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Guocong Song, Ye Li

Institutions: Motorola, Georgia Institute of Technology

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Asymptotic Throughput Analysis for Channel-Aware Scheduling*

Guocong Song† and Ye (Geoffrey) Li

The School of Electrical and Computer Engineering

Georgia Institute of Technology, Atlanta, GA 30332-0250

Email: {songgc, liye}@ece.gatech.edu

Abstract

In this paper, we provide an asymptotic performance analysis of channel-aware packet scheduling based on extreme value theory. We first address the average throughput of systems with a homogeneous average *signal-to-noise ratio* (SNR) and obtain its asymptotic expression. Compared to the exact throughput expression, the asymptotic one, which is applicable to a broader range of fading channels, is more concise and easier to get insights. Furthermore, we confirm the accuracy of the asymptotic results by theoretical analysis and numerical simulation. For a system with heterogeneous SNRs, normalized-SNR-based scheduling need to be used for fairness. We also investigate the asymptotic average throughput of the normalized-SNR-based scheduling and prove that the average throughput in this case is less than that in the homogeneous case with a power constraint.

Index Terms

Channel-aware scheduling, multiuser diversity, extreme value theory.

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† Corresponding author.

I. INTRODUCTION

Time-varying fading is an important characteristic of wireless channels. For a point-to-point link, using adaptive modulation and coding [1], [2], the transmitter can send more data at a higher transmission data rate when the channel quality is good. However, bandwidth efficiency with adaptive modulation and coding is still low during deep-fading periods. In [3], the authors have studied the sum capacity of uplink fading channels when the channel state information is known for the transmitters and the receiver. They have obtained two important results. First, the optimal strategy is to schedule only one user with the best channel condition. Second, the sum capacity increases with the number of users, which results from independent channel variations across users. Therefore, the above phenomenon is called *multiuser diversity*. The similar results in downlink fading channels are shown in [4]. The delay of channel-condition feedback is considered [5].

To obtain the multiuser diversity gain, adaptive modulation and channel-aware scheduling must be used. However, the channel variance and the opportunistic nature of channel-aware scheduling make throughput analysis very difficult. There are two major approaches to analyze the capacity of systems with multiuser diversity. One is exact analysis based on special functions. Capacity analyses for Rayleigh and Nakagami fading channels are addressed in [6] and [7], respectively. More specific cases that include constrained bandwidth, discrete transmission rates, and multiple cells are also considered in [8]. However, the results of these capacity analyses are too complicated to get insights and hardly extended to a general fading environment. Another major approach is asymptotic analysis based on extreme value theory or extreme order statistics [9], [10]. An asymptotic analysis for *signal-to-noise ratio* (SNR) for multiuser diversity is presented in [11], in which the log scaling law with the number of users of multiuser diversity is obtained for Rayleigh fading channels, and the scaling law for Rician fading channels is also given. In [12], asymptotic analysis of throughput with proportional fair scheduling is studied with an assumption that throughput is a linear function of SNR. To the best knowledge, however, these asymptotic analyses only propose scaling laws for asymptotic SNR rather than give rigorous convergence results. In addition, there are no asymptotically analytical results under such a more realistic assumption that throughput is a log function of SNR.

The rest of this paper is organized as follows. In Section II, we briefly describe the system model and the main results of extreme value theory used in this paper. In Section III, we introduce the exact throughput analysis for multiuser diversity. In Section IV, we investigate the asymptotic throughput for channels with a general fading distribution and confirm the accuracy of the asymptotic analysis by means of analytical and numerical results. In Section V, we analyze the average throughput of the normalized-SNR-based scheduling. Finally, conclusion remarks are presented in Section VI.

II. PROBLEM FORMULATION

In this section, we first describe the system model in our analysis and then briefly introduce the main results of extreme value theory useful to our asymptotic analysis.

A. System Model

Consider a shared downlink channel of a single-carrier system with a bandwidth B and M users. The downlink channel is time-slotted, and each time slot can adaptively be assigned to a user. It is assumed that the base station knows the channel state information of each user, and that continuous rate adaptation is applied in the downlink channel. Therefore, the current transmission data rate, R, depends on the instantaneous SNR, Γ. A tight *bit-error rate* (BER) approximation of *M-ary quadrature amplitude modulation* (MQAM) over the *additive white Gaussian noise* (AWGN) channel is present in [1]. This approximation leads to the relationship between throughput and SNR by

$$R = B\log_2(1+\beta\Gamma), \tag{1}$$

where β is a constant related to the targeted BER, that is, $\beta = -1.5/\ln(5\cdot\mathrm{BER})$. Although (1) is originally derived from MQAM, it can model continuous rate adaptation as well [1], [13]. If $\beta = 1$, (1) is just the Shannon capacity for the AWGN channel.

First, we assume that the all users experience statistically independent identical fading processes. The *max-sum-capacity* (MSC) scheduling [11], [14] is used in the system. The MSC scheduling is a channel-aware scheduling scheme that maximizes the total throughput in the system and works well in the homogeneous system. It assigns the channel to the user with the best channel condition on each time slot, which is described as

$$m = \arg\max_{i \in \mathcal{M}} \{\Gamma_i\}, \tag{2}$$

where $\mathcal{M} = \{1, 2, \cdots, M\}$ is the set of user indices, and Γ_i is the instantaneous SNR of user i.

We also consider a heterogeneous case in which different users have different average SNR values due to different path losses. For the purpose of fairness, the normalized-SNR-based scheduling [5], [6] is used. This scheduling makes decisions based on the normalized SNR rather than the absolute SNR values, which is expressed as

$$m = \arg\max_{i \in \mathcal{M}} \{\frac{\Gamma_i}{\gamma_i}\}, \tag{3}$$

where γ_i is the average SNR of user i; that is, $\mathbb{E}\{\Gamma_i\} = \gamma_i$[1]. It is obvious that the normalized-SNR-based scheduling is the same as the MSC scheduling for the homogeneous system.

B. Extreme Value Theory

Extreme value theory deals with asymptotic distributions of extreme values, such as maxima or minima. It can be used to analyze the performance of the above scheduling approaches. In this section, we will briefly introduce two major results of extreme value theory [9], [10] that are used in the analysis.

Let $\xi_1, \xi_2, \cdots, \xi_M$ be *independently identically distributed* (i.i.d.) random variables with distribution function $F(x)$, and $Z_M = \max_{i \in \mathcal{M}} \xi_i$. If there exist constants $a_M \in \mathbb{R}$, $b_M > 0$, and some non-degenerate distribution function H such that the distribution of $\frac{Z_M - a_M}{b_M}$ converges to H, then H belongs to one of the three standard extreme value distributions: Frechet, Webull, and Gumbel distributions.

[1] γ_i is the average over fast fading. However, it may still change due to path loss and shadowing, but its change rate is much slower than that of fast fading.

It is very interesting that there are only three possible non-degenerate limiting distributions for maxima. The distribution function of ξ_i, $F(x)$, determines the exact limiting distribution. Thus, if a distribution function $F(x)$ results in one limiting distribution for extremes, it is called that $F(x)$ belongs to the domain of attraction of this limiting distribution. The following lemma indicates a sufficient condition for a distribution function $F(x)$ belonging to the domain of attraction of the Gumbel distribution.

Lemma 1: Let $\xi_1, \xi_2, \cdots, \xi_M$ be i.i.d. random variables with distribution function $F(x)$. Define $\omega(F) = \sup\{x : F(x) < 1\}$. Assume that there is a real number x_1 such that, for all $x_1 \leq x < \omega(F)$, $f(x) = F'(x)$ and $F''(x)$ exist and $f(x) \neq 0$. If

$$\lim_{x \to \omega(F)} \frac{d}{dx} \left[\frac{1 - F(x)}{f(x)} \right] = 0, \tag{4}$$

then there exist constants a_M and $b_M > 0$ such that $\frac{Z_M - a_M}{b_M}$ uniformly converges in distribution to a normalized Gumbel random variable as $M \to \infty$. The normalizing constants a_M and b_M are determined by

$$a_M = F^{-1}\left(1 - \frac{1}{M}\right),$$
$$b_M = F^{-1}\left(1 - \frac{1}{Me}\right) - F^{-1}\left(1 - \frac{1}{M}\right),$$

where $F^{-1}(x) = \inf\{y : F(y) \geq x\}$.

For a random variable Z with the normalized Gumbel distribution, whose distribution function is $\exp[-\exp(-x)]$, $-\infty < x < \infty$, it follows that

$$\mathbb{E}\{Z\} = E_0,$$
$$\mathbb{V}ar\{Z\} = \frac{\pi^2}{6},$$

where $E_0 = 0.5772\cdots$ is the Euler constant [10].

In this paper, we intend to calculate the average throughput; thus, mean convergence is used extensively. However, convergence in distribution is not equivalent to moment convergence in general. The following lemma from [15] establishes the relation between convergence in distribution and moment convergence.

Lemma 2: If $\frac{Z_M - a_M}{b_M}$ converges in distribution to a random variable Z that has a non-degenerate distribution function, and if $\mathbb{E}\{[(Z_M)^-]^p\} < \infty$ for any positive real number p, where $(x)^- = \begin{cases} -x, \ x < 0, \\ 0, \quad \text{otherwise} \end{cases}$, then

$$\lim_{M \to \infty} \mathbb{E}\left(\frac{Z_M - a_M}{b_M}\right)^p = \mathbb{E}\{Z^p\},$$

provided $\mathbb{E}|Z|^p < \infty$.

Obviously, convergence in distribution for the maximum of *nonnegative* random variables results in moment convergence. With extreme value theory, we can study the asymptotic performance of channel-aware scheduling.

III. Exact Analysis for Throughput with Multiuser Diversity

In this section, we will briefly review the traditional exact analysis method for throughput and the challenges involved with it. Assume that all users' SNRs, $\{\Gamma_1, \Gamma_2, \cdots, \Gamma_M\}$, are i.i.d. nonnegative random variables with distribution function $F_\Gamma(\gamma)$.

According to the MSC scheduling, the base station schedules the user with the strongest channel condition. Therefore, the effective SNR at the transmitter, Γ_{eff}, is given by

$$\Gamma_{\text{eff}} = \max_{i \in \mathcal{M}} \Gamma_i, \tag{5}$$

and its distribution is

$$F_{\Gamma_{\text{eff}}}(\gamma) = F_\Gamma^M(\gamma).$$

The total throughput of the MSC scheduling is expressed as

$$\begin{aligned} R_{\text{total}} &= \max_{i \in \mathcal{M}} B \log_2(1 + \beta\Gamma_i) \\ &= B \log_2(1 + \beta\Gamma_{\text{eff}}). \end{aligned}$$

We are more interested in the average values of the effective SNR and the total throughput. Therefore, we can calculate the average SNR when the MSC scheduling is used as

$$\mathbb{E}\{\Gamma_{\text{eff}}\} = \int_0^\infty \gamma \, dF_{\Gamma_{\text{eff}}}(\gamma) \tag{6}$$

and the average total throughput as

$$\mathbb{E}\{R_{\text{total}}\} = B \int_0^\infty \log_2(1 + \beta\gamma)\, dF_{\Gamma_{\text{eff}}}(\gamma). \tag{7}$$

Multiuser diversity actually has the same mathematical model with selection diversity. Thus, the results related to selection diversity can be used for multiuser diversity analysis as long as the number of antennas is replaced with the number of users. In the case of Rayleigh fading, for instance, the distribution function of SNR for Rayleigh fading with average SNR γ_0 can be expressed as

$$F_\Gamma(\gamma) = 1 - \exp(-\frac{\gamma}{\gamma_0}). \tag{8}$$

According to the results in [16], we have

$$\mathbb{E}\{\Gamma_{\text{eff}}\} = \gamma_0 \sum_{i=1}^{M} \frac{1}{i}, \tag{9}$$

and

$$\mathbb{E}\{R_{\text{total}}\} = \frac{M}{\ln 2} \sum_{i=0}^{M-1} (-1)^{i+1} \begin{pmatrix} M-1 \\ i \end{pmatrix} \frac{e^{\frac{1+i}{\gamma_0}}}{i+1} E_i(-\frac{1+i}{\gamma_0}) \tag{10}$$

with

$$E_i(-x) = E_0 + \ln(x) + \sum_{i=1}^{\infty} \frac{(-1)^i x^i}{i!\ \ i}.$$

As seen from the above, the exact analysis of throughput analysis for Rayleigh fading channels is very complicated, and the exact result lacks insights as well. Moreover, it is really hard to obtain solutions for other fading distributions. Therefore, in the rest of the paper, we will provide simple results through asymptotic analysis.

IV. Asymptotic Throughput Analysis for General Channel Distributions

With extreme value theory, finding the limiting distribution of the maximum throughput is crucial to obtain the asymptotic throughput. In this section, we consider a general case. Mathematically, we study the limiting distribution of the throughput

$$R = T(\Gamma) = B \log_2(1 + \beta\Gamma),$$

given a SNR distribution, $F_\Gamma(\gamma)$. However, the complicated form of throughput distribution makes it very difficult to check condition (4) directly . Thus, we provide a simpler approach, which is stated in the following theorem for *limiting throughput distribution* (LTD).

LTD Theorem: Assume that all users' SNRs, $\{\Gamma_1, \Gamma_2, \cdots, \Gamma_M\}$, are i.i.d. nonnegative random variables with a distribution $F_\Gamma(\gamma)$ such that $\omega(F_\Gamma) = \infty$, and $f_\Gamma(\gamma) = F'_\Gamma(\gamma)$ as well as $F''_\Gamma(\gamma)$ exist and $f_\Gamma(\gamma) \neq 0$ for all $x_1 \leq x < \infty$, where x_1 is some real number. If

$$\lim_{\gamma\to\infty} \frac{d}{d\gamma}\left[\frac{1-F_\Gamma(\gamma)}{f_\Gamma(\gamma)}\right] = 0, \tag{11}$$

then the distribution of throughput, $F_R(r) = F_\Gamma(T^{-1}(r))$, belongs to the domain of the attraction of the Gumbel distribution. Furthermore, the normalizing constants are

$$a_M = B\log_2\left(1+\beta F_\Gamma^{-1}(1-\frac{1}{M})\right), \tag{12}$$

$$b_M = B\log_2\left(\frac{1+\beta F_\Gamma^{-1}(1-\frac{1}{Me})}{1+\beta F_\Gamma^{-1}(1-\frac{1}{M})}\right). \tag{13}$$

The proof is in Appendix A. The LTD theorem tells us that we do not have to check $F_R(r)$ directly, which is usually very complicated to find its limiting distribution. In addition, Lemma 2 leads to

$$\frac{\mathbb{E}\{R^{\text{hom}}_{\text{total}}\} - a_M}{b_M} \to E_0,$$

as $M \to \infty$, where $R^{\text{hom}}_{\text{total}}$ is the total throughput for the homogeneous scenario. For a large M, the average total throughput can be evaluated by using the following expression.

$$\mathbb{E}\{R^{\text{hom}}_{\text{total}}\} \approx a_M + E_0 b_M. \tag{14}$$

A. Example 1: Rayleigh Fading

The exponential distribution leads to

$$\frac{1-F_\Gamma(\gamma)}{f_\Gamma(\gamma)} = \gamma_0.$$

As a result, it follows that

$$\frac{d}{d\gamma}\left[\frac{1-F_\Gamma(\gamma)}{f_\Gamma(\gamma)}\right] = 0, \text{ for } \gamma > 0.$$

According to the results of extreme value theory in Section II-B, the exponential distribution is in the domain of attraction of the Gumbel distribution.

First, we study the asymptotic distribution for the effective SNR, Γ_{eff}, in (5). From Lemma 1, we have the normalizing constants for effective SNR shown as follows,

$$a_M = \gamma_0 \ln M,$$

$$b_M = \gamma_0.$$

As $M \to \infty$,

$$\frac{\mathbb{E}\{\Gamma_{\text{eff}}\} - \gamma_0 \ln M}{\gamma_0} \to E_0.$$

For a large M,

$$\mathbb{E}\{\Gamma_{\text{eff}}\} \approx \gamma_0(\ln M + E_0). \tag{15}$$

It is shown in [17] that

$$\frac{1}{2(M+1)} < \sum_{i=1}^{M} \frac{1}{i} - (\ln M + E_0) < \frac{1}{2M},$$

which implies that the difference between the exact value (9) and the asymptotic value (15) is very small even for a small M.

For throughput analysis, the normalizing constants for throughput are obtained from the LTD theorem as

$$a_M = B \log_2(1 + \beta\gamma_0 \ln M),$$
$$b_M = B \log_2\left(1 + \frac{\beta\gamma_0}{1 + \beta\gamma_0 \ln M}\right).$$

Consequently, the average throughput is given by

$$\begin{aligned}\mathbb{E}\{R_{\text{total}}\} &\approx a_M + E_0 b_M \\ &= B \log_2(1 + \beta\gamma_0 \ln M) + E_0 \cdot B \log_2\left(1 + \frac{\beta\gamma_0}{1 + \beta\gamma_0 \ln M}\right),\end{aligned} \tag{16}$$

where $\ln M$ is usually called the multiuser diversity gain for Rayleigh channels [11]. In contrast to (10), (16) provides a very simple approximation for the average throughput. The numerical results in Section IV-E will show that this approximation is very accurate even when M is small.

Note that as $M \to \infty$, $a_M \to \infty$, and $b_M \to 0$. Therefore, with a large M,

$$\mathbb{E}\{R_{\text{total}}\} \approx B \log_2(1 + \beta\gamma_0 \ln M)$$

is a rougher but simpler estimation for the average throughput.

B. Example 2: Nakagami Fading

The Nakagami distribution is frequently used to characterize the fading statistics of wireless channels in certain environments. Then, the cdf of the received SNR is given by

$$F_\Gamma(\gamma) = \Gamma_{(m,\frac{m}{\gamma_0})}(\gamma) = \int_0^\gamma \left(\frac{m}{\gamma_0}\right)^m \frac{t^{m-1}}{\Gamma(m)} e^{-\frac{m}{\gamma_0}t} dt, \tag{17}$$

where m is called the fading figure, which is defined as the ratio of the total power to the power of fading components, and $\Gamma(m)$ is the gamma function. In this subsection, we use the results of the LTD theorem to study the impact of Nakagami fading on throughput in the system with the MSC scheduling. Applying the results of extreme value theory in Section II-B and letting $u = \frac{m}{\gamma_0}$, we have

$$\begin{aligned}
&\lim_{\gamma\to\infty} \frac{d}{d\gamma}\left[\frac{1 - F_\Gamma(\gamma)}{f_\Gamma(\gamma)}\right] \\
&= \lim_{\gamma\to\infty} -\frac{[1 - F_\Gamma(\gamma)]}{f_\Gamma^2(\gamma)/f'_\Gamma(\gamma)} - 1 \\
&= \lim_{\gamma\to\infty} \frac{1 - \displaystyle\int_0^\gamma t^{m-1} e^{-ut} dt}{\dfrac{\gamma^m e^{-u\gamma}}{u\gamma - m + 1}} - 1 \\
&= 0 \quad \text{(by L'Hospital's rule).}
\end{aligned}$$

According to the results of extreme value theory in Section II-B and the LTD theorem, both $F_\Gamma(\gamma)$ and $F_R(r)$ belong to the domain of the attraction of the Gumbel distribution. Therefore, the average total throughput for the Nakagami fading can be given by

$$\begin{aligned}
\mathbb{E}\{R_{\text{total}}^{\text{hom}}\} &\approx B \log_2\left(1 + \beta F_\Gamma^{-1}(1 - \frac{1}{M})\right) + E_0 B \log_2\left(\frac{1 + \beta F_\Gamma^{-1}(1 - \frac{1}{Me})}{1 + \beta F_\Gamma^{-1}(1 - \frac{1}{M})}\right) \\
&= B \log_2\left(1 + \beta \Gamma^{-1}_{(m,\frac{m}{\gamma_0})}(1 - \frac{1}{M})\right) + E_0 B \log_2\left(\frac{1 + \beta \Gamma^{-1}_{(m,\frac{m}{\gamma_0})}(1 - \frac{1}{Me})}{1 + \beta \Gamma^{-1}_{(m,\frac{m}{\gamma_0})}(1 - \frac{1}{M})}\right),
\end{aligned} \tag{18}$$

where $\Gamma^{-1}_{(m,\frac{m}{\gamma_0})}(\gamma)$ is the inverse incomplete gamma function. Despite no closed form for it, the inverse incomplete gamma function is usually provided in common softwares, such as Matlab and Mathematica.

Actually, besides the Rayleigh and Nakagami distributions, the normal, Rician, and log-normal distributions, which are often used to describe the statistics of wireless channels, belong to the domain of the attraction of the Gumbel distribution [10].

C. Further Properties of Asymptotic Throughput

Assume that the fading distribution $F_\Gamma(\gamma)$ satisfies (11) and $F_\Gamma^{-1}(\gamma) \to \infty$ as $\gamma \to \infty$. Thus, as $M \to \infty$, $a_M \to \infty$. In addition, we prove in Appendix B that

$$\lim_{M\to\infty} \frac{b_M}{a_M} = 0. \tag{19}$$

Applying (19) to $F_\Gamma(\gamma)$ (a_M and b_M here are related to Γ_{eff} in (5)), we have

$$\lim_{M\to\infty} \frac{F_\Gamma^{-1}(1-\frac{1}{Me}) - F_\Gamma^{-1}(1-\frac{1}{M})}{F_\Gamma^{-1}(1-\frac{1}{M})} = 0. \tag{20}$$

From (20), we have the limit of b_M that is corresponding to the throughput shown as follows:

$$\begin{aligned}\lim_{M\to\infty} b_M &= \lim_{M\to\infty} B\log_2\left(\frac{1+\beta F_\Gamma^{-1}(1-\frac{1}{Me})}{1+\beta F_\Gamma^{-1}(1-\frac{1}{M})}\right)\\ &= \lim_{M\to\infty} B\log_2\left(\frac{F_\Gamma^{-1}(1-\frac{1}{Me})}{F_\Gamma^{-1}(1-\frac{1}{M})}\right)\\ &= \lim_{M\to\infty} B\log_2\left(\frac{F_\Gamma^{-1}(1-\frac{1}{Me}) - F_\Gamma^{-1}(1-\frac{1}{M})}{F_\Gamma^{-1}(1-\frac{1}{M})}+1\right)\\ &= 0. \end{aligned} \tag{21}$$

In fact, we have the same result for the Rayleigh fading case in Section IV-A. Thus, when the number of users M is very large, we have

$$\mathbb{E}\{R_{\text{total}}^{\text{hom}}\} \approx a_M = B\log_2\left(1+\beta F_\Gamma^{-1}(1-\frac{1}{M})\right), \tag{22}$$

which is a rough estimation for the average total throughput with a large M. According to (20), we have

$$F_\Gamma^{-1}(1-\frac{1}{M}) = \mathbb{E}\{\Gamma_{\text{eff}}\} + o(\mathbb{E}\{\Gamma_{\text{eff}}\}).$$

Thus, (22) can also be rewritten as

$$\mathbb{E}\{R_{\text{total}}^{\text{hom}}\} \approx B\log_2\left(1+\beta\left[\mathbb{E}\{\Gamma_{\text{eff}}\}+o(\mathbb{E}\{\Gamma_{\text{eff}}\})\right]\right),$$
$$\approx B\log_2\left(1+\beta\mathbb{E}\{\Gamma_{\text{eff}}\}\right). \tag{23}$$

The above equation means that the average throughput is approximately a function of the average effective SNR.

Lemma 2 also shows that for any positive real number p,

$$\lim_{M\to\infty}\mathbb{E}\left(\frac{R_{\text{total}}^{\text{hom}}-a_M}{b_M}\right)^p=\mathbb{E}\{Z^p\}, \tag{24}$$

where Z is a normalized Gumbel random variable. We consider $p=2$, and have

$$\lim_{M\to\infty}\mathbb{E}\left(\frac{R_{\text{total}}^{\text{hom}}-a_M}{b_M}\right)^2=E_0^2+\frac{\pi^2}{6}.$$

Thus, as $M\to\infty$,

$$\mathbb{V}ar\{R_{\text{total}}^{\text{hom}}\}\to\frac{\pi^2}{6}b_M^2.$$

Because of (21), $\mathbb{V}ar\{R_{\text{total}}^{\text{hom}}\}\to 0$, which indicates that this asymptotic analysis of average throughput is quite accurate. In addition, it follows that

$$\lim_{M\to\infty}\mathbb{E}\left(R_{\text{total}}^{\text{hom}}-a_M\right)^p=\lim_{M\to\infty}b_M\mathbb{E}\{Z^p\},$$
$$=0. \tag{25}$$

According to [15], (25) guarantees that $R_{\text{total}}^{\text{hom}}-a_M$ converges in probability[2] to 0.

D. Channel Access Probability and Average Throughput per User

The channel access probability P_i is the probability that user i obtains the channel to transmit data. In the homogeneous fading case, due to the the symmetry, each user has the same channel access probability; that is,

$$P_i=\frac{1}{M}.$$

[2]Assume X_n and X to be a random variable sequence and a random variable, if $\lim_{n\to\infty}\mathbb{P}\{|X_n-X|>\epsilon\}=0$ for any $\epsilon>0$, then we say that X_n converges in probability to X.

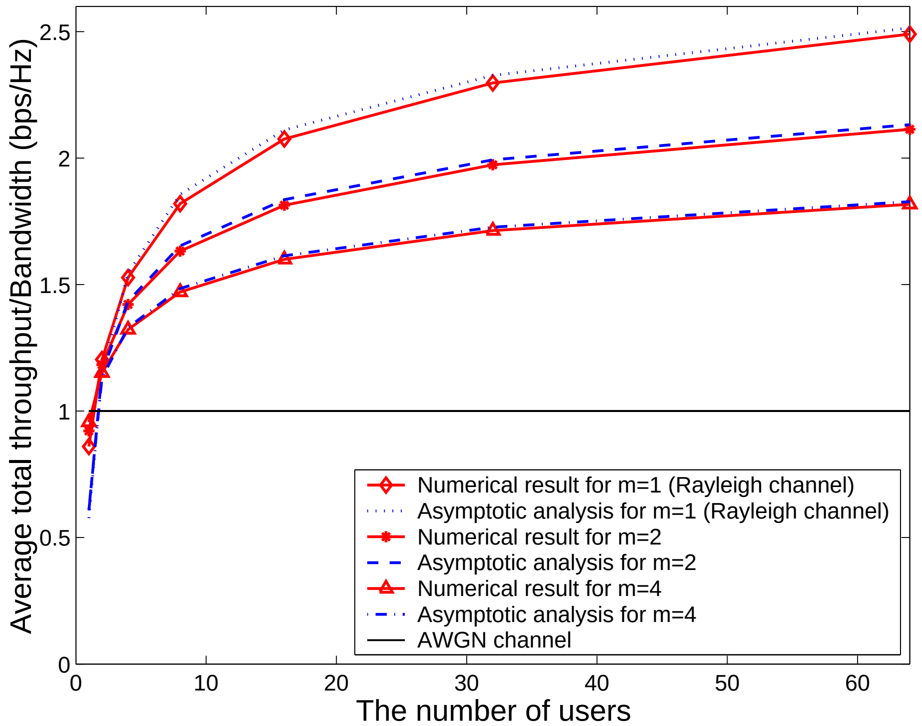

Fig. 1. Average throughput for different environments. $\beta\gamma_0 = 1$.

Therefore, the average throughput of user i with the scheduling, $\mathbb{E}\{R_i^{\text{s}}\}$, is given by

$$\mathbb{E}\{R_i^{\text{s}}\} = \frac{1}{M}\mathbb{E}\{R_{\text{total}}^{\text{hom}}\}.$$

E. Numerical Results

We assume that all users experience i.i.d. Nakagami fading. Let $\beta\gamma_0 = 1$. Figure 1 shows the average total throughput in the Nakagami fading channels with different values of m. For comparison, we also plot the average throughput in the AWGN channel with the same average SNR in Figure 1.

It is shown in Figure 1 that the asymptotic results are still accurate even if the number of users is small. The figure shows that the throughput increases with the number of users in the fading scenario with dynamic scheduling. As m increases, the fading fluctuation of the channel reduces, and the multiuser diversity gain is also diminished.

V. Throughput Analysis for Normalized-SNR-Based Scheduling

In the previous sections, we presented the asymptotic throughput analysis for the homogeneous fading case. In reality, the values of the average SNR of users vary according to their path losses and shadowing. Denote the average SNR of user i as γ_i. We consider a scenario in which different users have the same normalized SNR distribution $F(\gamma)$ but with different average SNR, γ_i's. We assume that $F(\gamma)$ satisfies $\omega(F) = \infty$ and (11).

Obviously, the MSC scheduling results in unfair channel access probabilities. When the normalized-SNR-based scheduling is used, the base station schedules the user with the largest normalized SNR to get the channel, which is mathematically expressed in (3). Define the effective normalized SNR at the transmitter as

$$\Gamma_{\text{eff}} = \max_{i \in \mathcal{M}} \frac{\Gamma_i}{\gamma_i}.$$

Because of the identical distribution of the normalized SNR, the previous results based on extreme value theory is still applicable to the effective normalized SNR, and all users have the same channel access probability as well; that is,

$$P_i = \frac{1}{M}.$$

Thus, the average throughput of user i can be expressed as

$$\mathbb{E}\{R_i^{\text{s}}\} = \frac{1}{M} \int_0^\infty B \log_2(1 + \beta\gamma_i\gamma) dF_{\Gamma_{\text{eff}}}(\gamma) \tag{26}$$

Recalling (7) and the LTD theorem, in the i.i.d. fading case if the distribution of SNR $F_\Gamma(\gamma)$ satisfies (11), then, with a large M,

$$\int_0^\infty \log_2(1 + \beta\gamma) dF_{\Gamma_{\text{eff}}}(\gamma) \approx \log_2\left(1 + \beta F_\Gamma^{-1}(1 - \frac{1}{M})\right) + E_0 \log_2\left(\frac{1 + \beta F_\Gamma^{-1}(1 - \frac{1}{Me})}{1 + \beta F_\Gamma^{-1}(1 - \frac{1}{M})}\right). \tag{27}$$

Comparing (26) and (27), we obtain the average throughput for user i as follows:

$$\mathbb{E}\{R_i^{\text{s}}\} \approx \frac{B}{M} \left\{ \log_2\left(1 + \beta\gamma_i F^{-1}(1 - \frac{1}{M})\right) + E_0 \log_2\left(\frac{1 + \beta\gamma_i F^{-1}(1 - \frac{1}{Me})}{1 + \beta\gamma_i F^{-1}(1 - \frac{1}{M})}\right) \right\},$$

for a large M. Therefore, with the normalized-SNR-based scheduling, each user obtains the same multiuser diversity gain as that in the homogeneous scenario and has the same channel access probability, but its average throughput depends on its average SNR.

Furthermore, we will compare the total throughput in the heterogeneous and homogeneous scenarios. We assume that

$$\gamma_0 = \frac{1}{M}\sum_{i=1}^{M}\gamma_i, \tag{28}$$

$$\sigma_\gamma^2 = \frac{1}{M}\sum_{i=1}^{M}(\gamma_i - \gamma_0)^2.$$

It can be proven that when the number of users M is large,

$$-\frac{B}{2\ln 2}\frac{\sigma_\gamma^2}{\gamma_0^2} \leq \mathbb{E}\{R_{\text{total}}^{\text{het}}\} - \mathbb{E}\{R_{\text{total}}^{\text{hom}}\} \leq 0. \tag{29}$$

This means that the homogeneous case leads to the maximum total throughput when (28) holds.

VI. Conclusion

Using extreme value theory, we have proposed an asymptotic average throughput analysis for the MSC scheduling with a general fading distribution, which not only has concise expressions, but also provides accurate results. This asymptotic analysis shows that the use of the simple scheduling techniques and the feedback of channel state information can significantly improve the bandwidth efficiency. We have also extended the analysis into a scenario in which different users experience different path losses. The results shows that the normalized-SNR-based scheduling can obtain the same multiuser diversity gain as that in the homogeneous case while maintaining access-time proportional fairness.

Appendix A
Proof of the LTD Theorem

Proof: According to the results of extreme value theory in Section II-B, we have to show that

$$\lim_{r\to\infty}\frac{d}{dr}\left[\frac{1-F_R(r)}{f_R(r)}\right] = 0,$$

if

$$\lim_{\gamma\to\infty}\frac{d}{d\gamma}\left[\frac{1-F_\Gamma(\gamma)}{f_\Gamma(\gamma)}\right]=0. \tag{A.1}$$

Since

$$\frac{1-F_R(r)}{f_R(r)}=\frac{1-F_\Gamma(T^{-1}(r))}{f_\Gamma(T^{-1}(r))\left(T^{-1}\right)'(r)},$$

we have

$$\begin{aligned}
&\frac{d}{dr}\left[\frac{1-F_R(r)}{f_R(r)}\right]\\
&=-1-\frac{[1-F_\Gamma(T^{-1}(r))]\left[f'_\Gamma(T^{-1}(r))((T^{-1})'(r))^2+f_\Gamma(T^{-1}(r))\left(T^{-1}\right)''(r)\right]}{\left[f_\Gamma(T^{-1}(r))\left(T^{-1}\right)'(r)\right]^2}\\
&=\underbrace{-1-\frac{[1-F_\Gamma(T^{-1}(r))]f'_\Gamma(T^{-1}(r))}{f^2_\Gamma(T^{-1}(r))}}_{\text{Part I}}-\underbrace{\frac{[1-F_\Gamma(T^{-1}(r))]\left(T^{-1}\right)''(r)}{f_\Gamma(T^{-1}(r))\left[\left(T^{-1}\right)'(r)\right]^2}}_{\text{Part II}}
\end{aligned} \tag{A.2}$$

Because $T^{-1}(r)$ is monotonically increasing with x and $T^{-1}(r)\to\infty$ as $r\to\infty$,

$$\lim_{r\to\infty}\frac{[1-F_\Gamma(T^{-1}(r))]f'_\Gamma(T^{-1}(r))}{f^2_\Gamma(T^{-1}(r))}=\lim_{\gamma\to\infty}\frac{[1-F_\Gamma(\gamma)]f'_\Gamma(\gamma)}{f^2_\Gamma(\gamma)}$$

It is easy to check that

$$\frac{d}{d\gamma}\left[\frac{1-F_\Gamma(\gamma)}{f_\Gamma(\gamma)}\right]=-1-\frac{[1-F_\Gamma(\gamma)]f'_\Gamma(\gamma)}{f^2_\Gamma(\gamma)}.$$

Thus, we have

$$\lim_{r\to\infty}\text{Part I}=\lim_{r\to\infty}\frac{d}{d\gamma}\left[\frac{1-F_\Gamma(\gamma)}{f_\Gamma(\gamma)}\right]. \tag{A.3}$$

Let $\tilde{T}^{-1}(r)=\frac{2^{\frac{x}{B}}}{\beta}$. Due to the fact that

$$\left(T^{-1}\right)''(r)=\frac{\ln 2}{B}\left(T^{-1}\right)'(r),$$

and $(\tilde{T}^{-1})'(r)=(T^{-1})'(r)$, it follows that

$$\begin{aligned}
\lim_{r\to\infty}\text{Part II}&=\lim_{r\to\infty}\frac{\ln 2[1-F_\Gamma(T^{-1}(r))]}{Bf_\Gamma(T^{-1}(r))\left(T^{-1}\right)'(r)} &\text{(A.4)}\\
&=\lim_{r\to\infty}\frac{\ln 2[1-F_\Gamma(T^{-1}(r))]}{Bf_\Gamma(T^{-1}(r))\left(\tilde{T}^{-1}\right)'(r)}. &\text{(A.5)}
\end{aligned}$$

Since $\tilde{T}^{-1}(r) = T^{-1}(r) + \frac{1}{\gamma}$ and $\tilde{T}^{-1}(r) \to \infty$ as $r \to \infty$,

$$\lim_{r\to\infty} \frac{1 - F_\Gamma(\tilde{T}^{-1}(r))}{f_\Gamma(\tilde{T}^{-1}(r))} = \lim_{r\to\infty} \frac{1 - F_\Gamma(T^{-1}(r))}{f_\Gamma(T^{-1}(r))},$$

if (A.1) holds. Thus, we have

$$\begin{aligned}
\lim_{r\to\infty} \text{Part II} &= \lim_{r\to\infty} \frac{\ln 2[1 - F_\Gamma(\tilde{T}^{-1}(r))]}{B f_\Gamma(\tilde{T}^{-1}(r)) \left(\tilde{T}^{-1}\right)'(r)} \\
&= \lim_{r\to\infty} \frac{\ln 2[1 - F_\Gamma(\tilde{T}^{-1}(r))]}{B f_\Gamma(\tilde{T}^{-1}(r)) \tilde{T}^{-1}(r) \frac{\ln 2}{B}} \\
&= \lim_{\gamma\to\infty} \frac{1 - F_\Gamma(\gamma)}{f_\Gamma(\gamma)\gamma} \qquad \text{(A.6)}
\end{aligned}$$

Combining (A.3) and (A.6), we obtain

$$\lim_{r\to\infty} \frac{d}{dr}\left[\frac{1 - F_R(r)}{f_R(r)}\right] = \lim_{\gamma\to\infty} \frac{d}{d\gamma}\left[\frac{1 - F_\Gamma(\gamma)}{f_\Gamma(\gamma)}\right] + \lim_{\gamma\to\infty} \frac{1 - F_\Gamma(\gamma)}{f_\Gamma(\gamma)\gamma}. \qquad \text{(A.7)}$$

According to L'Hospital's rule, for a function $g(x)$ such as $g(x) \to \infty$ as $x \to \infty$, if $\lim_{x\to\infty} g'(x) = 0$, then $\lim_{x\to\infty} \frac{g(x)}{x} = 0$. Equation (A.1) results in

$$\lim_{\gamma\to\infty} \frac{1 - F_\Gamma(\gamma)}{f_\Gamma(\gamma)\gamma} = 0,$$

Therefore, we obtain

$$\lim_{r\to\infty} \frac{d}{dr}\left[\frac{1 - F_R(r)}{f_R(r)}\right] = 0.$$

Since

$$\begin{aligned}
F_R^{-1}(x) &= T(F_\Gamma^{-1}(x)), \\
&= B \log_2(1 + \beta F_\Gamma^{-1}(x)),
\end{aligned}$$

we can obtain the normalizing constants (12) and (13) according to the results of extreme value theory in Section II-B. □

APPENDIX B
PROOF OF EQUATION (19)

Proof: Let $X \geq 0$ be a random variable with distribution function $F(x)$ and $\mathbb{E}\{X\}$ is finite. The expected residual life of X is given by

$$\begin{aligned} \mathsf{R}(t) &= \mathbb{E}\{X - t | X \geq t\} \\ &= \frac{1}{1 - F(t)} \int_t^{\infty} 1 - F(x) dx. \end{aligned}$$

Theorem 2.1.3 and Lemma 2.7.2 in [10] show that if $F(x)$ is in the domain of the Gumbel distribution,

$$b_M = \mathsf{R}(a_M), \tag{B.1}$$

and

$$\lim_{t \to \infty} \frac{\mathsf{R}(t)}{t} = 0. \tag{B.2}$$

Since a_M monotonically increases with M, (B.1) and (B.2) directly indicates that

$$\begin{aligned} \lim_{M \to \infty} \frac{b_M}{a_M} &= \lim_{M \to \infty} \frac{\mathsf{R}(a_M)}{a_M} \\ &= \lim_{t \to \infty} \frac{\mathsf{R}(t)}{t} \\ &= 0 \end{aligned}$$

□

APPENDIX C
PROOF OF INEQUALITY (29)

Proof: When the number of users M is large, we only consider use the first term, a_M, to evaluate the average throughput. Thus, the average total throughput in the heterogeneous scenario is given by

$$\begin{aligned} \mathbb{E}\{R_{\text{total}}^{\text{het}}\} &= \sum_{i=1}^{M} E\{R_i^{\text{s}}\} \\ &\approx \frac{B}{M} \sum_{i=1}^{M} \log_2 \left(1 + \beta \gamma_i F^{-1}(1 - \frac{1}{M})\right), \end{aligned}$$

and the average total throughput in the homogeneous scenario is

$$\mathbb{E}\{R_{\text{total}}^{\text{hom}}\} \approx B \log_2 \left(1 + \beta\gamma_0 F^{-1}(1 - \frac{1}{M})\right),$$

We obtain

$$\begin{aligned}\mathbb{E}\{R_{\text{total}}^{\text{het}}\} - \mathbb{E}\{R_{\text{total}}^{\text{hom}}\} &\approx \frac{B}{M}\sum_{i=1}^{M} \log_2 \left(\frac{1 + \beta\gamma_i F^{-1}(1 - \frac{1}{M})}{1 + \beta\gamma_0 F^{-1}(1 - \frac{1}{M})}\right) \\ &\to \frac{B}{M}\sum_{i=1}^{M} \log_2 \left(\frac{\gamma_i}{\gamma_0}\right), \quad \text{as } M \to \infty. \end{aligned} \tag{C.1}$$

(C.1) is valid since $F^{-1}(1 - \frac{1}{M}) \to \infty$ as $M \to \infty$.

With the following inequality,

$$x - \frac{1}{2}x^2 \le \ln(1 + x) \le x, \quad \text{for } x \ge 0, \tag{C.2}$$

we will consider the upper and lower bounds, respectively. For the upper bound, it follows from (C.1) and (C.2) that

$$\begin{aligned}\mathbb{E}\{R_{\text{total}}^{\text{het}}\} - \mathbb{E}\{R_{\text{total}}^{\text{hom}}\} &\le \frac{B}{\ln(2)M}\sum_{i=1}^{M}\left(\frac{\gamma_i}{\gamma_0} - 1\right) \\ &= 0.\end{aligned}$$

Similarly, the lower bound is given by

$$\begin{aligned}\mathbb{E}\{R_{\text{total}}^{\text{het}}\} - \mathbb{E}\{R_{\text{total}}^{\text{hom}}\} &> \frac{B}{\ln(2)M}\sum_{i=1}^{M}\left(\frac{\gamma_i}{\gamma_0} - 1\right) - \frac{1}{2\ln 2}\frac{B}{M}\sum_{i=1}^{M}\left(\frac{\gamma_i}{\gamma_0} - 1\right)^2 \\ &= 0 - \frac{B}{2\ln 2}\left[\frac{1}{M}\sum_{i=1}^{M}\left(\frac{\gamma_i}{\gamma_0}\right)^2 - 1\right] \\ &= -\frac{B}{2\ln 2}\frac{\sigma_\gamma^2}{\gamma_0^2}.\end{aligned}$$

□

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