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ATENA – A tool for engineering analysis of fracture in concrete

VLADIMIR CERVENKA, JAN CERVENKA and RADOMIR PUKL

Cervenka Consulting, Prague, Czech Republic
e-mail: cervenka@cervenka.cz

Abstract. Advanced constitutive models implemented in the finite element system ATENA serve as rational tools to explain the behaviour of connection between steel and concrete. Three nonlinear material models available in ATENA are described: crack band model based on fracture energy, fracture-plastic model with non-associated plasticity and microplane material model. Nonlinear simulation using these advanced constitutive models can be efficiently used to support and extend experimental investigations and to predict behaviour of structures and structural details.

Keywords. ATENA; analysis of concrete fracture; nonlinear simulation.

1. Introduction

Structural response of anchoring elements can be simulated by nonlinear finite element analysis. This is a general approach based on principles of mechanics and should provide an objective tool for all types of geometry, material properties and loading. Such simulation is recently used to supplement experimental investigations, where it significantly increases the value of experimental data. The goal of this approach is to provide a tool more general than simple design formulas, which are usually valid for very limited ranges of parameters. The scope of application for complex nonlinear analysis is aimed at the development of new technical solutions of anchors, special loading types and investigation of failure cases. It is not meant for normal design, which can be accomplished by simple design formulas.

An algorithm for nonlinear analysis is based on three basic parts: Finite element technique, constitutive model and nonlinear solution methods, which should compose a balanced approximation. Nevertheless, the constitutive models decide the material behaviour, and therefore they are treated more extensively here, while the finite elements and nonlinear solution are mentioned only briefly. With reference to research authorities in the field of concrete mechanics and materials, such as RILEM, FIB, FRAMCOS, it is recognized that the most important effects to be included in the constitutive model of concrete are tensile fracturing and compressive confinement.

Several constitutive models covering these effects are implemented in the computer code ATENA, which is a finite element package designed for computer simulation of concrete structures. The graphical user interface in ATENA provides an efficient and powerful environment for solving many anchoring problems. ATENA enables virtual testing of structures using computers, which is the present trend in the research and development world. Several practical examples of the utilization of ATENA for simulation of connections between steel and concrete are presented by Pukl *et al* (2001).

2. Material models

The program system ATENA offers a variety of material models for different materials and purposes (ATENA 2000). For metals, von Mises plasticity can be used, for rock and soil, Drucker-Prager plasticity with associated or non-associated flow rule is available, while for steel, reinforcement multilinear uniaxial model with cycling is determined. Nonlinear and contact springs for supports can be used, for interfaces Mohr-Coulomb friction is available. In some cases the use of isotropic elastic material law can be advantageous. Nevertheless, the most important material models in ATENA are the material models for concrete. These advanced models take into account all the important aspects of real material behaviour in tension and compression. Three nonlinear material models for concrete are available in ATENA: crack band model based on fracture energy, fracture-plastic model with non-associated plasticity, and microplane material model. These three material models are described bellow in greater detail.

2.1 *Crack band model*

The basic constitutive model in ATENA is based on the smeared crack concept and the damage approach. Concrete without cracks is considered, isotropic while concrete with cracks is orthotropic. The material axes of cracked concrete, the orthotropy axes, can be defined by two models: rotated or fixed cracks (refer ATENA (2000) or Cervenka (2000) for details). In the rotated crack model, the crack direction always coincides with the principal strain direction. In the fixed crack model the crack direction and the material axes are defined by the principal stress direction at the onset of cracking. In further analysis, this direction is fixed and cannot change. An important difference in the above approaches is in the shear model on the crack plane. In the fixed crack model, a strain field rotation generates shear stress on the crack plane. Consequently the model of shear becomes important. In the case of the rotated cracks a shear on the crack plane never appears and the shear model need not to be employed.

The stress response is based on the damage concept and is defined by means of the equivalent uniaxial stress–strain law. This law describes the development of distinct material variables and their damage and covers the complete material behaviour under monotonically increasing load including pre- and post-peak softening in compression and tension. In case of a uniaxial stress state it reflects the experimentally observed uniaxial behaviour. In a biaxial state, the equivalent strain is calculated using the current secant inelastic elastic modulus. In the uncracked concrete the material is considered isotropic and one elastic secant modulus is defined corresponding to the lowest compressive stress. In cracked concrete, which is orthotropic, two moduli are defined, the first one for compressive and the second one for tensile material axes respectively. The effect of stress on compressive and tensile strengths is considered by modifying the peak stresses using the failure functions based on Kupfer's experiments.

The method described above is applicable to the pre-peak response and unfortunately, cannot be simply extended to the post-peak range. It is known from material research that post-peak softening is structure-dependent and a simple strain-based model is not objective, but dependent on the finite element mesh due to strain localization in softening. In order to avoid this problem, localization limiters should be employed. Therefore, a fracture mechanics approach (see Margoldová *et al* 1998) based on the crack band model (Bažant & Oh 1983) and fracture energy is implemented. Such a model, substantially reduces the mesh sensitivity. In this model, discrete cracks and compression failure zones, which represent discontinuities, are modelled in the finite element displacement fields by means of strain localization within bands. The model is based on an assumption of equal energy dissipation. A unified approach is used for tensile and compressive softening. The behaviour of a crack in concrete is idealized by the model of a cohesive fictitious crack according to Hillerborg *et al* (1976) where the crack opening law is governed by three parameters: tensile strength, fracture energy, and the shape of the softening curve. The exponential shape experimentally derived by Hordijk (1991) is used for the descending branch, figure 1.

The unloading path of the stress–strain law is considered the origin. This is certainly an approximation, which can be accepted in the case of monotonic loading history. However, even if the load is increased monotonically, at certain material points the stresses are unloading. For example, in the process of crack formation distributed cracks are initiated in large material volumes, and then some cracks open while many other cracks close. Consequently, the unloading law is essential for strain localization. The unloading material modulus describes material damage due to mechanical loading. In this respect, the model described above is similar to the damage theory.

Shear of the cracked concrete is important for the fixed crack model, as already mentioned in the context of crack models. Many researchers have found, for example Rots (1988), that a simple reduction of shear resistance by a constant factor (a constant shear retention factor), does not work well. Therefore, the presented model utilizes the variable shear retention factor, in which the crack shear resistance decreases with the crack opening.

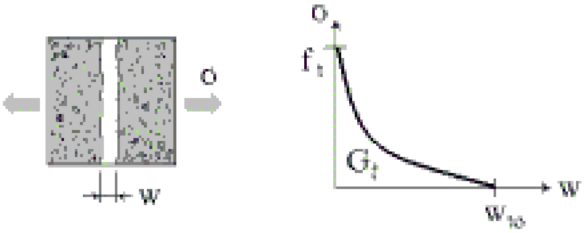
Decrease of compressive strength in the cracked concrete may be important in some types of failure. It was introduced by Collins (Vecchio & Collins 1986) and is now being used in design. This model describes a reduction of concrete compressive due to lateral cracking. In the present model the exponential formula based on Collin's experiments is employed. The amount of maximal reduction is taken as a parameter in order to enable a control of this effect.

The constitutive model described above can be used for plane stress analysis of normal, as well as high strength and steel fibre reinforced concrete. For these concrete types, special modifications of the descending branch are available.

2.2 *Fracture-plastic model*

This three-dimensional constitutive material model for concrete combines plasticity with fracture. For detailed description of the model please see our earlier papers (Cervenka *et al* 1998; Cervenka & Cervenka 1999; ATENA 2000). The fracture is modelled by an orthotropic smeared crack model based on Rankine tensile criterion. Hardening–softening plasticity model based on the Menétrey-William three-parameter failure surface is used to model concrete crushing (Menétrey & William 1995). The model presented differs from the other published formulations in its ability to handle physical changes like crack closure, and it is not restricted to any particular shape of hardening/softening laws. Also within the proposed approach it is possible to formulate the two models (i.e. plastic and fracture) entirely separately, and their combination can be provided for in a different algorithm or model.

The method of strain decomposition as it was introduced by de Borst (1986) is used to combine fracture and plasticity models together. Both models are developed within the framework of return mapping algorithm. This approach guarantees the solution for all magnitudes of strain increment. From an algorithmic point of view the problem is then transformed into



$$\frac{\sigma}{f_t} = (1 + c_1 \frac{w}{w_{\text{to}}})^3 \exp(-c_2 \frac{w}{w_{\text{to}}}) - \frac{w}{w_{\text{to}}}(1 + c_1^3)$$

$$\exp(-c_2)c_1 = 3,\ c_2 = 6.93,\ w_{\text{to}} = 5.14 G_f / f_t$$

Figure 1. Stress-crack opening law according to Hordijk (1991).

finding an optimal return point on the failure surface. The return-mapping algorithm for the plastic model is based on predictor–corrector approach as is shown in figure 2 (for details see Cervenka & Cervenka 1999).

The combined algorithm must determine the separation of strains into plastic and fracturing components, while it must preserve the stress equivalence in both models. The algorithm is based on a recursive iterative scheme. It can be shown that such a recursive algorithm cannot reach convergence in certain cases such as, for instance, softening and dilating materials. For this reason the recursive algorithm is extended by a variation of the relaxation method to stabilize convergence.

2.3 *Microplane model*

The basic idea of the microplane model is to abandon constitutive modelling in terms of tensors and their invariants and formulate the stress–strain relation in terms of stress and strain vectors on planes of various orientations in the material, now generally called the microplanes. This idea arose in G.I. Taylor's pioneering study from 1938 of hardening plasticity of poly-

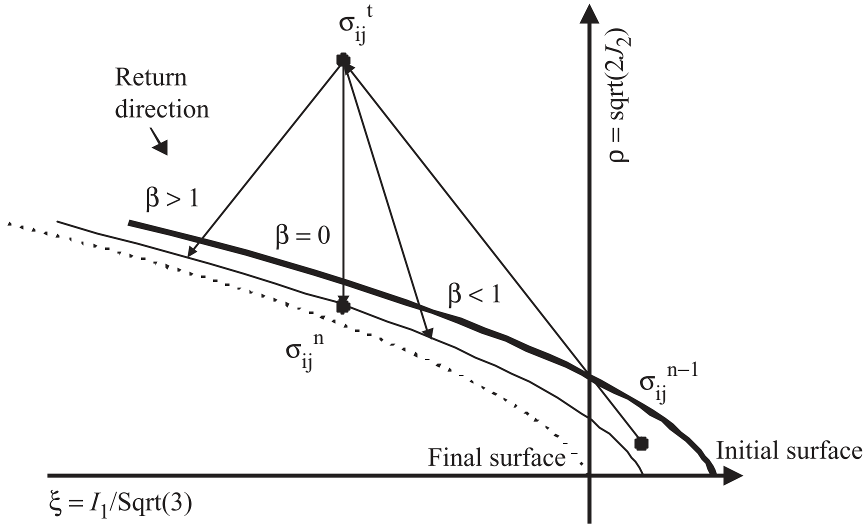


Figure 2. Plastic predictor–corrector algorithm according to Cervenka & Cervenka (1999).

crystalline metals. Proposing the first version of the microplane model, Bažant (1984), in order to model strain-softening, extended or modified Taylor's model in several ways (for details see Bažant *et al* 2000), of which the main one was the kinematic constraint between the strain tensor and the microplane strain vectors. Since 1984, there have been numerous improvements and variations of the microplane approach. A detailed overview of the history of the microplane model is given by Bažant *et al* (2000). Sketch of the fundamental concepts of the microplane model is shown in figure 3.

In the microplane model, the constitutive equations are formulated on a plane called the microplane with an arbitrary orientation characterized by its unit normal n_i. The kinematic constraint means that the normal strain ε_N and shear strains ε_M, ε_L on the microplane are calculated as the projections of the macroscopic strain tensor.

$$\begin{aligned}\varepsilon_N &= n_i n_j \varepsilon_{ij},\\ \varepsilon_M &= \tfrac{1}{2}\left(m_i n_j + m_j n_i\right)\varepsilon_{ij},\\ \varepsilon_L &= \tfrac{1}{2}\left(l_i n_j + l_j n_i\right)\varepsilon_{ij},\end{aligned}$$

where m_i and l_i are chosen orthogonal vectors lying in the microplane and defining the shear strain components.

The constitutive relations for the microplane strains and stresses can be generally stated as:

$$\begin{aligned}\sigma_N(t) &= F^t_{\tau=0}\left[\varepsilon_N(\tau), \varepsilon_M(\tau), \varepsilon_L(\tau)\right],\\ \sigma_M(t) &= G^t_{\tau=0}\left[\varepsilon_N(\tau), \varepsilon_M(\tau), \varepsilon_L(\tau)\right],\\ \sigma_L(t) &= H^t_{\tau=0}\left[\varepsilon_N(\tau), \varepsilon_M(\tau), \varepsilon_L(\tau)\right],\end{aligned}$$

where F, G and H are functionals of the history of the microplane strains in time t. A detailed

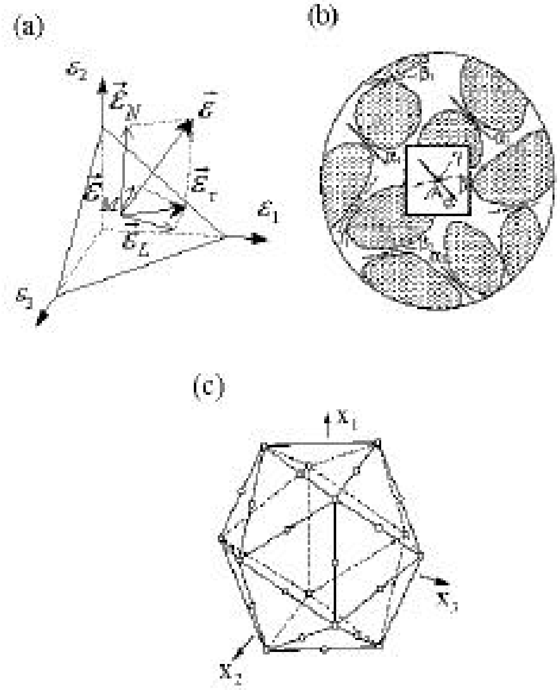


Figure 3. Fundamental concepts of the microplane model.

derivation of these functionals is given by Caner & Bažant (2000). The macroscopic stress tensor is obtained by the principle of virtual work that is formulated for a unit hemisphere Ω. After integration, the following expression for the macroscopic stress tensor is recovered:

$$\sigma_{ij} = \frac{3}{2\pi}\int_{\Omega} S_{ij}\,d\Omega \approx 6\sum_{\mu=1}^{N_m} w_i S_{ij}^{(\mu)},$$

$$S_{ij} = \sigma_N n_i n_j + (\sigma_M/2)\left(m_i n_j + m_j n_i\right) + (\sigma_L/2)\left(l_i n_j + l_j n_i\right),$$

where the integral is approximated by an optimal Gaussian integration formula for a spherical surface.

The microplane model M4 derived above is implemented into the finite element package ATENA. Details about the implementation and applications are given by Bažant *et al* (2001).

3. Software package ATENA

ATENA is a commercial finite element software package for nonlinear simulation of concrete and reinforced concrete structures. Based on advanced material models, as described above, it can be used for realistic simulation of structural response and behaviour.

ATENA works under the MS Windows operating system and its code is written in MS Visual C++. It heavily uses MFC and ATL libraries, thereby ensuring high productivity in code development and high compatibility with other third-party PC-based software. The code has object-oriented architecture. It is created in a hierarchical manner and each SW layer has its own DLL library (or libraries). Code and associated data are arranged in objects together (in i.e. C++ classes). ATENA system consists of several dynamically linked libraries (DLLs)

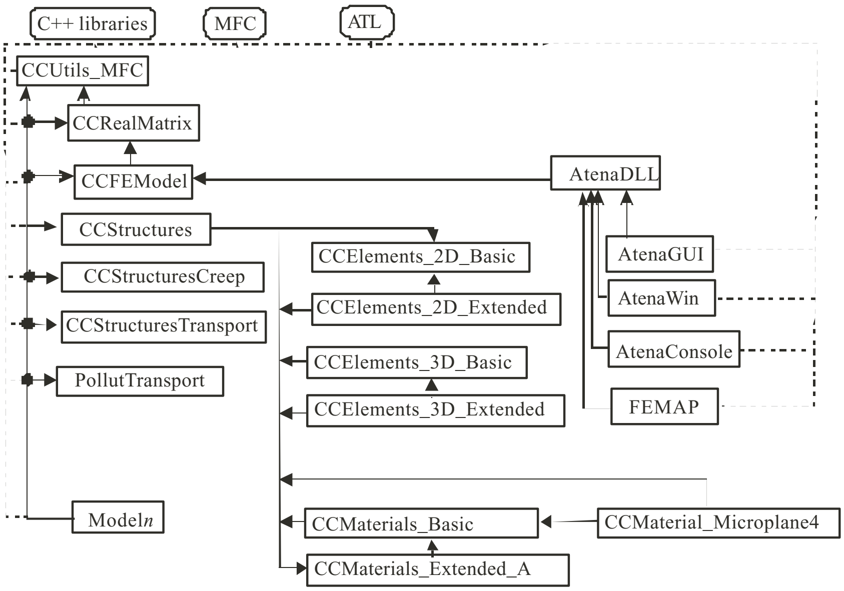


Figure 4. Layered structure of ATENA system.

and a few control programs (figure 4). It is believed that architecture and build up of ATENA system supersede usual finite element packages.

ATENA offers a user-friendly graphical interface, which enables efficient solving of engineering problems including anchoring technology and reinforcing of concrete (ATENA 2000). Native ATENA GUI is available for 2D and rotationally symmetrical problems. It supports the user during pre- and post-processing, and enables real-time graphical tracing and control during the analysis. ATENA pre-processing includes an automatic meshing procedure, which generates Q10, isoparametric quadrilateral and triangular elements. Reinforcement in ATENA can be treated as smeared reinforcement, reinforcing bars or prestressing cables. The discrete reinforcement is independent on the finite element mesh. Graphical post-processing can show cracks in concrete, with their thickness, shear and residual normal stresses. User-defined crack filter is available for obtaining of realistic crack patterns. Other important values (strains, stresses, deflections, forces, reactions etc.) can be represented graphically as rendered areas, isoareas, and isolines, in the form of vector or tensor arrow fields. All values can also be obtained in well-arranged numerical form. The interactive solution control window (figure 5) enables graphical as well as numerical monitoring of the actual task, and supports user interventions during the analysis (user interrupt, restart). For 3D pre- and post-processing, professional third party software FEMAP in combination with alphanumerical ATENA Console window is employed.

ATENA enables loading of the structure with various actions: body forces, nodal or linear forces, supports, prescribed deformations, temperature, shrinkage, pre-stressing. These loading cases are combined into load steps, which are solved utilizing advanced solution methods: Newton–Raphson, modified Newton–Raphson or arc-length. Secant, tangential or elastic material stiffness can be employed in particular models. Line-search method with optional parameters accelerates the convergence of solution, which is controlled by residua-based and energy-based criteria. This is only a concise survey of ATENA features. All of the described features support the user by engineering analysis of connections between steel and concrete and computer simulation of its behaviour.

4. Conclusions

The nonlinear finite element package ATENA is based on advanced constitutive models. Crack band approach employed for tensile and compressive softening avoids the finite element mesh sensitivity of solution.

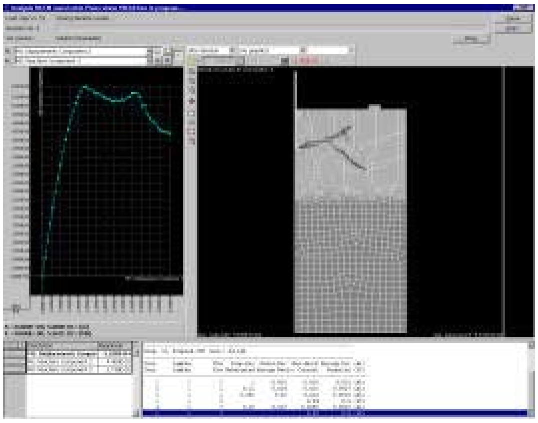


Figure 5. ATENA real-time graphical window.

ATENA is able to predict and explain behaviour of steel reinforcement as well as steel anchors in concrete structures in a consistent way. It can be effectively used to support and extend experimental investigations for innovative solutions in the field of connections between steel and concrete.

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