Autoepistemic Description Logics

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Abstract

We present Autoepistemic Description Logics (ADLs), in which the language of Description Logics is augmented with modal operators interpreted according to the nonmonotonic logic *MKNF.* We provide decision procedures for query answering in two very expressive ADLs. We show their representational features by addressing defaults, integrity constraints, role and concept closure. Hence, ADLs provide a formal characterization of a wide variety of nonmonotonic features commonly available in frame-based systems and needed in the development of practical applications.

1 Introduction

Description Logics (DL) have been studied in the past years to provide a formal characterization of frame-based systems. However, while the fragment of first-order logic which characterizes the most popular constructs of these languages has been clearly identified (see for example [Woods & Schmolze, 1992]), there is not yet consensus on the features of frame-based systems that cannot be formalized in a classical first-order setting. In fact, frame systems, as well as DL-based systems [Brachman *et al,* 1990, MacGregor, 1988], admit forms of nonmonotonic reasoning, such as defaults and closed world reasoning, and procedural features, e.g. rules. These issues have been addressed in the recent literature (see for example [Baader & Hollunder, 1995, Donini *et al.,* 1992, Donini, Nardi, & Rosati, 1995, Padgham & Zhang, 1993, Quantz & Royer, 1992]), but the proposals typically capture one of the above mentioned aspects.

In addition-, most implementations of DL-based systems are object-centered, which enables them to perform efficient reasoning on the properties of individuals. Such behaviour can be naturally justified if one can restrict the reasoning to the individuals that are known to the knowledge base (i.e. individuals that have an explicit name). Based on this intuition, an epistemic extension of DLs with a modal operator K, interpreted in terms of minimal knowledge has been proposed in [Donini *et a*/., 1992]. In that formalism one can express a form of closed-world reasoning, as well as integrity constraints in the form of epistemic queries (as proposed in [Reiter, 1990]); in addition, by admitting a simple form of epistemic sentences in the knowledge base, one can formalize the so-called procedural rules.

In this paper we propose a new framework of Autoepistemic Description Logics (ADLs) which follows the lines of [Donini *et al.,* 1992], extending it in two respects: we rely on the nonmonotonic modal logic *MKNF* [Lifschitz, 1994] and we consider several kinds of epistemic sentences to be used in the knowledge base. In *MKNF* one can formalize Default Logic, Autoepistemic Logic, Circumscription and Logic Programming, i.e. many of the best known formalisms for nonmonotonic reasoning. With *MKNF* we can naturally extend the previous approach to modal DLs, by introducing a second modal operator interpreted as autoepistemic assumption. Moreover, reasoning methods are available for deduction in propositional *MKNF* [Rosati, 1997].

As for the representational features of the framework, we show that ADLs are able to capture a large variety of non-first-order features. In addition to procedural rules and epistemic queries, the formalism accounts for defaults, integrity constraints inside the KB, role and concept closure, which are addressed in the paper. Moreover, the whole representational power of *MKNF* becomes available, thus making it feasible to consider new features, like autoepistemic reasoning, that are not implemented in current DL-based systems.

As for reasoning in ADLs we define methods for query answering which provide sound and complete reasoning procedures in DL-based systems admitting the above mentioned non-first-order features. It turns out that the proposed deduction methods constitute interesting decidable extensions of propositional nonmonotonic reasoning.

Based on the above considerations we argue that ADLs can indeed be considered as a unified framework for the logical reconstruction of frame-based systems.

The paper is organized as follows. We first present the extension of DLs obtained by adding the epistemic operators of *MKNF.* We then discuss the representational features of ADLs by considering several forms of nonmonotonic reasoning and integrity constraints. Finally, we address reasoning in these logics by providing reasoning methods for two rather general cases.

2 Autoepistemic Description Logics

Autoepistemic Description Logics (ADLs) are defined as an extension of DLs, in which the modal operators K and A are allowed in the formation of concept and role expressions. The meaning of modal sentences is given according to the logic *MKNF* [Lifschitz, 1994, Lin & Shoham, 1992],

Let $\mathcal{DL}$ be a generic description logic. Then, $\mathcal{DLK_{NF}}$ stands for the description logic $\mathcal{DL}$ augmented with the modal operators **K** and **A**. We say that C is a $\mathcal{DLK_{NF}}$-*concept* if C is a concept expression of $\mathcal{DLK_{NF}}$. Analogously, R is a $\mathcal{DLK_{NF}}$-*role* if R is a role expression in $\mathcal{DLK_{NF}}$.

Below we present the epistemic DL $\mathcal{ALCK_{NF}}$, namely we refer to the DL $\mathcal{ALC}$, although some of the results concern more expressive DLs. The syntax of $\mathcal{ALCK_{NF}}$ is as follows:

$$\begin{array}{lll} C & ::= & \top \mid \bot \mid C_a \mid C_1 \sqcap C_2 \mid C_1 \sqcup C_2 \mid \neg C \mid \\ & & \exists R.C \mid \forall R.C \mid \mathbf{K}C \mid \mathbf{A}C \\ R & ::= & P \mid \mathbf{K}P \mid \mathbf{A}P \end{array}$$

where C_a denotes an atomic concept, C (possibly with a subscript) denotes a concept, P denotes an atomic role, arid R (possibly with a subscript) denotes a role.

The semantics is obtained by interpreting concepts and roles on *MKNF* structures. With respect to the original semantics for *MKNF* in the first-order case, we introduce two changes: (i) following the approach of [Reiter, 1990, Donini *et al.*, 1992] the semantics of ADLs is based on the *Rigid Term Assumption*: for every interpretation the mapping from the individuals into the domain elements is fixed; (ii) the semantics of ADLs is also based on the following *Common Domain Assumption*: in each model, every interpretation is defined over the same, fixed, countable-infinite domain of individuals Δ. Hence, we define an *epistemic interpretation* as a triple $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ where $\mathcal{I}$ is a $\mathcal{DL}$-interpretation (a possible world) and $\mathcal{M}, \mathcal{N}$ are sets of interpretations defined over the domain Δ.

Atomic concepts and roles are interpreted as subsets of Δ and $\Delta \times \Delta$, respectively. $\top$ is interpreted as Δ and $\bot$ as $\emptyset$. Non-epistemic concepts and roles are given the standard semantics of DLs; conversely epistemic sentences are interpreted on epistemic interpretations, as follows.

$$\begin{array}{rcl} (\neg C)^{\mathcal{I},\mathcal{M},\mathcal{N}} & = & \Delta \setminus (C)^{\mathcal{I},\mathcal{M},\mathcal{N}} \\ (C_1 \sqcap C_2)^{\mathcal{I},\mathcal{M},\mathcal{N}} & = & (C_1)^{\mathcal{I},\mathcal{M},\mathcal{N}} \cap (C_2)^{\mathcal{I},\mathcal{M},\mathcal{N}} \\ (C_1 \sqcup C_2)^{\mathcal{I},\mathcal{M},\mathcal{N}} & = & (C_1)^{\mathcal{I},\mathcal{M},\mathcal{N}} \cup (C_2)^{\mathcal{I},\mathcal{M},\mathcal{N}} \\ (\exists R.C)^{\mathcal{I},\mathcal{M},\mathcal{N}} & = & \{d \in \Delta \mid \exists d'.(d,d') \in (R)^{\mathcal{I},\mathcal{M},\mathcal{N}} \\ & & \text{and } d' \in (C)^{\mathcal{I},\mathcal{M},\mathcal{N}}\} \\ (\forall R.C)^{\mathcal{I},\mathcal{M},\mathcal{N}} & = & \{d \in \Delta \mid \forall d'.(d,d') \in (R)^{\mathcal{I},\mathcal{M},\mathcal{N}} \\ & & \text{implies } d' \in (C)^{\mathcal{I},\mathcal{M},\mathcal{N}}\} \\ (\mathbf{K}C)^{\mathcal{I},\mathcal{M},\mathcal{N}} & = & \bigcap_{\mathcal{J}\in\mathcal{M}} (C)^{\mathcal{J},\mathcal{M},\mathcal{N}} \\ (\mathbf{A}C)^{\mathcal{I},\mathcal{M},\mathcal{N}} & = & \bigcap_{\mathcal{J}\in\mathcal{N}} (C)^{\mathcal{J},\mathcal{M},\mathcal{N}} \\ (\mathbf{K}P)^{\mathcal{I},\mathcal{M},\mathcal{N}} & = & \bigcap_{\mathcal{J}\in\mathcal{M}} (P)^{\mathcal{J},\mathcal{M},\mathcal{N}} \\ (\mathbf{A}P)^{\mathcal{I},\mathcal{M},\mathcal{N}} & = & \bigcap_{\mathcal{J}\in\mathcal{N}} (P)^{\mathcal{J},\mathcal{M},\mathcal{N}} \end{array}$$

For example, an individual $d \in \Delta$ is an instance of a concept $\mathbf{K}C$ (i.e. $d \in (\mathbf{K}C)^{\mathcal{I},\mathcal{M},\mathcal{N}}$) iff $d \in C^{\mathcal{J},\mathcal{M},\mathcal{N}}$ for all interpretations $\mathcal{J} \in \mathcal{M}$. In other words, an individual is "known" to be an instance of a concept if it belongs to the concept interpretation of every possible world in $\mathcal{M}$. An individual $d \in \Delta$ is an instance of a concept $\mathbf{A}C$ (i.e. $d \in (\mathbf{A}C)^{\mathcal{I},\mathcal{M},\mathcal{N}}$) iff $d \in C^{\mathcal{J},\mathcal{M},\mathcal{N}}$ for all interpretations $\mathcal{J} \in \mathcal{N}$. In other words, an individual is "assumed" to be an instance of a concept C if it belongs to C in all possible worlds of $\mathcal{N}$. Similarly, an individual $d \in \Delta$ is an instance of a concept $\exists \mathbf{K}R.\top$ iff there is an individual $d' \in \Delta$ such that $(d, d') \in R^{\mathcal{J},\mathcal{M},\mathcal{N}}$ for all $\mathcal{J} \in \mathcal{M}$.

The truth of inclusion statements in an epistemic interpretation $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ is defined in terms of set inclusion: $C \sqsubseteq D$ is satisfied in $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ iff $(C)^{\mathcal{I},\mathcal{M},\mathcal{N}} \subseteq (D)^{\mathcal{I},\mathcal{M},\mathcal{N}}$. Assertions are interpreted in terms of set membership: $C(a)$ is satisfied in $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ iff $a \in (C)^{\mathcal{I},\mathcal{M},\mathcal{N}}$ and $R(a, b)$ is satisfied in $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ iff $(a, b) \in (R)^{\mathcal{I},\mathcal{M},\mathcal{N}}$. A $\mathcal{DLK_{NF}}$-*knowledge base* Ψ is defined as a pair $\Psi = \langle \mathcal{T}, \mathcal{A} \rangle$, where $\mathcal{T}$ (called *TBox*) is a finite set of inclusion statements (intensional knowledge) of the form $C \sqsubseteq D$, where C, D are $\mathcal{DLK_{NF}}$-concepts, and $\mathcal{A}$ (called the *ABox*) is a finite set of membership assertions (extensional knowledge) of the form $C(a)$ or $R(a, b)$, where C is a $\mathcal{DLK_{NF}}$-concept, R is a $\mathcal{DLK_{NF}}$-role, and a, b are individuals in Δ. We call $\mathcal{O}_\Sigma$ the set of individuals occurring in Σ.

An inclusion $C \sqsubseteq D$ is *satisfied* by a structure $(\mathcal{M}, \mathcal{N})$ (denoted by $(\mathcal{M}, \mathcal{N}) \models \Psi$) iff each interpretation $\mathcal{I} \in \mathcal{M}$ is such that $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ satisfies $C \sqsubseteq D$. An assertion $C(a)$ (resp. $R(a, b)$) is satisfied by $(\mathcal{M}, \mathcal{N})$ (denoted by $(\mathcal{M}, \mathcal{N}) \models \Psi$) iff each interpretation $\mathcal{I} \in \mathcal{M}$ is such that $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ satisfies $C(a)$ (resp. $R(a, b)$). A $\mathcal{DLK_{NF}}$-knowledge base Ψ is *satisfied* by a structure $(\mathcal{M}, \mathcal{N})$ (denoted by $(\mathcal{M}, \mathcal{N}) \models \Psi$) iff each interpretation $\mathcal{I} \in \mathcal{M}$ is such that every sentence (inclusion or membership assertion) of Ψ is true in the epistemic interpretation $(\mathcal{I}, \mathcal{M}, \mathcal{N})$.

A set of interpretations $\mathcal{M}$ is a *model* for Ψ iff the structure $(\mathcal{M}, \mathcal{M})$ satisfies Ψ and, for each set of interpretations $\mathcal{M}'$, if $\mathcal{M} \subset \mathcal{M}'$ then $(\mathcal{M}', \mathcal{M})$ does not satisfy Ψ. Roughly speaking, such a preference semantics gives a minimal knowledge interpretation to the modality **K**, while the operator **A** is interpreted in terms of autoepistemic assumption (see [Lifschitz, 1994, Lin & Shoham, 1992] for further details).

The $\mathcal{DLK_{NF}}$-knowledge base Ψ is *satisfiable* if there exists a model for Ψ, *unsatisfiable* otherwise. Ψ logically implies an inclusion assertion $C \sqsubseteq D$ (where C, D are $\mathcal{DLK_{NF}}$-concepts), written $\Psi \models C \sqsubseteq D$, if $C \sqsubseteq D$ is true in every model for Ψ. Analogously, *instance checking* in Σ of a membership assertion $C(a)$ (where C is a $\mathcal{DLK_{NF}}$-concept and $a \in \mathcal{O}_\Sigma$) is defined as follows: $\Psi \models C(a)$ iff $C(a)$ is satisfied by every model of Ψ.

3 Reconstruction of frame-based systems

In this section we show that the expressive capabilities of ADLs allow for the reconstruction of several nonmonotonic features of KR systems. In particular, we focus on: defaults, integrity constraints, role and concept closure.

Defaults. Some studies on formalizing defaults in frame-based systems in DLs [Baader & Hollunder, 1995, Quantz & Royer, 1992] propose the extension of DLs through the use of Default Logic. We argue that, in order to provide a unified framework to the formalization of several forms of KB closure, it is more convenient to treat defaults as epistemic sentences. A first attempt in this direction is presented in [Donini, Nardi, & Rosati, 1995], where it is shown that one can translate defaults in a logic of minimal knowledge, but based on a different (and less intuitive) semantics for the modal operator K. In the following, we only address the "closed" semantics for defaults, but it is worth noticing that also different forms of "open" semantics (see e.g. [Kaminski, 1995]) can be formalized in ADLs. We call *$\mathcal{DL}$-default* a default rule of the form $d = \frac{\alpha:\beta_1,\ldots,\beta_n}{\gamma}$, where α, β_i, γ are $\mathcal{DL}$-concepts and $n \geq 0$.

The semantics proposed in [Baader & Hollunder, 1995] for defaults in $\mathcal{DL}$-KBs restricts the application of defaults only to the individuals explicitly mentioned in the ABox. Notice that this semantics can be viewed as the natural extension of the semantics of procedural rules given in [Donini *et al.*, 1992], where rules are applied only to the known individuals in the KB. Under this assumption, we are able to translate default rules in terms of $\mathcal{DLK}_{\mathcal{NF}}$ inclusions in the framework of ADLs. To this purpose, the translation of defaults into *MKNF* [Lifschitz, 1994] provides a modular and faithful translation of default rules into $\mathcal{DLK}_{\mathcal{NF}}$. More specifically, a $\mathcal{DL}$-default d is translated as:

$$\tau_{DK}(d) = \mathbf{K}I \sqsubseteq \neg\mathbf{K}\alpha \sqcup \mathbf{A}\neg\beta_1 \sqcup \ldots \sqcup \mathbf{A}\neg\beta_n \sqcup \mathbf{K}\gamma$$

where I is an atomic concept not appearing in the KB.

Let $(\Sigma, \mathcal{D})$ be a pair such that $\Sigma = \langle \mathcal{T}, \mathcal{A}\rangle$ is a $\mathcal{DL}$-KB and $\mathcal{D}$ is a set of $\mathcal{DL}$-defaults. Then, $\tau_{DK}(\Sigma, \mathcal{D}) = \langle \mathcal{T}', \mathcal{A}'\rangle$, where $\mathcal{T}' = \mathcal{T} \cup \{\tau_{DK}(d) : d \in \mathcal{D}\}$, $\mathcal{A}' = \mathcal{A} \cup \{I(a) : a \in \mathcal{O}_\Sigma\}$, i.e. for each individual $a \in \mathcal{O}_\Sigma$, the assertion $I(a)$ is added to $\mathcal{A}$. The condition $\mathbf{K}I$ corresponds to adding an extra prerequisite for the application of the default, which expresses the fact that the individual must be in $\mathcal{O}_\Sigma$. Such an extra condition is needed to realize the closed semantics for prerequisite-free defaults. Indeed, without imposing the prerequisite J, a prerequisite-free default would be applicable to *each* individual of Δ. The resulting modular translation based on τ_{DK} is faithful.

Theorem 3.1 *Given a $\mathcal{DL}$-KB with defaults $\langle \Sigma, \mathcal{D}\rangle$, where Σ is a $\mathcal{DL}$-KB and $\mathcal{D}$ is a set of $\mathcal{DL}$-defaults, the $\mathcal{DLK}_{\mathcal{NF}}$ KB $\tau_{DK}(\Sigma, \mathcal{D})$ is such that $\langle \Sigma, \mathcal{D}\rangle \models C(a)$ iff $\tau_{DK}(\Sigma, \mathcal{D}) \models C(a)$ for each $\mathcal{DL}$-concept C and each $a \in \mathcal{O}_\Sigma$.*

Integrity Constraints. In this section we study the problem of representing integrity constraints (IC) in ADLs. [Reiter, 1990] pointed out the *epistemic* nature of ICs : they are not statements about the world, they are statement about what the KB is said to know. Generally speaking, the satisfaction of ICs in Reiter's approach is checked in the following way: let $\mathcal{P}$ be a property that the KB must satisfy. Find a suitable epistemic query $Q_\mathcal{P}$ formalizing $\mathcal{P}$. Then, check whether the KB (interpreted under a closure assumption) entails $Q_\mathcal{P}$.

Previous work [Donini *et al.*, 1992] has shown that Reiter's approach can be realized in $\mathcal{DL}$-KBs by endowing the *query* language with epistemic abilities. Nevertheless, it would be very desirable to express ICs as any other piece of information on the domain of interest, i.e. as sentences *inside* the KB. The difficulty that arises is precisely in the formalization of the notions of closure underlying these forms of integrity constraints. Notably, ICs do not add "objective information": rather, they impose conditions on the consistency of the KB. This is true in the case of first-order KBs, or KBs with a single model as in the case under consideration. In the case of KBs with multiple models, ICs can be viewed as an *a fortiori* check which establishes which of the models are actually allowed. In particular, many forms of ICs impose properties that must hold for the *known* individuals of the KB. The modal operator K appears as an appropriate way to formalize this intuition. Moreover, conditions imposed by ICs are consistency conditions which *cannot change the content of the KB.* In other words, augmenting the KB with ICs can only have one of the following two possible effects: either the model of the KB remains unchanged (it satisfies the ICs), or the KB becomes inconsistent (since its model does not satisfy the ICs). As we shall see, the modal operator A turns out to be well-suited to this purpose.

We now show that the combination of the modalities K and A provides for the formalization in ADLs of sophisticated constraints on the KB content.

Example 1. Let us consider the IC "each known person must be known to be either male or female", which is meant to avoid any situation where an individual has been added to the KB without specifying her/his sex. One might attempt to formalize it through the $\mathcal{DLK}_{\mathcal{NF}}$ inclusion $I_1 = \mathbf{K}\mathtt{person} \sqsubseteq (\mathbf{K}\mathtt{male} \sqcup \mathbf{K}\mathtt{female})$. However, this formalization is incorrect, since such an assertion *forces* in the KB the knowledge about the sex of each known person. Instead, the meaning of the IC is to *check* whether the sex of each known person is known to the KB. This difference can be better explained as follows. Suppose Σ contains only one assertion, $\mathtt{person}(\mathtt{Bob})$. Of course, Σ does not satisfy the IC. Now, if we add I_1 to Σ, we obtain two models for $(\Sigma, \{I_1\})$: one in which $\mathtt{male}(\mathtt{Bob})$ holds, and another in which $\mathtt{female}(\mathtt{Bob})$ holds. On the contrary, we would like $(\Sigma, \{I_1\})$ to be inconsistent, since Σ does not satisfy the IC. The solution to the above problem lies in the use of the autoepistemic belief operator $\mathbf{A}$. Indeed, if we add to a KB the $\mathcal{DLK}_{\mathcal{NF}}$ inclusion $I_1' = \mathbf{K}\mathtt{person} \sqsubseteq (\mathbf{A}\mathtt{male} \sqcup \mathbf{A}\mathtt{female})$ we formalize the intended meaning of the IC. The difference between I_1 and I_1' lies in the fact that I_1' does not force any new knowledge on known persons. In our example, since $\mathtt{person}(\mathtt{Bob})$ holds, the assertion $\mathbf{A}\mathtt{male} \sqcup \mathbf{A}\mathtt{female}(\mathtt{Bob})$ must hold. Now, since there is no reason to conclude either $\mathtt{male}(\mathtt{Bob})$ or $\mathtt{female}(\mathtt{Bob})$ from Σ, the autoepistemic beliefs $\mathbf{A}\mathtt{male}(\mathtt{Bob})$ and $\mathbf{A}\mathtt{female}(\mathtt{Bob})$ are not consistent with the objective knowledge of Σ, therefore $\mathbf{A}\mathtt{male} \sqcup \mathbf{A}\mathtt{female}(\mathtt{Bob})$ is inconsistent with Σ. □

We remark that there is a precise correspondence between the A operator and Moore's *L* operator of Autoepistemic Logic [Rosati, 1997]. From this correspondence, the above example can also be understood as a variation of the "classical" inconsistent autoepistemic theory $\{L\varphi\}$. Thus, the idea in the formalization of ICs is precisely to represent an IC as a believed sentence: if such a belief is not "supported" by the objective knowledge, then an inconsistency arises.

Example 2. The IC "Each known employee must have a known social security number, which must be known to be valid" can be correctly formalized by the set of $\mathcal{ALCK}_{\mathcal{NF}}$ assertions $I_2 = \{\neg\mathbf{K}\mathtt{emp} \sqcup \exists\mathbf{K}\mathtt{SSN}.\mathbf{A}\mathtt{valid}(a) \mid a \in \mathcal{O}_\Sigma\}$. In fact, it can be shown that an $\mathcal{ALCK}_{\mathcal{NF}}$ ABox does not satisfy the IC iff $\Sigma \cup I_2$ is inconsistent. □

Role and concept closure. Finally, we show how two particular forms of closed-world reasoning, namely role closure and concept closure, can be nicely formalized in the framework of $\mathcal{DLK}_{\mathcal{NF}}$. These kinds of closure appear as very useful tools in knowledge representation.

Closure on roles is available both in CLASSIC [Brachman *et al,* 1990] and in LOOM [MacGregor, 1988]. The idea is to restrict universal role quantifications to the known individuals filling the role in the KB.

Example (Role closure). Let Ψ_1 be the following $\mathcal{ALC}$-KB: $\{\mathtt{CHILD(Ann,Marc)}, \mathtt{CHILD(Ann,Paula)}\}$, where doctor is an abbreviation for the concept $\mathtt{d} \sqcap \neg\mathtt{l} \sqcap \mathtt{rich}$, and lawyer is an abbreviation for $\mathtt{l} \sqcap \mathtt{rich}$ (expressing that doctors and lawyers are disjoint concepts, and both are rich). Now, suppose we want to formalize the property: "one of the *known* children of Ann is *known* to be a doctor, and another one is *known* to be a lawyer". One would like to conclude that "all known children of Ann are known to be rich". It turns out that the correct formalization is provided by the use of both modalities **A** and **K**. Formally: let $B = \exists\mathbf{A}\mathtt{CHILD}.\mathbf{K}\mathtt{doctor} \sqcap \exists\mathbf{A}\mathtt{CHILD}.\mathbf{K}\mathtt{lawyer(Ann)}$. Then, $\Psi_1 \cup \{B\} \models \forall\mathbf{K}\mathtt{CHILD}.\mathbf{K}\mathtt{rich(Ann)}$. □

Epistemic operators make it natural to extend the notion of closure to concept expressions.

Example (Concept closure). Let Ψ_2 be the following $\mathcal{ALCK}$-KB: $\{\mathtt{doctor(Paula)}, \mathtt{lawyer(Marc)}, \forall\mathtt{CHILD.hasBlueEyes(Ann)}\}$. Suppose we want to add to Ψ_2 the following informal property $\mathcal{P}$: "One of Ann's children is one of the *known* doctors". Now, since Paula is the only known doctor, we want to be able to conclude that Paula is one of Ann's children, and hence $\Psi_2 \cup \{\mathcal{P}\} \models \mathtt{hasBlueEyes(Paula)}$. This can be obtained by formalizing $\mathcal{P}$ through the assertion $\exists\mathbf{K}\mathtt{CHILD}.\mathbf{A}\mathtt{doctor(Ann)}$, since it can be shown that all models for the KB $\Psi_2 \cup \{\mathcal{P}\}$ either Paula or Marc is one of Ann's children. Therefore, $\Psi_2 \cup \{\mathcal{P}\} \models \mathtt{hasBlueEyes(Paula)}$. □

4 Reasoning in ADLs

In this section we study reasoning in ADLs. First, we study $\mathcal{DLK}_{\mathcal{NF}}$*-simple KBs*, i.e. $\mathcal{DLK}_{\mathcal{NF}}$-knowledge bases in which there is no occurrence of epistemic operators in the scope of quantifiers. We prove that decidability is preserved in simple theories. Furthermore, in many cases the worst-case complexity of deduction is not affected by such a nonmonotonic extension. We also provide an algorithm for instance checking in $\mathcal{DLK}_{\mathcal{NF}}$-simple KBs which is parametric wrt the DL in which the KB is expressed. As we shall see, such an algorithm allows for reasoning in DLs with (closed) defaults.

Then, we address *subjectively quantified* $\mathcal{ALCK}_{\mathcal{NF}}$*-ABoxes*, i.e. $\mathcal{ALCK}_{\mathcal{NF}}$-ABoxes in which occurrences of epistemic operators in the scope of quantifiers are allowed (under some restrictions). We prove that instance checking in such ABoxes is decidable. This result allows us to prove decidability of reasoning in DLs with features like role and concept closure and integrity constraints, which can be expressed in $\mathcal{ALCK}_{\mathcal{NF}}$-ABoxes with quantifying-in, as shown in the previous section.

Notably, in the case of $\mathcal{ALCK}_{\mathcal{NF}}$ we can easily reduce a simple KB to a subjectively quantified ABox. On the other hand, the method for reasoning on simple KBs is applicable to *any* non-modal DL for which a procedure for instance checking is available.

4.1 Reasoning without quantifying-in

We start by defining the notion of $\mathcal{DLK}_{\mathcal{NF}}$-simple KBs.

Definition 4.1 *A* $\mathcal{DLK}_{\mathcal{NF}}$*-concept C is* simple *iff there are no occurrences in C of an epistemic operator in the scope of quantifiers.*

Definition 4.2 *A* $\mathcal{DLK}_{\mathcal{NF}}$-simple KB *is a pair* (Σ, Γ) *such that* $\Sigma = \langle \mathcal{T}, \mathcal{A} \rangle$ *is a* $\mathcal{DL}$*-KB and* Γ *is a set of* $\mathcal{DLK}_{\mathcal{NF}}$-simple inclusions, *i.e. inclusions of the form* $\mathbf{K}C \sqsubseteq D$*, where D is a* $\mathcal{DLK}_{\mathcal{NF}}$*-simple concept and C is a* $\mathcal{DL}$*-concept such that* $\Sigma \not\models \top \sqsubseteq C$.

As in [Donini *et al.*, 1992], the condition $\Sigma \not\models \top \sqsubseteq C$ corresponds to a "closed" semantics for the $\mathcal{DLK}_{\mathcal{NF}}$-simple inclusion $\mathbf{K}C \sqsubseteq D$, in the sense that it corresponds to consider the application of such axioms only to the individuals in $\mathcal{O}_\Sigma$. This is precisely due to the minimal knowledge semantics of the modal operator **K**: in fact, due to the form of $\mathcal{DLK}_{\mathcal{NF}}$-simple inclusions, there is no way to force the property $\mathbf{K}C$ on individuals $\notin \mathcal{O}_\Sigma$ (since $\Sigma \not\models \top \sqsubseteq C$). Now, from the definition of model in $\mathcal{DLK}_{\mathcal{NF}}$ (that is, from the minimal knowledge semantics of the operator **K**) it is easy to see that only sets of $\mathcal{DL}$-interpretations in which $\neg\mathbf{K}C$ holds for each individual not in $\mathcal{O}_\Sigma$ are preferred. That is, the application on such individuals of the epistemic inclusion $\mathbf{K}C \sqsubseteq D$ has no effect. Therefore, the following property holds.

Lemma 4.3 *Let* $\Sigma = (\langle \mathcal{T}, \mathcal{A} \rangle, \Gamma)$ *be a* $\mathcal{DLK}_{\mathcal{NF}}$*-simple KB. Let* $\Sigma' = \langle \mathcal{T} \cup \Gamma, \mathcal{A} \rangle$*, and let* $\Sigma'' = \langle \mathcal{T}, \mathcal{A} \cup \{\neg\mathbf{K}C \sqcup D(a) | \mathbf{K}C \sqsubseteq D \in \Gamma \text{ and } a \in \mathcal{O}_\Sigma\} \rangle$*. Then, the sets of models of* Σ' *and* Σ'' *coincide.*

We now present a method for computing instance checking in $\mathcal{DLK}_{\mathcal{NF}}$-simple KBs, based on the procedure defined for propositional *MKNF* theories (see [Rosati, 1997]). However, the extension to the case of ADLs requires several preliminary notions. In particular, we have to define consistency of a partition of assertions wrt a $\mathcal{DLK}_{\mathcal{NF}}$-simple KB.

Definition 4.4 *Let C be a* $\mathcal{DLK}_{\mathcal{NF}}$*-concept expression. Let P, N be sets of* $\mathcal{DLK}_{\mathcal{NF}}$*-assertions of the form* $\mathbf{K}D(a), \mathbf{A}D(a)$*, such that* $P \cap N = \emptyset$*. Then,* $C(a)(P, N)$ *is the assertion obtained by substituting with* $\top$ *each occurrence in C of a concept D which is not within the scope of a modal operator, and such that* $D(a) \in P$*, and with* $\bot$ *each occurrence in C of a concept D which is not within the scope of a modal operator, and such that* $D(a) \in N$.

Informally, $C(a)(P,N)$ is the assertion representing the evaluation of $C(a)$ wrt the guess of the modal subformulas of $C(a)$ according to the partition (P,N). Notice that, if each modal subexpression in $C(a)$ appears either in P or in N, then $C(a)(P,N)$ is a non-modal assertion. We denote $P^K = \{C(a)(P,N) | \mathbf{K}C \in P \text{ and } a \in \mathcal{O}_\Sigma\}$, $P^A = \{C(a)(P,N) | \mathbf{A}C \in P \text{ and } a \in \mathcal{O}_\Sigma\}$, and $\mathbf{A}P^K = \{\mathbf{A}C(a) | C(a) \in P^K\}$.

In order to reason with $\mathcal{DLK_{NF}}$-simple KB we need an effective method for identifying models. Let C be a $\mathcal{DLK_{NF}}$-concept. The set of *modal atoms* of C, denoted as $MA(C)$, is the set of role subexpressions or concept subexpressions of C of the form $\mathbf{K}C'$ or $\mathbf{A}C'$. The set of modal atoms of an inclusion $C \sqsubseteq D$ is the union of the sets $MA(C)$ and $MA(D)$.

Definition 4.5 *Let γ be a $\mathcal{DLK_{NF}}$-inclusion. Let $\mathcal{O}$ be a set of individuals. Let $MA(\gamma)$ be the set of modal atoms in γ. Then, the set of* instances of $MA(\gamma)$ in $\mathcal{O}$, *denoted as $MI(\gamma,\mathcal{O})$, is the set $\{C(a) | a \in \mathcal{O} \text{ and } C \in MA(\gamma)\}$. Moreover, if Γ is a set of $\mathcal{DLK_{NF}}$-inclusions, then $MA(\Gamma,\mathcal{O}) = \bigcup_{\gamma\in\Gamma} MA(\gamma,\mathcal{O})$.*

Definition 4.6 *Let (Σ,Γ) be a $\mathcal{DLK_{NF}}$-simple KB. Let (P,N) be a partition of $MI(\Gamma,\mathcal{O}_\Sigma)$. Then, (P,N) is consistent with (Σ,Γ) iff the following conditions hold:*
i. for each $\gamma \in \Gamma$ and for each $a \in \mathcal{O}_\Sigma$, $\gamma(a)(P,N) \equiv \top$;
ii. the $\mathcal{DL}$-knowledge base $\langle\mathcal{T}, \mathcal{A} \cup P^K\rangle$ is satisfiable;
iii. the $\mathcal{DL}$-knowledge base $\langle\mathcal{T}, \mathcal{A} \cup P^A\rangle$ is satisfiable;
iv. for each $\mathbf{K}C(a) \in N$, $\Sigma \cup P^K \not\models C(a)(P,N)$;
v. for each $\mathbf{A}C(a) \in N$, $\Sigma \cup P^A \not\models C(a)(P,N)$.

Notice that in the above definition both P^K and P^A are sets of $\mathcal{DL}$-assertions, since (P,N) is a partition of $MI(\Gamma,\mathcal{O}_\Sigma)$. Therefore, $\langle\mathcal{T}, \mathcal{A} \cup P^K\rangle$ and $\langle\mathcal{T}, \mathcal{A} \cup P^A\rangle$ are $\mathcal{DL}$-KBs.

Lemma 4.7 *Let (P,N) be defined as above. (P,N) identifies a model for $\langle\mathcal{T} \cup \Gamma, \mathcal{A}\rangle$ iff*
i. (P,N) is consistent with (Σ,Γ);
ii. $\Sigma \cup P^K \models P^A$;
iii. for each partition (P',N') of $MI(\Gamma',\mathcal{O}_\Sigma)$, where $\Gamma' = \Gamma \cup \{\mathbf{A}P^K\}$ either (a) (P',N') is not consistent with (Σ,Γ) or (b) $\Sigma \cup P^K \not\models P'^K$ or (c) $\Sigma \cup P'^K \models P^K$ or (d) $\Sigma \cup P^K \not\models P'^A$.

The proof follows from Lemma 4.3 and properties of the logic *MKNF* [Rosati, 1997]. Based on the above lemma, we can provide a procedure for establishing whether a partition (P,N) identifies a model for $\langle\mathcal{T} \cup \Gamma, \mathcal{A}\rangle$.

Figure 1 reports the algorithm Simple-Not-Entails for computing instance checking in $\mathcal{DLK_{NF}}$-simple KBs. Notice that, in the case when the query *C(a)* is non-modal, the method can be realized using a procedure for computing instance checking in *VC.*

Based on the algorithm Simple-Not-Entails, one can prove decidability of instance checking in $\mathcal{DLK_{NF}}$-simple KBs.

Theorem 4.8 *Let (Σ,Γ) be a $\mathcal{DLK_{NF}}$-simple KB. Let $C(a)$ be a $\mathcal{DL}$-assertion. The problem $(\Sigma,\Gamma) \models C(a)$ is decidable iff instance checking in $\mathcal{DL}$ is decidable.*

We can provide a computational characterization of the instance checking problem in $\mathcal{DLK_{NF}}$-simple KBs. In particular, the above theorem implies that adding $\mathcal{ALCK_{NF}}$-simple inclusions to an $\mathcal{ALC}$-KB with empty TBox $\Sigma = \langle\emptyset, \mathcal{A}\rangle$ does not increase the worst-case complexity of instance checking of an $\mathcal{ALCK}$-assertion, which is PSPACE-complete in the case of a non-modal Σ [Donini *et al.*, 1992]. The same result holds in the case of an $\mathcal{ALC}$-KB $\Sigma = \langle\mathcal{T}, \mathcal{A}\rangle$ with $\mathcal{ALCK_{NF}}$-simple inclusions (i.e. instance checking is EXPTIME-complete as in the non-modal case).

Notably, it is easy to see that the translation $\tau_{DK}(\Sigma,\mathcal{D})$ of a $\mathcal{DL}$-KB with defaults $(\Sigma,\mathcal{D})$ is equivalent to the $\mathcal{DLK_{NF}}$-simple KB (Σ,Γ), in which Γ is the set $\{\tau_{DK}(d) : d \in \mathcal{D}\}$. Therefore, from Theorem 3.1 it follows that in many cases adding defaults to a $\mathcal{DL}$-KB does not increase the complexity of deduction: in particular, reasoning in $\mathcal{ALC}$-ABoxes with defaults is PSPACE-complete.

Algorithm Simple-Not-Entails($\mathcal{DL}$, (Σ,Γ), $C(a)$)
Input: description logic $\mathcal{DL}$,
$\mathcal{DLK_{NF}}$-simple KB $(\Sigma = \langle\mathcal{T},\mathcal{A}\rangle, \Gamma)$,
$\mathcal{DLK}$-assertion (query) $C(a)$;
Output: **true** if $(\Sigma,\Gamma) \not\models C(a)$, **false** otherwise.
begin
$MA(\Gamma)$ = set of modal atoms in Γ;
$MI(\Gamma,\mathcal{O}_\Sigma) = \{D(a) | a \in \mathcal{O}_\Sigma \text{ and } D \in MA(\Gamma)\}$;
if there exists partition (P,N) of $MI(\Gamma,\mathcal{O}_\Sigma)$
such that
(P,N) identifies a model of $\langle\mathcal{T} \cup \Gamma, \mathcal{A}\rangle$ **and**
$\Sigma \cup P^K \not\models C(a)$
then return true
else return false
end

Figure 1: Algorithm Simple-Not-Entails.

4.2 Reasoning with quantifying-in

We start by defining the notion of subjectively quantified $\mathcal{ALCK_{NF}}$-ABoxes.

Definition 4.9 *A subjectively quantified $\mathcal{ALCK_{NF}}$-assertion is an $\mathcal{ALCK_{NF}}$-assertion $C(a)$ where C is a concept expression of $\mathcal{ALCK_{NF}}$ in which each quantified subexpression is of the form $\exists P.D'$, $\forall P.D'$, $\exists \mathbf{M}P.\mathbf{M}'D$, $\forall \mathbf{M}P.\mathbf{M}'D$, where $\mathbf{M},\mathbf{M}' \in \{\mathbf{K},\mathbf{A}\}$, D' is an $\mathcal{ALC}$-concept, P is an atomic role.*

An $\mathcal{ALCK_{NF}}$-ABox composed of subjectively quantified assertions is called *subjectively quantified $\mathcal{ALCK_{NF}}$-ABox*. The method for reasoning on subjectively quantified ABoxes is based on a tableaux calculus following the lines of [Donini *et al,* 1996], where special closure conditions are defined to enforce the preference criterion on the models represented by the branches of the tableau. For ADLs, the lifting from propositional logic to DLs raises several difficulties that are addressed in the sequel. In particular, we sketch a calculus for characterizing the models of a subjectively quantified $\mathcal{ALCK_{NF}}$-ABox in terms of $\mathcal{ALC}$-KBs, to which the procedure for query answering defined in [Donini *et al,* 1992] can be applied.

The tableaux rules include the standard rules for an S5 tableau [Fitting, 1983] for handling propositional connectives and epistemic expressions of the form $\mathbf{K}C$, $\mathbf{A}C$ and $\neg\mathbf{K}C$, $\neg\mathbf{A}C$. The tableaux calculus for generating the models for subjectively quantified $\mathcal{ALC}$-KBs behaves as usual, except for the fact that each rule is applied only to epistemic prefixed formulas.

First, we define the notion of modal atoms of a subjectively quantified $\mathcal{ALCK}_{\mathcal{NF}}$-assertion $C(x)$ as the set of assertions $C'(x)$, where C' is a subexpression of C of the form $\mathbf{K}C''$ or $\mathbf{A}C''$ or $\exists \mathbf{M}P.\mathbf{M}'C''$ or $\forall \mathbf{M}P.\mathbf{M}'C''$, where $\mathbf{M}, \mathbf{M}' \in \{\mathbf{K}, \mathbf{A}\}$. The set of modal atoms $MA(\Sigma)$ of a subjectively quantified $\mathcal{ALCK}_{\mathcal{NF}}$-ABox Σ is the union of the sets of modal atoms of all the assertions it contains.

A branch $\mathcal{B}$ is a set of prefixed formulas of the form $\langle w : C(x)\rangle$. The tableau for Σ starts with the set $\{\langle 1 : \mathbf{K}A\rangle \mid \text{assertion } A \in \Sigma\}$. $\mathcal{O}_{\mathcal{B}}$ is the set of individuals in $\mathcal{B}$.

We concentrate on the rules that handle subjectively quantified expressions, and on a rule that saturates the branch wrt modal atoms of E. We phrase them using K; the cases arising from the use of both modalities K and A are analogous. The rules are as follows:

$\forall$-rule: if $\langle w : \forall \mathbf{K}P.\mathbf{K}C(x)\rangle \in \mathcal{B}$, then for each $\langle 1 : \mathbf{K}P(x,y)\rangle \in \mathcal{B}$ add $\langle 1 : \mathbf{K}C(y)\rangle$ to $\mathcal{B}$.

$\exists$-rule: if $\langle w : \exists \mathbf{K}P.\mathbf{K}C(x)\rangle \in \mathcal{B}$ and there is no y such that both $\langle 1 : \mathbf{K}P(x,y)\rangle$ and $\langle 1 : \mathbf{K}C(y)\rangle \in \mathcal{B}$, then add $\langle 1 : \mathbf{K}P(x,z)\rangle$ and $\langle 1 : \mathbf{K}C(z)\rangle$ to $\mathcal{B}$, where $z \in \mathcal{O}_{\mathcal{B}} \cup \{\iota\}$ and $\iota \notin \mathcal{O}_{\mathcal{B}}$.

***mcut*-rule:** if $\mathbf{K}C(x) \in MA(\Sigma)$ then add $\langle 1 : \mathbf{K}C(x)\rangle$ or $\langle 1 : \neg\mathbf{K}C(x)\rangle$ to $\mathcal{B}$, if neither is present in $\mathcal{B}$.

A branch is *completed* if no rule is applicable to it; a branch is *open* if there is no pair of prefixed formulas in $\mathcal{B}$ of the form $\langle w : C(x)\rangle$ and $\langle w : \neg C(x)\rangle$.

An open completed branch *B* does not always represent a model. To select models according to the preference criterion, one needs to characterize the objective knowledge associated with *B*. In particular, one has to distinguish between the objective knowledge implied by K-prefixed and A-prefixed modal atoms [Rosati, 1997]. To this aim, we remark that the *mcut*-rule forces a partition on the modal atoms of Σ in $\mathcal{B}$. We call $(P_{\mathcal{B}}, N_{\mathcal{B}})$ such a partition. Based on Def. 4.4, we define the following $\mathcal{ALC}$-ABoxes:

$$OBJ_K(\mathcal{B}) = \{C(x)(P_{\mathcal{B}}, N_{\mathcal{B}}) \mid \mathbf{K}C(x) \in P_{\mathcal{B}}\}$$
$$OBJ_A(\mathcal{B}) = \{C(x)(P_{\mathcal{B}}, N_{\mathcal{B}}) \mid \mathbf{A}C(x) \in P_{\mathcal{B}}\}$$

We can now define the notion of preferred branch.

Definition 4.10 *A branch $\mathcal{B}$ of the tableau for Σ is preferred iff $\mathcal{B}$ is open and completed, $OBJ_K(\mathcal{B}) \models OBJ_A(\mathcal{B})$ and, for each open and completed branch $\mathcal{B}'$ of the tableau for $\Sigma \cup \{\mathbf{A}C(x) \mid C(x) \in OBJ_K(\mathcal{B})\}$, either $OBJ_K(\mathcal{B}) \not\models OBJ_K(\mathcal{B}')$ or $OBJ_K(\mathcal{B}') \models OBJ_K(\mathcal{B})$ or $OBJ_K(\mathcal{B}) \not\models OBJ_A(\mathcal{B}')$.*

The above notion of preferred branch allows us to identify *all* the models of Σ, up to renaming of individuals in $\mathcal{O}_{\mathcal{B}} - \mathcal{O}_{\Sigma}$.

Theorem 4.11 *Let Σ be a subjectively quantified $\mathcal{ALCK}_{\mathcal{NF}}$-ABox. Then, a branch $\mathcal{B}$ of the tableau for Σ is preferred iff there exists a model $\mathcal{M}$ for Σ and a mapping $\mu : \Delta \to \Delta$ such that $\mathcal{M} = \{\mathcal{I} \mid \mathcal{I} \models \mu(OBJ_K(\mathcal{B}))\}$. Moreover, let $C(a)$ be an $\mathcal{ALCK}$-assertion. Then, $\Sigma \models C(a)$ iff there exists a preferred branch $\mathcal{B}$ of the tableau for Σ such that $OBJ_K(\mathcal{B}) \not\models C(a)$.*

It can be shown that the tableaux method above outlined always terminates. Moreover, since $OBJ_K(\mathcal{B}) \not\models C(a)$ can be checked by the algorithm presented in [Donini *et al.*, 1992], we have proved decidability of the instance checking problem for subjectively quantified $\mathcal{ALCK}_{\mathcal{NF}}$-ABoxes.

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