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ORIGINAL ARTICLE

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Atmospheric angular momentum time-series: characterization of their internal noise and creation of a combined series

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Abstract The atmosphere induces variations in Earth rotation. These effects are classically computed using the "angular momentum approach". In this method, the variations in Earth rotation are estimated from the variations in the atmospheric angular momentum (AAM). Several AAM time-series are available from different meteorological centers. However, the estimation of atmospheric effects on Earth rotation differs when using one atmospheric model or the other. The purpose of this work is to build an objective criterion that justifies the use of one series in particular. Because the atmosphere is not the only cause of Earth rotation variations, this criterion cannot rely only on a comparison of AAM series with geodetic data. Instead, we determine the quality of each series by making an estimation of their noise level, using a generalized formulation of the "three-cornered hat method". We show the existence of a link between the noise of the AAM series and their correlation with geodetic data: a noisy series is usually less correlated with Earth orientation data. As the quality of the series varies in time, we construct a combined AAM series, using time-dependent weights chosen so that the noise level of the combined series is minimal. To determine the influence of a minimal noise level on the correlation with geodetic data, we compute the correlation between the combined series and Earth orientation data. We note that the combined series is always amongst the best correlated series, which confirms the link established before. The quality criterion, while totally independent of Earth orientation observations, appears to be physically convincing when atmospheric and geodetic data are compared.

Keywords Earth rotation · Atmospheric angular momentum · Data combination · Three-cornered hat method

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1 Introduction

The rotational motion of the Earth is not regular. Fluctuations are due to the presence of the Moon, the Sun and, to a lesser extent, the other planets, which apply a gravitational torque on the Earth's equatorial bulge. On the other hand, large-scale variations in the distribution of masses in the external fluid layers (the atmosphere and oceans) also induce variations in Earth rotation (e.g., Barnes et al. 1983; Gross et al. 2003). In addition, the response of the Earth to these external forcings is influenced by its internal constitution.

The variations in Earth rotation are divided among three different motions: the nutation and the polar motion, which together contain all the variations in the direction of the rotation axis, and the length-of-day (LOD) variations, which occur because of rotational speed fluctuations. Nutations are defined as the retrograde quasi-diurnal variations (from -1.5 to -0.5 cycles per sideral day) in the terrestrial reference frame (TRF) and are treated, by convention, in the celestial reference frame (CRF), where they have periods greater than two days. The polar motion is defined from the TRF and contains all the variations with frequencies that are not in the retrograde quasi-diurnal band [see, e.g., Brzezinski (1992) for an explanation of the conventional separation between nutation and polar motion].

Because the motions of the Moon, Sun and planets are mostly periodic, with periods greater than a few days in the CRF, their gravitational torque mainly gives rise to nutations, and they nearly have no effect on polar motion. On the other hand, the main motions of the masses in the oceans and atmosphere have long periods (e.g., annual and semi-annual) in the TRF, and create the polar motion. The atmosphere and oceans also induce variations in the LOD.

This paper is concerned with the estimation of atmospheric effects on polar motion and LOD. These can be computed with two equivalent methods. The first one consists in estimating the torques produced by the atmosphere acting on the Earth and is called the "torque approach" (e.g., Barnes et al. 1983, de Viron et al. 2005). The second method, classically used, is the "angular momentum approach" (for recent papers using this method, see e.g. Kouba and Vondrák 2005, Kouba 2005). This method considers that, as far as the atmospheric effects on Earth rotation are concerned, the combined solid Earth/atmosphere system is isolated. Its angular momentum is thus conserved so that variations in the Earth angular momentum are associated with opposite variations in the atmospheric angular momentum (AAM).

In order to evaluate atmospheric effects on Earth rotation, one generally uses the *effective* AAM (EAAM) functions, also called atmospheric excitation functions, introduced by Barnes et al. (1983), rather than the AAM themselves. Both represent essentially the same information, except for a scaling factor: the EAAM are obtained from the AAM by convolution of a transfer function accounting for the non-rigidity of the Earth. Because the atmosphere induces variations in Earth rotation by an exchange of angular momentum, comparison (and in particular the correlation) of the EAAM series with their geodetic equivalent leads to an estimation of the atmospheric effects on Earth rotation.

The atmospheric excitation functions are commonly split into two parts: the first one represents a rigid rotation of the atmosphere with the Earth and is called the "pressure term", and the second one is the angular momentum due to the motions in the atmosphere relative to the Earth and is called the "wind term".

Several meteorological centers run their models, developed in the context of weather forecasting, to compute their EAAM time-series. The International Earth Rotation and Reference Systems Service (IERS) Special Bureau for the Atmosphere (Salstein et al. 1993) provides the data coming from five meteorological analyses: the European Centre for Medium-Range Weather Forecasts (ECMWF), the United Kingdom Meteorological Office (UKMO), the Japanese Meteorological Agency (JMA), the National Center for Environmental Prediction (NCEP), and the National Center for Environmental Prediction/ National Center for Atmospheric Research (NCEP/NCAR) reanalysis. Unfortunately, as we show in Sect. 4, when the correlation between the atmospheric excitation, provided by these centers, and geodetic data is computed, the results differ from one atmospheric model to another. The estimation of atmospheric effects on Earth rotation thus depends on the atmospheric series chosen.

Until now, the choice of one series in particular relies only on technical criteria, like the length of the time-series or the regularity of its sampling, which imply that the NCEP/NCAR reanalysis time-series is often used (e.g., Nastula and Salstein 1999; Kolaczek et al. 2003). It must be noted that the NCEP/NCAR, as a reanalysis, is more consistent over a long time period than the other analyses. This model is also used when the ocean is part of the study, because it is necessary to work with the same atmospheric model as the one used for the ocean forcing, which is most of the time the NCEP/NCAR reanalysis (e.g., Gross et al. 2003).

However, as far as we know, no studies has shown that the NCEP/NCAR reanalysis series has the best quality, so that, until now, the choice to work with this series is completely arbitrary. It thus seems necessary to find an objective criterion to either justify the use of the reanalysis series or determine which other one should be used. This is the purpose of this paper.

2 Problem definition

At first glance, one could think that the atmospheric series that has the best quality should be the one that best explains the variations in Earth rotation and thus is the better correlated with geodetic observations. However, the atmosphere is not the only cause of Earth rotation variations (the oceans and hydrology also play a role). A better correlation of one atmospheric series with the Earth orientation observations does not necessarily imply that the series has a better quality: the better correlated series might overestimate the atmospheric effect, which makes the overall budget worse when the oceans and hydrology are taken into account.

In order to consistently compare the atmospheric and Earth rotation observations, the effects of the oceans and hydrology must be subtracted from the geodetic observations and the atmospheric data must be compared to the residuals. However, we did not make this kind of residual analysis for several reasons. The main reason is that the oceanic models are not independent of the atmospheric models, the latter being used as input forcing. To make a meaningful residual analysis, the AAM time-series should be compared to the geodetic observations from which the oceanic angular momentum from an oceanic model forced by the corresponding atmospheric model would have been subtracted. The problem is that, for several atmospheric models, no oceanic model forced by them exists, with most of the oceanic models being forced by the NCEP/NCAR reanalysis data. If an oceanic model forced by the NCEP/NCAR reanalysis is used, the atmospheric series that will best explain the residuals would likely be the NCEP/NCAR reanalysis and this method will thus not be objective.

Another reason is that, presently, the quality of the modeling of these fluids is such that adding the ocean contribution does not completely remove the discrepancies between the observations of Earth rotation and the joint effects of the atmosphere and ocean (e.g., Gross et al. 2003). In some cases, subtracting the contribution of the geodetic observations from the ocean observations even increases the variance of the residuals (Holme and de Viron 2005).

In this work, we propose an alternative approach that does not rely on a comparison with Earth orientation data. We propose to determine the quality of the AAM series by making a comparison of the series with each other. For this purpose, we use the so-called "three-cornered hat" method, developed in the context of the characterization of frequency standard stability (Gray and Allan 1974). When at least three time-series of the same process are available, this method provides an estimation of their individual noise level. Its main interest is that, unlike other classical methods (e.g., weighted

Table 1 Atmospheric angular momentum (AAM) data for different models

Model	Covered period	Sampling period (h)	Missing data (%)
ECMWF	1/1/1988–4/1/2000	6	13
UKMO	27/11/1986–4/1/2000	6	8
JMA	1/4/1993–30/12/2000	6	4
NCEP	1/7/1976–13/3/1997	24	7
	22/4/1997–4/1/2000	6	
NCEP/NCAR reanalysis	1/1/1948–31/8/2003	6	0

least-squares, Kalman filter, etc.), it gives information about the noise without making any assumption about the signal statistics.

However, the three-cornered hat method, in its classical formulation, has some limitations: it makes the hypothesis that there is no correlation between the noise of the series, which can lead to meaningless results. To avoid these problems, we use a generalized formulation of the method which takes the correlations into account. This generalization was first proposed by Tavella and Premoli (1991) and then further developed by Premoli and Tavella (1993), Tavella and Premoli (1994) and Galindo et al. (2001).

As we will show, the noise level of the series, estimated with the generalized three-cornered hat method, varies with time to a very large extent. None of the series has always the lowest noise level. This provides the motivation to generate a series by combination of the five time-series available, which would present, at each epoch, a noise level as low as possible. The combined series allows us to take advantage of the information included in each series, with the weight of a series in the combination depending on its quality.

Such a combination of time-series with weights determined by the three-cornered hat method has already been done for Earth orientation data (e.g., Gambis 2002), which used the classical method of Gray and Allan (1974), and by Chin et al. (2005), which used a generalized three-cornered hat method, also taking the correlations into account, but different from the one we use in this paper. The differences between the methods are described in Sect. 5 of this paper.

3 Data used

3.1 Atmospheric data sets

The EAAM time-series are provided by the IERS Special Bureau for the Atmosphere (SBA). Each series is obtained as output of a particular meteorological model. The data coming from five meteorological models (ECMWF, UKMO, JMA, NCEP and the reanalysis made by the NCEP/NCAR centers) that assimilate a variety of observations as input data, are given separately for the wind and pressure terms and split into the North and South Hemisphere contribution.

The pressure term is given both in the inverted barometer (IB) and in the non-inverted barometer (NIB) approximations. These two approximations describe two different simplified reactions of the ocean under variations in atmospheric pressure. The IB approximation assumes that the total pressure on the oceanic floor is constant so that a larger atmospheric pressure is compensated by a lower water level. This approximation is known to be mostly valid at periods long enough for the ocean mass to readjust (longer than a few days). The second approximation (NIB) considers that the ocean is completely decoupled from the atmosphere so that it does not react to the atmospheric pressure changes. This approximation is mainly used when dealing with short-period variations in the atmosphere. In this work, we focus on the long-period variations so we will use only the IB version of the pressure term.

The angular momentum is a vectorial quantity and is given in terms of three spatial components: the Z-component is directed along the mean principal axis of maximum inertia, the X- and Y-components are in the equatorial plane, the first one being in the direction of the Greenwich meridian and the second one such as to complete the right-handed triad. The period covered for each series as well as the sampling period and the percentage of missing data are given in Table 1.

We use the three-cornered hat method (Gray and Allan 1974; Tavella and Premoli 1991), which requires the availability of at least three different series at each time. We thus use the data only in the period from 1988 to 2000. From 1 January 1988 to 1 April 1993, we deal with the four time-series: ECMWF, UKMO, NCEP and NCEP/NCAR reanalysis, while from 1 April 1993 to 1 January 2000, the JMA data are used as well.

First we pre-processed the data as follows: because we focus on the low frequencies (periods longer than one day), we averaged the data over each day. Then, we had to correct jumps (sharp changes in the mean) in the data. The jumps are due to changes in the model used by the meteorological centers, often being the ocean-bottom topography or the resolution. In order to use the three-cornered hat method, it is important to correct them because the method relies on an estimation of the variance of the series and such jumps give rise to an increase in the variance, which will be interpreted as an increase in the noise level. In this case, the correction was quite easy because the jumps were so big that the highest value of the time series before the jump was lower than the lowest value of the time series after the jump. We only had to estimate the mean of the two parts of the series and

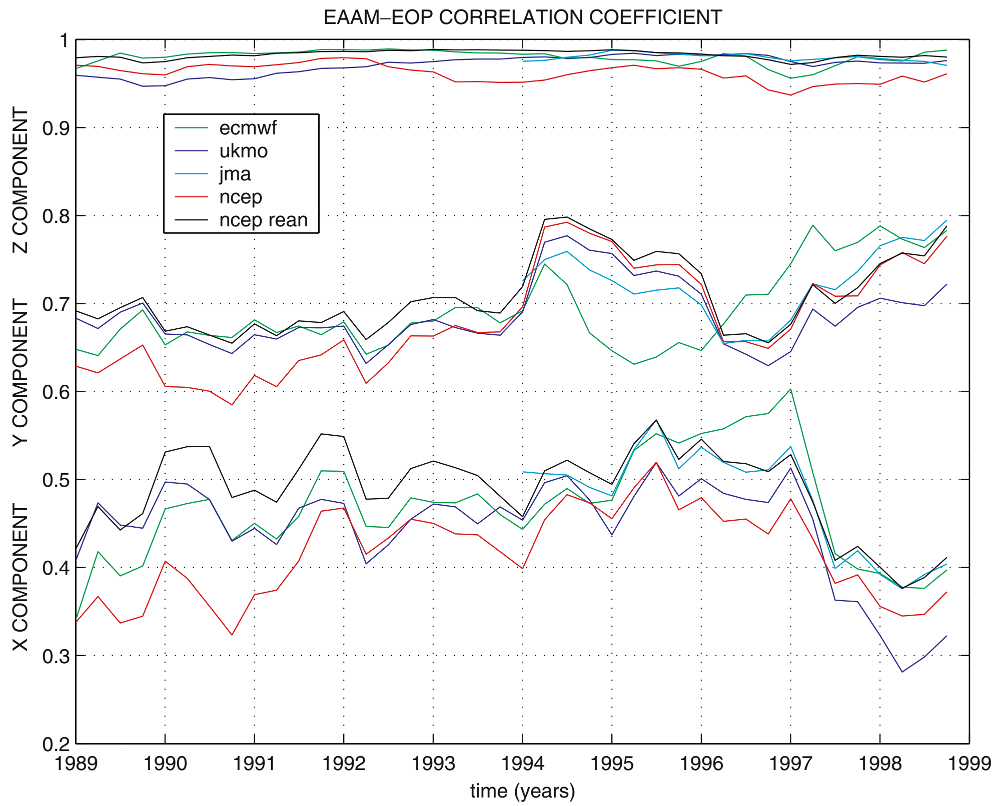

Fig. 1 Time-variation of the correlation coefficient between the total atmospheric angular momentum (AAM) series (wind + IB-pressure terms) and Earth orientation data

subtract it. Lastly, we removed a trend in the time series with a moving average technique.

3.2 Geodetic data sets

Several Earth orientation data series are available. Those series result from various combinations of the data obtained with different techniques of Earth orientation measurement. The data series used in this paper is the COMB2002 series developed by Gross (2003). This series is the result of the combination of data obtained from the following techniques of Earth orientation measurement: laser ranging to the Moon or artificial satellites, very long baseline interferometry (VLBI), and the global positioning system (GPS). These series were combined using the Kalman filter method. The data are sampled with a period of 1 day and there are no missing data.

4 Correlation between atmospheric and geodetic data

In order to estimate the atmospheric effects on Earth rotation, atmospheric data are compared to Earth orientation data using the relation (Brzezinski 1992; Gross 1992):

$$\mathbf{p}(t) + \frac{i}{\sigma_0}\frac{d\mathbf{p}}{dt} = \chi(t), \quad (1)$$

where $\mathbf{p}(t)$ is the Earth orientation measurement (the position of the celestial ephemeris pole), σ_0 is the complex Chandler frequency, i is the imaginary unit and χ is the EAAM. The left-hand side of Eq. (1) is sometimes called the "geodetic excitation". In the following, we use the acronym AAM in the place of EAAM.

In order to get a first comparison between the AAM series, we estimated the correlation coefficient between the total AAM, wind plus IB-pressure terms, and the geodetic excitation. The time variation was obtained using a time window of 2 years moving by 3 months. The results are presented in Fig. 1.

The correlation level is very different for the three spatial components. The correlation is of about 97% for the Z-component, while only 70 and 45% for Y- and X-component, respectively. This can be explained by the fact that for the Z-component, the atmosphere is the main cause of variations in the angular momentum of the Earth, while for the X- and Y-components, the ocean plays an important role. To generate variations in the angular momentum of the Earth in a particular direction, there must be displacements of matter or changes in the surface pressure in a plane perpendicular to this direction. For the Z-component, the variations in the angular momentum are due to variations in the direction of the parallels. In the atmosphere, such motions are generated by the zonal winds, which dominate the atmospheric circulation, while in the ocean, the continents are barriers to large global scale motions in this direction.

The variations in the X- and Y-components are due to displacements between the North- and South-pole, motions that occur in the ocean water, as well as in the atmosphere; explaining the importance of the oceans in polar motion excitation. The difference in the correlation between the X- and Y-components can be explained by the fact that for the X-component, the AAM has roughly the same amplitude as the oceanic angular momentum (OAM), while for the Y-component, the amplitude of the AAM is very much greater than the OAM. This explains why the correlation for the AAM only with Earth orientation data is greater for Y than for X.

The main interest of Fig. 1 for our study is that it shows the differences between the atmospheric analyses and their impact on the estimation of atmospheric effects on Earth rotation. In 1991, for example, the NCEP model gives a correlation of about 35 % for the X-component while 50% is obtained from the NCEP/NCAR reanalysis series. These discrepancies between the different meteorological models are the motivation of our work.

5 The three-cornered hat method

To determine the quality of the AAM series, we have to estimate their noise levels. Because the atmospheric processes are complex, no stochastic modeling of their noise level has been made, so that we choose to use the three-cornered hat method which allows an estimation of the noise level of the series only by comparing them against each other.

From a mathematical point of view, if $\{X^i\}_{i=1,\ldots,N}$ are N time series, the three-cornered hat method splits them into two terms:

$$X^i = S + \epsilon^i, \quad \forall i = 1, \ldots, N, \tag{2}$$

where S is defined as the part of the series that is common to all of them and ϵ^i is what remains, i.e., the part particular to the given series. With this definition, the three-cornered hat method relies on an important hypothesis, which states that S is the signal and ϵ^i is the noise. Using this hypothesis, information about the noises can be obtained by taking differences between the series.

The three-cornered hat method allows an estimation of the variance of the individual noise of each series, under some assumptions on the correlations between those noises. Different formulations of the three-cornered hat method are used, depending on the assumptions made on the correlations between the noises.

In the case of the AAM series used in this paper, the noise is due to the differences in the modeling of the atmospheric circulation from one meteorological center to the other. Although each center works independently, the time series produced cannot be considered as uncorrelated: they mostly use the same dynamical equations and the same data for assimilation. However, we do not know how much the noises of the series are correlated.

5.1 Classical method

The classical three-cornered hat method, introduced by Gray and Allan (1974), relies on the assumption that the noises of the series are not correlated. Although we do not expect that the noises of the AAM series fulfill this condition, we briefly present this method because it is the most commonly used and it gives the basis to present the generalized method.

The classical method takes the difference between every set of two series and then calculates their variance. For three time-series, under the assumption that the noises of the series are not correlated, we get the linear system:

$$\begin{cases} \mathrm{Var}(X^1 - X^2) = \mathrm{Var}(\epsilon^1 - \epsilon^2) = \mathrm{Var}(\epsilon^1) + \mathrm{Var}(\epsilon^2) \\ \mathrm{Var}(X^1 - X^3) = \mathrm{Var}(\epsilon^1 - \epsilon^3) = \mathrm{Var}(\epsilon^1) + \mathrm{Var}(\epsilon^3) \\ \mathrm{Var}(X^2 - X^3) = \mathrm{Var}(\epsilon^2 - \epsilon^3) = \mathrm{Var}(\epsilon^2) + \mathrm{Var}(\epsilon^3) \end{cases} \tag{3}$$

as the covariances have been assumed to be zero. Solving this system then gives an estimate of the individual noise variances. When more than three time series are available, the linear system in Eq. (3) is overdetermined and can be solved by a least-squares method, for instance.

However, this classical method sometimes gives meaningless results: the estimated variances can become negative. An overdetermined system, giving rise to an inconsistent result, is an indication that the problem is ill-formulated (mainly because of the assumption that the noises are not correlated).

5.2 Generalized method

A generalized method, which does not make the assumption of zero correlation, has been developed by Tavella and Premoli (1991, 1994), Premoli and Tavella (1993) and Galindo et al. (2001). We present the key features of this method.

5.2.1 Statement of the problem

Let $\{X^i\}_{i=1,\ldots,N}$ be a set of N time-series. We take the difference between each series and one of them, arbitrary chosen as the reference:

$$Y^{iN} \equiv X^i - X^N = \epsilon^i - \epsilon^N, \quad i = 1, \ldots, N-1, \tag{4}$$

where X^N is the reference series. As demonstrated by Tavella and Premoli (1994), the results are independent of the choice of one series or another. In the work presented here, because it is the most often used series, we take the NCEP/NCAR reanalysis series as reference.

The covariance matrix of the series of differences is then computed:

$$\mathbf{D}^{ij} \equiv \mathrm{Cov}(Y^{iN}, Y^{jN}), \quad i, j = 1, \ldots, N-1. \tag{5}$$

Because of the bilinearity property of the covariance, this matrix can be expressed by:

$$\mathbf{D}^{ij} = \mathbf{R}^{ij} - \mathbf{R}^{iN} - \mathbf{R}^{jN} + \mathbf{R}^{NN}, \quad i, j = 1, \ldots, N-1, \tag{6}$$

where we have introduced the $N \times N$ covariance matrix of the individual noises $\mathbf{R}^{ij} = \mathrm{Cov}(\epsilon^i, \epsilon^j)$ of which the elements are the unknowns of the problem. The matrices $\mathbf{R}$ and $\mathbf{D}$, being real and symmetric by definition, the number of independant parameters is $N(N+1)/2$ for $\mathbf{R}$ and $(N-1)N/2$ for $\mathbf{D}$. Equation (6) is underdetermined with $(N-1)N/2$ equations and $(N+1)N/2$ unknowns. It thus has N free parameters that we can choose to be $\mathbf{R}^{1N}, \ldots, \mathbf{R}^{NN}$, i.e., the covariances between the noise of each series and the noise of the reference series, and in terms of which the solution for the other unknowns is given by:

$$\mathbf{R}^{ij} = \mathbf{D}^{ij} + \mathbf{R}^{iN} + \mathbf{R}^{jN} - \mathbf{R}^{NN}, \quad i, j = 1, \ldots, N-1. \tag{7}$$

5.2.2 Choice of the free parameters

An important constraint on the solution domain for the free parameters is that the estimated covariance matrix $\mathbf{R}$ must be definite positive. Tavella and Premoli (1994) showed that this condition is satisfied if and only if its determinant is strictly positive:

$$\mathrm{Det}(\mathbf{R}) > 0. \tag{8}$$

This condition restricts the solution domain for the free parameters, but is not sufficient to determine them.

To fix the value of those parameters, an hypothesis on the correlations is introduced: while the classical method made the hypothesis of uncorrelated series, the generalized method assumes that the right solution is the one with the smallest correlation between the noises of the different series (as small as the constraint on the solution domain allows it). The parameters are thus chosen in such a way that the sum of the estimated correlations between all the time series is minimal, taking the constraint in Eq. (8) on the solution domain into account.

Using this generalized method, we implicitly make the assumption that the time series are only poorly correlated. Mathematically, the constraint on the correlations comes to numerically minimize the following objective function:

$$F(\mathbf{R}^{1N}, \ldots, \mathbf{R}^{NN}) = \sum_{i<j}^{N} \frac{[\mathbf{R}^{ij}(\mathbf{R}^{1N}, \ldots, \mathbf{R}^{NN})]^2}{[\mathbf{R}^{ii}(\mathbf{R}^{1N}, \ldots, \mathbf{R}^{NN})][\mathbf{R}^{jj}(\mathbf{R}^{1N}, \ldots, \mathbf{R}^{NN})]}. \tag{9}$$

A description of the algorithm used for the minimization can be found in Galindo et al. (2001). When the free parameters have been estimated, the $\mathbf{R}$ matrix is completely determined by Eq. (7).

In Galindo et al. (2001), the method was tested on simulated data. It took two sets of four time-series, poorly correlated for the first set and strongly correlated for the second set, and computed the complete covariance matrix by taking the differences between their time-series and then using the generalized three-cornered hat method. This matrix was then compared with the true covariance matrix estimated directly from the simulated data to test the quality of the estimation by the generalized three-cornered hat method. The results were that, for the poorly correlated set, the estimation of the variances was quite good while the covariances, which are very small relative to the variances, were always underestimated. For the strongly correlated data set, both the variances and the covariances were largely underestimated, which verifies that this method cannot be used when the hypothesis of low correlation is not fulfilled. This generalized method allows a good estimation of the variances of weakly correlated time series but not of their covariances.

Because the noises of the AAM time series are correlated, we did not use the classical method but rather the generalized method just described. However, using this method, we make the assumption that the correlations between the noises of the AAM series are low.

5.3 Other generalized methods

Another generalized method has been used by Chin et al. (2005). Their method relies on another hypothesis on the correlations between the noises. To solve the underdetermined system in Eq. (6), they make the assumption that at least N correlations are zero. This assumption makes sense in their work, where they combine Earth orientation parameters from different measurement techniques: they assume that the series from different techniques are uncorrelated, while keeping the correlation of the series obtained from the same technique. With the AAM data used in our paper, we cannot assume that some series are uncorrelated. As such, the generalized technique of Chin et al. (2005) cannot be used in this paper.

6 Noise-level estimation

With the generalized three-cornered hat method, we determine the individual noise variance of the series produced by the five centers for the three spatial components of the AAM, separately for the wind and IB-pressure terms and for the sum of these two terms (the total excitation function). In order to obtain the noise variances as a function of time, we use a moving-window technique. We choose a 2-year length window. After some tests on the time step by which the window is moved, we found that 3 months are sufficient to obtain the main variabilities during the year. Our results are presented in Figs. 2, 3 and 4.

The noise variance of a time series gives some indication of its quality because it reflects the noise level of the series. We can thus interpret our results in terms of quality: a high-quality time-series will have a low noise variance. However, the method we used does not allow an estimation of the absolute noise level of one series in particular, the results only make sense when all the series are considered together, the noise levels estimated are relative; they have no physical meaning by themselves.

As we can see in Figs. 2, 3 and 4, the relative qualities of the series are variable in time and we cannot decide

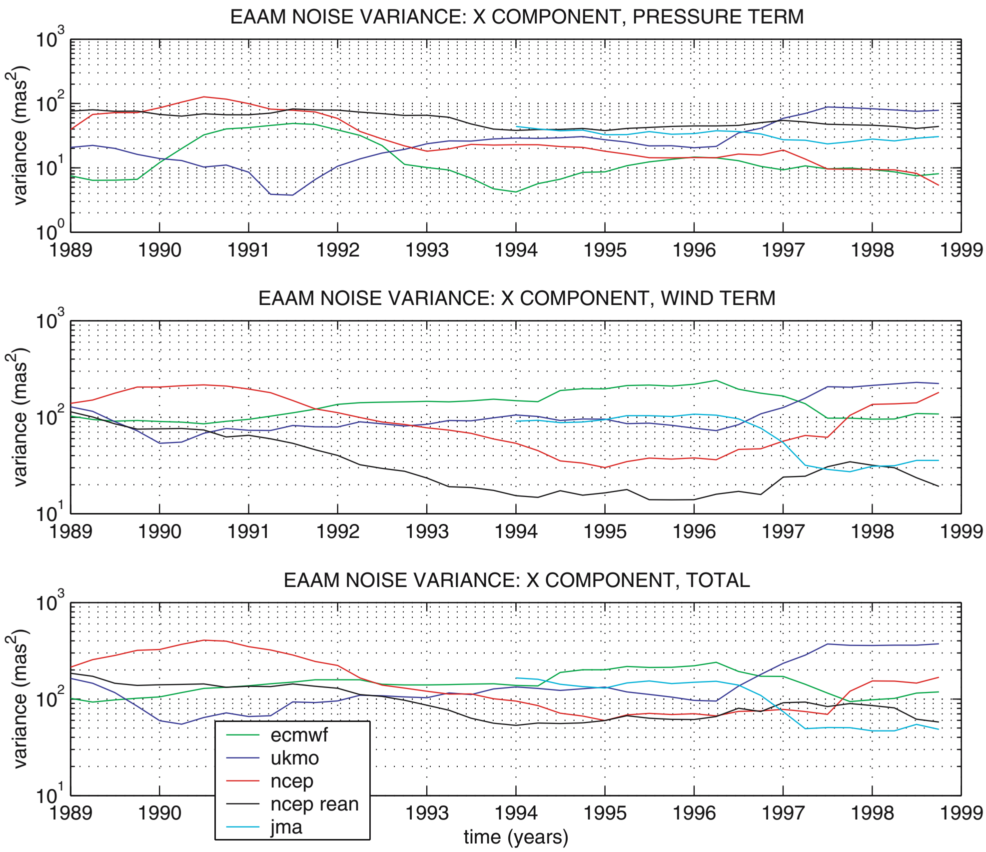

Fig. 2 Noise variance of the AAM series, X-component

from these results which series must be used because none of them has the lowest noise level for all times. Moreover, some series that are quite good during certain periods can suddenly become one order of magnitude worse than the others. We also notice that the noise variance is different for the wind and pressure term.

We want to determine whether the noise level of an AAM series has an influence on its correlation with geodetic data. For this purpose, we compare Fig. 1, representing the correlation of each AAM series with Earth orientation data, and Figs. 2, 3 and 4. In Figs. 2, 3 and 4, only the third plot, showing the noise level of the total excitation function, must be considered. By comparing Fig. 1 with Figs. 2, 3 and 4, we notice that there seems to be a link between a high noise level in the AAM time series and a low correlation coefficient with Earth orientation data. It can be clearly seen on a few examples, like the NCEP series before 1992 for the X- and Y-components, the ECMWF series between 1994 and 1997 for the three components (but especially for the Y-component), and the UKMO series after 1997 for the Y-component and before 1992 for the Z-component.

More quantitatively, we compute the correlation coefficient between the noise variance of the AAM series (shown on the third plot of Figs. 2, 3 and 4) and their time-variable correlation with geodetic data (shown in Fig. 1). The correlation coefficients are given in Table 2. Coefficients in bold fonts are significant at least at the 95% level. Most of them are negative, indicating that a higher noise level in the AAM series coincides with a lower correlation. This is not surprising and validates our estimation of the noise level.

We conclude that the correlation of an AAM series with Earth orientation data is very often related to its noise variance. When the noise variance of an AAM series increases, in most of the cases, its correlation with geodetic data decreases, which indicates that the method we propose is physically consistent.

7 A combined AAM time series

As we showed in Sect. 6, the noise level of the AAM series varies so much in time that we cannot decide which series should be used. To solve this problem, we make a combination of the AAM series, taking their quality into account, such that the combined series has a noise level as low as possible.

From the mathematical point of view, we want to create a time-series $X(t)$ by combination of the five existing ones $\{X^i\}_{i=1,\ldots,5}$:

$$X(t) = \sum_{i=1}^{5} w_i(t) X_i(t), \tag{10}$$

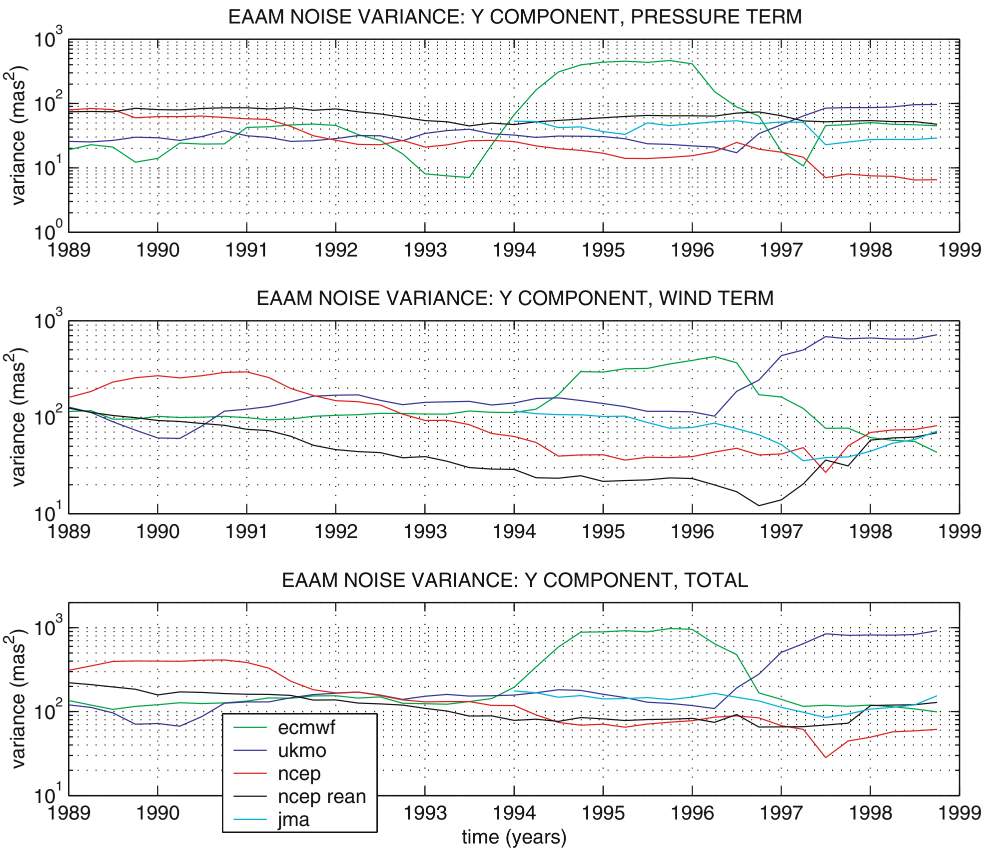

Fig. 3 Noise variance of the AAM series, Y-component

where $w_i(t)$ is the time-dependant weight associated with the $X_i(t)$ series and reflecting its quality at that time. These weights are normalized in such a way that $\sum_{i=1}^{5} w_i = 1$.

We determine the weights by requiring that the noise variance of the combined series be minimal. Because the generalized three-cornered hat method assumes that the correlations between the noises are low, we neglect these correlations in the calculation of the weights. This approximation is encouraged by the fact that the generalized method, as explained before, does not give a good estimation for the correlations.

The condition of a minimal noise variance for the combined series, together with the approximation that consists of neglecting the correlations, gives the following normalized weights as a solution:

$$w_i = \frac{\frac{1}{\mathrm{Var}(\epsilon_i)}}{\sum_{j=1}^{5} \frac{1}{\mathrm{Var}(\epsilon_j)}}. \tag{11}$$

The weight associated with a time series is inversely proportional to its noise variance. Strictly, this formula is valid only for uncorrelated time series. By using it, we make an approximation. In our computations, the correlations are taken into account to get good estimations of the variances but their own values, which are not reliable, are not used.

We make two different combinations of the AAM series. For the first one, we make a direct combination of the total excitation series. For the second one, we independently combine the time series for the IB-pressure term and the time-series for the wind term. These combined series for the wind and pressure terms are then summed to form the new total excitation series. The second combination method is more satisfactory from the physical point of view because, as noticed in Sect. 6, the noise variances are different for the wind and pressure terms.

The combined series has, by design, a lower noise level than each individual series. We want to determine the influence of a lower noise level in the AAM series on the correlation with Earth orientation observations. Table 3 shows the correlation coefficient between the atmospheric series and the geodetic excitation. The combined series obtained from a direct combination of the total atmospheric excitation are labeled "combined series" and those for which the IB-pressure and wind terms were combined independently "combined series PW".

The combined series is well correlated with Earth orientation data, thus confirming the link we established before between the noise level and the correlation with geodetic data. For the X-component, they are better correlated than the ECMWF, UKMO and NCEP series, but are less correlated than JMA and NCEP/NCAR reanalysis series. However, for the

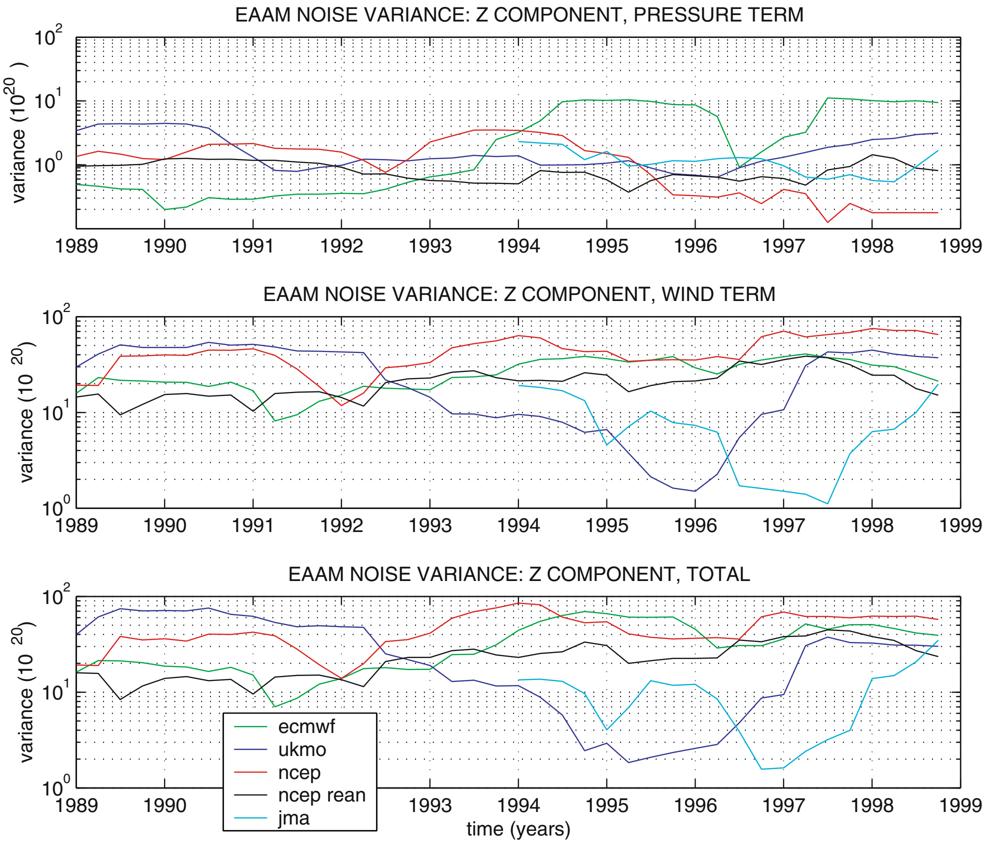

Fig. 4 Noise variance of the AAM series, Z-component

Table 2 Correlation coefficients between the noise variance of the AAM series and their correlation with Earth orientation data

Model	Correlation coefficient (%)		
	X-component	Y-component	Z-component
ECMWF	**80**	−37	−43
UKMO	**−78**	11	**−95**
JMA	**82**	−13	−54
NCEP	**−65**	**−77**	**−81**
NCEP/NCAR reanalysis	−2	−40	−12

Bold fonts indicate the coefficients which are significant at least at the 95% level

Y- and Z-components, the combined series are slightly better correlated. The high correlation between the combined AAM series and the geodetic data indicates that our combination technique, whereas based on purely statistical considerations, is also physically meaningful, and that the combination does not decrease the correlation with Earth orientation data.

The time variation of the correlation coefficient is represented in Figs. 5 and 6. The combined series present a good correlation at each epoch; they are always amongst the best correlated series. The combined series are very close to the NCEP/NCAR reanalysis series. This suggests that, if one of the five series had to be chosen, it should probably be this one. This would also be encouraged by the fact that the NCEP/NCAR reanalysis series has a rather low noise level. We also note on Figs. 5 and 6 that the two kinds of combined series present a similar correlation.

We make an additional comparison of the AAM series with the Earth orientation observations by computing their mean squared error (MSE). The results are shown in Table 4. The MSE is a measure of the distance between two series so that the lower the MSE, the closer the AAM to the Earth orientation series. By comparing Tables 3 and 4, we note that they give the same information: when an AAM series is well correlated to the geodetic data, the MSE is low.

8 Conclusions

The atmosphere exchanges angular momentum with the solid Earth, perturbing its rotation. Several meteorological centers provide AAM data. We compared the atmospheric time series by computing their correlation with Earth

Table 3 Mean correlation between atmospheric and geodetic excitation functions over the period from 1/4/1993 to 1/1/2000

Model	Correlation coefficient (%)		
	X-component	Y-component	Z-component
ECMWF	46.57	72.59	97.29
UKMO	41.96	70.16	97.62
JMA	47.97	72.75	97.23
NCEP	42.91	73.74	95.40
NCEP/NCAR reanalysis	48.20	74.23	97.88
Combined series	46.57	74.51	98.02
Combined series PW	46.58	74.46	97.85

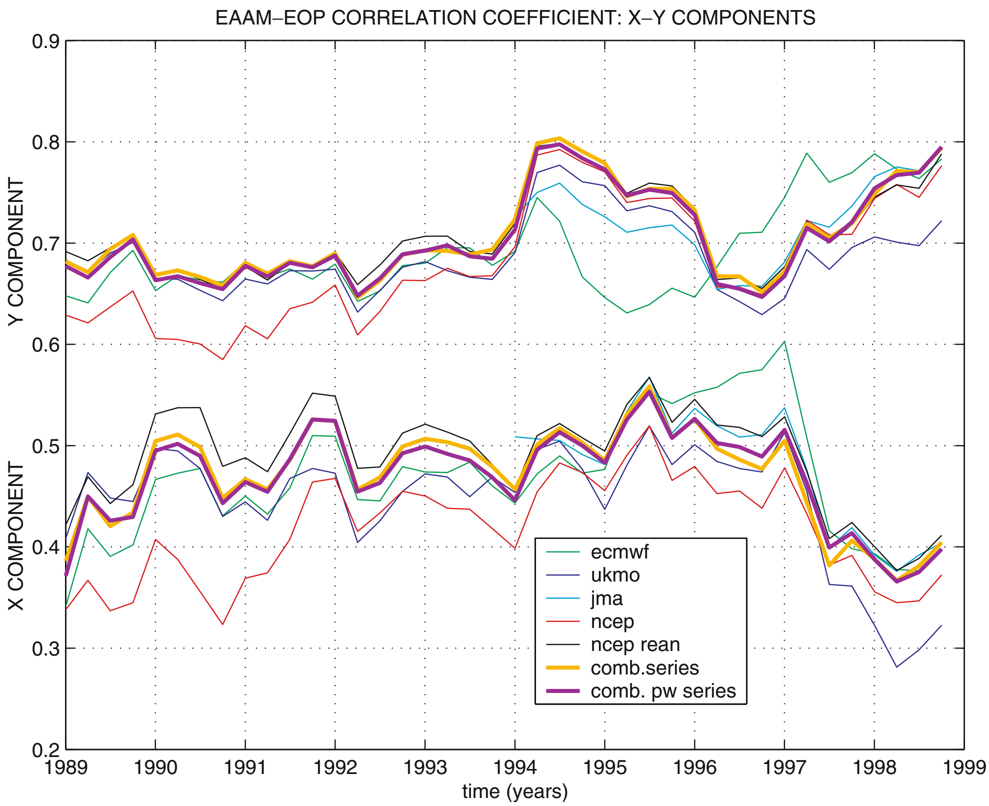

Fig. 5 Time-variation of the correlation coefficient between the AAM series (including the combined series) and Earth orientation data for the X- and Y-components

orientation data and showed the differences that occur from one AAM series to the other.

Considering the differences between the atmospheric models, our purpose was to build a criterion that would allow a less arbitrary choice of one AAM series. We built a quality criterion independent of Earth orientation observations: we estimated the noise level of the AAM series with a generalized three-cornered hat method.

We then compared the time variable noise level with the correlation with the geodetic excitation. We made the following conclusion: when the noise of an atmospheric series is higher than the others, its correlation with the geodetic data is usually lower. Our quality criterion, while being totally independent, seems to be consistent with Earth orientation data.

Because we showed that none of the series has the lowest noise level for all times, we decided to construct a combined series by making a weighted average of the existing ones, the weights being chosen so that the series has a noise variance as low as possible. We estimated a posteriori the correlation between the combined series and Earth orientation data to determine the influence of a minimal noise level on the correlation with geodetic data. We noted that the combined series presented a good correlation at each epoch. Both ways of producing the time series (combination of the total AAM series and combination of the pressure and wind terms independently) are always close to the best correlated series. We also showed that the individual combination of the wind and pressure terms was not better than the direct combination.

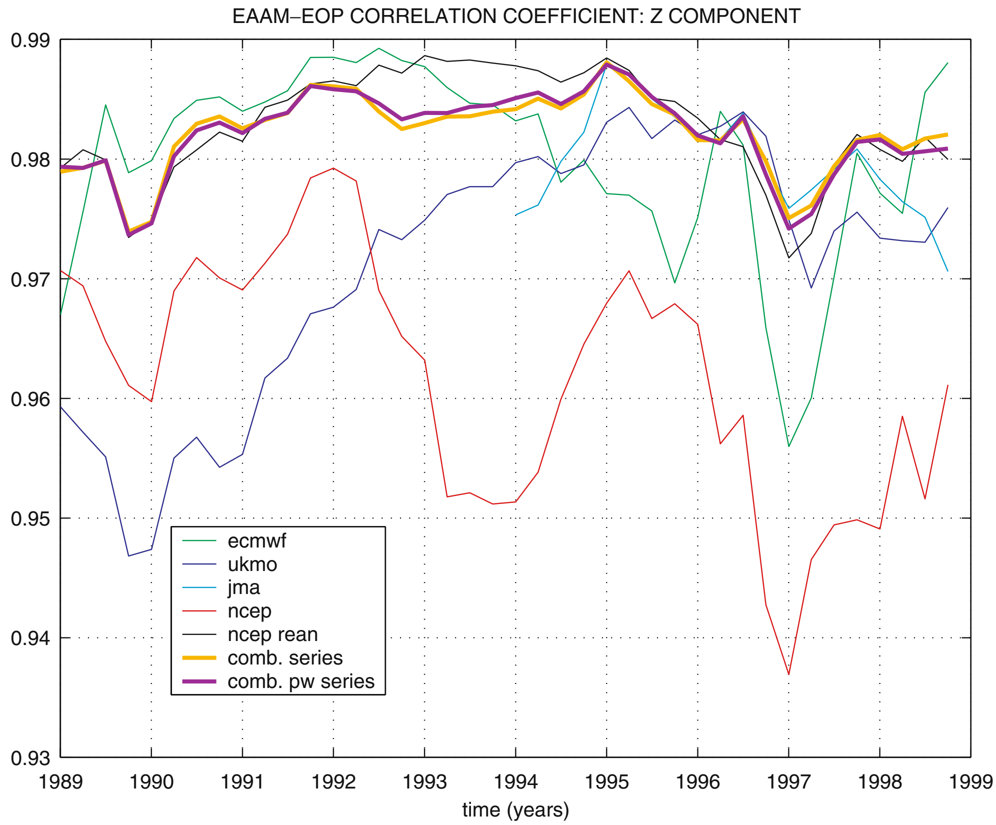

Fig. 6 Time-variation of the correlation coefficient between the AAM series (including the combined series) and Earth orientation data for the Z-component

Table 4 Mean squared error (MSE) between atmospheric and geodetic excitation functions over the period from 1/4/1993 to 1/1/2000

Model	Mean squared error		
	X-component (10^2mas^2)	Y-component (10^2mas^2)	Z-component (10^{-20})
ECMWF	15.4	14.9	110.8
UKMO	17.1	20.4	101.2
JMA	14.8	13.5	116.2
NCEP	17.3	13.8	179.2
NCEP/NCAR reanalysis	14.5	13.1	93.0
Combined series	14.7	13.7	84.3
Combined series PW	14.8	13.0	91.2

The advantage of using a combined series is that it offers a time series with a good correlation with Earth orientation for all times, while the individual series can present a good correlation at certain time step and a bad correlation at others. However, if a single time series must be chosen, then the NCEP/NCAR reanalysis time series seems to be the most appropriate because its noise is usually low and its correlation to geodetic data is close to that of the combined series.

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