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ATTITUDE CONTROL OF A SPACECRAFT  
WITH A STRAPDOWN INERTIAL REFERENCE  
SYSTEM AND ONBOARD COMPUTER

*by John Hrastar*

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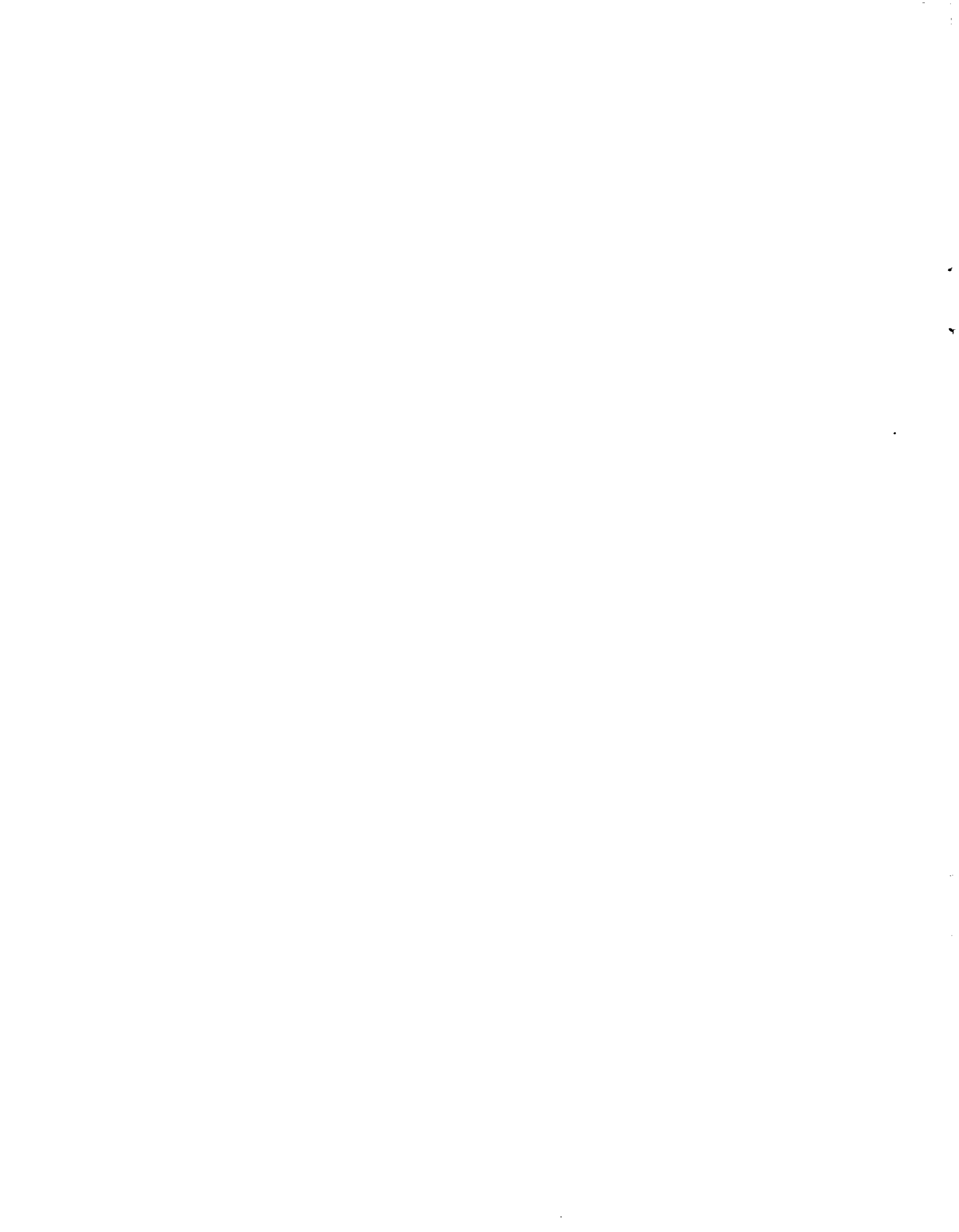
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16. Abstract			
<p>A strapdown inertial reference system and an onboard digital computer can be used to change the attitude of a spacecraft over large angles in an arbitrary direction.</p> <p>A study was conducted with an advanced Orbiting Astronomical Observatory (OAO) using a three-axis control law previously proven globally stable. The control system can reorient the spacecraft to an arbitrary inertial attitude with a single command by operating the three momentum wheels simultaneously. Reorientation, therefore, becomes a simple extension of a hold or pointing mode. The time required for reorientation with this system is considerably shorter than the time required for a series of single-axis slews. The spacecraft attitude and control law are continuously updated by the onboard computer's using information provided by the strapdown inertial reference system. The use of a computer and inertial reference system with characteristics of systems presently under development demonstrates the feasibility of orientation with such a reference system.</p> <p>The basic system is not limited to the OAO but may be adapted for other three-axis-stabilized spacecraft.</p>			
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# ATTITUDE CONTROL OF A SPACECRAFT WITH A STRAPDOWN INERTIAL REFERENCE SYSTEM AND ONBOARD COMPUTER

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## INTRODUCTION

In recent years, the microminiaturization of electronic components has made possible the manufacture and use of very small, general purpose (GP) digital computers, which can be included aboard spacecraft. One of the primary uses for such a computer is data processing. Another important use is in conjunction with the attitude control system of the spacecraft. The advent of the small computer has made it possible to consider relatively complex control laws for the spacecraft attitude control system. All the effects of dynamic coupling and large-angle nonlinearities may be considered at the start of the design procedure. Small-angle limits need not restrict the control system design. Thus an integrated, three-axis control system may be designed in place of three small-angle single-axis systems.

Although a great deal of work remains in this area, Mortensen (Reference 1) and Meyer (Reference 2) have proposed three-axis control laws.

Meyer defines the direction cosine matrix relating the spacecraft axes to a fixed inertial reference as the output of the attitude control system. This matrix is multiplied by a reference matrix, and the control law is formulated from the product or error matrix. The control law he assumes is shown to be asymptotically stable in the large. The attitude matrix is assumed to be known at all times. It may be determined by some type of inertial sensor, e.g., star trackers or gyros. The matrix formulation and multiplication and the control law formulation require a computer.

In an earlier work, using the Cayley-Rodrigues parameters, Mortensen proposed a control law under the assumption of a particular formulation for the spacecraft kinematics. These are also known as Euler-Rodrigues parameters (by Roberson) and the Gibbs vector components (Reference 3). Mortensen assumes the parameters defining the body orientation to be known and shows the control using these parameters to be asymptotically stable in the large. The parameters are not easily measured but may be determined by solving the first-order differential equations in the parameters and body rates. This method is similar to Meyer's method except that the attitude matrix is never formed explicitly. The control law is formed directly from the kinematic parameters. The computation of the kinematic parameters and the control law formulation require a computer.

Control laws of these types have three characteristics that should prove very valuable in future spacecraft:

(1) Stability for any attitude. Thus, the desired attitude of a spacecraft is unrestricted and may be determined from considerations other than stability.

(2) Three-axis reorientation capability. Slewing all three axes simultaneously results in a rapid reorientation.

(3) Simplicity. Since a three-axis reorientation is simply an extension of a hold or point operation, mode switching may be simplified.

This report is a result of a conceptual study for an advanced Orbiting Astronomical Observatory (OAO) spacecraft. The spacecraft considered was similar to the present OAO but included an extended telescope. The basic assumptions underlying the study with respect to the attitude control system were the presence of a GP, onboard computer and a very precise, strapdown inertial reference unit (IRU). The primary purpose of the study was to determine the feasibility of a three-axis control system's pointing and reorienting the spacecraft. Use of the computer and IRU was assumed, and implementation of the type of control law proposed by Mortensen was considered. Although an advanced OAO is the specific spacecraft considered, the principles may be applied to other spacecraft that require a pointing and slew capability.

The control law is formulated along with the algorithm used to generate this control law within the onboard computer. The input to the onboard computer is the pulse output of the strapdown IRU. The spacecraft dynamics and the onboard computer were simulated on a digital computer to check the effect of sampling interval, slew angle, quantization level, and other variables. These simulations gave good results for the parameters of this study. A computer and IRU presently under development are acceptable for the type of spacecraft studied.

The simulation was also used to check the dynamic response of the spacecraft to a reorientation maneuver. The spacecraft responded well. If time is an important factor, this type of control law is superior to the more conventional systems.

The in-flight reprogramming capability of the onboard computer gives this system a great deal of flexibility. This is evidenced by the ability of the spacecraft to slew about an arbitrary axis through the center of mass as discussed in a later section.

It is concluded that this type of system is feasible for an observatory type of spacecraft and offers a number of advantages over conventional systems.

## **CONTROL LAW FORMULATION**

It is necessary to know the spacecraft attitude at all times to effect a three-axis reorientation. Knowledge of the orthogonal transformation matrix relating the spacecraft body axes to a known, fixed inertial frame is therefore required. Explicit determination of the entire matrix is not necessary as long as the three independent parameters of the kinematic representation are known.

One method of parameterizing this matrix is by the Cayley-Rodrigues parameters  $a$ ,  $\beta$ , and  $\gamma$ . See Equations A17, Appendix A. In terms of these three independent parameters, the matrix\* (Reference 3) is

$$\bar{\bar{A}} = \frac{1}{1 + a^2 + \beta^2 + \gamma^2} \begin{bmatrix} 1 + a^2 - \beta^2 - \gamma^2 & 2(a\beta + \gamma) & 2(a\gamma - \beta) \\ 2(a\beta - \gamma) & 1 - a^2 + \beta^2 - \gamma^2 & 2(\beta\gamma + a) \\ 2(a\gamma + \beta) & 2(\beta\gamma - a) & 1 - a^2 - \beta^2 + \gamma^2 \end{bmatrix} \quad (1)$$

Continuous knowledge of  $a$ ,  $\beta$ , and  $\gamma$  determines body attitude at all times with respect to the inertial frame. The parameters are not easily directly measured but may be determined by continuously solving the kinematic differential equation relating  $\dot{a}$ ,  $\dot{\beta}$ , and  $\dot{\gamma}$  to the body rates  $\omega_x$ ,  $\omega_y$ , and  $\omega_z$ .

$$\begin{bmatrix} \dot{a} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 + a^2 & a\beta - \gamma & a\gamma + \beta \\ a\beta + \gamma & 1 + \beta^2 & \beta\gamma - a \\ a\gamma - \beta & \beta\gamma + a & 1 + \gamma^2 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad (2)$$

Using these parameters, Mortensen has proposed the following control law (Reference 1):

$$\begin{aligned} u_x &= k_{21}\omega_x + k_1(1 + a^2 + \beta^2 + \gamma^2)a, \\ u_y &= k_{22}\omega_y + k_1(1 + a^2 + \beta^2 + \gamma^2)\beta, \\ u_z &= k_{23}\omega_z + k_1(1 + a^2 + \beta^2 + \gamma^2)\gamma, \end{aligned} \quad (3)$$

and

where  $u_x$ ,  $u_y$ , and  $u_z$  are control torques and  $k_1$ ,  $k_{21}$ ,  $k_{22}$ , and  $k_{23}$  are constant gains. He shows the system with this control law to be asymptotically stable in the large with respect to the origin ( $a = \beta = \gamma = 0$ ). The dynamic and control equations are developed in Appendix A. This control law is analogous to a conventional law in that the torque is a function (although nonlinear) of rate and position. For small angles (i.e., the spacecraft body axes near the desired attitude), the following approximations (Reference 3) hold:

$$\begin{aligned} a &\approx \frac{\theta_1}{2}, \\ \beta &\approx \frac{\theta_2}{2}, \\ \gamma &\approx \frac{\theta_3}{2}, \end{aligned} \quad (4)$$

and

where  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  are Euler angles.

\*Capital letters with double overbars are square matrices; those with single overbars are column matrices.

Therefore, for small angles, the control reduces to a simple linear rate plus position control.

$$\begin{aligned} u_x &\approx k_{21}\omega_x + k_3\theta_1, \\ u_y &\approx k_{22}\omega_y + k_3\theta_2, \\ u_z &\approx k_{23}\omega_z + k_3\theta_3, \end{aligned} \tag{5}$$

and

with  $k_3 = k_1/2$ .

A control law of this type, globally stable for any attitude, includes the ability to point and slew without the necessity for mode switching. The inertial reference is defined as  $\alpha = \beta = \gamma = 0$ . When a new reference is desired, e.g., pointing at a different star, the transformation relating the present position to the new position is determined. The corresponding parameters  $\alpha$ ,  $\beta$ , and  $\gamma$  are determined and sent up to the spacecraft. The spacecraft, using this control law, acts to null  $\alpha$ ,  $\beta$ , and  $\gamma$  and thereby moves to the new attitude. This is a reorientation or three-axis slew.

By means of the strapdown IRU and GP computer,  $\alpha$ ,  $\beta$ , and  $\gamma$  may be continuously updated to allow use of this control law. (See Figure 1.)

### ATTITUDE REFERENCE ALGORITHM

The sensor to be used for operational control of the spacecraft during a reorientation is an IRU composed of three inertial gyros. This is a strapdown system with the gyros operating in a pulse rebalance loop. They are continuously nulled by a series of pulses the size and rate of which provide

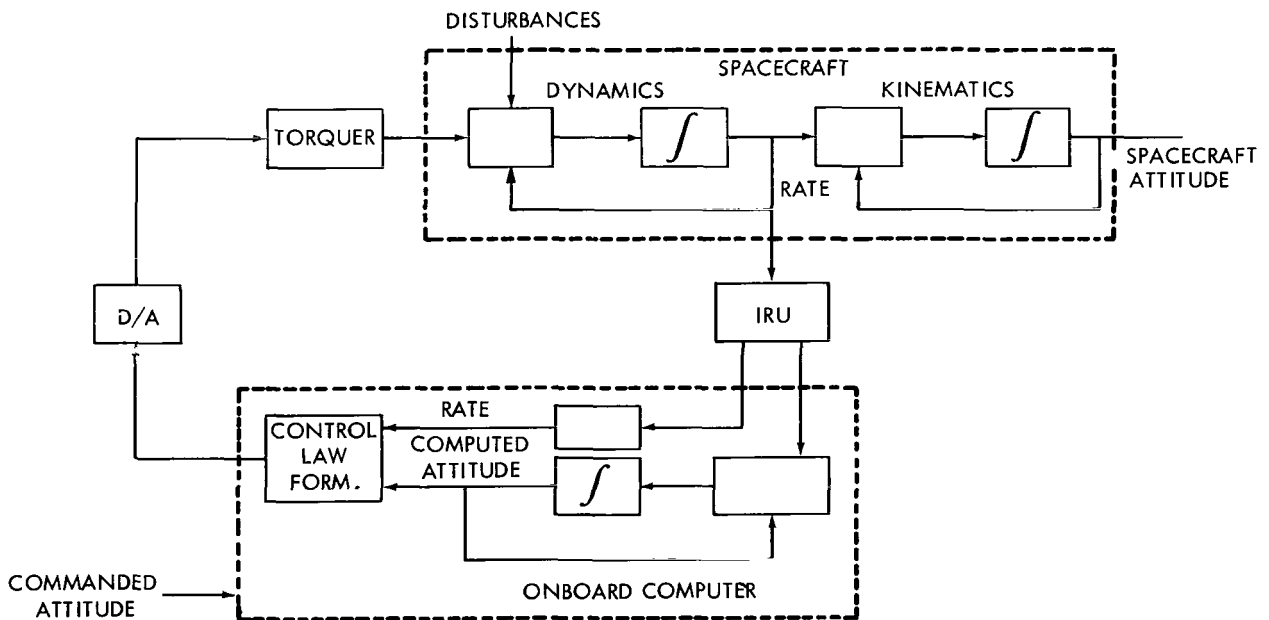


Figure 1—Control system block diagram.



position and rate information. References 4 and 5 describe this type of gyro. The information from the IRU must be used in conjunction with the computer to update the parameters  $\alpha$ ,  $\beta$ , and  $\gamma$  through-out a reorientation. This updating may be done by solving Equation 2.

The body rates are not known accurately enough for Equation 2 to be solved in its present form. Instead an integration algorithm based on an incremental angle may be used (Reference 5).

A Taylor series expansion may be used to update  $\alpha$ ,  $\beta$ , and  $\gamma$ . The value at time  $t + h$  may be determined from the value at time  $t$  and the information received from the gyro during the sampling interval  $h$ . Only the parameter  $\alpha(t)$  will be considered here, although similar expressions exist for  $\beta(t)$  and  $\gamma(t)$ .

$$\alpha(t + h) = \alpha(t) + \dot{\alpha}(t)h + \frac{1}{2}\ddot{\alpha}(t)h^2 + \frac{1}{6}\dddot{\alpha}(t)h^3 + \dots \quad (6)$$

If only a first-order expansion is used and if a substitution for  $\dot{\alpha}(t)$  is made from Equation 2,

$$\alpha(t + h) = \alpha(t) + \frac{[1 + \alpha^2(t)]\Delta_x}{2} + \frac{[\alpha(t)\beta(t) - \gamma(t)]\Delta_y}{2} + \frac{[\alpha(t)\gamma(t) + \beta(t)]\Delta_z}{2}, \quad (7)$$

where  $\Delta_i = \omega_i(t)h$ ,  $i = x, y$ , and  $z$ . The  $\Delta_i$  terms represent the angular output of the  $i$ th gyro during the interval  $h$ , i.e., a number of pulses of weight  $q$  ( $q$  is the gyro quantization level, 2.4" of arc in the IRU slew mode). The updating therefore may be done on an incremental angle basis. If the second-order terms of the Taylor series are kept, terms on the order of  $h^2$  appear. Appendix B lists the second-order expansions for all three parameters.

Updating the parameters in this manner allows the control law to be updated at each sampling point also. The body rates used in the control law are simply  $\Delta_i/h$ .

A study was conducted to determine the feasibility of this type of algorithm. A digital computer simulation of the system was used to help evaluate the errors. The attitude error was defined as the difference between the true spacecraft attitude and the attitude computed by means of the Taylor expansion. The spacecraft dynamics and true attitude were computed by means of a four-point Runge-Kutta routine. The integration step was kept much smaller than the sampling interval  $h$ .

The results of the study are as follows:

(1) The error in the update algorithm increases with an increase in the sampling interval  $h$ . This is not a serious constraint because even relatively long sampling intervals gave good results. With a second-order Taylor expansion and a 5-s sampling interval, the errors at the end of a 60° slew were on the order of 14" of arc.

(2) The error in the update algorithm increases with the magnitude of the slew angle for the longer sampling intervals (2 s and greater). For a sampling interval of 100 ms, the error is apparently independent of the magnitude of the slew angle even for very long slews (165°). It remains on the order of 2.4" of arc, which is the quantization level of the gyros. For a sampling interval of 1 s, the error for a 165° slew is only about 5.3" of arc with a second-order Taylor expansion.

(3) For moderate sampling intervals and slew angles (up to 2 s and 90°), the attitude error is primarily a function of the gyro quantization level and is on this order of magnitude. This also holds true for shorter slews (30°) with longer sampling intervals (5 s).

(4) The computer round-off error may be kept negligible by the use of a 36 bit, double-precision word. The computer considered in the study (see the following section) has the ability to operate on either a single- (18 bits) or double-precision basis.

The estimated times for the update computations are shown in Table 1. These times are based on the computer described in the following section and are compatible with the sampling intervals investigated.

These results indicate that use of this type of algorithm for updating the spacecraft attitude parameters is feasible.

Table 1—Estimated time for the update computation.

Taylor series expansion	Single-precision (ms)	Double-precision (ms)
First-order	2	7
Second-order	4	14

## COMPUTER

A candidate for the computer required for attitude updating and control law formulation is the onboard processor being developed at GSFC.\* This computer is being developed primarily as a data processor but appears to have the capacity and capability for handling the control equations. Some of the features of this computer are

- (1) Postlaunch reprogramming capability. The control laws may be changed in flight if desired.
- (2) Low power. The central processing unit requires 5 W.
- (3) Add time of 7.5  $\mu$ s.
- (4) Maximum use (80%) of integrated circuits.

## INERTIAL REFERENCE UNIT

The IRU being developed by MIT for the OAO will apparently meet the attitude reference requirements. Three operational modes are available: hold, slew, and I-stab. The primary concern of this report, three-axis reorientation, requires use of the slew mode. For this study the processing being

\*Taylor, T., et. al., "A General Purpose On Board Processor for Scientific Spacecraft," NASA/Goddard Space Flight Center Document X-562-67-202, July 1967.

developed for the early models of the IRU was not considered. The only concern was the basic performance of the gyros.

Some of the important characteristics of the IRU in its slew mode are a resolution of 2.4" of arc/pulse, a maximum rate of 480" of arc/s, a pulse rate of 200 pps, a compensated drift of 20" of arc/orbit (12" of arc/hr), and a torquing error less than 0.01%.

## NORMAL REORIENTATION

The system dynamic, kinematic, and control equations were simulated on a digital computer. The primary objective of the simulation was to determine performance with respect to a slew or reorientation. The equations reduce to approximately linear equations for small angles (hold or pointing). Thus it was not necessary to consider the hold or pointing modes of operation.

The angle  $\phi$  (Appendix A) is a good scalar representation of error, i.e., the difference between the actual body orientation and the desired orientation (Reference 2). This is true because  $\alpha$ ,  $\beta$ , and  $\gamma$  are all zero if and only if  $\phi$  is zero. Therefore,  $\phi$  was chosen as the primary performance index. For a single-axis slew,  $\phi$  is equivalent to the Euler angle around the slew axis.

The only requirements for stability are that the position gain  $k_p$  and rate gains  $k_x$ ,  $k_y$ , and  $k_z$  be positive and nonzero. There are no other requirements with respect to magnitude or linearity. Therefore, once the stability criteria are met, the designer may set the gains to meet other criteria such as speed of response, damping, etc.

### Response

Probably the most important criterion for a normal reorientation is the time. The shorter the reorientation time, the longer the experimenting time. Another criterion, maintenance of a stable slew axis, is discussed in a following section. The time criterion was used in making several computer runs.

The time for a slew was defined as the interval which extended from the time when initial conditions were read into the onboard computer to the time when the norm reached its minimum value. The norm, defined as

$$\text{NORM} = + \left( \omega_x^2 + \omega_y^2 + \omega_z^2 + \phi^2 \right)^{1/2}, \quad (8)$$

was usually assigned a minimum of  $10^{-4}$ . Since the norm is always positive, the rates were required to be below  $10^{-4}$  rad/s and  $\phi$  was required to be below  $10^{-4}$  rad (0.34' of arc) before a slew was considered complete. Therefore, slew or reorientation time as defined here includes settling time.

Since a slew is basically a momentum exchange, the gains were set to result in a maximum wheel speed during the slew. The rate gains were set to provide good damping at the end of the slew. Both type A and type B wheels were used. (See Figure 2.) The characteristics were a stall torque of 0.27 N-m (0.20 ft-lb), a maximum momentum of 13.6 N-m-s (10.0 ft-lb-s), and a time constant  $T_m$  of 50 s.

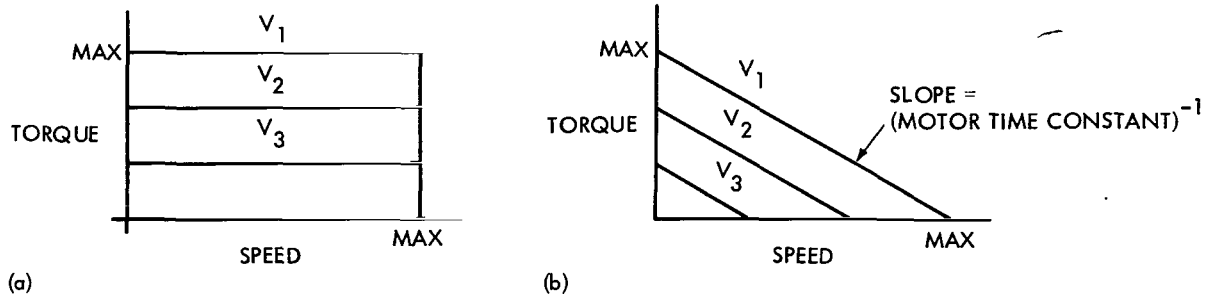


Figure 2—Reaction wheel torque speed curve, (a) type A wheel and (b) type B wheel.

To compare a three-axis reorientation with the more conventional method using three single-axis slews, a number of runs were made with the slew axis eigenvector aligned with a control axis. (See Equations A17, Appendix A.) This is equivalent to a conventional single-axis slew.

Figure 3 shows the response for four single-axis and two multiaxis slews [eigenvector (0.5, 0.5, 0.707)]. The maximum slew rate for the single-axis slew is about  $0.25 \times 10^{-2}$  rad/s. This is determined by the 13.6 N-m-s maximum wheel momentum and the  $5420 \text{ kg-m}^2$  body inertia (the same for all axes). The maximum slew rate for the multiaxis slew occurs when all three wheels are saturated. It is about  $0.43 \times 10^{-2}$  rad/s. Thus all three wheels are being used to reduce  $\phi$ . The single-axis and multiaxis slews in Figure 3 are not directly comparable because the final orientations are different. A comparison will be made later in the report.

Figure 4 is a plot of total slew time for slews of various lengths. The curves all converge to about 160 s for a  $2^\circ$  slew. This is apparently due to the end effects, i.e., acceleration at the start and settling at the completion of the slew. The slope of the linear portion of the single-axis curves is  $(0.25 \times 10^{-2})^{-1}$  s/rad. This is determined by the maximum wheel momentum. All the points checked for multiaxis slews were below the time required for equivalent length single-axis slews.

Three points were checked both ways to compare the time required for reorientation by means of a multiaxis slew with the time required for the same reorientation by means of three single-axis slews. See points D, E, and F on Figure 4. Table 2 summarizes the results.

In the table,  $T_1$  is the total time for three single-axis slews. This time is determined by using the Euler angles and single-axis slew curves of Figure 4. The time to complete the same reorientation using a multiaxis slew is  $T_3$ . The ratio  $T_1/T_3$  shows the multiaxis reorientation to be more than twice as fast as the conventional slew. This should be expected for at least two reasons: The end effects, which are independent of the length of the slew, occur three times for three slews and only once for a single slew, and all the wheels contribute simultaneously during a multiaxis slew. This contribution causes the total rate by which the error is being reduced to be higher than for a single-axis slew.

The slew time using a type B motor was slightly longer in each case due to the reduced torque available for acceleration and deceleration. The slopes of the linear portions are the same since the maximum wheel speeds are the same.

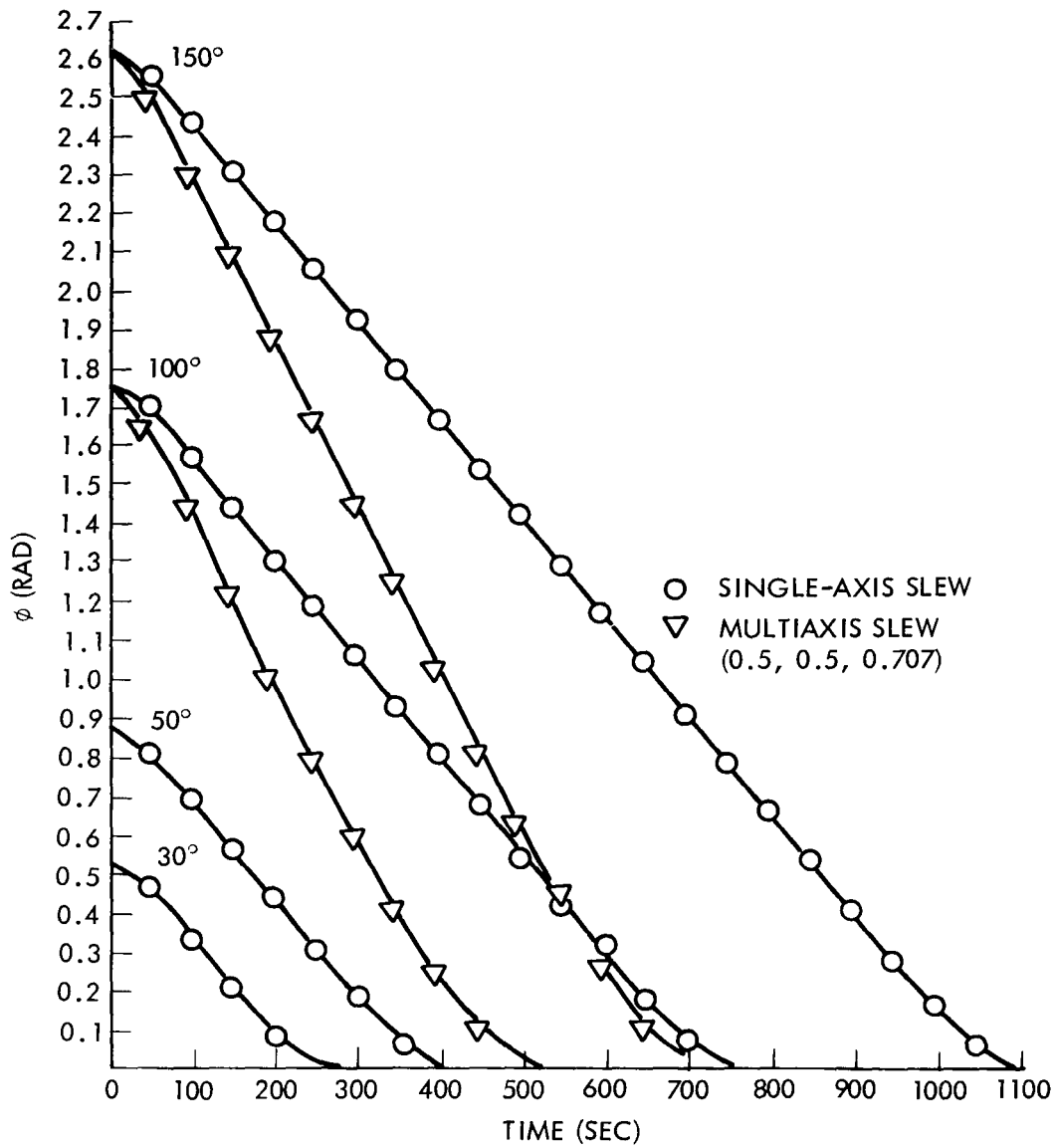


Figure 3-Slew response.

### Accuracy

In a hold or point mode the accuracy is primarily a function of the sensors. When the spacecraft is holding on the experiment telescope or roll axis star tracker, there is no problem. When it is holding on the IRU, the primary error will be due to the gyro drift. This is expected to be about 12" of arc/hr. The desired accuracy determines the update frequency when the spacecraft is holding on the IRU.

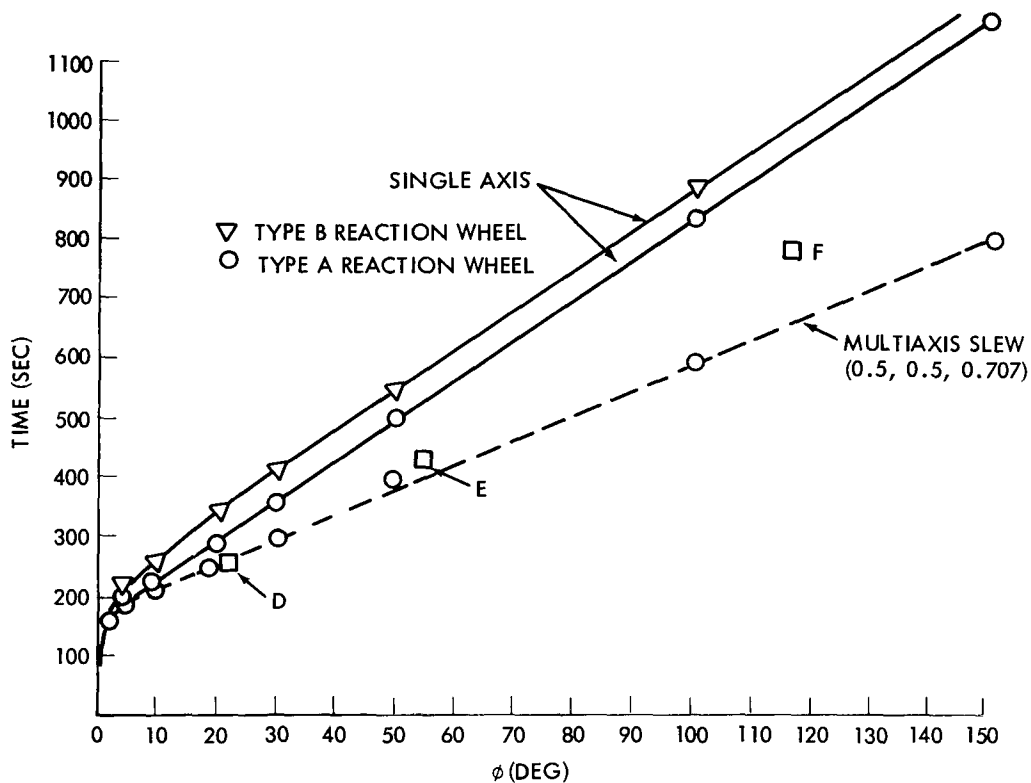


Figure 4—Slew time.

Table 2—Reorientation times.

	Euler angles (rad, deg)			Eigenvector comp.			$\phi$ (rad, deg)	$T_1$ (s)	$T_3$ (s)	$\frac{T_1}{T_3}$
	$\theta_1$	$\theta_2$	$\theta_3$	$C_x$	$C_y$	$C_z$				
D	0.1745, 10	0.2745, 15.7	0.1745, 10	0.5187	0.6797	0.5187	0.3797, 21.8	700	260	2.69
E	0.523, 30	0.523, 30	0.523, 30	0.6546	0.3782	0.6546	0.9722, 55.7	1080	420	2.57
F	1.045, 60	1.045, 60	1.045, 60	0.6946	0.1869	0.6946	2.0327, 116.5	1680	790	2.13

There are three sources of error present during a reorientation. They are the attitude updating, gyro drift, and the torquing inaccuracy of the gyros. For a sampling interval of 100 ms, the attitude update error may be kept on the order of the gyro quantization level, which is 2.4" of arc. The reorientation error due to gyro drift will be small because most slews will take less than 20 min to complete.

The compensated drift during this time is only 3 to 4" of arc. There is an error accumulation due to the torque rebalance loop when the gyro is being torqued. The torque rebalance pulses will contain some errors in size and shape. This will cause an attitude error buildup over a long slew. The torquing inaccuracy is estimated to be 0.01%. This represents an error of  $0.0165^\circ$  (1' of arc) over a  $165^\circ$  slew. Therefore, this is the major attitude error in a large angle reorientation.

The accuracy of the system need not be as high as that required for fine pointing. The objective of the reorientation is the acquisition of a new experiment star. Therefore, the primary factor in the determination of the required slew accuracy is the field of view of the experiment telescope or the roll axis star tracker. If the total reorientation error is well within the field of view of the sensor, it is feasible to reorient the spacecraft by the method described. With an experiment that has a field of view  $\pm 4'$  of arc, it does appear feasible to move from fine pointing to fine pointing by means of a single large-angle three-axis reorientation.

### **Disturbances**

The effect of environmental disturbance torques during a slew is to increase the total momentum of the system. The reorientation trajectory with disturbances will in general be different from the undisturbed trajectory. However, the final reference as stored in the computer and IRU is not affected; the system will null at the same reference as the undisturbed system. The time for the reorientation will depend on the magnitude and direction of the disturbances and may be shorter or longer than that required for an undisturbed reorientation. Although an extensive torque disturbance analysis was not conducted, some computer runs were made with disturbance torques. No gross changes in system performance were noted. The additional time required for one  $165^\circ$  slew was on the order of 1 min with disturbances on the order of 0.0001 to 0.0005 N-m (1000 to 5000 dyn-cm).

### **FIXED-AXIS REORIENTATION**

When the spacecraft is reoriented from one inertial attitude to another, as discussed in the previous section, the primary criteria are time and accuracy. Once the attitude parameters are loaded in the computer, the spacecraft follows a trajectory determined in part by the controller gains to the new attitude. In this case, the instantaneous axis of rotation (the +1 eigenvector of the attitude reference matrix) is not inertially fixed but moves throughout the reorientation time.

There are times when it is important to maintain rotation about a fixed, although arbitrary, axis. This would be the case in an acquisition mode in which it is necessary to slew about an axis so that a particular sensor will pick up a particular star. The axis of rotation must remain inertially fixed; i.e., the rotation eigenvector must remain fixed, to assure the sensor will eventually have the star in its field of view. This may be accomplished by commanding a reorientation around this eigenvector and limiting the motor torques and maximum momenta by the ratio of the eigenvector components. The body rates are then proportional to the eigenvector components, and the total angular velocity vector remains collinear with the slew axis (eigenvector) throughout the entire slew.

A computer simulation confirmed the stability of the eigenvector position for this type of reorientation.

## DISCUSSION

This study was not a general study of various control laws and/or kinematic representations. The choice of Mortensen's control law with the Euler-Rodrigues parameters appeared to be natural for this problem. Further study may show it is advantageous to use Meyer's method when the entire transformation matrix is updated. Basically there is little difference between the two approaches. It may be shown that the control laws are very similar.

These types of control laws may be used with other types of sensors. In particular, the attitude matrix defined by Meyer as the position output of the system is independent of the sensors used to describe its elements. Star trackers, as well as gyros, may be used. However, it appears that a strap-down IRU as considered here offers the most advantages for large-angle reorientations. The use of gimballed star trackers has two disadvantages. Due to the limited gimbal rotation, large-angle reorientations cannot be completed without switching trackers during the slew. This introduces additional constraints on the reorientation maneuver. Secondly, the attitude matrix is more complex because of the presence of the trigonometric functions resulting from the tracker gimbals.

Because of the gyro drift it seems necessary for the spacecraft to hold its position during the experiment time by means of optical trackers, either the experiment telescope or the roll axis star tracker. Holding the spacecraft position with an optical tracker requires a relatively simple control system and does not require a computer. Therefore, during this time the computer may be free to process the experiment data. During a reorientation when no experiment is being carried out, the computer may be used completely for the control system. Although the computer-controlled system may easily handle both the hold and reorientation mode, it may be more efficient to use it in this manner.

The in-flight programing capability of the computer may be used to advantage. The control gains may be set initially to carry out a constant eigenvector slew for initial acquisitions. Once this is complete, the gains may be changed to allow a more rapid slew for normal reorientations.

## CONCLUSIONS

A computer control system that performs as well during a slew as during pointing is feasible for an advanced OAO. With an onboard digital computer and strapdown IRU, the system may complete large-angle three-axis reorientations with the same control law used for pointing. The computer speed is such that a second-order Taylor series may be used to update the spacecraft attitude during a reorientation. The errors due to the attitude updating may be kept on the order of the gyro quantization level. The primary error in a large-angle reorientation is due to the gyro torquing inaccuracy. This error, however, is not prohibitive.

A GP computer now under development, the On Board Processor being developed at GSFC, appears to meet the computational requirements.

An IRU being developed for GSFC by MIT appears capable of meeting the sensor requirements.



A three-axis slew is superior to three single-axis slews when time is important. A three-axis slew was shown to be more than twice as fast as three single-axis slews for the reorientations that were checked. It is also simpler and requires fewer commands at the start of the slew.

If the proper gain settings are used, a slew may be maintained about any constant, arbitrary axis.

It is possible to use a conventional linear rate and position control when pointing and to employ the computer primarily for reorientation. The computer could then be used for data processing when the spacecraft is in a pointing mode.

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## Appendix A

### DYNAMIC AND STABILITY ANALYSIS

The spacecraft dynamic and control equations are developed in this appendix. Although Mortensen did not use reaction wheels for control, the method for proving stability closely parallels his development (Reference 1).

If the control torque on the spacecraft is assumed to be developed by reaction wheels, the dynamic equations (Reference 3) are

$$\begin{aligned} \dot{H}_x + (I_z - I_y)\omega_y\omega_z + (\omega_y H_{wz} - \omega_z H_{wy}) &= M_x - \dot{H}_{wx}, \\ \dot{H}_y + (I_x - I_z)\omega_x\omega_z + (\omega_z H_{wx} - \omega_x H_{wz}) &= M_y - \dot{H}_{wy}, \end{aligned} \quad (A1)$$

and

$$\dot{H}_z + (I_y - I_x)\omega_x\omega_y + (\omega_x H_{wy} - \omega_y H_{wx}) = M_z - \dot{H}_{wz},$$

where  $H_x$ ,  $H_y$ , and  $H_z$  are the spacecraft momentum about  $x$ ,  $y$ , and  $z$  (principal axes) minus wheel momentum ( $H_i = I_i\omega_i$ ,  $i = x, y$ , and  $z$ );  $I_x$ ,  $I_y$ , and  $I_z$ , the spacecraft inertia about  $x$ ,  $y$ , and  $z$ ;  $H_{wx}$ ,  $H_{wy}$ , and  $H_{wz}$ , the wheel momentum about  $x$ ,  $y$ , and  $z$ ; and  $M_x$ ,  $M_y$ , and  $M_z$ , the external torques about  $x$ ,  $y$ , and  $z$ . Assume  $M_x = M_y = M_z = 0$ . During a slew the external torques are small compared to the control torques, and the assumption should be valid. If the equation is put into matrix form,

$$\begin{bmatrix} \dot{H}_x \\ \dot{H}_y \\ \dot{H}_z \end{bmatrix} = \begin{bmatrix} 0 & \frac{-H_{Tz}}{I_y} & \frac{H_{Ty}}{I_z} \\ \frac{H_{Tz}}{I_x} & 0 & \frac{-H_{Tx}}{I_z} \\ \frac{-H_{Ty}}{I_x} & \frac{H_{Tx}}{I_y} & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} - \begin{bmatrix} \dot{H}_{wx} \\ \dot{H}_{wy} \\ \dot{H}_{wz} \end{bmatrix}, \quad (A2)$$

where  $H_{Ti} = H_i + H_{wi}$ ,  $i = x, y$ , and  $z$ . If Equation A2 is combined with Equation 2,

$$\begin{bmatrix} \dot{H}_x \\ \dot{H}_y \\ \dot{H}_z \\ \dot{a} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} 0 & \frac{-H_{Tz}}{I_y} & \frac{H_{Ty}}{I_z} & 0 & 0 & 0 \\ \frac{H_{Tz}}{I_x} & 0 & \frac{-H_{Tx}}{I_z} & 0 & 0 & 0 \\ \frac{-H_{Ty}}{I_x} & \frac{H_{Tx}}{I_y} & 0 & 0 & 0 & 0 \\ \left(\frac{1+a^2}{2I_x}\right) & \left(\frac{a\beta}{2I_y}\right) & \left(\frac{a\gamma}{2I_z}\right) & 0 & \frac{H_z}{2I_z} & \frac{-H_y}{2I_y} \\ \left(\frac{a\beta}{2I_x}\right) & \left(\frac{1+\beta^2}{2I_y}\right) & \left(\frac{\beta\gamma}{2I_z}\right) & \frac{-H_z}{2I_z} & 0 & \frac{H_x}{2I_x} \\ \left(\frac{a\gamma}{2I_x}\right) & \left(\frac{\beta\gamma}{2I_y}\right) & \left(\frac{1+\gamma^2}{2I_z}\right) & \frac{H_y}{2I_y} & \frac{-H_x}{2I_x} & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \\ a \\ \beta \\ \gamma \end{bmatrix} - \begin{bmatrix} \dot{H}_{wx} \\ \dot{H}_{wy} \\ \dot{H}_{wz} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (A3)$$

If a control law of the type shown in Equations 3 and a reaction wheel with a torque speed curve as shown in Figure 2(a) is assumed, the control equations are

$$\begin{aligned}
\dot{H}_{wx} &= k_x H_x + k_p \left(\frac{1+a^2}{2}\right)a + k_p \left(\frac{\beta a}{2}\right)\beta + k_p \left(\frac{a\gamma}{2}\right)\gamma, \\
\dot{H}_{wy} &= k_y H_y + k_p \left(\frac{a\beta}{2}\right)a + k_p \left(\frac{1+\beta^2}{2}\right)\beta + k_p \left(\frac{\beta\gamma}{2}\right)\gamma, \\
\dot{H}_{wz} &= k_z H_z + k_p \left(\frac{a\gamma}{2}\right)a + k_p \left(\frac{\beta\gamma}{2}\right)\beta + k_p \left(\frac{1+\gamma^2}{2}\right)\gamma,
\end{aligned} \quad (A4)$$

and

$$\dot{H}_{wz} = k_z H_z + k_p \left(\frac{a\gamma}{2}\right)a + k_p \left(\frac{\beta\gamma}{2}\right)\beta + k_p \left(\frac{1+\gamma^2}{2}\right)\gamma,$$

where  $k_x$ ,  $k_y$ , and  $k_z$  are rate gains and  $k_p$  is position gain.

The total system is ninth order and requires nine state variables for a complete description of the trajectory in state space. The state variables are the three body momenta, three parameters specifying body position, and the three total momenta. The equilibrium position of the spacecraft can be seen to be independent of the system total momenta. Thus there is a three-dimensional subspace every point of which is an equilibrium point for the spacecraft. Therefore, for spacecraft stability analysis the six-dimensional vector representing the body position and rate may be used in place of the nine-dimensional system vector. The total momentum in this formulation may be considered a time-varying parameter.

$$\begin{bmatrix} \dot{H}_x \\ \dot{H}_y \\ \dot{H}_z \\ \dot{a} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} -k_x & \frac{-H_{Tz}}{I_y} & \frac{H_{Ty}}{I_z} & -k_p \left( \frac{1+a^2}{2} \right) & -k_p \left( \frac{\beta a}{2} \right) & -k_p \left( \frac{\alpha \gamma}{2} \right) \\ \frac{H_{Tz}}{I_x} & -k_y & \frac{-H_{Tx}}{I_z} & -k_p \left( \frac{\alpha \beta}{2} \right) & -k_p \left( \frac{1+\beta^2}{2} \right) & -k_p \left( \frac{\beta \gamma}{2} \right) \\ \frac{-H_{Ty}}{I_x} & \frac{H_{Tx}}{I_y} & -k_z & -k_p \left( \frac{\alpha \gamma}{2} \right) & -k_p \left( \frac{\beta \gamma}{2} \right) & -k_p \left( \frac{1+\gamma^2}{2} \right) \\ \frac{1+a^2}{2I_x} & \frac{a\beta}{2I_y} & \frac{\alpha\gamma}{2I_z} & 0 & \frac{H_z}{2I_z} & \frac{-H_y}{2I_z} \\ \frac{a\beta}{2I_x} & \frac{1+\beta^2}{2I_y} & \frac{\beta\gamma}{2I_z} & \frac{-H_z}{2I_z} & 0 & \frac{H_x}{2I_x} \\ \frac{\alpha\gamma}{2I_x} & \frac{\beta\gamma}{2I_y} & \frac{1+\gamma^2}{2I_z} & \frac{H_y}{2I_y} & \frac{-H_x}{2I_x} & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \\ a \\ \beta \\ \gamma \end{bmatrix} \quad (\text{A5})$$

This is in the form

$$\dot{\bar{X}} = \bar{F}(X)\bar{X} \quad (\text{A6})$$

where  $\bar{X} = [H_x, H_y, H_z, a, \beta, \gamma]^T$  and  $\bar{F}(X)$  is a nonlinear system matrix. Define the matrices  $\bar{K}$  and  $\bar{R}$  as

$$\bar{K} = \text{diag} [-k_x, -k_y, -k_z, 0, 0, 0]$$

and

$$\bar{R} = \bar{F} - \bar{K}. \quad (\text{A7})$$

Choose as a candidate  $V$  function the quadratic form

$$V = \bar{X}^T \bar{G} \bar{X} \quad (\text{A8})$$

where

$$\bar{G} = \text{diag} \left[ \frac{1}{I_x}, \frac{1}{I_y}, \frac{1}{I_z}, k_p, k_p, k_p \right].$$

Since  $\bar{G}$  is a constant, it follows that

$$\dot{V} = \dot{\bar{X}}^T \bar{G} \bar{X} + \bar{X}^T \bar{G} \dot{\bar{X}}. \quad (\text{A9})$$

If Equations A7 are substituted into A6 and then into A9, Equation A9 becomes

$$\dot{V} = \bar{X}^T \bar{R}^T \bar{G} \bar{X} + \bar{X}^T \bar{K}^T \bar{G} \bar{X} + \bar{X}^T \bar{G} \bar{R} \bar{X} + \bar{X}^T \bar{G} \bar{K} \bar{X}. \quad (\text{A10})$$

Note that  $\overline{\overline{GR}}$  is skew symmetric and that  $\overline{\overline{G}} = \overline{\overline{G}}^T$ . Therefore,

$$\overline{\overline{GR}} = -(\overline{\overline{GR}})^T = -\overline{\overline{R}}^T \overline{\overline{G}}^T = -\overline{\overline{R}}^T \overline{\overline{G}}. \quad (\text{A11})$$

This reduces Equation A10 to

$$\dot{V} = \overline{\overline{X}}^T \overline{\overline{K}}^T \overline{\overline{G}} \overline{\overline{X}} + \overline{\overline{X}}^T \overline{\overline{G}} \overline{\overline{K}} \overline{\overline{X}}. \quad (\text{A12})$$

$$\overline{\overline{K}}^T \overline{\overline{G}} = \overline{\overline{K}}^T \overline{\overline{G}}^T = (\overline{\overline{G}} \overline{\overline{K}})^T = \overline{\overline{G}} \overline{\overline{K}} \quad (\text{A13})$$

is also true. Therefore,

$$\dot{V} = 2\overline{\overline{X}}^T \overline{\overline{G}} \overline{\overline{K}} \overline{\overline{X}} = -2\left(\frac{k_x}{I_x} H_x^2 + \frac{k_y}{I_y} H_y^2 + \frac{k_z}{I_z} H_z^2\right). \quad (\text{A14})$$

Thus, if

$$k_x > 0, \quad k_y > 0, \quad \text{and} \quad k_z > 0, \quad (\text{A15})$$

$\dot{V}$  is negative semidefinite. Since

$$V = \frac{H_x^2}{I_x} + \frac{H_y^2}{I_y} + \frac{H_z^2}{I_z} + k_p \alpha^2 + k_p \beta^2 + k_p \gamma^2 \quad (\text{A16})$$

is positive definite ( $k_p > 0$ ), the origin is stable. It may be shown that the origin is not only stable but asymptotically stable; i.e.,  $V \rightarrow 0$  as  $t \rightarrow \infty$ .

Since the relations

$$\begin{aligned} \alpha &= C_x \tan \frac{\phi}{2}, \\ \beta &= C_y \tan \frac{\phi}{2}, \end{aligned} \quad (\text{A17})$$

and

$$\gamma = C_z \tan \frac{\phi}{2},$$

are valid (Reference 3), where  $C_x$ ,  $C_y$ , and  $C_z$  are the components of the +1 eigenvector of the orthogonal transformation parameterized by  $\alpha$ ,  $\beta$ , and  $\gamma$  and  $\phi$  is the angle around the +1 eigenvector that the body frame is rotated from the inertial frame, Equation A16 may be written as

$$V = \frac{H_x^2}{I_x} + \frac{H_y^2}{I_y} + \frac{H_z^2}{I_z} + k_p \tan^2 \frac{\phi}{2}. \quad (\text{A18})$$

Therefore, for  $V$  to be bounded it is necessary that the body angular momentum be bounded and that  $\phi$  be less than  $\pi$ . The total system momentum must be within the total capacity of the wheels. It

will be stored in the wheels at the completion of the reorientation. The latter constraint ( $\phi < \pi$ ) is not a practical restriction on the system since it is easily satisfied. With the boundedness of  $V$  established, it may be concluded that  $V$  is a valid Lyapunov function and the stability conclusions are valid.

Asymptotic stability in the large has not yet been proven in general when a type B reaction wheel is used. However, two important special cases may be shown to be asymptotically stable. One is the case in which the total system momentum is zero. The other is the linearized system near the origin.

When a type B reaction wheel is used, the additional back emf terms  $-H_{wi}/T_m$ ,  $i = x, y$ , and  $z$ , are added to the respective control equations (Equations A4). When the total system momentum is zero,  $H_i = -H_{wi}$ . Thus, the system is reduced to sixth order and is easily proven stable in a manner analogous to that used in the preceding proof.

When the system equations are linearized near the origin, the result is three uncoupled linear second-order systems. These are easily proven stable by conventional methods. Any stored momentum represents a shift to a new origin.





## Appendix B

### PARAMETER UPDATE EQUATIONS

The equations for updating the parameters  $a$ ,  $\beta$ , and  $\gamma$  by means of a second-order Taylor series are

$$\begin{aligned} a_{n+1} = & a_n + \frac{1}{2} \left[ (1 + a_n^2) \Delta_x + (a_n \beta_n - \gamma_n) \Delta_y + (a_n \gamma_n + \beta_n) \Delta_z \right] \\ & + \frac{1}{4} \left[ a_n (1 + a_n^2) \Delta_x^2 + \beta_n (a_n \beta_n - \gamma_n) \Delta_y^2 + \gamma_n (a_n \gamma_n + \beta_n) \Delta_z^2 \right. \\ & + (2a_n^2 \beta_n + \beta_n - a_n \gamma_n) \Delta_x \Delta_y + (2a_n^2 \gamma_n + \gamma_n + a_n \beta_n) \Delta_x \Delta_z \\ & \left. + (2a_n \beta_n \gamma_n + \beta_n^2 - \gamma_n^2) \Delta_y \Delta_z \right], \end{aligned}$$

$$\begin{aligned} \beta_{n+1} = & \beta_n + \frac{1}{2} \left[ (a_n \beta_n + \gamma_n) \Delta_x + (1 + \beta_n^2) \Delta_y + (\beta_n \gamma_n - a_n) \Delta_z \right] \\ & + \frac{1}{4} \left[ a_n (a_n \beta_n + \gamma_n) \Delta_x^2 + \beta_n (1 + \beta_n^2) \Delta_y^2 + \gamma_n (\beta_n \gamma_n - a_n) \Delta_z^2 \right. \\ & + (2a_n \beta_n^2 + a_n + \beta_n \gamma_n) \Delta_x \Delta_y + (2a_n \beta_n \gamma_n - a_n^2 + \gamma_n^2) \Delta_x \Delta_z \\ & \left. + (2\beta_n^2 \gamma_n + \gamma_n - a_n \beta_n) \Delta_y \Delta_z \right], \end{aligned}$$

and

$$\begin{aligned} \gamma_{n+1} = & \gamma_n + \frac{1}{2} \left[ (a_n \gamma_n - \beta_n) \Delta_x + (\beta_n \gamma_n + a_n) \Delta_y + (1 + \gamma_n^2) \Delta_z \right] \\ & + \frac{1}{4} \left[ a_n (a_n \gamma_n - \beta_n) \Delta_x^2 + \beta_n (\beta_n \gamma_n + a_n) \Delta_y^2 + \gamma_n (1 + \gamma_n^2) \Delta_z^2 \right. \\ & + (2a_n \beta_n \gamma_n + a_n^2 - \beta_n^2) \Delta_x \Delta_y + (2a_n \gamma_n^2 + a_n - \beta_n \gamma_n) \Delta_x \Delta_z \\ & \left. + (2\beta_n \gamma_n^2 + \beta_n + a_n \gamma_n) \Delta_y \Delta_z \right]. \end{aligned}$$

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