spin) fermionic fields, this has been done so far for $\mathcal{N} = 1$. It would then be interesting to explore the properties of this sort of representations of super-BMS$_3$ for an extended number of fermionic generators, as in the case discussed in this work.

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A Conventions

We choose the orientation to be such that $\varepsilon_{012} = 1$. The Minkowski metric η_{ab} is assumed to be non-diagonal, so that $\eta_{01} = \eta_{10} = \eta_{22} = 1$, while the remaining components vanish. In the spinorial representation, the generators of SO(2, 1) are given by $(J_a)^{\alpha}_{\beta} = \frac{1}{2}(\Gamma_a)^{\alpha}_{\beta}$, where the matrices Γ_a fulfill the Clifford algebra, $\{\Gamma_a, \Gamma_b\} = 2\eta_{ab}$. They are chosen in terms of the Pauli matrices σ_i, so that

$$\Gamma_0 = \frac{1}{\sqrt{2}}(\sigma_1 + i\sigma_2) \,, \qquad \Gamma_1 = \frac{1}{\sqrt{2}}(\sigma_1 - i\sigma_2) \,, \qquad \Gamma_2 = \sigma_3 \,, \tag{A.1}$$

with

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{A.2}$$

Spinors ψ^{α} are labeled according to $\alpha = +, -$, and we define the Majorana conjugate as $\bar{\psi}_{\alpha} = \psi^{\beta} C_{\beta\alpha}$, where $C_{\alpha\beta}$ stands for the charge conjugation matrix, given by

$$C_{\alpha\beta} = C^{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} , \tag{A.3}$$

so that $C^{\alpha\beta}$ is the inverse. The charge conjugation matrix then fulfills $C^T = -C$, and $(C\Gamma_a)^T = C\Gamma_a$.

In the fundamental representation of $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{U}(1)$, the generators of $\mathrm{SL}(2, R)$ and the U(1) generator are given by L_m, with $m = -1, 0, 1$, and T, respectively. They are chosen as

$$L_{-1} = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} ; \qquad L_0 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} ; \qquad L_1 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} ; \qquad T = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} . \tag{A.4}$$

Therefore, the nonvanishing components of the trace of quadratic products of them become $\operatorname{tr}(L_1 L_{-1}) = -1$, $\operatorname{tr}\left(L_0^2\right) = \frac{1}{2}$ and $\operatorname{tr}\left(T^2\right) = -2$.

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