

ATOMIC OSCILLATOR STRENGTHS—III

ALKALI-LIKE SPECTRA

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Summary

Oscillator strengths are calculated for dipole transitions in the first spectra of Na, K, Rb and Cs, and in the second spectra of Mg, Ca, Sr and Ba. Allowance is made for the effect of spin-orbit interaction, which produces perturbation of the energy levels and also configuration mixing.

The 2S – 2D quadrupole transitions are also considered.

1. *Introduction.* In continuation of this series (1), (2) of papers concerning absolute f -values of atomic spectrum lines, we study a number of one-electron spectra of astrophysical and laboratory interest. It is common in astrophysical literature to find f -values for the alkalis calculated by means of the coulomb approximation (3) (CA) combined with LS-coupling relative line strengths. There can be little doubt that for one-electron spectra the CA is at its best for providing radial wavefunctions, but there have been few studies of the effects of spin-orbit coupling on the relative and absolute line strengths.

Departures from a 2 : 1 intensity ratio in the doublets of the principal series of the alkali spectra have long been known (4), (5). Fermi (6) showed that configuration mixing by spin-orbit coupling is able to explain at least part of this effect. Stone (7) has shown that even if mixing is neglected the wavefunctions for the different J states in a configuration are sufficiently different, again because of spin-orbit interaction, that large departures from LS-coupling relative strengths are to be expected. In this paper we investigate the result of including both effects of spin-orbit coupling.

2. *Theory.* Spin-orbit interaction in a one-electron spectrum mixes together all terms arising from configurations with the same l (6), (8). The total wavefunction $\Phi_i(r)$ of a level $n_i l S L J$ is a mixture of pure eigenfunctions, $\phi(r)$ of all levels having the same $l S L J$:

$$\Phi_i(r) = a_i \phi_i(r) + \sum_{j \neq i} b_j \phi_j(r). \quad (1)$$

To first order $a_i = 1$ and, adopting the notation of Condon & Shortley (8)

$$b_j = \frac{(n_i l S L J | H^I | n_j l S L J)}{E_i - E_j} \quad (2)$$

where H^I is the spin-orbit Hamiltonian. The matrix elements of H^I are

$$(\mathbf{L} \cdot \mathbf{S}) \int_0^\infty \xi(r) \phi_i(r) \phi_j(r) dr$$

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where

$$\xi(r) = \frac{1}{2m^2c^2} \left(\frac{1}{r} \frac{dV(r)}{dr} \right)$$

and $V(r)$ is the potential.

With

$$(\mathbf{L} \cdot \mathbf{S}) = \frac{\hbar^2}{2} \{ J(J+1) - S(S+1) - L(L+1) \}$$

we have

$$\Phi_i(r) = \phi_i(r) + \frac{\hbar^2}{2} \{ J(J+1) - S(S+1) - L(L+1) \} \sum_{j \neq i} \phi_j(r) \times \frac{\int_0^\infty \xi(r) \phi_i(r) \phi_j(r) dr}{(E_i - E_j)}. \quad (3)$$

Following the method suggested by Fermi (6) we note that the term splitting $\Delta_{n_i l_i}$ can be found from

$$\Delta_{n_i l_i} = \hbar^2(l_i + \frac{1}{2}) \int_0^\infty \xi(r) \phi_i^2(r) dr. \quad (4)$$

Also, the one-electron Schrodinger equation for the $n_i l_i$ electron is

$$\frac{d^2 \phi_i(r)}{dr^2} - \left\{ V(r) + \frac{l_i(l_i+1)}{r^2} - E_i \right\} \phi_i(r) = 0. \quad (5)$$

For small r both $V(r)$ and $l_i(l_i+1)/r^2$ are much greater than E_i , so the eigenfunctions are related to one another by

$$\phi_j(r) = K_{ji} \phi_i(r) \text{ for small } r \quad (6)$$

where K_{ji} is a constant. As the integral in equation (4) is determined by the value of $\phi_i(r)$ at small r we have, to a good approximation,

$$K_{ji} = \left(\frac{\Delta_{n_j l_j}}{\Delta_{n_i l_i}} \right)^{1/2} \quad (7)$$

Substituting equations (4), (6) and (7) in equation (3) we have

$$\Phi_i(r) = \phi_i(r) + \frac{J(J+1) - S(S+1) - L(L+1)}{2L+1} \sum_{j \neq i} \frac{(\Delta_{n_i l_i} \cdot \Delta_{n_j l_j})^{1/2}}{E_i - E_j} \phi_j(r). \quad (8)$$

We define

$$I_{ab} = \int_0^\infty \phi_a(r) \phi_b(r) r dr$$

$$\delta_{ab} = (\Delta_{n_a l_a} \Delta_{n_b l_b})^{1/2}$$

and

$$T_a = \frac{J(J+1) - S(S+1) - L(L+1)}{2L+1}$$

for a level in $n_a l_a$.

Then

$$\int_0^\infty \Phi_i(r) \Phi_k(r) r dr = I_{ik} + T_i \sum_{j \neq i} \frac{\delta_{ij}}{E_i - E_j} I_{jk} + T_k \sum_{j \neq k} \frac{\delta_{kj'}}{E_k - E_j} I_{j'i} + T_i T_k \sum_{j \neq i} \sum_{j' \neq k} \frac{\delta_{ij} \delta_{kj'}}{(E_i - E_j)(E_k - E_{j'})} I_{jj'} \quad (9)$$

In order to calculate oscillator strengths we use the formula (9)

$$gf_{ik} = \frac{2}{3} |(E_i - E_k)| (2L_i + 1)(2L_k + 1) L_{>} W^2(L_i L_i L_k L_k; 01) \times \left\{ \int_0^\infty \Phi_i(r) \Phi_k(r) r dr \right\}^2 \quad (10)$$

where $L_{>}$ is the greater of L_i and L_k , and W is the Racah coefficient.

3. *Wavefunctions.* The wavefunctions $\phi_i(r)$ are found from the solution of Schrodinger's equation written in the form

$$\frac{d^2 \phi_i(r)}{dr^2} - \left\{ V(r) + \frac{l_i(l_i + 1)}{r^2} - E_i + \xi(r)(\mathbf{L} \cdot \mathbf{S}) \right\} \phi_i(r) = 0. \quad (11)$$

The spin-orbit interaction energy $\xi(r)(\mathbf{L} \cdot \mathbf{S})$ is significant only for small r . Very close to the nucleus a relativistic form must be used. In order to obtain realistic wavefunctions for small r we use the potential suggested in the first paper (1), which puts

$$\begin{aligned} -V(r) &= \frac{2Z}{r} \Psi(x) + \frac{2(Z-N)}{R_0} - \frac{1}{16\pi^2} \quad \text{for } r \leq R_0 \\ &= \frac{2(Z-N)}{r} \quad \text{for } r \geq R_0 \end{aligned} \quad (12)$$

where R_0 is the radius of the Thomas-Fermi-Dirac ion and the potential inside this ion has the form $\Psi(x)$. x is related to r by $x = r/\alpha\mu$, where $\mu = 0.88534Z^{-1/3}$ and α is a scaling factor to be found by making the observed (spectroscopic) E_i a true eigenvalue of equation (11).

If equation (12) accurately represents the real potential we should expect to find the same value of α for the two J states of a term. As an example we consider $6p^2P$ of Cs I, which has $\Delta_{6p} = 554.1 \text{ cm}^{-1}$, making the E_i differ by 2.8 per cent between $J = 1/2$ and $J = 3/2$. Solution of equation (5) with potential (12) results in the two α 's differing by 1.48 per cent, but with inclusion of spin-orbit interaction as in equation (11) the α 's differ by only 0.18 per cent.

Values of the radial integrals I_{ik} were calculated from these wavefunctions. However, because $\xi(r)$ rapidly becomes small as r increases, and because contributions to I_{ik} come predominantly from large r , the radial integrals are determined almost completely from the region where the potential is coulombic; i.e. we may use equation (5) with $V(r) = -(Z-N)/r$.

Stone (7) (see Introduction) compared his calculated f -values with ones found from the CA, and found good agreement for all except $6s-7p$ and $6s-8p$. He claimed that the latter discrepancies were caused by incorrect allowance for spin-orbit interaction in the CA. This, however, is not the correct explanation. In fact there is almost complete cancellation of the positive and negative contributions to the radial integrals for $5s-7p$ and $6s-8p$, so I_{ik} is extremely sensitive to the values of the E_{ik} . Table I shows the percentage difference between Stone's calculated binding energies and the observed levels. Also shown are his gf -values and those from the CA. It can be seen that where Stone's energies are in agreement with experiment, then his f -values agree with the CA, and vice versa. By adopting Stone's E_i we can almost exactly reproduce his f -values from the CA. We conclude that the CA is preferable for these sensitive transitions, but note that in any case

TABLE I
gf for Cs I

Level	Level error (per cent)	<i>gf</i> 6S _{1/2} -nP	
		Stone	CA
7P _{1/2}	1.29	0.00284	0.00671
7P _{3/2}	0.15	0.0174	0.0174
8P _{1/2}	1.06	0.000317	0.00134
8P _{3/2}	0.23	0.00349	0.0046

we cannot hope to obtain very accurate radial integrals for such cases by any straightforward theoretical method.

As a result, and to reduce computing time, we adopted the asymptotic expansion of the coulomb wavefunction* (equation (5) of the first paper (I)) to give $\phi_i(r)$. Extensive calculations of *f*-values were then undertaken, using equations (9) and (10), and we present some of the more useful results here.

It is worthwhile to note that we have neglected the fact that in the alkali-like spectra the *nd* and *nf* doublet levels are sometimes inverted. This effect is very small and is probably due to configuration interaction with an excited core electron. It is unlikely to significantly affect the transition probabilities. Whenever the doublets are inverted we have assumed zero splitting.

4. Quadrupole transitions. If we define

$$I'_{ab} = \int_0^{\infty} \phi_a(r) \phi_b(r) r^2 dr$$

then we can use equation (9), with I'_{ab} substituted for I_{ab} , to calculate quadrupole integrals. The 2S - n^2D series arising from the ground state are of interest as they have been observed in the laboratory (IO), (II). From Condon & Shortley (8) we have

$$gf_{ik} = 0.333 \times 10^{-6} (|E_i - E_k|)^3 S (I'_{ik})^2$$

where

$$S = 4/5 \text{ for } {}^2D_{5/2} - {}^2S_{1/2}$$

$$S = 8/15 \text{ for } {}^2D_{3/2} - {}^2S_{1/2}.$$

5. *The tables of gf-values.* In Tables II to IX we tabulate *gf*-values for those transitions in which allowance for spin-orbit interaction has proved necessary. For example, in Na I only the *np* terms are significantly split, and we tabulate *s-p* and *p-d* transitions, whereas in Cs I both the *np* and the *nd* terms are split, so it is necessary also to include *d-f* transitions. Very few *f-g* transitions are of astrophysical interest so we will not consider them here.

In Tables X and XI we give *gf*-values for the 2S - 2D forbidden lines.

6. Discussion

6.1 *Dipole transitions.* It will be of little value to make a detailed comparison of our calculated *f*-values with available experimental determinations in the

* In the case of the lowest *d* state in Sr II and Ba II the coulomb wavefunction was normalized to agree with the STFD (I) wavefunction for large *r*.

TABLE II
gf-values for Na I

s → *p* transitions

<i>J</i> - <i>J'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>
3-3	1.24	0	6.22 -1	3-4	2.00 -2	9.71 -3	3-5	3.28 -3	1.57 -3
3-6	1.06	-3	5.02 -4	4-3	6.55 -1	3.25 -1	4-4	1.91 0	9.53 -1
4-5	5.13	-2	2.51 -2	4-6	1.04 -2	5.04 -3	5-3	5.38 -2	2.70 -2
5-4	1.24	0	6.18 -1	5-5	2.50 0	1.25 0	5-6	8.28 -2	4.07 -2
6-3	1.71	-2	8.62 -3	6-4	9.22 -2	4.64 -2	6-5	1.81 0	9.01 -1
6-6	3.07	0	1.54 0	7-3	7.78 -3	4.12 -3	7-4	2.88 -2	1.49 -2
7-5	1.26	-1	6.41 -2	7-6	2.36 0	1.18 0	8-3	4.37 -3	2.23 -3
8-4	1.35	-2	6.85 -3	8-5	3.95 -2	1.99 -2	8-6	1.60 -1	7.99 -2
9-3	2.71	-3	1.37 -3	9-4	7.61 -3	3.84 -3	9-5	1.85 -2	9.29 -3
9-6	4.92	-2	2.47 -2	10-3	1.80 -3	9.13 -4	10-4	4.77 -3	2.41 -3

p → *d* transitions

<i>J</i> - <i>J'</i>	3/2 - 5/2	3/2 - 3/2	1/2 - 3/2	<i>n</i> - <i>n'</i>	3/2 - 5/2	3/2 - 3/2	1/2 - 3/2	<i>n</i> - <i>n'</i>
3-3	3.04	0	3.34 -1	1.68 0	3-4	3.51 -1	3.90 -2	1.96 -1
3-5	1.09	-1	1.21 -2	6.11 -2	3-6	4.88 -2	5.42 -3	2.81 -2
3-7	2.67	-2	2.97 -3	1.51 -2	4-3	7.04 -1	7.82 -2	3.90 -1
4-4	3.41	0	3.79 -1	1.88 0	4-5	5.10 -1	5.67 -2	2.83 -1
4-6	1.76	-1	1.96 -2	9.92 -2	4-7	8.40 -2	9.33 -3	4.71 -2
5-3	9.51	-4	1.06 -4	6.05 -4	5-4	1.38 0	1.53 -1	7.63 -1
5-5	3.80	0	4.22 -1	2.10 0	5-6	6.20 -1	6.89 -2	3.47 -1
5-7	2.25	-1	2.50 -2	1.26 -1	6-3	1.78 -4	1.98 -5	1.15 -4
6-4	4.89	-3	5.43 -4	2.97 -3	6-5	2.01 0	2.23 -1	1.11 0
6-6	4.21	0	4.68 -1	2.33 0	6-7	7.15 -1	7.94 -2	3.97 -1

TABLE III
gf-values for K I

s → *p* transitions

<i>J</i> - <i>J'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>
4-4	1.32	0	6.57 -1	4-5	2.21 -2	9.86 -3	4-6	4.33 -3	1.84 -3
4-7	1.61	-3	6.63 -4	4-8	7.90 -4	3.20 -4	4-9	4.56 -4	1.82 -4
5-4	6.74	-1	3.29 -1	5-5	1.98 0	9.89 -1	5-6	4.38 -2	2.00 -2
5-7	8.64	-3	3.79 -3	5-8	3.14 -3	1.34 -3	5-9	1.52 -3	6.34 -4
6-4	6.31	-2	3.16 -2	6-5	1.27 0	6.22 -1	6-6	2.57 0	1.29 0
7-4	2.05	-2	1.03 -2	7-5	1.06 -1	5.34 -2	7-6	1.83 0	9.00 -1
8-4	9.56	-3	4.87 -3	8-5	3.40 -2	1.73 -2	8-6	1.44 -1	7.29 -2
9-4	5.28	-3	2.77 -3	9-5	1.59 -2	8.23 -3	9-6	4.56 -2	2.34 -2
10-4	2.92	-3	2.11 -3	10-5	8.45 -3	5.20 -3	10-6	2.07 -2	1.19 -2
11-4	2.12	-3	1.21 -3	11-5	5.55 -3	3.01 -3	11-6	1.21 -2	6.38 -3

TABLE III (continued)

 $p \rightarrow d$ transitions

$J-J'$	$3/2 - 5/2$	$3/2 - 3/2$	$1/2 - 3/2$		$3/2 - 5/2$	$3/2 - 3/2$	$1/2 - 3/2$
$n-n'$				$n-n'$			
4-3	3.22 0	3.56 -1	1.79 0	4-4	3.05 -3	3.39 -4	2.66 -3
4-5	7.11 -3	7.90 -4	3.40 -3	4-6	6.29 -3	6.99 -4	2.83 -3
5-3	9.25 -1	1.03 -1	5.16 -1	5-4	4.41 0	4.90 -1	2.44 0
5-5	2.58 -2	2.87 -3	1.62 -2	5-6	1.60 -4	1.78 -5	1.38 -6
6-3	4.47 -2	4.97 -3	2.43 -2	6-4	1.86 0	2.07 -1	1.05 0
6-5	5.53 0	6.14 -1	3.07 0	6-6	5.45 -2	6.06 -3	3.49 -2
7-3	1.30 -2	1.44 -3	7.03 -3	7-4	1.23 -1	1.37 -2	6.66 -2
7-5	2.77 0	3.08 -1	1.55 0	7-6	6.60 0	7.33 -1	3.65 0
8-3	5.79 -3	6.43 -4	3.14 -3	8-4	3.77 -2	4.19 -3	2.03 -2
8-5	2.04 -1	2.27 -2	1.21 -1	8-6	3.68 0	4.09 -1	2.07 0
9-3	3.17 -3	3.52 -4	1.71 -3	9-4	1.74 -2	1.93 -3	9.33 -3
9-5	6.42 -2	7.13 -3	3.51 -2	9-6	2.85 -1	3.17 -2	1.55 -1

TABLE IV

 gf -values for Rb I $s \rightarrow p$ transitions

$J-J'$	$1/2 - 3/2$	$1/2 - 1/2$		$1/2 - 3/2$	$1/2 - 1/2$		$1/2 - 3/2$	$1/2 - 1/2$
$n-n'$			$n-n'$			$n-n'$		
5-5	1.37 0	6.81 -1	5-6	3.88 -2	1.31 -2	5-7	9.46 -3	2.81 -3
5-8	3.93 -3	1.09 -3	5-9	2.06 -3	5.42 -4	5-10	1.24 -3	3.09 -4
6-5	7.34 -1	3.33 -1	6-6	2.02 0	1.02 0	6-7	6.27 -2	2.24 -2
6-8	1.44 -2	4.49 -3	6-9	5.73 -3	1.64 -3	6-10	2.95 -3	7.79 -4
7-5	6.46 -2	3.26 -2	7-6	1.35 0	6.26 -1	7-7	2.60 0	1.32 0
7-8	9.17 -2	3.42 -2	7-9	2.16 -2	7.12 -3	7-10	8.71 -3	2.61 -3
8-5	2.07 -2	1.09 -2	8-6	1.03 -1	5.42 -2	8-7	1.93 0	9.03 -1
9-5	9.51 -3	5.25 -3	9-6	3.24 -2	1.78 -2	9-7	1.36 -1	7.34 -2
10-5	5.04 -3	3.19 -3	10-6	1.48 -2	8.79 -3	10-7	4.20 -2	2.41 -2
11-5	1.78 -3	4.09 -3	11-6	6.29 -3	7.63 -3	11-7	1.66 -2	1.50 -2

 $p \rightarrow d$ transitions

$J-J'$	$3/2 - 5/2$	$3/2 - 3/2$	$1/2 - 3/2$		$3/2 - 5/2$	$3/2 - 3/2$	$1/2 - 3/2$
$n-n'$				$n-n'$			
5-4	2.63 0	2.92 -1	1.47 0	5-5	1.10 -1	1.22 -2	3.68 -2
5-6	8.41 -2	9.34 -3	3.36 -2	5-7	5.01 -2	5.56 -3	2.00 -2
5-8	3.18 -2	3.53 -3	1.16 -2	6-4	6.43 -1	7.14 -2	3.86 -1
6-5	3.61 0	4.01 -1	2.02 0	6-6	9.05 -2	1.01 -2	2.28 -2
6-7	7.57 -2	8.41 -3	2.51 -2	6-8	4.50 -2	5.00 -3	1.61 -2
7-4	5.65 -2	6.28 -3	3.08 -2	7-5	1.26 0	1.40 -1	7.63 -1
7-6	4.53 0	5.03 -1	2.55 0	7-7	6.49 -2	7.21 -3	1.40 -2
7-8	5.98 -2	6.64 -3	1.78 -2	8-4	1.77 -2	1.97 -3	9.41 -3
8-5	1.40 -1	1.56 -2	7.79 -2	8-6	1.87 0	2.08 -1	1.14 0
9-4	8.17 -3	9.08 -4	4.23 -3	9-5	4.70 -2	5.22 -3	2.54 -2
9-6	2.26 -1	2.51 -2	1.26 -1	10-4	4.55 -3	5.06 -4	2.29 -3
10-5	2.26 -2	2.51 -3	1.18 -2	10-6	7.81 -2	8.67 -3	4.17 -2

TABLE V
gf-values for Cs I

s → *p* transitions

<i>J</i> - <i>J'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>	<i>J</i> - <i>J'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>	<i>J</i> - <i>J'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>
6-6	1.42	0	6.94 - 1	6-7	5.21	-2	9.98 - 3	6-8	1.42	-2	2.01 - 3
6-9	6.21	-3	7.55 - 4	6-10	3.37	-3	3.69 - 4	6-11	2.08	-3	2.07 - 4
6-12	1.40	-3	1.25 - 4	7-6	7.89	-1	3.06 - 1	7-7	2.05	0	1.04 0
7-8	7.34	-2	1.51 - 2	7-9	1.81	-2	2.55 - 3	7-10	7.50	-3	8.39 - 4
8-6	6.85	-2	3.42 - 2	8-7	1.43	0	5.87 - 1	8-8	2.62	0	1.36 0
8-9	1.02	-1	2.33 - 2	8-10	2.53	-2	4.10 - 3	9-6	2.19	-2	1.17 - 2
9-7	1.05	-1	5.84 - 2	9-8	2.02	0	8.50 - 1	9-9	3.18	0	1.66 0
10-6	9.93	-3	5.79 - 3	10-7	3.29	-2	1.98 - 2	10-8	1.37	-1	8.00 - 2
11-6	5.18	-3	3.53 - 3	11-7	1.48	-2	9.80 - 3	11-8	4.14	-2	2.68 - 2

p → *d* transitions

<i>J</i> - <i>J'</i>	3/2 - 5/2	3/2 - 3/2	1/2 - 3/2	<i>n</i> - <i>n'</i>	<i>J</i> - <i>J'</i>	3/2 - 5/2	3/2 - 3/2	1/2 - 3/2	<i>n</i> - <i>n'</i>
6-5	9.21	-1	9.58 - 2	5.69 - 1	6-6	1.13	0	1.36 - 1	4.54 - 1
6-7	3.29	-1	3.79 - 2	1.43 - 1	6-8	1.45	-1	1.62 - 2	6.17 - 2
6-9	7.96	-2	8.61 - 3	3.08 - 2	7-5	9.26	-2	8.15 - 3	7.12 - 2
7-6	1.24	0	1.27 - 1	8.05 - 1	7-7	1.17	0	1.45 - 1	4.47 - 1
7-8	3.64	-1	4.25 - 2	1.51 - 1	7-9	1.69	-1	1.90 - 2	6.84 - 2
8-5	8.91	-3	6.34 - 4	6.06 - 3	8-6	3.13	-1	2.89 - 2	2.36 - 1
8-7	1.57	0	1.59 - 1	1.04 0	9-5	2.18	-3	8.79 - 5	1.16 - 3
9-6	5.19	-2	4.56 - 3	3.42 - 2	9-7	5.54	-1	5.17 - 2	4.15 - 1
10-5	9.69	-4	3.71 - 6	2.16 - 4	10-6	1.93	-2	1.39 - 3	1.05 - 2
10-7	1.03	-1	9.07 - 3	6.54 - 2	11-5	4.83	-5	4.55 - 5	5.43 - 4
11-6	6.92	-3	1.01 - 3	6.97 - 3	11-7	3.37	-2	4.12 - 3	2.75 - 2
12-5	3.07	-5	1.32 - 5	1.91 - 4	12-6	4.15	-3	5.27 - 4	3.39 - 3

d → *f* transitions

<i>J</i> - <i>J'</i>	5/2 - 7/2	5/2 - 5/2	3/2 - 5/2	<i>n</i> - <i>n'</i>	<i>J</i> - <i>J'</i>	5/2 - 7/2	5/2 - 5/2	3/2 - 5/2	<i>n</i> - <i>n'</i>
5-4	2.50	0	1.25 - 1	2.04 0	5-5	9.10	-1	4.55 - 2	7.53 - 1
5-6	4.46	-1	2.23 - 2	3.66 - 1	6-4	4.92	0	2.46 - 1	4.31 0
6-5	3.37	-1	1.68 - 2	2.23 - 1	7-4	1.12	0	5.56 - 2	1.02 0
7-5	7.51	0	3.75 - 1	6.53 0	8-4	9.21	-2	4.61 - 3	7.82 - 2
8-5	2.39	0	1.20 - 1	2.18 0	9-4	2.82	-2	1.41 - 3	2.32 - 2

TABLE VI
gf-values for Mg II

s → *p* transitions

<i>J</i> - <i>J'</i>	1/2 - 3/2	1/2 - 1/2		1/2 - 3/2	1/2 - 1/2		1/2 - 3/2	1/2 - 1/2		
<i>n</i> - <i>n'</i>			<i>n</i> - <i>n'</i>			<i>n</i> - <i>n'</i>				
3-3	1.20	0	5.97	-1	3-4	8.74	-5	9.06	-5	
3-6	6.96	-4	3.89	-4	4-3	5.56	-1	2.75	-1	
4-5	1.89	-3	7.49	-4	4-6	7.19	-5	6.24	-5	
5-4	1.06	0	5.24	-1	5-5	2.42	0	1.21	0	
6-3	2.28	-2	1.13	-2	6-4	1.20	-1	6.01	-2	
7-3	1.06	-2	5.66	-3	7-4	4.06	-2	2.10	-2	
8-3	6.03	-3	3.11	-3	8-4	1.97	-2	9.98	-3	
								3-5	9.07	-4
								4-4	1.83	0
								5-3	6.68	-2
								5-6	7.31	-3
								6-5	1.54	0
								7-5	1.69	-1
								8-5	5.78	-2

p → *d* transitions

<i>J</i> - <i>J'</i>	3/2 - 5/2	3/2 - 3/2	1/2 - 3/2		3/2 - 5/2	3/2 - 3/2	1/2 - 3/2		
<i>n</i> - <i>n'</i>				<i>n</i> - <i>n'</i>					
3-3	3.78	0	4.20	-1	1.87	0	3-4	1.37	-1
3-5	2.51	-2	2.79	-3	1.45	-2	3-6	7.83	-3
3-7	3.51	-3	3.90	-4	2.12	-3	4-3	1.07	0
4-4	4.45	0	4.94	-1	2.46	0	4-5	2.89	-1
4-6	6.98	-2	7.76	-3	4.09	-2	4-7	2.75	-2
5-3	2.67	-2	2.97	-3	1.44	-2	5-4	2.13	0
5-5	5.38	0	5.98	-1	2.98	0	5-6	4.14	-1
5-7	1.13	-1	1.26	-2	6.39	-2	6-3	7.53	-3
6-4	5.27	-2	5.86	-3	2.84	-2	6-5	3.17	0
								3.52	-1
								1.76	0

TABLE VII
gf-values for Ca II

s → *p* transitions

<i>J</i> - <i>J'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>	<i>J</i> - <i>J'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>	<i>J</i> - <i>J'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>
4-4	1.34	0	6.63 - 1	4-5	8.32	-4	1.17 - 4	4-6	1.74	-5	6.81 - 5
4-7	5.86	-5	8.10 - 5	5-4	6.23	-1	3.00 - 1	5-5	1.96	0	9.77 - 1
5-6	3.29	-3	9.04 - 4	5-7	3.56	-6	2.24 - 5	6-4	7.77	-2	3.81 - 2
6-5	1.14	0	5.54 - 1	6-6	2.54	0	1.27 0	6-7	8.13	-3	2.73 - 3
7-4	2.68	-2	1.32 - 2	7-5	1.33	-1	6.61 - 2	7-6	1.63	0	7.94 - 1
7-7	3.10	0	1.55 0	8-4	1.20	-2	7.24 - 3	8-5	4.43	-2	2.43 - 2
8-6	1.81	-1	9.45 - 2	8-7	2.10	0	1.04 0	9-4	7.05	-3	3.83 - 3
9-5	2.18	-2	1.14 - 2	9-6	6.24	-2	3.19 - 2	9-7	2.32	-1	1.16 - 1

p → *d* transitions

<i>J</i> - <i>J'</i>	3/2 - 5/2	3/2 - 3/2	1/2 - 3/2	<i>n</i> - <i>n'</i>	<i>J</i> - <i>J'</i>	3/2 - 5/2	3/2 - 3/2	1/2 - 3/2	<i>n</i> - <i>n'</i>
4-3	3.10	-1	3.43 - 2	1.70 - 1	4-4	3.10	0	3.46 - 1	1.69 0
4-5	3.99	-1	4.41 - 2	2.23 - 1	4-6	1.30	-1	1.43 - 2	7.28 - 2
4-7	5.70	-2	6.26 - 3	3.63 - 2	5-3	6.32	-3	7.28 - 4	3.15 - 3
5-4	1.10	0	1.23 - 1	6.06 - 1	5-5	3.70	0	4.13 - 1	2.02 0
5-6	5.37	-1	5.95 - 2	2.98 - 1	5-7	1.81	-1	2.00 - 2	1.08 - 1
6-3	3.23	-3	3.76 - 4	1.68 - 3	6-4	5.50	-4	7.78 - 5	1.18 - 4
6-5	1.85	0	2.06 - 1	1.02 0	6-6	4.24	0	4.73 - 1	2.30 0
6-7	6.42	-1	7.11 - 2	3.68 - 1	7-3	1.96	-3	2.34 - 4	1.05 - 3
7-4	4.13	-4	5.66 - 5	1.49 - 4	7-5	1.27	-4	5.56 - 6	3.00 - 4
7-6	2.57	0	2.86 - 1	1.41 0	7-7	4.73	0	5.28 - 1	2.60 0

d → *f* transitions

<i>J</i> - <i>J'</i>	5/2 - 7/2	5/2 - 5/2	3/2 - 5/2	<i>n</i> - <i>n'</i>	<i>J</i> - <i>J'</i>	5/2 - 7/2	5/2 - 5/2	3/2 - 5/2	<i>n</i> - <i>n'</i>
3-4	8.93	-1	4.46 - 2	7.66 - 1	3-5	4.14	-1	2.07 - 2	3.55 - 1
3-6	2.24	-1	1.12 - 2	1.92 - 1	3-7	1.36	-1	6.80 - 3	1.15 - 1
4-4	6.32	0	3.16 - 1	5.48 0	4-5	9.44	-3	4.72 - 4	9.15 - 3
4-6	7.36	-3	3.68 - 4	5.79 - 3	4-7	1.18	-2	5.89 - 4	9.16 - 3
5-4	1.47	0	7.35 - 2	1.28 0	5-5	8.53	0	4.26 - 1	7.38 0
5-6	1.80	-1	9.01 - 3	1.60 - 1	6-4	5.76	-2	2.88 - 3	4.95 - 1
6-5	3.03	0	1.52 - 1	2.63 0	6-6	1.02	+1	5.11 - 1	8.84 0
7-4	1.59	-2	7.96 - 4	1.36 - 2	7-5	1.21	-1	6.06 - 3	1.04 - 1

TABLE VIII
gf-values for Sr II

s → *p* transitions

<i>J</i> - <i>J'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>	<i>J</i> - <i>J'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>	<i>J</i> - <i>J'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>
5-5	1.47	0	7.16 -1	5-6	1.12	-2	1.38 -3	5-7	1.64	-3	2.57 -5
5-8	4.82	-4	1.19 -6	6-5	7.31	-1	3.15 -1	6-6	2.10	0	1.04 0
6-7	1.53	-2	1.84 -3	6-8	1.55	-3	2.04 -5	7-5	8.53	-2	3.97 -2
7-6	1.30	0	5.74 -1	7-7	2.67	0	1.33 0	7-8	2.32	-2	4.05 -3
8-5	2.85	-2	1.44 -2	8-6	1.37	-1	6.91 -2	8-7	1.81	0	8.09 -1
9-5	1.05	-2	1.07 -2	9-6	4.09	-2	2.99 -2	9-7	1.74	-1	1.06 -1

p → *d* transitions

<i>J</i> - <i>J'</i>	3/2 - 5/2	3/2 - 3/2	1/2 - 3/2	<i>n</i> - <i>n'</i>	<i>J</i> - <i>J'</i>	3/2 - 5/2	3/2 - 3/2	1/2 - 3/2	<i>n</i> - <i>n'</i>
5-4	4.44	-1	4.87 -2	2.30 -1	5-5	3.07	0	3.47 -1	1.58 0
5-6	4.98	-1	5.44 -2	2.72 -1	5-7	1.82	-1	1.96 -2	1.02 -1
5-8	7.87	-2	8.31 -3	6.44 -2	6-4	1.95	-4	6.67 -5	3.04 -5
6-5	1.01	0	1.14 -1	5.26 -1	6-6	3.45	0	3.93 -1	1.74 0
6-7	5.89	-1	6.49 -2	3.20 -1	6-8	2.04	-1	2.20 -2	1.41 -1
7-4	1.01	-6	7.00 -6	5.34 -5	7-5	2.34	-3	1.04 -4	4.38 -3
7-6	1.56	0	1.77 -1	8.05 -1	8-4	3.45	-5	2.75 -6	1.73 -5
8-5	5.89	-4	1.52 -5	7.33 -4	8-6	1.41	-2	9.95 -4	1.50 -2

d → *f* transitions

<i>J</i> - <i>J'</i>	5/2 - 7/2	5/2 - 5/2	3/2 - 5/2	<i>n</i> - <i>n'</i>	<i>J</i> - <i>J'</i>	5/2 - 7/2	5/2 - 5/2	3/3 - 5/2	<i>n</i> - <i>n'</i>
4-4	2.50	0	1.25 -1	2.08 0	4-5	1.03	0	5.17 -2	8.69 -1
4-6	5.35	-1	2.66 -2	4.45 -1	4-7	3.16	-1	1.58 -2	2.53 -1
5-4	4.92	0	2.46 -1	4.29 0	5-5	3.32	-1	1.66 -2	2.53 -1
5-6	2.41	-1	1.21 -2	1.90 -1	5-7	1.52	-1	7.59 -3	1.15 -1
6-4	1.08	0	5.38 -2	9.59 -1	6-5	7.25	0	3.62 -1	6.30 0
7-4	9.17	-2	4.58 -3	7.91 -2	7-5	2.24	0	1.12 -1	1.99 0
8-4	2.84	-2	1.42 -3	2.41 -2	8-5	2.05	-1	1.03 -2	1.76 -1

TABLE IX
gf-values for Ba II

s → *p* transitions

<i>J</i> - <i>J'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>	1/2 - 3/2	1/2 - 1/2	<i>n</i> - <i>n'</i>		
6-6	1.56	0	7.35 - 1	6-7	2.43 - 2	3.90 - 4	6-8	5.49 - 3	3.51 - 6		
7-6	8.31 - 1	2.90 - 1	7-7	2.18	0	1.06	0	7-8	2.83 - 2	3.18 - 4	
8-6	9.33 - 2	4.11 - 2	8-7	1.43	0	5.35 - 1	8-8	2.75	0	1.36	0
9-6	2.08 - 2	2.87 - 2	9-7	1.21 - 1	9.63 - 2	9-8	1.88	0	8.46 - 1		
10-6	1.30 - 2	1.03 - 2	10-7	4.61 - 2	2.86 - 2	10-8	1.93 - 1	9.58 - 2			
11-6	7.77 - 3	5.51 - 3	11-7	2.28 - 2	1.35 - 2	11-8	6.52 - 2	3.28 - 2			

p → *d* transitions

<i>J</i> - <i>J'</i>	3/2 - 5/2	3/2 - 3/2	1/2 - 3/2	<i>n</i> - <i>n'</i>	3/2 - 5/2	3/2 - 3/2	1/2 - 3/2				
6-5	6.92 - 1	6.97 - 2	3.48 - 1	6-6	3.59	0	4.12 - 1	1.71	0		
6-7	4.75 - 1	4.94 - 2	2.73 - 1	6-8	1.32 - 1	1.31 - 2	1.47 - 1				
6-9	7.38 - 2	7.23 - 3	5.98 - 2	6-10	4.42 - 2	4.21 - 3	3.24 - 2				
7-5	1.28 - 2	1.87 - 3	2.59 - 3	7-6	1.42	0	1.56 - 1	7.31 - 1			
7-7	4.16	0	4.84 - 1	1.92	0	7-8	5.17 - 1	5.50 - 2	3.98 - 1		
7-9	1.97 - 1	2.03 - 2	1.29 - 1	7-10	9.90 - 2	9.84 - 3	6.14 - 2				
8-5	2.11 - 3	4.48 - 4	3.70 - 4	8-6	7.40 - 3	1.62 - 3	1.82 - 5				
8-7	2.18	0	2.43 - 1	1.12	0	8-8	4.49	0	5.28 - 1	2.35	0
8-9	6.92 - 1	7.48 - 2	3.69 - 1	8-10	2.53 - 1	2.62 - 2	1.33 - 1				

d → *f* transitions

<i>J</i> - <i>J'</i>	5/2 - 7/2	5/2 - 5/2	3/2 - 5/2	<i>n</i> - <i>n'</i>	5/2 - 7/2	5/2 - 5/2	3/2 - 5/2		
5-4	1.74 + 1	9.28 - 1	1.58 + 1	5-5	9.86 - 1	3.01 - 2	6.04 - 1		
5-6	5.73 - 3	1.56 - 4	1.51 - 6	5-7	1.32 - 2	1.77 - 3	1.81 - 2		
5-8	2.42 - 2	2.01 - 3	2.16 - 2	6-4	8.39 - 1	3.36 - 2	6.53 - 1		
6-5	7.23	0	3.81 - 1	6.40	0	6-6	1.80 - 1	4.96 - 3	1.23 - 1
6-7	9.08 - 4	4.70 - 5	2.21 - 4	6-8	3.83 - 3	4.80 - 4	2.98 - 3		
7-4	2.15 - 1	8.68 - 3	1.71 - 1	7-5	1.88	0	9.01 - 2	1.57	0
8-4	4.91 - 2	2.21 - 3	4.01 - 2	8-5	1.20 - 2	1.36 - 3	1.70 - 2		
9-4	2.22 - 2	9.87 - 4	1.70 - 2	9-5	3.52 - 3	4.23 - 4	5.59 - 3		
10-4	1.72 - 2	3.11 - 4	5.13 - 3	10-5	2.92 - 4	4.85 - 4	7.06 - 3		

TABLE X

gf for $^2S-^2D$ in the alkalis

$J-J'$	$1/2 - 5/2$	$1/2 - 3/2$		$1/2 - 5/2$	$1/2 - 3/2$		$1/2 - 5/2$	$1/2 - 3/2$
$n-n'$			$n-n'$			$n-n'$		
Na I								
3-3	2.38 -6	1.59 -6	3-4	6.12 -7	4.08 -7	3-5	2.48 -7	1.66 -7
3-6	1.27 -7	8.47 -8	3-7	7.35 -8	4.92 -8	3-8	4.72 -8	3.14 -8
3-9	3.20 -8	2.14 -8	3-10	2.26 -8	1.52 -8	3-11	1.68 -8	1.12 -8
3-12	1.28 -8	8.55 -9						
K I								
4-3	1.42 -6	1.61 -6	4-4	1.06 -7	7.05 -8	4-5	4.91 -9	3.27 -9
4-6	4.83 -12	3.22 -12	4-7	5.93 -10	3.96 -10	4-8	7.63 -10	6.39 -10
4-9	1.04 -9	6.92 -10	4-10	7.51 -11	6.31 -11			
Rb I								
5-4	2.06 -6	1.37 -6	5-5	5.82 -10	3.87 -10	5-6	1.89 -8	1.26 -8
5-7	2.20 -8	1.46 -8	5-8	1.80 -8	1.20 -8	5-9	1.38 -8	7.29 -9
5-10	1.05 -8	6.99 -9	5-11	7.85 -9	5.25 -9			
Cs I								
6-5	8.74 -7	5.58 -7	6-6	3.54 -7	2.73 -7	6-7	2.28 -7	1.65 -7
6-8	1.33 -7	9.38 -8	6-9	8.25 -8	5.74 -8	6-10	5.48 -8	3.74 -8
6-11	3.81 -8	2.55 -8	6-12	2.77 -8	1.81 -8			

TABLE XI

gf for $^2S-^2D$ in the once-ionized alkaline earths

$J-J'$	$1/2 - 5/2$	$1/2 - 3/2$		$1/2 - 5/2$	$1/2 - 3/2$		$1/2 - 5/2$	$1/2 - 3/2$
$n-n'$			$n-n'$			$n-n'$		
Mg II								
3-3	1.77 -5	1.17 -5	3-4	1.47 -6	7.74 -7	3-5	3.07 -7	2.05 -7
3-6	1.00 -7	6.67 -8	3-7	4.26 -8	2.84 -8	3-8	2.24 -8	1.48 -8
Ca II								
4-3	6.18 -8	4.04 -8	4-4	1.39 -5	7.35 -6	4-5	3.25 -6	2.16 -6
4-6	1.28 -6	8.47 -7	4-7	6.45 -7	4.26 -7	4-8	3.76 -7	2.48 -7
Sr II								
5-4	1.96 -7	1.10 -7	5-5	1.13 -5	7.77 -6	5-6	3.44 -6	2.24 -6
5-7	1.51 -6	9.87 -7	5-8	8.33 -7	5.28 -7	5-9	5.00 -7	3.18 -7
Ba II								
6-5	1.47 -8	5.56 -9	6-6	1.15 -5	7.98 -6	6-7	3.61 -6	2.04 -6
6-8	1.39 -6	8.68 -7	6-9	7.56 -7	4.57 -7	6-10	4.61 -7	2.72 -7

alkalis, primarily because of the very large disagreements among the observed data.

The f -values calculated here for the leading line of the principal series are in good agreement (better than 10 per cent) with accurate lifetime measurements in the alkali spectra. Apart from cases where cancellation in the transition integral is important we would expect other f -values to be of comparable accuracy.

Application of the calculated oscillator strengths to lines observed in the solar spectrum (12) shows general good quantitative agreement.

The doublet intensity ratios in the principal series of the alkalis have been observed and we will discuss Cs I here. Kvater & Meister (13) observed the doublet ratio up to 11 2P using the 'hook' method. Sambursky (4) also made measurements on these lines but Kvater & Meister give cogent reasons for rejecting most of his results. In Table XII we list the doublet ratio

$$R = \frac{gf(^2P_{3/2}-^2S_{1/2})}{gf(^2P_{1/2}-^2S_{1/2})}$$

as measured and calculated by various methods.

TABLE XII
Doublet ratio R for Cs I

	Kvater & Meister	Stone	Warner	CA
$6s\ ^2S-6p\ ^2P$	2.03	2.06	2.04	2.06
– $7p\ ^2P$	4.29	6.12	5.22	3.09
– $8p\ ^2P$	8.0	11.1	7.03	3.43
– $9p\ ^2P$	8.1	17.2	8.22	3.58
– $10p\ ^2P$	8.1	21.5	9.14	3.66
– $11p\ ^2P$	8.0	28.7	10.05	3.72

The column headed CA gives R calculated by the coulomb approximation without allowance for configuration mixing. It can be seen that the best agreement with observation is given by the calculations performed according to the theory given in Section 2.

A recent paper by Gallagher (18) has given f -values for the once-ionized alkaline earths, measured with a lifetime technique (the Hanle-effect). In Table XIII we compare these experimental results with the calculations of this paper.

TABLE XIII
Comparison of experimental and theoretical gf -values

		gf_{exp}	gf_{theor}
Mg II	$3\ ^2P_{3/2}-3\ ^2S_{1/2}$	1.28 ± 0.06	1.20
Ca II	$4\ ^2P_{3/2}-4\ ^2S_{1/2}$	1.32 ± 0.04	1.34
	– $3\ ^2D_{5/2}$	0.32 ± 0.04	0.310
	– $3\ ^2D_{3/2}$	0.035 ± 0.004	0.0343
Sr II	$5\ ^2P_{3/2}-5\ ^2S_{1/2}$	1.42 ± 0.06	1.47
	– $4\ ^2D_{5/2}$	0.57 ± 0.12	0.444
	– $4\ ^2D_{3/2}$	0.064 ± 0.12	0.0487
	$5\ ^2P_{1/2}-5\ ^2S_{1/2}$	0.68 ± 0.03	0.716
	– $4\ ^2D_{3/2}$	0.33 ± 0.06	0.230
Ba II	$6\ ^2P_{3/2}-6\ ^2S_{1/2}$	1.48 ± 0.10	1.56
	– $5\ ^2D_{5/2}$	0.84 ± 0.09	0.692
	– $5\ ^2D_{3/2}$	0.10 ± 0.01	0.0697
	$6\ ^2P_{1/2}-6\ ^2S_{1/2}$	0.70 ± 0.05	0.735
	– $5\ ^2D_{3/2}$	0.42 ± 0.04	0.348

There is general good agreement. The small systematic differences for transitions to the lowest nd state in Sr II and Ba II is probably connected with the non-coulombic nature of these levels.

6.2 *Quadrupole transitions.* Our calculated f -values for the first members of the ${}^2S-n^2D$ series in the alkali spectra are factors of 2 to 3 smaller than those measured by Prokofiev (14), whereas the calculations by Stevenson (15) using Hartree fields were factors of 1.5 to 2.5 larger than the measured values. On the other hand, for Ba II we are about 40 per cent larger than the values calculated by Garstang & Hill (16). It would be valuable to have further measurements on the quadrupole transitions in one-electron spectra.

Our f -values give a qualitative account of some observed peculiarities. Thus in Cs I the ${}^2S-n^2D$ was readily observed up to $n = 10$ by Schrum *et al.* (11) and the decrease in intensity along the series was slow. On the other hand, Prokofiev (14) points out that in Rb I the $5\ {}^2S-5\ {}^2D$ doublet must have a very much lower f -value than $5\ {}^2S-4\ {}^2D$ because the former is not detectable. Both of these features are seen in our calculated f -values. Finally, Prokofiev (14) finds that the f -value for $6\ {}^2S-6\ {}^2D$ in Cs I is about one half that for $6\ {}^2S-5\ {}^2D$, in agreement with our calculated ratio of 0.44.

Osterbrock (17) pointed out that the $4\ {}^2S-4\ {}^2D$ doublet of Ca II should be detectable in the solar spectrum, and this has been confirmed (12). However, we see from Table XI that the $4\ {}^2S-4\ {}^2D$ doublet (at 1758.75 and 1759.35 Å) has an f -value about 200 times greater than $4\ {}^2S-3\ {}^2D$, and we may expect to find this line in rocket spectra of the Sun. The $3\ {}^2S-3\ {}^2D$ doublet of Mg II at 1398.78 Å may also be detectable.

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