Attitude estimation on $SO(3)$ based on direct inertial measurements

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***Abstract*— This paper considers the question of obtaining high quality attitude estimates from typical low cost inertial measurement units for applications in control of unmanned aerial vehicles. A nonlinear complementary filter exploiting the structure of Special Orthogonal Group $S0(3)$ is proposed. The filter is expressed explicitly in terms of direct and untreated measurements. For a typical low cost inertial measurement where two inertial direction are measured (gravity and magnetic field), it is shown that the filter is well conditioned. If only a single direction is available (typically the gravity) associated with gyros measurements, it is shown that the full gyros bias vector is correctly estimated and that the estimated orientation converges to a set consistent with the measurements. Experimental results, conducted on a the HoverEye© UAV, demonstrate the efficiency of the proposed filter.**

I. INTRODUCTION

A crucial problem in controlling an unmanned aerial vehicle (UAV) is the estimation of position and orientation, or pose, of the airframe. A single sensor is not sufficient to directly measure the orientation and the position of the vehicle with respect to the inertial frame. In practice, it is necessary to fuse data from different sensors to obtain a robust estimate of the pose that best exploits the specific properties of each sensor (inertial or absolute information, accuracy, bandwidth, noise level, etc.) Orientation estimation is an important step in the design of autonomous flying robots since estimated angles of attitude are used in the so-called lowest control-loop to stabilize the attitude. Historically, applications in unmanned aerial vehicles have addressed this issue by using high quality sensor systems. For example the YAMAHA RMAX radio controlled helicopters use high quality optical gyros for their stability augmentation systems. However, military specification inertial measurement units are expensive, often subject to export restrictions, and not suitable for commercial applications.

More recently, the focus on new low cost aerial robotic systems, has lead to a strong interest in attitude estimation algorithms. Traditional linear Kalman filter techniques [9] (including EKF techniques [5], [7]) have proved difficult to apply robustly to applications with low quality sensor systems [8]. The inherent non-linearity of the system and non-Gaussian noise encountered in practice can lead to very poor behaviour of such filters. More sophisticated stochastic filtering techniques such as particle filters and even unscented filters place too much computational load on the low cost processing systems associated with the commercial applications considered. The majority of filtering techniques used and implemented to estimate the attitude of UAVs are based on linear complementary filters with frequency domain analysis [1]; a tradeoff between a good short term precision given by gyroscopic integration and reliable long term accuracy provided by accelerometers. Most existing applications use single state complementary filtering on Euler angle decomposition of rotation matrix [10]. A fully integrated filter for the rotation in terms of quaternion representation was developed in [12], [11], [4]. Existing complementary filters use a calculation of the actual orientation, derived directly from measurements, to drive the error signal for the filter. This introduces an additional algebraic operation into the filter implementation that increases the computational load and is prone to poor numerical conditioning.

In this paper, we study the design of a nonlinear complementary filter on $SO(3)$ based directly on measurements. We consider the question of filter design with the goal of preserving the natural Lie group structure of the evolution of attitude on $SO(3)$. The measurements considered are directly coupled to those available for the experimental HoverEye© vehicle, however, these measurements are characteristics of almost all low cost inertial units used in UAVs. We show that a complimentary filter can be well posed in terms of direct measurements and prove that it is asymptotically stable in the case where at least two inertial measurements are available in addition to full measurement of the angular rates of the system. The filter includes an adaptive compensation of bias terms that are always present in the output of the angular velocity measurements due to the technology used in the solid state gyrometers. The filter remains well defined even if only a single inertial measurement is available. This is important, since many practical systems can provide a good (low frequency) estimate of the direction of gravity but are not equipped with magnetometers, vision system or alternative sensor system to provide a second inertial direction. The proposed filter has several practical advantages over existing filters associated with implementation and low-sensitivity to noise. The filter performance is verified by experimental results.

II. PRELIMINARY MATERIAL

A. Notation and mathematical identities

The special orthogonal group is denoted $SO(3)$. The associated Lie-algebra is the set of anti-symmetric matrices

$$\mathfrak{so}(3) = \{A \in \mathbb{R}^{3\times 3} \mid A = -A^T\}$$

For any two matrices $A, B \in \mathbb{R}^{n\times n}$ then the Lie-bracket is $[A, B] = AB - BA$. Let $\Omega \in \mathbb{R}^3$ then define

$$\Omega_\times = \begin{pmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{pmatrix}.$$

For any $v \in \mathbb{R}^3$ then $\Omega_\times v = \Omega \times v$ is the vector cross product. The operator vex : $\mathfrak{so}(3) \to \mathbb{R}^3$ denotes the inverse of the $\Omega_\times$ operator

$$\begin{aligned} \text{vex}\,(\Omega_\times) &= \Omega, & \Omega &\in \mathbb{R}^3. \\ \text{vex}(A)_\times &= A, & A &\in \mathfrak{so}(3) \end{aligned}$$

For any two matrices $A, B \in \mathbb{R}^{n\times n}$ the Euclidean matrix inner product and Frobenius norm are defined

$$\langle\langle A, B\rangle\rangle = \text{tr}(A^T B) = \sum_{i,j=1}^{n} A_{ij}B_{ij}$$

$$||A|| = \sqrt{\langle\langle A, A\rangle\rangle} = \sqrt{\sum_{i,j=1}^{n} A_{ij}^2}$$

The following identities are used in the paper

$$\begin{aligned} (Rv)_\times &= Rv_\times R^T, & R \in SO(3), v \in \mathbb{R}^3 \\ (v \times w)_\times &= [v_\times, w_\times] & v, w \in \mathbb{R}^3 \\ v^T w = \langle v, w\rangle &= \frac{1}{2}\langle\langle v_\times, w_\times\rangle\rangle, & v, w \in \mathbb{R}^3 \\ v^T v = |v|^2 &= \frac{1}{2}||v_\times||^2, & v \in \mathbb{R}^3 \\ \langle\langle A, v_\times\rangle\rangle &= 0, & A = A^T \in \mathbb{R}^3, v \in \mathbb{R}^3 \end{aligned}$$

The following notation for frames of reference is used

- $\mathcal{A}$ denotes an inertial (fixed) frame of reference.
- $\mathcal{B}$ denotes a body-fixed-frame of reference.
- $\mathcal{E}$ denotes the estimator frame of reference.

Let π_a, π_s denote, respectively, the anti-symmetric and symmetric projection operators in square matrix space

$$\pi_a(H) = \frac{1}{2}(H - H^T), \qquad \pi_s(H) = \frac{1}{2}(H + H^T).$$

B. Complimentary filtering on $SO(3)$

The system considered is the kinematics of the orientation matrix

$$\dot{R} = R\Omega_\times \tag{1}$$

Recall the dynamics of direct and passive complementary filters, proposed in [4]

$$\dot{\hat{R}}_d = (R_y\Omega + \hat{R}_d\omega)_\times \hat{R}_d = \hat{R}_d(\tilde{R}_d\Omega + \omega)_\times \tag{2}$$

$$\dot{\hat{R}}_p = (\hat{R}_p\Omega + \hat{R}_p\omega)_\times \hat{R}_p = \hat{R}_p(\Omega + \omega)_\times \tag{3}$$

where $\hat{R}_d$ (resp. $\hat{R}_p$) denote the estimate of the body fixed rotation matrix R using the direct (resp. passive) nonlinear complementary filter. The matrix R_y is a reconstruction of the present rotation matrix based on the direct measurements. The error matrix $\tilde{R}_d = \hat{R}_d^T R_y$ (resp. $\tilde{R}_p = \hat{R}_p^T R_y$) is a noisy measurement of the attitude of the body fixed frame $\mathcal{B}$ with respect to the estimator frame $\mathcal{E}$. The term $\omega = \omega(\hat{R}_{(.)}, R) \in \mathcal{E}$ is the innovation or correction term in the filter. In [4], the innovation term is chosen as follows

$$\omega = \pi_a(\tilde{R}),$$

to insure the convergence of $\tilde{R}_p$ (resp. $\tilde{R}_d$) to the identity matrix. Note that from the Eq. 2, the implementation

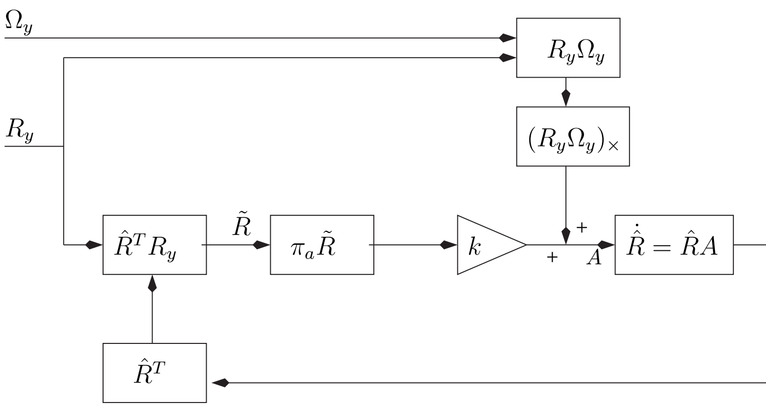

Fig. 1. Block diagram of the direct complementary filter on $SO(3)$.

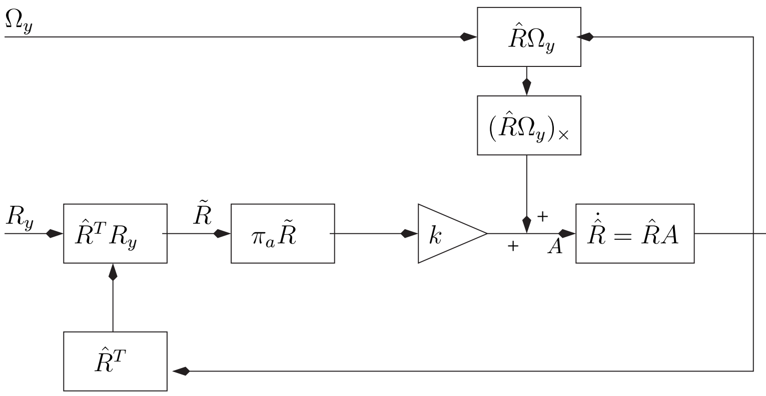

Fig. 2. Block diagram of the passive complementary filter on $SO(3)$.

of direct complementary filter requires the reconstructed rotation R_y both in the definition of the error term for the innovation ω and in the correction for the feed-forward compensation of the angular velocity Ω. The passive filter (Eq. 3) avoids using the measure R_y in the feed-forward term of the filter and is more robust to high frequency sensor noise as a consequence. Nevertheless, both methods require reconstruction of R_y from direct sensor outputs to define the error term $\tilde{R}$. The reconstruction of R_y introduces possible poor numerical conditioning and amplification of noise into the filter. It is desirable to consider filters that avoid the reconstruction process entirely. The direct filter cannot be reframed in terms of explicit sensor errors as the measurement $\tilde{R}_d$ is used as a frame transform as well as as an error term in the filter implementation. For this reason we consider only the passive filter in the sequel.

C. Measurements

Consider that the body is equipped with a strap-down IMU composed of 3-axis rate gyroscopes, 3-axis accelerometers, and 3-axis magnetometers. The rate gyros will measure the rotational velocities, so the angle of rotation can be obtained through integration of the sensor signal. Due to the integration process, even the smallest constant bias error will lead to divergence of the angle estimate, a property known as drift. As a consequence the filter must also accurately estimate the gyro bias in all three axes of the gyroscopes. Denote by Ω^y the biased gyro measurements [10], [12] measured in the body fixed frame $\mathcal{B}$

$$\Omega^y = \Omega + b + \mu$$

where Ω denotes the true value and μ denotes a Gaussian noise process. The term $b := b(t)$ denotes a deterministic gyro bias vector that is slowly varying with time.

The accelerometers measure the non-gravitational linear accelerations of the system.

$$a = -R^T(g_0 - \dot{v})$$

where $\dot{v}$ is the inertial frame acceleration of the body-fixed frame and g_0 denotes the gravitational field, normally $g_0 = |g_0|e_3$ where $|g_0| \approx 9.8$ is the gravitational constant. Thus, the vector

$$^{B}a_y = \frac{a}{|a|}$$

is a reasonable low-frequency estimate of the direction of gravity in the body fixed frame ($^{B}a_y \approx -R^T e_3$), assuming that the platform motion satisfies $E[\dot{v}] = 0$.

The magnetometers provide measurements of the magnetic field

$$m = R^{T\,A}m$$

where ^{A}m is the inertial magnetic field vector and $m := {}^{B}m$ is the representation of this vector in the body-fixed-frame $\mathcal{B}$.

Together the measured vectors $a = {}^{B}a$ and $m = {}^{B}m$ can be used to construct an estimate of the rotation R given that ^{A}g and ^{A}m are known *a-priori*,

$$R_y = \arg\min_{R\in SO(3)} \left(\lambda_1|^{A}a - Ra|^2 + \lambda_2|^{A}m - Rm|^2\right).$$

The weights λ_1 and λ_2 are chosen depending on the relative confidence in the gravitational and magnetometer readings. The minimisation is often solved in a suboptimal manner where the constraints are applied in sequence, thus, two degrees of freedom in the rotation matrix are resolved by the acceleration readings and the final degree of freedom is resolved using the magnetometer. As a consequence of the above process the measurement R_y is difficult to characterise. Moreover, if the magnetometer readings are unavailable or useless (due to localised magnetic fields) then the optimisation process is ill-defined.

III. Error based complimentary filter design

In this section, we reformulate the passive complementary filter in terms of measurement errors.

Consider multiple measures $v_i \in \mathbb{R}^3$ associated with several ($n \geq 2$) not collinear measured directions v_{0i}, $i = 1, 2, \ldots n$ associated with n inertial direction. Let $\hat{v}_i$ be the estimates of v_i

$$\hat{v}_i = \hat{R}^T v_{0i}$$

Remark 3.1: A low cost IMU will measure vectors a and m representing the gravitational and magnetic vector fields respectively.

$$a = R^T a_0, \qquad m = R^T m_0$$

where $a_0 = -e_3$. △

For a single direction, the natural error is

$$E_v = 1 - \cos(\angle\hat{v}, v) = 1 - \langle\hat{v}, v\rangle$$

which yields

$$E_v = 1 - \mathrm{tr}(\bar{R}v_0v_0^T)$$

where $\bar{R} = \hat{R}R^T$. The term $\bar{R}$ does not have a nice interpretation in terms of mapping coordinate. It is due to the scalar product

$$\langle\hat{v}, v\rangle = v_0^T\bar{R}v_0$$

However, if $\bar{R} = I$, then $\hat{R}$ converges to R. For multiple measures, the cost function my written as follows

$$E_{v_1} = 1-\langle\hat{v}_1, v_1\rangle,\ E_{v_2} = 1-\langle\hat{v}_2, v_2\rangle, \ldots\ E_{v_n} = 1-\langle\hat{v}_n, v_n\rangle,$$

This leads to the following global cost

$$E_{\text{mes}} = \sum_{i=1}^{n} k_i E_{v_i} = \sum_{i=1}^{n} k_i - \mathrm{tr}(\bar{R}Q_0) \tag{4}$$

where

$$Q_0 = \sum_{i=1}^{n} k_i v_{0i}v_{0i}^T, \quad k_i > 0$$

is a positive definite matrix ($Q_0 > 0$) if $n > 2$ (or positive semi-definite for $n \leq 2$). The weights k_i are very important in the proposed scheme. They are chosen depending on the relative confidence in the measured directions.

A. Estimation from the measurements of multiple and different directions

The proposed cost on $\bar{R}$ leads to a filter on $\bar{R} \in SO(3)$. For biased gyros measurements, the passive complementary dynamics are specified by [4]:

$$\begin{aligned}\dot{\hat{R}} &= \hat{R}(\Omega^y - \hat{b} + \omega)_\times, \\ &= \hat{R}(\Omega - \tilde{b} + \omega)_\times,\end{aligned} \tag{5}$$

where $\tilde{b} = b - \hat{b}$ represents the difference between the bias and its estimate vector. Note that the extension of our previous work [4] proposed in this paper consists in a reformulation of the passive complementary filter in terms of measurement errors that avoids any algebraic reconstruction of the attitude. This leads to an observer on $SO(3)$ suitable for implementation on embedded hardware and provides good attitude estimates as well as estimating the gyro biases on-line. Design of the innovation ω and the bias dynamics $\dot{\hat{b}}$ are described in the following theorem:

Theorem 3.2: Consider the rotation kinematics Eq. 1. Assume that there are two or more, ($n \geq 2$) vectorial measures v_i available. Consider the passive nonlinear complementary filter defined by Eq. 5 along with innovation ω and adaptation dynamics

$$\omega = -k_P \left(\sum_{i=1}^{n} k_i v_i \times \hat{v}_i \right), \quad k_P > 0, \tag{6a}$$

$$\dot{\hat{b}} = -\frac{k_I}{k_P}\omega, \quad k_I > 0 \tag{6b}$$

Then, for any initial condition such that

$$E_{\text{mes}}(0) < 2\sum_{i=1}^{n} k_i, \;\; k_I > \frac{|\tilde{b}(0)|^2}{2\sum_{i=1}^{n} k_i - E_{\text{mes}}(0)} \tag{7}$$

The solution $(\hat{R}, \hat{b})$ converges asymptotically to $(R(t), b)$.

Proof: Define a candidate Lyapunov function by

$$V = \sum_{i=1}^{n} k_i - \text{tr}(\bar{R}Q_0) + \frac{1}{k_I}\tilde{b}^2 = E_{\text{mes}} + \frac{1}{k_I}\tilde{b}^2$$

The derivative of V is given by

$$\begin{aligned} \dot{V} &= -\,\text{tr}\left(\dot{\bar{R}}Q_0\right) - \frac{2}{k_I}\tilde{b}^T\dot{\hat{b}} \\ &= -\text{tr}\left(\hat{R}(\tilde{b} + k_P\omega)_\times R^T Q_0\right) - \frac{2}{k_I}\tilde{b}^T\dot{\hat{b}} \end{aligned}$$

It is straightforward to verify that $\tilde{b}^T\dot{\hat{b}} = \frac{1}{2}\langle\langle \tilde{b}_\times, \dot{\hat{b}}_\times \rangle\rangle$ and that

$$\forall R \in SO(3), \forall v \in \mathbb{R}^3, \; Rv_\times R^T = (Rv)_\times$$

The derivative of the candidate Lyapunov function becomes

$$\begin{aligned} \dot{V} &= -\text{tr}\left((\hat{R}\omega + \hat{R}\tilde{b})_\times \bar{R}Q_0\right) - \frac{1}{k_I}\langle\langle \tilde{b}_\times, \dot{\hat{b}}_\times \rangle\rangle \\ &= -\text{tr}\left((\hat{R}\omega + \hat{R}\tilde{b})_\times \left(\pi_a(\bar{R}Q_0) + \pi_s(\bar{R}Q_0)\right)\right) \\ &\qquad - \frac{1}{k_I}\langle\langle \tilde{b}_\times, \dot{\hat{b}}_\times \rangle\rangle \end{aligned}$$

where $\pi_a\left(\bar{R}Q_0\right)$ and $\pi_s\left(\bar{R}Q_0\right)$ denote the anti-symmetric part and the symmetric part of the matrix $\left(\bar{R}Q_0\right)$ respectively.

Knowing that for any symmetric matrix H_s and any skew-symmetric H_a, $\text{tr}(H_s H_a) = 0$, the derivative of the Lyapunov function V becomes

$$\begin{aligned} \dot{V} &= \langle\langle (\hat{R}\omega)_\times, \pi_a\left(\bar{R}Q_0\right)\rangle\rangle + \langle\langle (\hat{R}\tilde{b})_\times, \pi_a\left(\bar{R}Q_0\right)\rangle\rangle \\ &\qquad - \frac{1}{k_I}\langle\langle \tilde{b}_\times, \dot{\hat{b}}_\times \rangle\rangle \end{aligned} \tag{8}$$

At this stage of the proof, it is convenient to rewrite Eq. 6a in more adequate expression that recapture the error matrix $\bar{R} = \hat{R}R_y^T$. Noting that for any u, $v \in \mathbb{R}^3$

$$(u \times w)_\times = (u_\times w_\times - w_\times u_\times) = wu^T - uw^T$$

one can express the equation of the innovation term, Eq. 6a, as follows

$$\omega = -k_P \text{vex}\left(\sum_{i=1}^{n} \frac{k_i}{2}(v_i\hat{v}_i{}^T - \hat{v}_i v_i^T) \right) \tag{9}$$

It is straight forward to show that Eq. 9, can be expressed as follows:

$$\omega = -k_P\hat{R}^T\text{vex}\left(\pi_a\left(\bar{R}Q_0\right)\right), \tag{10}$$

Introducing now, the expressions of ω (Eq. 10) and of $\dot{\hat{b}}$ (Eq. 6b) in the time derivative of the Lyapunov function V, Eq. 8, it yields

$$\dot{V} = -k_P||\pi_a\left(\bar{R}Q_0\right)||^2 \tag{11}$$

The Lyapunov function derivative is negative semi-definite. It is equal to zero when

$$\pi_a\left(\bar{R}Q_0\right) = 0.$$

This implies that for $\dot{V} = 0$ one has

$$\bar{R}Q_0 = Q_0\bar{R}^T. \tag{12}$$

It remains to prove that $\bar{R}$ (or $\tilde{R}$) converges to identity matrix.

Since $\bar{R}$ is a real matrix, the eigenvalues and eigenvectors of $\bar{R}$ verify

$$\bar{R}^T x_k = \lambda_k x_k \text{ and } x_k^H\bar{R} = \lambda_k^H x_k^H \tag{13}$$

where λ_k^H (for $k = 1\ldots3$) represents the complex conjugate of the eigenvalue λ_k and x_k^H represents the Hermitian transpose of the eigenvector x_k associated to λ_k. Combining Eq. 12 and Eq. 13, it follows

$$\begin{aligned} x_k^H\bar{R}Q_0x_k &= \lambda_k^H x_k^H Q_0 x_k \\ x_k^H Q_0\bar{R}^T x_k &= \lambda_k x_k^H Q_0 x_k = \lambda_k^H x_k^H Q_0 x_k \end{aligned}$$

Note that for $n \geq 3$, $Q_0 > 0$ is positive definite and $x_k^H Q_0 x_k > 0$, $\forall k = \{1, 2, 3\}$. One has $\lambda_k = \lambda_k^H$ for all k. In the case when $n = 2$, it is simple to verify that two of the three eigenvalues are real. Consequently, the three eigenvalues of $\bar{R}$ are real. Knowing that $\bar{R}$ is an orientation matrix and that eigenvalues of an orthogonal matrix must be

$$\text{eig}(\bar{R}) = (1,\; \cos(\theta) + i\sin(\theta),\; \cos(\theta) - i\sin(\theta)),$$

it yields that $\theta = 0$ or $\theta = \pm\pi$. The second possibility is excluded by the constraint on the initial conditions. Consequently $\bar{R}$ converges to the identity matrix I_3 and therefore $\tilde{R}$ converges to I_3.

To address the problem of gyroscope bias error estimation we appeal to LaSalle principle. The invariant set is contained in the set defined by the condition $\pi_a(\bar{R}Q_0) = 0$. Recalling Eq. 6b, it follows that $\dot{\hat{b}} = 0$ on the invariant set and therefore the gyros bias estimation error converges to a constant value $\tilde{b}^{\infty}$. Recalling the derivative of $\bar{R}$, one gets

$$\dot{\bar{R}} = (\hat{R}\omega + \hat{R}\tilde{b})_{\times}\bar{R}$$

Using the fact that $\bar{R} \equiv I_3$ and that $\omega \equiv 0$ on the invariant set, one has

$$\hat{R}\tilde{b}^{\infty} = 0 \Rightarrow \tilde{b}^{\infty} = 0$$

From the above discussion $\tilde{b}$ converges asymptotically towards zero. ■

Remark 3.3: If $n = 3$, the weights $k_i = 1$ and the measured directions are orthogonal ($v_i^T v_j = 0, \forall i \neq j$) then $Q_0 = I_3$. The cost function ϵ becomes

$$E_{\text{mes}} = 3 - \text{tr}(\bar{R}Q_0) = \text{tr}(I_3 - \bar{R})$$

Knowing that $\text{tr}(\bar{R}) = \text{tr}(R\hat{R}^T) = \text{tr}(\hat{R}^T R) = \text{tr}(\tilde{R})$, we can rewrite E_{mes} as follows

$$E_{\text{mes}} = \text{tr}(I_3 - \tilde{R})$$

which is the original cost considered in [4]. In that case, the correction term and the adaptation dynamics are those proposed in [4] and correspond to

$$\omega = -k_P\hat{R}^T\text{vex}\left(\pi_a\bar{R}\right) = k_P\text{vex}\left(\pi_a\tilde{R}\right) \quad (14)$$

$$\dot{\hat{b}} = -\frac{k_I}{k_P}\omega \quad (15)$$

△

Remark 3.4: The situation most commonly encountered in practice involves measurements a and m representing the gravitational and magnetic vector fields. The cost function ϵ is then

$$E_{\text{mes}} = k_1(1 - \langle\hat{a}, a\rangle) + k_2(1 - \langle\hat{m}, m\rangle)$$

The weights k_1 and k_2 are introduced in order to consider the confidence that we have for each measure. Generally it is preferable to have $k_1 >> k_2$, this is due to the fact that magnetometers readings are more susceptible to perturbations caused by the electrical engines. △

B. Estimation from the measurements of a single direction

Let a be a measured body fixed frame direction associated with a single inertial direction a_0

$$a = R^T a_0$$

Let $\hat{a}$ be an estimate

$$\hat{a} = \hat{R}^T a_0$$

The natural error is

$$E_a = 1 - \text{tr}(\bar{R}a_0a_0^T)$$

Corollary 3.5: Consider the observers defined by Eq. 5 along with the correction term ω and dynamics of $\hat{b}$ are given by Eqn's 6a-6b respectively (with equivalent form the innovation term ω, Eq. 10). Then, if the signal $\Omega_{\times}a$ is persistently exciting,

$$\Omega_{\times}a \neq 0,$$

and for any initial condition such that

$$E_a(0) < 2 \quad \text{and} \quad k_I > \frac{\tilde{b}(0)^2}{2 - E_a(0)}$$

The passive filter based measurements, insure that the estimates $\hat{a}$ will converge to the true values of a and guarantee the convergence of $\tilde{b}$ towards zero.

Proof: Let us define the following storage function for passive and non-passive complementary filters

$$V_a = E_a + \frac{1}{k_I}\tilde{b}^2$$

The derivative of V_a using the dynamics of passive filter is given by

$$\dot{V} = -k_P||(a \times \hat{R}^T a_0)_{\times}||^2 = -2k_P|a \times \hat{a}|^2$$

From the last equation where the storage function is negative semi-definite, standard Lyapunov argument concludes that the cross product $(a \times \hat{a})$ tends to zero and therefore the estimate $\hat{a}$ will converge to the true value a.

To address the problem of the convergence of $\tilde{b}$ and $\tilde{R}$ we appeal to LaSalle principle. The invariant set is contained in the set defined by the condition $\hat{a} \equiv a$ and then $\omega \equiv 0$ on the invariant set.

Recalling (6b), it follows that $\dot{\hat{b}} = 0$ on the invariant set and therefore the gyros bias estimation error converges to a constant vector. As a is a unit vector, the convergence of $\hat{a}$ towards a means that $E_a \to 0$ and consequently that $\dot{E}_a \to 0$. Deriving E_a, it yields

$$\tilde{b}_{\times}a \equiv 0,$$

in the invariant set. Therefore, as $\tilde{b}$ converges to a constant vector and a is continuously varying due to the persistent excitation of Ω, it follows that $\tilde{b}$ converges to zero and therefore $\hat{b}$ converges to b. ■

IV. Experimental results

In this section we present experimental results that demonstrate the performance of the proposed filter.

Experiments have been done on the VTOL UAV HoverEye© developed by Bertin Technologies. The VTOL belongs to the class of the 'sit on tail' ducted fan VTOL UAV, like the iSTAR9 and Kestrel developed respectively by Allied Aerospace [3] and Honeywell [2]. It was equipped with a low-cost IMU (Inertial Measurement Unit) which consists of 3-axis accelerometers and 3-axis gyroscopes. Magnetometers have not been integrated in the UAV due to perturbations caused by electrical engines. Due to the discrete data acquisition and in order to preserve the evolution of the estimated matrix $\hat{R}$ on the $SO(3)$ manifold, the proposed observer has been implemented in the experiments in discrete time using exact integration of passive complementary filters dynamics (Eq. 5) and Euler integration of the bias estimation dynamics in equation (6b). More precisely, if we denote the sample time by T, rewrite $\dot{\hat{R}} = \hat{R}\hat{\Omega}_{\times}$ and assume that $\hat{\Omega}(t) = \hat{\Omega}^k$ for

Fig. 3. The VTOL UAV HoverEye© of Bertin Technologies.

$t \in [kT, (k+1)T]$, then we have the discrete time model of the observer

$$\hat{R}^{k+1} = \hat{R}^k A^k, \quad \hat{b}^{k+1} = \hat{b}^k + T k_I \pi_a(\bar{R}^k Q_0)$$

where $A^k = \exp(\hat{\Omega}^k_\times)$ has closed form solution given by Rodrigue's formula (see equation [6])

$$A^k_k = I - \hat{\Omega}^k_\times \frac{\sin(|\hat{\Omega}^k|T)}{|\hat{\Omega}^k|} + (\hat{\Omega}^k_\times)^2 \frac{1-\cos(|\hat{\Omega}^k|T)}{|\hat{\Omega}^k|^2} \quad (16)$$

For the experiments the gains of the proposed filter has been chosen as follows; $k_P = 1\text{rd.s}^{-1}$ and $k_I = 0.3\text{rd.s}^{-1}$. Before integrating the algorithm on HoverEye©, a preliminary calibration of sensors was made. First, the accelerometers bias were calibrated in order to align the direction of the gravity provided by the IMU with the direction normal to the plane of the propellers. The experimental results, shown in figures 4-5, were realized as follows; the vehicle is remote controlled by an operator performing quasi stationary flight in an indoor environment. First, the vehicle is located on the ground, initially headed toward $\psi(0) = 0$. After take off, the vehicle is stabilized in hovering condition, around a fixed heading which remains near the initial one. Then, the operator engages a $\simeq 90^{\circ}$-left turn manoeuvre, comes back to the initial heading, then makes a $\simeq 90^{\circ}$-right turn manoeuvre and comes back again around the initial heading. After landing, the vehicle is placed by hand at its initial pose such that final and initial attitudes are the identical. Figure 4 presents the

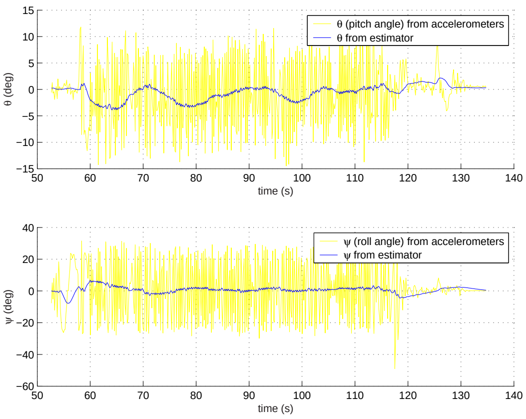

Fig. 4. Estimation results of the Pitch and roll angles.

measured pitch and roll angles (ϕ, θ) from accelerometers and the estimated values from the proposed estimator. One can observe that the efficiency of the proposed filter in recovering the pitch and roll angles. Figure 5 presents

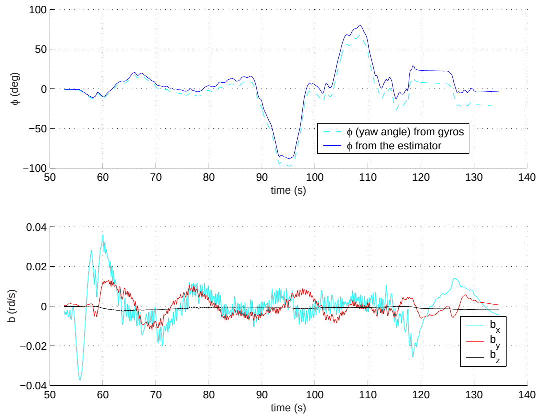

Fig. 5. Gyros bias estimation and influence of the observer on yaw angle.

the gyros bias estimation and the predicted yaw angle (ϕ) from an open loop integration of the gyroscopes. One can observe that the estimator succeeds in recovering the initial yaw orientation of the microdrone after landing manoeuvre despite only utilising gyros and accelerometers measurements.

V. Concluding Remarks

In this paper we have discussed the problem of orientation extraction and gyro bias estimation from passive complementary filter based measurements developed directly in the special orthogonal group SO(3). Some advantages of passive complementary filter based measurements have been presented and discussed. Experimental results have been provided as complement of the theoretical approach.

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