The 2nd Joint International Conference on Multibody System Dynamics
May 29-June 1, 2012, Stuttgart, Germany

Automatic Extraction of DH Parameters of Serial Manipulators using Line Geometry

Rajeevlochana C.G.*, Subir K. Saha*, Shivesh Kumar#

*Department of Mechanical Engineering
Indian Institute of Technology Delhi
New Delhi, 110016, India
rajeevlochan.iitd@gmail.com,
saha@mech.iitd.ac.in

#Department of Mechanical Engineering
National Institute of Technology
Karnataka
Suratkal, 575025, India
shivesh.mecheng@gmail.com

ABSTRACT

Serial manipulators/robots are used extensively in industries to perform various tasks such as pick-and-place operation, painting, arc-welding, assembly of components etc. To perform tasks accurately, exact kinematic parameters of the manipulator are required. Note that these parameters are generally represented using the well-known Denavit-Hartenberg (DH) parameters. Typically, a set of nominal DH parameters are provided by the robot manufacturers, which may not be exact due to assembly errors etc. Hence, there is a need to know them exactly. In this paper, a novel analytic method is proposed to extract DH parameters of a robot manipulator. For this, each joint axis of a manipulator consisting of a direction and a point on it are provided as input to the proposed algorithm. The exact DH parameters are then extracted recursively from base link to the end-effector using the concepts of Plücker coordinates and Dual Vector Algebra. The algorithm has been implemented as an addin/plugin inside Autdoesk Inventor CAD software, which determines the DH parameters of a serial manipulator from its CAD model.

1. INTRODUCTION

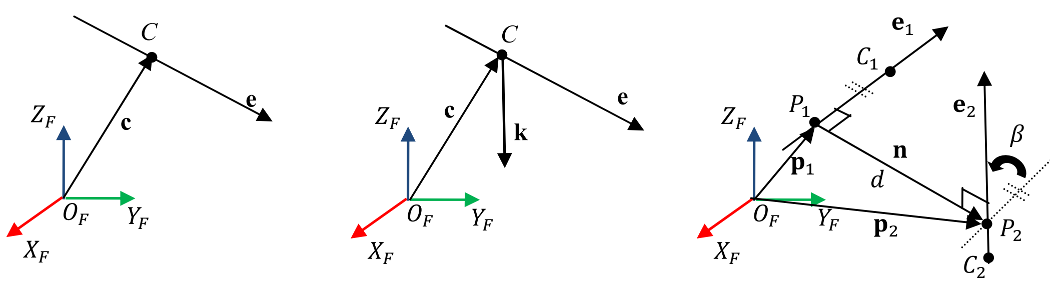
Serial manipulators/robots usually have a poor accuracy as compared to their repeatability. The reason could be a non-exact kinematic model, due to which a manipulator reaches configuration A instead of the desired configuration B. Since the repeatability is higher, the manipulator reaches configurations closer to A whenever it is asked to reach configuration B. The inaccuracy in kinematic model could be due to manufacturing errors, assembly issues, wear and tear, permanent bending of the links due to fatigue, etc. Note here that the well-known Denavit-Hartenberg (DH) parameters [4] are typically used to define the robot kinematics. Over the years, researchers have proposed various methods to determine these DH parameters. For example, [1] and [2] identified them directly, whereas [11] conceptualized and identified what the authors call S-Model parameters from which DH parameters are determined.

Since the DH parameters are the most common and standard way of representing a robot architecture, some of the important methods to find them are described here. In [2], it requires the location and orientation of the joint axes as inputs, which are determined by rotating one joint at a time and locking the others. Such rotation will result in a circular motion by the end-effector. Hence, by tracing the end-effector positions one can determine the circle and its axis which is the axis of the revolute joint causing the circular motion. These joint axes are then used to determine the exact DH parameters of the serial manipulator at hand using Vector Algebra. In [1], the DH parameters were determined by applying the methodology introduced in [11], which uses the plane of rotation and the centre of rotation. Further it is extended to use the radius of rotation and a plane translated along the axis of rotation. The idea developed by [6] was also used in [1] where an extra parameter (β) was introduced to deal with parallel or near parallel joint axes.

In this paper, we propose a novel analytical methodology to extract the DH parameters of a serial manipulator with revolute joints only using line geometry, Plücker coordinates and Dual Vector Algebra. It requires a point on the joint axes and its direction. It then determines whether two consecutive joint axes are parallel, intersecting or skewed [7]. It then analytically determines the DH parameters using the Plücker coordinates [3] and the Vector Algebra. Implementation of the proposed methodology is simpler and elegant compared to the one reported in [2]. It determines one set of DH parameters at a time in a recursive manner from the base link to the end-effector, which is more efficient and elegant than determination using Paul's backwards multiplication technique [1]. The paper is organized as follows: Section 2 introduces the concept of line geometry, followed by Plücker coordinates and Dual Vector Algebra. Section 3 then gives the proposed analytical methodology. An implementation of the proposed methodology is presented in Section 4, followed by conclusions in Section 5.

2. LINE GEOMETRY

A straight line in the 3D space can be represented using a unit vector (**e**) to represent its direction and a position vector (**c**) to represent a point on the line, both defined in a specific coordinate frame of reference (say, Frame F), as shown in Figure 1(a). Alternatively, Plücker coordinates [3] can be used to represent the same line using its unit directional vector (**e**) and the moment vector (**k**), as shown in Figure 1(b). Equation (1) defines the Plücker coordinates, where **s** is the 6-dimensional column vector.



(a) Line in 3D space (b) Plücker coordinates representing a line (c) Two lines in 3D space

Figure 1: Line geometry in 3D space

$$\mathbf{s} = \begin{bmatrix} \mathbf{e} \\ \mathbf{c} \times \mathbf{e} \end{bmatrix} = \begin{bmatrix} \mathbf{e} \\ \mathbf{k} \end{bmatrix} \tag{1}$$

In Equation (1), **e** is the 3-dimensional unit vector parallel to the line, and **c** is the 3-dimensional position vector of the point C on the line. The 3-dimensional vector $\mathbf{k} = \mathbf{c} \times \mathbf{e}$, then corresponds to the moment vector.

2.1. Pair of Straight Lines

Two lines in the 3D space are intersecting, parallel, collinear or skewed to each other. An elegant method, as reported in [7], to determine their relationship using the concepts of Dual Vector Algebra [5] is used here. Note that the Plücker coordinates used to represent a line can be expressed as a dual vector ($\hat{\mathbf{s}}$), which has a real part (**e**) and dual part (**k**), i.e.,

$$\hat{\mathbf{s}} = \mathbf{e} + \varepsilon\mathbf{k} \tag{2}$$

where $\varepsilon^2 = 0$.

For any two lines represented as dual vectors, say $\hat{\mathbf{s}}_1$ and $\hat{\mathbf{s}}_2$, line dot product, as illustrated in Figure 1(c), results in a dual number whose dual part indicates if the lines are skewed or not. For non-skewed lines, the real part determines if the lines are parallel or intersecting. Consider this result of the line dot product [7] as given below:

$$\begin{aligned} \hat{\mathbf{s}}_1.\hat{\mathbf{s}}_2 &= (\mathbf{e}_1 + \varepsilon\mathbf{k}_1).(\mathbf{e}_2 + \varepsilon\mathbf{k}_2) \\ &= \mathbf{e}_1.\mathbf{e}_2 + \varepsilon(\mathbf{e}_1.\mathbf{k}_2 + \mathbf{k}_1.\mathbf{e}_2) \end{aligned} \tag{3}$$

$$= \cos(\beta) + \varepsilon[-\mathrm{d}\sin(\beta)]$$

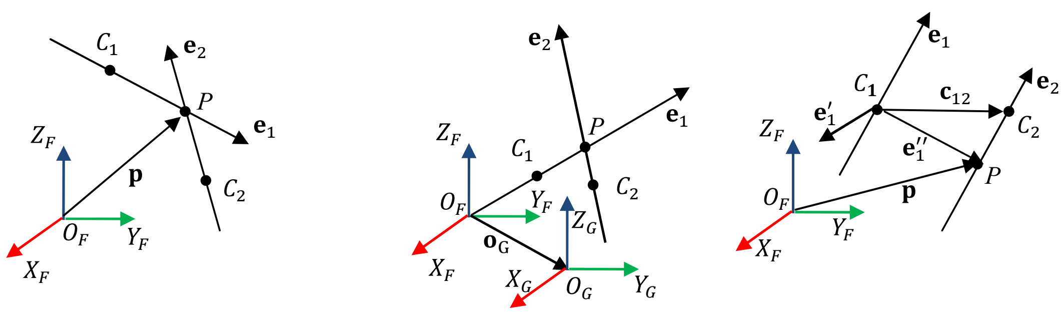
where β is the angle between the two lines, and d is the distance measured along common normal.

In our proposed methodology, the above concept of line dot product is used to determine the relationship between two successive joint axes of the serial manipulator at hand.

2.2. Intersecting Lines

Plücker coordinates of two lines $\mathbf{s_1}$ and $\mathbf{s_2}$ are used here to find their intersection point (P), as shown in Figure 2(a). The intersection point denoted with a vector **p** as reported in [3] is as below:

$$\mathbf{p} = \frac{\mathbf{k}_1 \times \mathbf{k}_2}{\mathbf{e}_2.\mathbf{k}_1} \text{ or } \frac{\mathbf{k}_2 \times \mathbf{k}_1}{\mathbf{e}_1.\mathbf{k}_2} \text{ for } \mathbf{e}_1.\mathbf{k}_2 - \mathbf{e}_2.\mathbf{k}_1 = 0 \tag{4}$$



(a) Intersecting lines (b) One of the intersecting lines passing through origin (c) Parallel lines

Figure 2: Intersecting and parallel lines

The expression for vector **p** is more elegant and compact compared to the one using Line Geometry or Vector Algebra. If either of the line passes through the origin O_F the moment **k** of that line becomes null (**0**), and hence the intersection point P cannot be determined using Equation 4. Hence, a novel method is proposed to overcome this inability by following the steps below. This is illustrated in Figure 2(b).

1. Find a point O_G at unit distance from O_F along the common normal of $\mathbf{e}_1$ and $\mathbf{e}_2$, i.e.,

$$[\mathbf{o}_\mathrm{G}]_\mathrm{F} = [\mathbf{e}_1]_\mathrm{F} \times [\mathbf{e}_2]_\mathrm{F} \tag{5}$$

2. Define a new coordinate frame (Frame G) with origin at O_G, and the coordinate axes parallel to those in Frame F. The transformation from Frame F to Frame G is a pure translation with no rotation.
3. Vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ denoting the directions are invariant after transformation from Frame F to Frame G, i.e.,

$$[\mathbf{e}_1]_\mathrm{G} = [\mathbf{e}_1]_\mathrm{F} \; ; \; [\mathbf{e}_2]_\mathrm{G} = [\mathbf{e}_2]_\mathrm{F} \tag{6}$$

4. Vectors $\mathbf{c}_1$ and $\mathbf{c}_2$ are transformed to Frame G i.e.,

$$[\mathbf{c}_1]_\mathrm{G} = [\mathbf{c}_1]_\mathrm{F} - [\mathbf{o}_\mathrm{G}]_\mathrm{F} \; ; \; [\mathbf{c}_2]_\mathrm{G} = [\mathbf{c}_2]_\mathrm{F} - [\mathbf{o}_\mathrm{G}]_\mathrm{F} \tag{7}$$

5. Vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ being the moments are transformed to Frame G [5], i.e.,

$$[\mathbf{k}_1]_G = [\mathbf{k}_1]_F + [\mathbf{o}_G]_F \times [\mathbf{e}_1]_F \;;\; [\mathbf{k}_2]_G = [\mathbf{k}_2]_F + [\mathbf{o}_G]_F \times [\mathbf{e}_2]_F \tag{8}$$

6. Intersection point $[\mathbf{p}]_G$ is then found in Frame G as

$$[\mathbf{p}]_G = \frac{[\mathbf{k}_1]_G \times [\mathbf{k}_2]_G}{[\mathbf{e}_2]_G . [\mathbf{k}_1]_G} \text{ or } \frac{[\mathbf{k}_2]_G \times [\mathbf{k}_1]_G}{[\mathbf{e}_1]_G . [\mathbf{k}_2]_G} \tag{9}$$

7. Intersection point $[\mathbf{p}]_F$ is next found in Frame F as

$$[\mathbf{p}]_F = [\mathbf{o}_G]_F + [\mathbf{p}]_G \tag{10}$$

The above approach is used to find the intersection point of two successive joint axes of a serial manipulator, which plays an important role in the extraction of the DH parameters.

2.3. Parallel Lines

Between two parallel lines ($\mathbf{s}_1$and $\mathbf{s}_2$), as shown in Figure 2(c), no unique solution exists for a line that is a common normal. One of the solutions can be determined by putting a condition that it passes through an arbitrary point C_1. To achieve this, a vector ($\mathbf{e}_1'$) normal to $\mathbf{c}_{12}$ and $\mathbf{e}_1$ is determined as given below:

$$\mathbf{e}_1' = \mathbf{c}_{12} \times \mathbf{e}_1 \tag{11}$$

where vectors $\mathbf{c}_{12}$ and $\mathbf{e}_1$are indicated in Figure 2(c). Moreover, a unit vector ($\mathbf{e}_1''$) normal to $\mathbf{e}_1$ and $\mathbf{e}_1'$ is determined as follows:

$$\mathbf{e}_1'' = \frac{\mathbf{e}_1 \times \mathbf{e}_1'}{|\mathbf{e}_1||\mathbf{e}_1'|} \tag{12}$$

Next, a new set of Plücker coordinates $\mathbf{s}_1''$ defining a new line perpendicular to $\mathbf{s}_1$ is introduced below:

$$\mathbf{s}_1'' = \begin{bmatrix} \mathbf{e}_1'' \\ \mathbf{c}_1 \times \mathbf{e}_1'' \end{bmatrix} \tag{13}$$

Using the lines $\mathbf{s}_1''$ and $\mathbf{s}_2$ one can now find the point of intersection *P* or vector $\mathbf{p}$ using the methodology presented in Section 2.2.

2.4. Skewed Lines

Between two skewed lines ($\mathbf{s}_1$and $\mathbf{s}_2$), a unique solution exists that is normal to both the lines, as shown in Figure 1(c). These points P_1 and P_2 on lines $\mathbf{s}_1$and $\mathbf{s}_2$ respectively are determined using the Plücker coordinates [3] i.e.,

$$\mathbf{p}_1 = \left[\frac{\mathbf{k}_2 . \mathbf{n} - \cos(\beta)\, \mathbf{k}_1 . \mathbf{n}}{\sin(\beta)}\right] \mathbf{e}_1 + \mathbf{e}_1 \times \mathbf{k}_1 \tag{14}$$

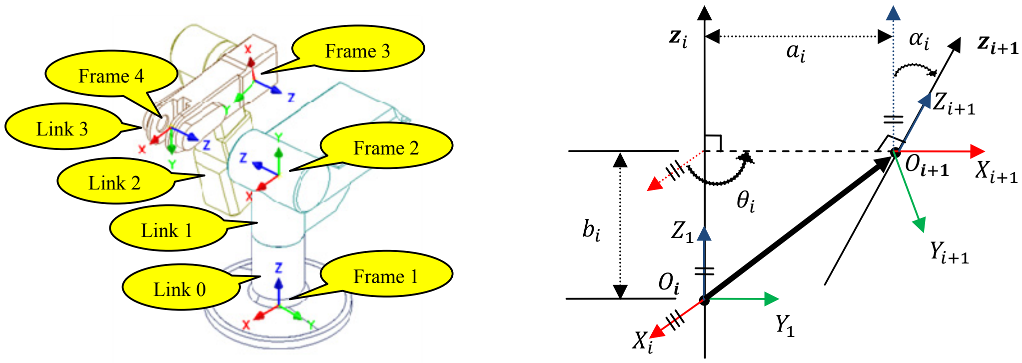
$$\mathbf{p}_2 = \left[\frac{-\mathbf{k}_1 . \mathbf{n} + \cos(\beta)\, \mathbf{k}_2 . \mathbf{n}}{\sin(\beta)}\right] \mathbf{e}_2 + \mathbf{e}_2 \times \mathbf{k}_2 \tag{15}$$

where $\mathbf{n} = \mathbf{e}_1 \times \mathbf{e}_2$ is the common normal and β is the angle between the lines as indicated in Figure 1(c). Both the vectors $\mathbf{p}_1$and $\mathbf{p}_2$ are used to extract the DH parameters for two successive skewed joint axes.

3. DENAVIT-HARTENBERG (DH) PARAMETERS

The links of a serial manipulator usually coupled with single-degree-of-freedom revolute or prismatic joints. A coordinate frame is attached to each link, as shown in Figure 3(a). The transformation between the Frame *i*+1 attached to Link *i* and the Frame *i* attached to Link *i*-1 can be represented using four Denavit-Hartenberg(DH) parameters [4], where *i* is the index of the link. Due to the space limitation, only the definitions of the parameters illustrated in Figure 3(b) are given below, whereas the details about the rules of attaching frames are available in [10]:

a. Joint Offset (b_i): Distance between X_i and X_{i+1} along Z_i
b. Joint Angle (θ_i): Angle between X_i and X_{i+1} about Z_i
c. Link Length (a_i): Distance between Z_i and Z_{i+1} along X_{i+1}
d. Twist Angle (α_i): Angle between Z_i and Z_{i+1} about X_{i+1}



(a) Coordinate frames attached to each link (b) Two successive coordinate frames

Figure 3: Representation of DH parameters

3.1. Extraction of DH Parameters

The proposed analytical method to extract the DH parameters of an *n*-degree-of-freedom serial manipulator is presented here. For the sake of simplicity in calculations, the frame attached to the base link (Link 0) is identical to the fixed coordinate frame (Frame F) such that points O_1 is coincident with O_F, and the first joint axis (Z_1) is along the Z_F. For all the subsequent joints, the joint axes, consisting of unit vector ($\mathbf{z}_i$) and the vector $\mathbf{c}_i$ representing a point on the axis need to be provided as input with respect to Frame F. The DH parameters are then extracted recursively from base link (Link 0) to the end-effector (Link *n*).

For the first step, i.e., when i = 1, the unit vectors $\mathbf{z}_1$ and $\mathbf{z}_2$ corresponding to first two joint axes are known as they are provided as input. The origin ($\mathbf{o}_1$) of Frame 1 is also known from the assumption and a vector to a point ($\mathbf{c}_2$) on second axis is known as input. For each subsequent step *i*, the two joint axes are represented as Plücker coordinates ($\mathbf{s}_i$ and $\mathbf{s}_{i+1}$) as given below:

$$\mathbf{s}_i = [\mathbf{z}_i \quad \mathbf{o}_i \times \mathbf{z}_i]^T \ ; \ \mathbf{s}_{i+1} = [\mathbf{z}_{i+1} \quad \mathbf{c}_{i+1} \times \mathbf{z}_{i+1}]^T \tag{16}$$

where Frame *i* is known from input if *i* = 1, else it is known from previous step.
The two axes at hand could be intersecting, parallel or skewed. Dual Vector Algebra, as explained in Section 2, is used to determine their relationship. Possible cases are explained below:

a) Intersecting Axes: The origin ($\mathbf{o}_{i+1}$) of Frame *i*+1 is the point of intersection of lines $\mathbf{s}_i$ and $\mathbf{s}_{i+1}$ as explained in Section 2.2. The vector $\mathbf{x}_{i+1}$ is the common normal to $\mathbf{z}_i$ and $\mathbf{z}_{i+1}$ and one of the two possible expression is given below:

$$\mathbf{x}_{i+1} = \mathbf{z}_i \times \mathbf{z}_{i+1} \tag{17}$$

where $\mathbf{x}_{i+1}$ is the unit vector along X_{i+1}

b) Parallel Axes: A line ($\mathbf{s}_i''$) is determined which is a common normal to both the lines $\mathbf{s}_i$ and $\mathbf{s}_{i+1}$, as explained in Section 2.3. The origin ($\mathbf{o}_{i+1}$) of Frame 2 is then determined as the point of intersection of lines $\mathbf{s}_i''$ and $\mathbf{s}_{i+1}$. The vector $\mathbf{x}_{i+1}$, a unit vector from O_i to O_{i+1}, is then given as:

$$\mathbf{x}_{i+1} = \frac{\mathbf{o}_{i+1} - \mathbf{o}_i}{|\mathbf{o}_{i+1} - \mathbf{o}_i|} \tag{18}$$

c) Skewed Axes: The common normal of two skewed lines ($\mathbf{s}_i$ and $\mathbf{s}_{i+1}$) is determined using the methodology of Section 2.4. The point of its intersection with $\mathbf{s}_i$ is $\mathbf{o'}_i$ and that with $\mathbf{s}_{i+1}$ is the origin ($\mathbf{o}_{i+1}$) of Frame i+1. The vector $\mathbf{x}_{i+1}$ is then given as:

$$\mathbf{x}_{i+1} = \frac{\mathbf{o}_{i+1} - \mathbf{o'}_i}{|\mathbf{o}_{i+1} - \mathbf{o'}_i|} \tag{19}$$

Once the origin ($\mathbf{o}_{i+1}$) and vector $\mathbf{x}_{i+1}$ are found, the Frame i+1 attached to Link i is completely known by determining $\mathbf{y}_{i+1}$ as:

$$\mathbf{y}_{i+1} = \mathbf{z}_{i+1} \times \mathbf{x}_{i+1} \tag{20}$$

where $\mathbf{y}_{i+1}$ is a unit vector along Y_{i+1}.

Now both the Frames i and i+1 are known. The DH parameters associated with the transformation between them as illustrated in Figure 3(b), are given below:

First, the joint offset b_i is found as

$$\mathrm{b}_i = (\mathbf{o}_{i+1} - \mathbf{o}_i).\mathbf{z}_i \tag{21}$$

Next, the joint angle θ_i is obtained, i.e.,

$$\theta_i = atan2((\mathbf{x}_i \times \mathbf{x}_{i+1}).\mathbf{z}_i, \mathbf{x}_i.\ \mathbf{x}_{i+1}) \tag{22}$$

The link length a_i and twist angle α_i are finally given by

$$\mathrm{a}_i = (\mathbf{o}_{i+1} - \mathbf{o}_i).\mathbf{x}_i \tag{23}$$

$$\alpha_i = atan2((\mathbf{z}_i \times \mathbf{z}_{i+1}).\mathbf{x}_{i+1}, \mathbf{z}_i.\ \mathbf{z}_{i+1}) \tag{24}$$

The above procedure is repeated until the last joint ($i = n$). Note that for the last joint, the point O_{n+1} needs to be provided as input. For that, it is assumed that $\mathbf{z}_{n+1}$ is parallel to $\mathbf{z}_n$ [1, 2]. If the point O_{n+1} lies on last joint axis, a unique solution for $\mathbf{x}_{n+1}$ cannot be found. In that case, Frame n+1 should be provided as input and its treatment is excluded here to keep the approach simple.

The complete working of the proposed methodology as a generic algorithm is shown in Figure 4(a), which calls different functions, as shown in Figures 4(b) to 4(f).

4. IMPLEMENTATION IN AUTODESK INVENTOR

The proposed analytical method has been implemented as an addin/plugin inside Autodesk Inventor software. The CAD assembly of a serial manipulator, as shown in Figure 5 was imported inside the Inventor environment. The base of the robot is placed such that the frame attached to it is identical to the world coordinate system. Assembly constraints are then defined between the links/parts of the manipulator in sequence from the base to the end-effector.

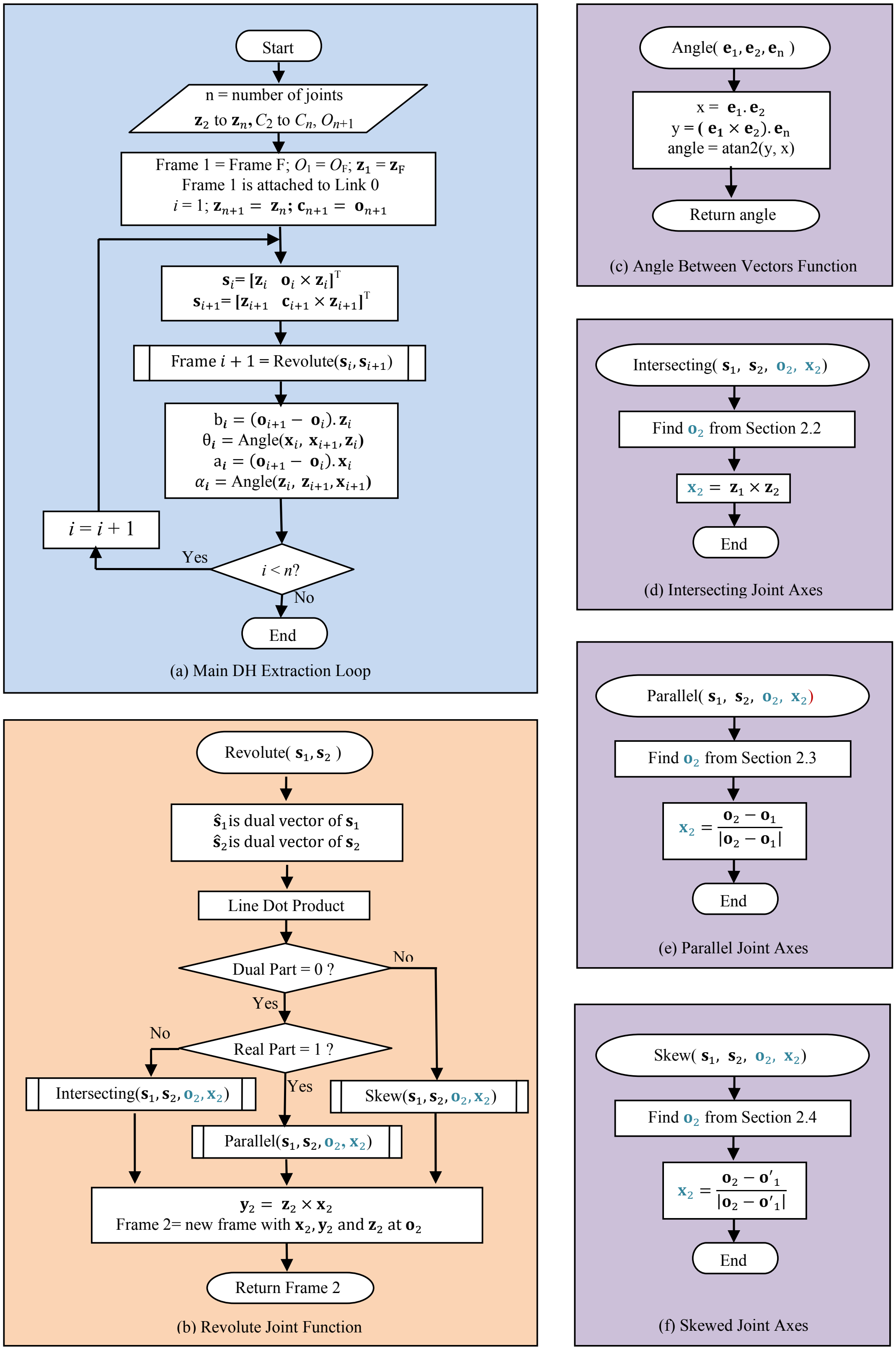


Figure 4: Algorithm for the extraction of DH parameters

The addin developed using Visual C# reads the CAD assembly data using the Application Programming Interface (API) of Autodesk Inventor and lists the CAD parts and the assembly constraints between them. In Autodesk Inventor, “Insert” assembly constraint corresponds to a revolute joint and is applied between circles of equal radii located on two mating links/parts. Using Inventor API, the properties of each circle and hence the associated revolute joint can be extracted into the addin. The normal of each circle as a unit vector and its center, both in the world coordinate system, are determined. These are then provided as input to the novel algorithm and the DH parameters are determined as explained in Section 3.

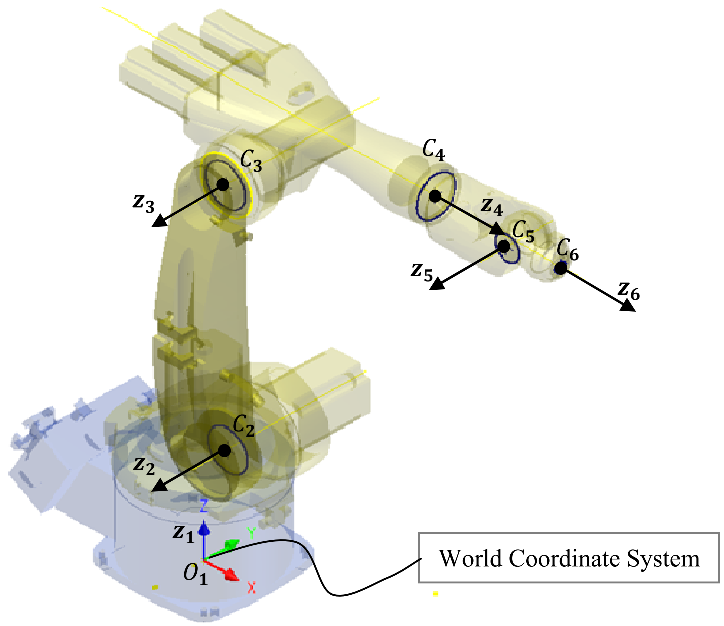


Figure 5: Determination of circles corresponding to revolute (insert) joints in Autodesk Inventor

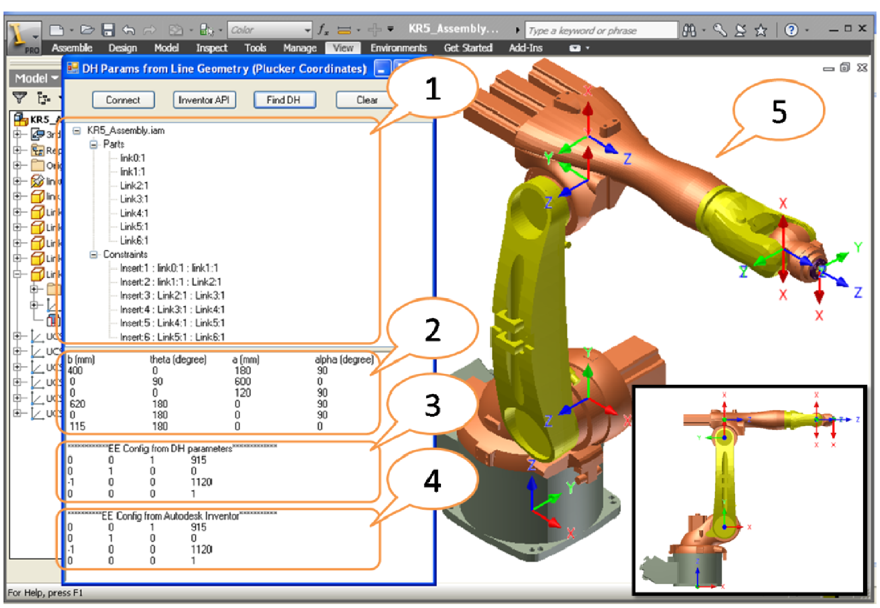


Figure 6: DH parameters extraction from the CAD model of KUKA KR5

The links/parts and assembly constraints of the CAD model of KUKA KR5 manipulator is listed in Figure 6 (callout 1). Its exact DH parameters as extracted by the addin, is shown in Figure 6 (callout 2).

The transformation of frame attached to the end-effector with respect to the base achieved by multiplying the transformation matrices corresponding to each DH parameter set (callout 3 of Figure 6) matches exactly with the transformation matrix determined directly between these two frames using Inventor API (callout 4 of Figure 6). The coordinate frames determined were inserted into the CAD model as shown in Figure 6 (callout 5) which help in better visualization and understanding of the DH parameters. The DH parameters also match with those mentioned in the company's specifications of KUKA KR5 [8]. Similarly, the DH parameters of PUMA 500 industrial robot/manipulator was extracted and verified with its specifications available in literature [9].

5. CONCLUSIONS

In this paper, a novel analytical method to extract the Denavit-Hartenberg (DH) parameters is proposed. The method requires the joint axes and a point on it as input and uses the concept of Plücker coordinates to represent the joint axes of a serial manipulator. Dual Vector Algebra was then used to determine the relationship between two successive joint axes and then the DH parameters were determined using Vector Algebra, in a forward recursion from base link to the end-effector. The methodology was used as an addin developed for Autodesk Inventor, which extracts DH parameters from CAD assembly of industrial manipulators/robots. The DH parameters of KUKA KR5 and PUMA 500 manipulators were determined from their CAD models and were validated with the existing results. The methodology can be practically used to determine the exact DH parameters an actual manipulator by tracing a point on the end-effector while rotating one joint at a time. The exact parameters determined can then be used to make modifications in its controller or be used as nominal parameters for the error estimation during calibration.

ACKNOWLEDGEMENT

The research work reported here was carried out under a project sponsored by BARC/BRNS, India. The authors sincerely acknowledge their financial support.

REFERENCES

[1] Abderrahim, M.; Whittaker, A. R.: Kinematic Model Identification of Industrial Manipulators. Robotics and Computer Integrated Manufacturing, Vol. 16, pp. 1-8, 2000.

[2] Barker, L. K.: Vector-Algebra Approach to Extract Denavit-Hartenberg Parameters of Assembled Robot Arms. Technical Paper #2191, NASA, 1983.

[3] Bauchau, O.A.: Flexible Multibody Dynamics. Dodrecht: Springer, 2011.

[4] Denavit, J; Hartenberg, R. S.: A Kinematic Notation for Lower-pair Mechanisms Based on Matrices. ASME Journal of Applied Mechanisms, Vol. 22, No. 2, pp. 215–221, 1955.

[5] Fischer, I.S.: Dual-Number Methods in Kinematics, Statics and Dynamics, Boca Raton, CRC Press, 1998.

[6] Hayati, S.; Mirmirani M.: Improving absolute accuracy of robot manipulators. International Journal on Robotic Systems, Vol 2(4), pp. 394-423, 1985.

[7] Ketchel, J. S.; Larochelle, P. M.: Collision Detection of Cylindrical Rigid Bodies Using Line Geometry. Proceedings of ASME International Design Engineering Technical Conferences & Computers and Information in Engineering Conference (IDETC/CIE), pp. 1-15, 2005.

[8] KUKA Robotics website, http://www.kuka-robotics.com

[9] PUMA webpage, http://www.cs.cmu.edu/~deadslug/puma.html

[10] Saha, S.K.; Introduction to Robotics. New Delhi: Tata McGraw-Hill, 2008

[11] Stone, H. W.; Sanderson, A. C.; Neuman, C. P.: Arm Signature Identification. IEEE International Conference on Robotics and Automation, pp. 41-48, 1986.