Availability models for protection techniques in WDM networks

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***Abstract*—This paper deals with the most common protection schemes in WDM optical networks and it provides for each scheme an algebraic analysis of the availability of protected optical connections. We consider single or multiple link failure scenarios: a link failure affects all the optical connections routed on that link. Availability models and formulas are followed by some numerical examples that allow us to compare the different availability degree granted by each protection technique, and to verify the accuracy of the presented formulas, even when an approximation term is introduced. In conclusion we apply the Monte Carlo approach to the previous cases in order to verify our theoretical analysis.**

***Index Terms*—Protection techniques, WDM networks, link failure, system availability.**

I. Introduction

The recent advances in photonic technologies and the growing telecommunication traf£c demand paved the road for the deployment of WDM networks able to transmit very high bit rate on each single £ber. In such networks, also a few seconds interruption could mean a huge waste of data: this is why protection techniques have become so important. In this paper we focus our attention on the path protection strategy, which consists of providing a backup path for each working path on an end-to-end basis (1:1). More generally, this paradigm can be extended to the M:N case, i.e. N optical working connections routed between a source and a destination node are protected by M backup alternative paths. So, path protection techniques analyzed in this paper comprise not only the widely used 1:1 approach in both dedicated and shared scenarios [1], but also, to obtain a wider and more complete analysis, some valuable cases in the more generic N:M approach.

Clearly any of previous approaches are of great bene£t to network integrity. Anyway the major concern to a service provider is customers' request to have some performance factors strictly guaranteed which are associated to Availability and Reliability (A&R). Connection availability is usually de£ned in Service Level Agreement (SLA) along with revenues and penalties.

The majority of publications on availability analysis focused on the quanti£cation of the path availability for ring based networks [2], [3]. As far as mesh networks using different protection strategies are concerned, a recent body of research investigates availability-ef£cient routing methods [4], [5], the effect of availability on network capacity [6], and proposes some functional modeling description of WDM networks [7].

In this work, our aim is to provide a rigorous algebraic approach for A&R analysis in WDM network protection scheme referring to the combinatorial and probability theories. We will exploit general network cases that allow us to compare the performance of the most common protection techniques from an availability point of view. Although the application is focused on WDM network, the reported formulas and results can be extended to any kind of circuit-switched networks.

The rest of the paper is organized as follows: in section II we illustrate some principles of the theory of the reliability and availability, that will be applied in our analytical models; in section III we present an algebraic approach to analyze the performances of the most common protection schemes. In section IV we report some numerical examples to compare the availability degree provided by the different protection techniques and in section V some tests based on the Monte Carlo approach are carried out to simulate failure scenarios and validate our analytical models. Finally in section VI we draw some conclusions.

II. Theory of the reliability and availability

The theory of reliability and availability analyzes the structure of a system based on a set of distinct subsystems, connected to obtain an intended function. Reliability (R) is de£ned as the probability that a system will perform its intended function for a speci£ed period of time under a given set of conditions, while the availability (A) is the probability that a system is available for use at a given time. Roughly, availability may be viewed as the fraction of time that a system is in an operational state independently of how many times it was previously broken and repaired. So, if we assume that systems, components and subsystems are not repairable, we will refer to reliability, while the availability is a typical feature of restorable systems.

We may express reliability in terms of a random variable T, *the time to system failure*. $R(t)$ is de£ned as the probability that a system operates without failure for a length of time t: $R(t) = P\{T > t\}$ The mean value of the random variable

T is the *Mean Time To failure* (*MTTF*). Another important parameter, strictly related to reliability, is the failure rate $z(t)$. $z(t)dt$ is the probability of a failure in the interval $(t,\, t+dt)$ given that the system has not yet failed in $T = t$ (for more details see [8]):

$$z(t)dt = P\{t < T \leq t + dt | T > t\} \tag{1}$$

For repairable system a fundamental quantity of interest is the *Availability* ($A(t)$). The most common relation used to obtain availability is:

$$A = \frac{MTBF}{MTBF + MTTR} \tag{2}$$

where $MTTR$ is the *Mean Time To Repair* and $MTBF$ is is the *Mean Time Between Failure*. Generally a system is made up of functional elements characterized by *MTBF* and *MTTR* which are assigned by the manufacturer.

In order to obtain the availability of a system, the following scheme must be observed: a) system decomposition in functional elements; b) development of the mathematical model which considers the relation among the subsystems; c) availability evaluation of every subsystems using the reliability values of each element; d) availability evaluation of the whole system under study.

In this paper the system under study is a WDM network topology supporting optical connections, while the subsystem are the optical connections routed on this topology. Each optical connection is subdivided in a set of components, the optical links (or cable), whose availability (reliability) is £xed a priori; each cable consists of a given number of optical £bers and each £ber is equipped with a certain number of optical channels (wavelengths). On the contrary nodes are modelled as utterly reliable components (for more considerations, see [9]). therefore we take into account only the link failure event, which affects all the £bers and optical channels on that link. While modeling the behavior of $z(t)$, it is important to choose the £ttest distribution to describe the performance of the component under study. As far as the optical cable is concerned we assume that the *failure rate* is time independent ($z(t) = \lambda = const$). This assumption implies that: (a) the rate of failure of a system is constant and independent of its age and (b) the failures are independent to each other. In the case of optical cables, the primary concerns are undoubtedly with random failures and this approximation is acceptable.

Our notation allows us to analyze reliability and availability in the same way:

- if E_i is the event $\{T_i > t\}$, that is *the i-th element is still operating at the time t*, and H the event $\{T > t\}$, that is *the system is still operating at the time t*,, then $P(E_i)$ represents the i-th element reliability and $P(H)$ the system reliability;
- otherwise, if E_i is the event *the i-th element is operating at the time t independently of what occurred previously*, then $P(E_i)$ is the i-th element availability and $P(H)$ the system availability.

From now on we consider systems composed of n elements (subsystem or components). The series and parallel connections are two fundamental models that describe the relation among functional subsystem and we will exploit them in the following. The equations that provides the availability of a parallel or series system are well-known and proven and can be found in [8].

III. Availability in WDM networks

In this section we provide algebraic formulas to express the availability (or reliability) of the WDM network system and of the optical connections (subsystems), trying to model different protected scenarios. We begin the investigation with the dedicated 1:1 case, then we generalize this technique, £rst increasing the number of working paths sharing the unique backup path (1:2, 1:N cases), and then increasing the number of shared spare paths to 2 and 3 (we provide formulas in 2:N case, while for 3:N case we report only numerical results). Afterwards we analyze protection strategies based on a single working path and an increasing number of backup paths (2:1, M:1). It is worth noting that it is dif£cult to apply M:N protection strategies in mesh networks with low connectivity index, especially for high values of N+M, because all the N+M paths have to to follow disjoint routes.

Finally we focus our attention on the shared mesh path protection, a 1:1 strategy in which more than one source-destination pair are involved. In this kind of protection the spare lightpaths of link disjoint working paths can share backup capacity

From now on we refer only to availability parameter, since the mathematical approach is the same as in reliability case, as previously explained. The following models and formulas are obtained enumerating all the favorable cases (the system is working) and summing their probabilities (see for example [10]).

A. Dedicated protection (1:1)

In the 1:1 technique (Fig. 1) the backup lightpath tis used when a failure occurs on the link disjoint working lightpath.

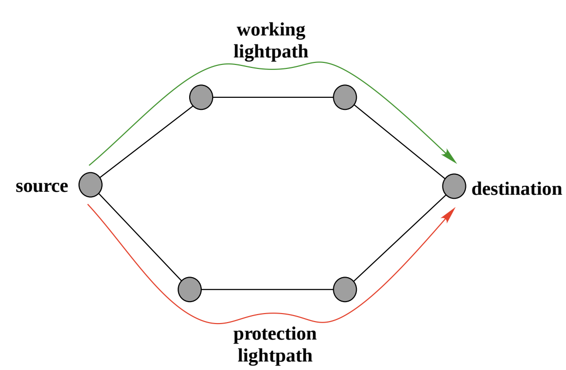

Fig. 1. Dedicated protection

The system availability is given by the union of two disjoint events: in the £rst the working path is available (event E_w);

in the second the working is not available (event $\overline{E_w}$), but the spare is available and can be used (event E_p):

$$P(H) = P\left\{E_w \cup \left[\overline{E_w} \cap E_p\right]\right\} \tag{3}$$

$$A = A_w + (1 - A_w)A_p = A_w + A_p - A_w A_p \tag{4}$$

Another possible technique is the 1+1 approach: the same connection can be routed simultaneously on the two disjoint lightpath (working and protection); the destination node chooses the signal with higher quality SNR. The probability of H is obtained by applying the parallel scheme:

$$P(H) = P\left\{E_w \cup E_p\right\} \tag{5}$$

so the availability A of the connection is the same as (4), since E_w and E_p are not time dependent. The availability analysis of dedicated protection is widely studied in [4].

B. (1:2) protection

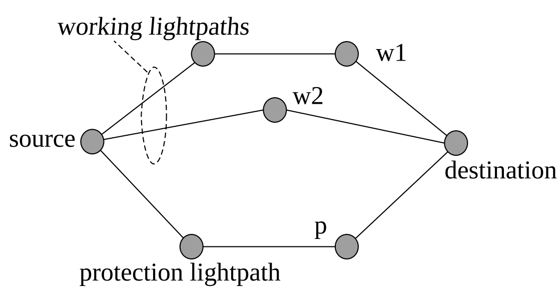

Fig. 2. 1:2 protection

The system in the Fig. 2 shows two connections protected by a single backup lightpath. The probability that the two working connections can be routed successfully (system availability) is obtained by the union of three disjoint events:

$$\begin{aligned} P(H) = P\{&[E_{w1} \cap E_{w2}] \cup \\ &\cup \left[E_{w1} \cap \overline{E_{w2}} \cap E_p\right] \cup \left[\overline{E_{w1}} \cap E_{w2} \cap E_p\right]\} \end{aligned} \tag{6}$$

$$\begin{aligned} A &= A_{w1}A_{w2} + A_{w1}(1 - A_{w2})A_p + (1 - A_{w1})A_{w2}A_p = \\ &= A_{w1}A_{w2} + A_{w1}A_p + A_{w2}A_p - 2A_{w1}A_{w2}A_p \end{aligned} \tag{7}$$

The availability of the £rst subsystem (the connection $w1$ protected) is given by:

$$P_{ss1}(H) = P\left\{E_{w1} \cup \left[\overline{E_{w1}} \cap E_p \cap E_{w2}\right]\right\} \tag{8}$$

$$\begin{aligned} A_{ss1} &= A_{w1} + (1 - A_{w1})A_pA_{w2} = \\ &= A_{w1} + A_pA_{w2} - A_{w1}A_pA_{w2} \end{aligned} \tag{9}$$

We can obtain the availability of the connection $w2$ in the same way.

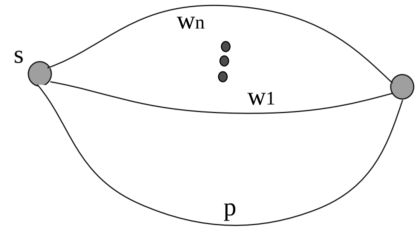

Fig. 3. 1:N protection

C. (1:N) protection

Now we we can extend the previous case to the general case of N working lightpaths (as shown in Fig. 3). The system availability is expressed by:

$$P(H) = \left\{\bigcap_{j=1}^{n} E_{w_j} \cup \left[\bigcup_{i=1}^{n}\left(\bigcap_{j=1}^{n} E_{w_{j\neq i}}\right) \cap \overline{E_{w_i}} \cap E_p\right]\right\} \tag{10}$$

$$A = (1 - nA_p)\prod_{j=1}^{n} A_{w_j} + \sum_{i=1}^{n}\left[\left(\prod_{j=1}^{n} A_{w_{j\neq i}}\right)A_p\right] \tag{11}$$

The availability of the generic connection k is given by the union of two disjoint events: (a) the working path of k is operating or (b) all the other working paths and the protection path are available:

$$P_{ssk}(H) = P\left\{E_{w_k} \cup \left[E_p \cap \left(\bigcap_{j=1}^{n} E_{w_{j\neq k}}\right) \cap \overline{E_{w_k}}\right]\right\} \tag{12}$$

$$A_{ssk} = A_{w_k} + A_p(1 - A_{w_k})\left(\prod_{j=1}^{n} A_{w_{j\neq k}}\right) \tag{13}$$

D. (2:N) protection

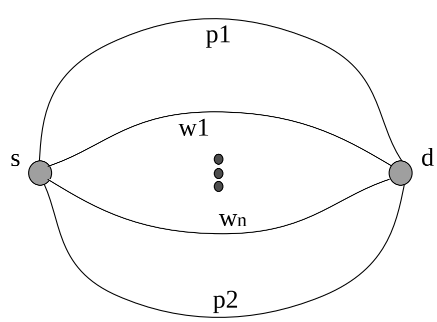

Fig. 4. 2:N protection

This protection scheme requires two protection lightpaths and $N > 2$ working lightpaths. The 2:N scheme is showed in Fig. 4. In case of failures the protection lightpaths are assigned giving higher priority to the connection with lower index (p_1 is the £rst to be used, then p_2). Availability is obtained by the disjoint union of $\begin{pmatrix} 2+N \\ N \end{pmatrix}$ events:

$$P(H) = P\left\{E_a \cup E_b \cup E_c \cup E_d\right\} \tag{14}$$

where: $E_a = \bigcap_{i=1}^{n} E_{w_i}$

$$E_b = \bigcup_{i=1}^{n} \overline{E_{w_i}} \left(\bigcap_{j=1}^{n} E_{w_{j\neq i}} \right) \cap E_{p1}$$

$$E_c = \bigcup_{i=1}^{n} \overline{E_{w_i}} \left(\bigcap_{j=1}^{n} E_{w_{j\neq i}} \right) \cap \overline{E_{p_1}} \cap E_{p_2}$$

$$E_d = \bigcup_{i=2}^{n} \bigcup_{j=1}^{i-1} \overline{E_{w_i}} \cap \overline{E_{w_j}} \cap \left(\bigcap_{k=1}^{n} E_{w_{k\neq j\neq i}} \right) \cap E_{p1} \cap E_{p_2}$$

$$\begin{aligned} A = & \prod_{i=1}^{n} A_{w_i} + \\ & + \sum_{i=1}^{n} (1 - A_{w_i}) \left(\prod_{j=1}^{n} A_{w_{j\neq i}} \right) (A_{p_1} + A_{p_2} - A_{p_1} A_{p_2}) + \\ & + \sum_{i=2}^{n} \sum_{j=1}^{i-1} (1 - A_{w_i}) \left(1 - A_{w_j}\right) \left(\prod_{k=1}^{n} A_{w_{k\neq j\neq i}} \right) A_{p_1} A_{p_2} \end{aligned} \tag{15}$$

The availability of a generic protected optical connection i, belonging to the 2:N scheme, is not reported for brevity.

The same analysis has been carried out in the 3:N case, and we will show the numerical values in section IV.

E. (2:1) protection

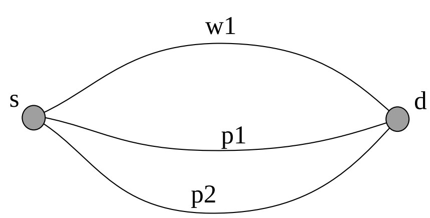

Fig. 5. 2:1 protection

In this scheme two disjoint backup lightpaths protect a single working lightpath. The 2:1 protection (Fig. 5) can be used to prevent double link failure. The path p_1 is used when the working path is out of service, while the path p_2 is used only when both w_1 and p_1 are out of service. The equations (16) and (17) refer to the system availability that coincides to the single connection availability.

$$P(H) = P\left\{E_{w_1} \cup \left[\overline{E_{w_1}} \cap E_{p_1}\right] \cup \left[\overline{E_{w_1}} \cap \overline{E_{p_1}} \cap E_{p_2}\right]\right\} \tag{16}$$

$$A = A_{w_1} + (1 - A_{w_1}) A_{p_1} + (1 - A_{w_1}) (1 - A_{p_1}) A_{p_2} \tag{17}$$

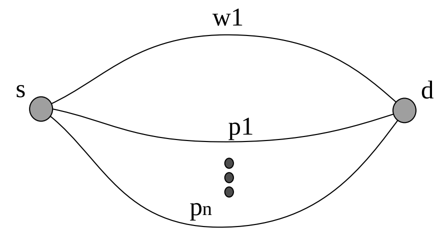

Fig. 6. M:1 protection

F. (M:1) protection

Let us now consider a generic number M of spare lightpaths (Fig. 6). Thanks to spare capacity redundancy, up to N link failures can be recovered. Equations (18) and (19) express either system and single connection availability:

$$P(H) = P\left\{ E_{w_1} \cup \left[\bigcup_{i=1}^{n} \overline{E_{w_1}} \cap \left(\bigcap_{j=1}^{i-1} \overline{E_{p_j}} \right) \cap E_{p_i} \right] \right\} \tag{18}$$

$$A = A_{w_1} + (1 - A_{w_1}) \sum_{i=1}^{n} A_{p_i} \prod_{j=1}^{i-1} \left(1 - A_{p_j}\right) \tag{19}$$

In conclusion, we have shown how the availability of a M:N system can be analytically obtained for speci£c values of M and N. However, a general equation having N and M as parameters can not be written in a closed form.

G. Shared protection

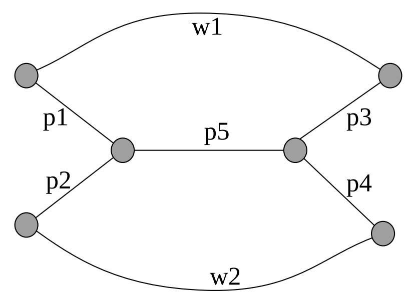

Fig. 7. Shared protection

We call shared (mesh) protection a 1:1 protection strategy, in which spare paths associated to disjoint working paths having different source and or destination can share optical channels. This strategy allows a relevant saving of network capacity, without affecting network ability to face any kind of single failure. Let us develop the analysis of this case using to an example. The two working connections $w1$ and $w2$ showed in Fig. 7 are protected by two protection lightpaths ($p1$ and $p2$) which share an optical channel. The system availability is the probability that both connections are routed successfully and it can be obtained by the union of three disjoint events (equations (20) and (21)).

$$P\left(H\right) = P\left\{\left[E_{w1} \cap E_{w2}\right] \cup E_a \cup E_b\right\} \tag{20}$$

where: $E_a = E_{w1} \cap \overline{E_{w2}} \cap (E_{p2} \cap E_{p5} \cap E_{p4})$
$E_b = \overline{E_{w1}} \cap E_{w2} \cap (E_{p1} \cap E_{p5} \cap E_{p3})$

$$A = A_{w1}A_{w2} + A_{w1}(1 - A_{w2})A_{p2}A_{p5}A_{p4} + + (1 - A_{w1})A_{w2}A_{p1}A_{p5}A_{p3} \tag{21}$$

To evaluate the availability of a single connection, we have to distinguish different double-link failure scenarios. For instance, even though the lightpath $w2$ and the optical channel $p2$ fail, the connection 1 can be routed successfully. So the £rst subsystem (protected connection 1) is characterized by the following availability:

$$P_{ss1}(H) = P\{E_{w1} \cup E_\alpha \cup E_\beta \cup E_\gamma\} \tag{22}$$

where: $E_\alpha = \overline{E_{w1}} \cap (E_{p1} \cap E_{p5} \cap E_{p3}) \cap E_{w2}$
$E_\beta = \overline{E_{w1}} \cap (E_{p1} \cap E_{p5} \cap E_{p3}) \cap \overline{E_{w2}} \cap \overline{E_{p2}}$
$E_\gamma = \overline{E_{w1}} \cap (E_{p1} \cap E_{p5} \cap E_{p3}) \cap \overline{E_{w2}} \cap E_{p2} \cap \overline{E_{p4}}$

$$\begin{aligned} A_{ss1} &= A_{w1} + (1 - A_{w1})A_{p1}A_{p5}A_{p3}A_{w2} + \\ &+ (1 - A_{w1})A_{p1}A_{p5}A_{p3}(1 - A_{w2})(1 - A_{p2}) + \\ &+ (1 - A_{w1})A_{p1}A_{p5}A_{p3}(1 - A_{w2})A_{p2}(1 - A_{p4}) \end{aligned} \tag{23}$$

The need to consider all the possible multiple-failures combinations makes the problem intractable if the topology is not as simple as that in Fig. 7. So, in order to avoid the cumbersome extension of previous exact formulas, we can apply an approximate the solution by neglecting multiple failure scenarios. This is equivalent to consider only terms in which $(1 - A_{wi})$ appears at the £rst order, neglecting higher-order terms, It can be proven that the second order terms are always absent even without the approximation, except when the spare path is totally shared (but this case coincides with the 1:N case). In the next section we will show by numerical examples that the approximated formula converges to the real availability values, for highly available components (rare-event approximation). The approximated availability of connection 1 is calculated in (24) and (25).

$$P_{ss1}(H) \cong P\left\{E_{w1} \cup \left[\overline{E_{w1}} \cap (E_{p1} \cap E_{p5} \cap E_{p3}) \cap E_{w2}\right]\right\} \tag{24}$$

$$A_{ss1} \cong A_{w1} + (1 - A_{w1})A_{p1}A_{p5}A_{p3}A_{w2} \tag{25}$$

H. Shared protection: extended analysis m$\times(1:1)$

In this paragraph we extend our analysis to a topology comprising m protected working connections whose protection lightpaths share some optical channels. The scheme described is showed in Fig. 8. The system availability formulas (equations (26) and (27)) are obtained neglecting multiple-failure cases.

$$P(H) = P\left\{\bigcap_{j=1}^{m} E_{w_j} \cup \left[\bigcup_{i=1}^{m}\left(\bigcap_{k=1}^{m} E_{w_{k\neq i}}\right) \cap \overline{E_{w_i}} \cap E_{p_z}\right]\right\} \tag{26}$$

dove: $z = \alpha, \beta, \gamma, \delta, \ldots$

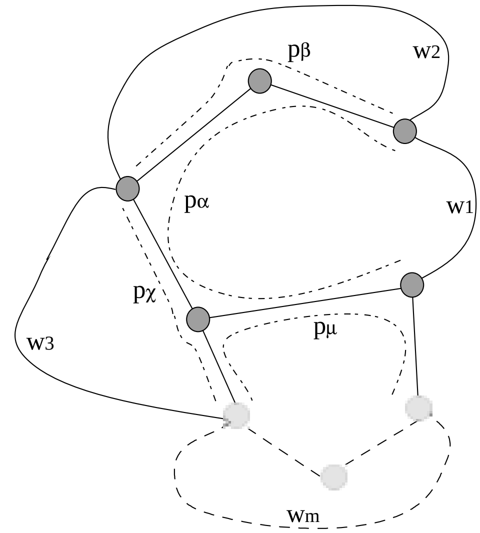

Fig. 8. m*(1:1) shared protection

$$A = \prod_{j=1}^{m} A_{w_j} + \sum_{i=1}^{m}\left(\prod_{k=1}^{m} A_{w_{k\neq i}}\right)(1 - A_{w_i})A_{p_z} \tag{27}$$

Also in the formulas (28) and (29) we use the single failure approximation.

$$P_{ssk}(H) \cong P\left\{E_{w_k} \cup \left[\left(\bigcap_{i=1}^{m} E_{w_{i\neq k}}\right) \cap \overline{E_{w_k}} \cap E_{p_{zk}}\right]\right\} \tag{28}$$

$$A_{ssk} \cong A_{wk} + (1 - A_{w_k})A_{p_{zk}}\left(\prod_{i=1}^{m} A_{w_{i\neq k}}\right) \tag{29}$$

Thanks to the approximation, the previous two formulas express the availability of a connection k as a function of the availability of w_k and of the availability of all the working paths, having the associated spare path which shares one or more channels with the spare path p_k. So A_{ssk} is in¤uenced by the length of its spare path, but not by the effective number of shared spare channels. In other words, if a spare path p_k has a length of 5 hops and it shares channels with other 3 spare paths (associated to distinct working channel), A_{ssk} is £xed by equation (29) and we are not interested on the number of effectively shared channels, that can vary from 1 to 5.

IV. Numerical results

In this section we analyze the protection techniques through numerical examples. We assume each working lightpaths w_i is composed of a single hop (channel) with unavailability 10^{-4}. Each protection lighpath consists in a series of 3 links (each with unavailability 10^{-4}). The total spare path availability is $A_p = A_{pi}^3 = (1 - U_i)^3 = (1 - 10^{-4})^3$. The reported numerical values refer to the availability of a single protected connection.

A. (M:N) protection

1) (1:N), (2:N), (3:N) protection: The unavailability values in the Fig 9(a) are obtained by applying respectively equation

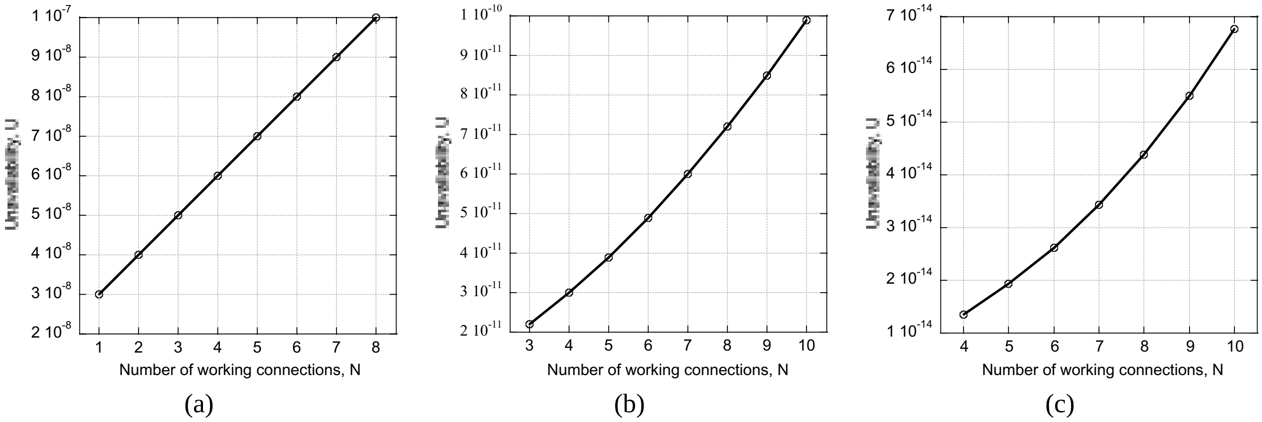

Fig. 9. Unavailability of 1:N (a), 2:N (b), 3:N (c) protection

(4) and (13) for 1:N (a). For 3:N and 2:N cases similar equations have been applied not shown in the paper for brevity.

Fig. 9(a) shows that unavailability get worse for increasing values of N with a linear slope of about 1×10^{-8} per N increment. By comparing these values to those of Fig. 9(b), we can observe that a connection protected by the 2:N technique is always more available than by 1:N for any value of N. This is simply because dedicated protection recovers any kind of single failure while the 2:N protection recovers any kind of single and double failures by means of two backup paths. The same consideration can be drawn for the 3:N case. Obviously, these large improvements of availability are paid in terms of a much higher amount of physical resources (cables and £bers) For 2:N and 3:N techniques, we notice that connection unavailability increases more than linearly with N. By comparing the three graphs we can conclude that the slope of the function $U(N)$, which is constant when single failures are recoverable, tends to increase more and more rapidly as the number of recoverable failures increase.

2) (M:1) protection: The values of Table I are obtained applying the equation (19). The unavailability values decrease very rapidly adding protection lightpaths, since a greater number of connection failures can be recovered.

TABLE I
UNAVAILABILITY IN THE M:1 PROTECTION

Protection technique	Unavailability $U_{ss_1} = 1 - A_{ss_1}$
$(2:1)$	$8,99825 \times 10^{-12}$
$(3:1)$	$2,77556 \times 10^{-15}$
$(4:1)$	$1,11022 \times 10^{-16}$

We can conclude that the availability degree in M:N protection strategies is primarily determined by M, again corresponding to the number of simultaneously recoverable failures. The number N of working paths sharing the backup paths has instead a marginal effect compared to M. It is important to observe that the ratio M/N is not a signi£cant parameter to compare the availability degree of different protection strategies: M is much more signi£cant. For example let us compare the unavailability of 3:4 and 2:1 strategies: by comparing the results of M/N ratio (respectively O.75 and 2), 2:1 would appear to provide a better degree of availability. Actually, 3:4 provides protection against any three link failure, achieving a higher level of unavailability. From Table I and from Fig. 9(c) we see that 2:1 unavailability is $\approx 9 \times 10^{-12}$, while in 3:4 case unavailability is $\approx 1.2 \times 10^{-14}$.

B. Shared protection

In the paragraph III-G we have obtained the single connection availability using either an exact (equation (23)) and an approximated approach (equation (25)). Table II shows the accuracy of our approximations; In the £rst row we present an unrealistic case for optical networks, in which link unavailability is exaggerated on purpose. In the second row we exploited more realistic unavailability values. We can observe that in the £rst case the percentage error of the approximated result is quite small while in the second is totally negligible. Since the corresponding equations are equal due to the approximation, then the unavailability value of the approximated case is equal to the unavailability of the 1:2 protection shown in Fig. 9(b).

TABLE II
SHARED PROTECTION UNAVAILABILITY

Unavail	U_{ssi} exact	U_{ssi} approx	% Error
$U_i = 0.1$	3.30049×10^{-2}	3.439×10^{-2}	4.2
$U_i = 10^{-4}$	3.9992×10^{-8}	3.9994×10^{-4}	5×10^{-3}

To con£rm the validity of the approximations for high availability values, we propose two other topologies shown in Fig. 10. In both cases we calculated the unavailability of the protected connection 1. Also in this situation we examine the two cases with unrealistic and realistic unavailability values. These results are shown in table III and con£rm the previous conclusions.

1) m(1:1) shared protection:* The values in Fig. 11 refer to the equation (29). The graph displays the unavailability of a generic connection k. We consider that its spare path is composed of 5 (straight line) or 7 (dotted line) links, and we

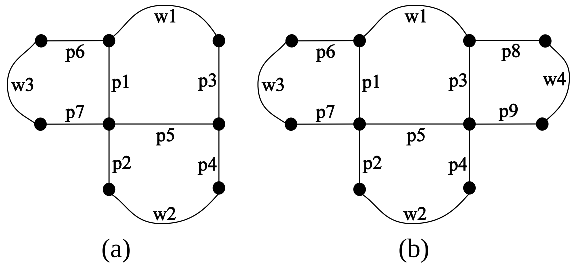

Fig. 10. (a) three and (b) four shared protected connections

TABLE III
CONNECTION UNAVAILABILITY IN FIG. 10 TOPOLOGIES

Topol	U_i	U_{ssi} exact	U_{ssi} approx	% Error
Fig. 10.a	0.1	1.50654×10^{-1}	1.66009×10^{-1}	10.2
Fig. 10.b	10^{-4}	4.9986×10^{-8}	4.999×10^{-8}	8×10^{-3}
Fig. 10.a	0.1	1.77069×10^{-1}	1.94462×10^{-1}	9.8
Fig. 10.b	10^{-4}	5.9979×10^{-8}	5.9985×10^{-8}	10^{-2}

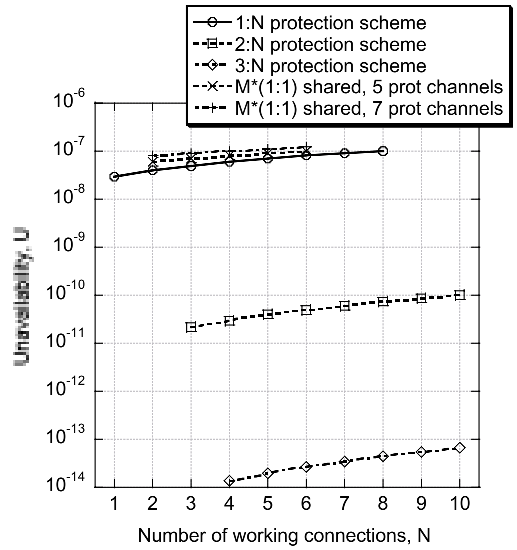

Fig. 12. Unavailability comparison of various protection schemes

vary the number of working-spare couples that share a backup channel with the backup path of w_k. As already observed

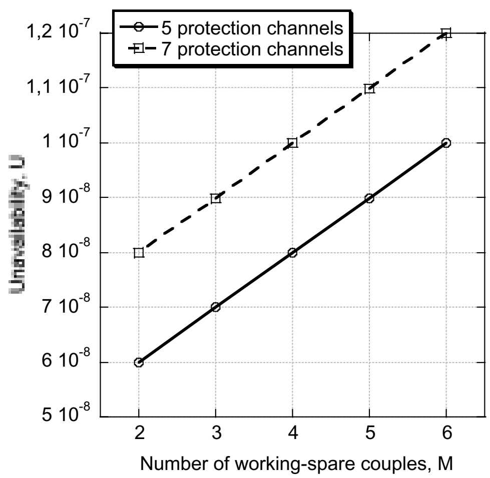

Fig. 11. Unavailability of the m*(1:1) shared protection

in protection technique for single fault recovery, the Fig. 11 shows a linear increase of connection unavailability, associated to the increase of the number of connections sharing backup capacity. It can also be seen that, in terms of availability performance, increasing the length of the protection lightpath by x hops is exactly equivalent to increasing the number of sharing connections by x.

C. Final comparison

In the previous sections we have separately studied the different protection approaches. Now we can jointly compare the performances of the various approaches (Fig. 12).

As already explained, unavailability improves sensibly when the protection scheme is able recover multiple failures: this behavior is evident in Fig. 12. The shared protection and 1:N protection techniques, able to prevent only in single failure scenarios, give similar unavailability results. The availability advantages of survivability to multiple failures should be compared keeping the cost of the resource-consuming protection techniques into account. A further work is currently under study to investigating the capacity vs availability trade-off.

V. MONTE CARLO APPROACH

We have developed a simple Monte-Carlo (MC) approach simulator to verify some aspects of our theoretical analysis. The equations we introduced in section III are obtained by considering a constant failure rate $z(t) = \lambda$, since it is proved to be a correct model for a component like an optical link (for instance, see [11], [12], [13], [14]). As previously said, formulas are not time dependent and in some cases simultaneous events cannot be described. For example, in the 1:2 protection scenario (see Fig. 2) in case of failures affecting both the working lightpaths, according to equation (8), both the connections are blocked, even though the protection path is available and thus one at least could be recovered. By means of MC we attempt to solve this problem introducing a random failure sequence that allow us to protect at least one of the failed connections.

The procedure to evaluate the connections unavailability by means of MC method is the following. A given network topology is decomposed in separated subnetworks each one representing a connection, with its working and spare paths (possibly more than one). Links that are shared by many backup ligthpaths appear replicated in the subnetworks corresponding to all the sharing connections. The unavailability of each link is simulated exploiting the generation of random numbers uniformly distributed in the range $[0, 1]$. If the number returned by random generator is lower than A_i, we assign the value *one* to the link. Viceversa, if the number is above the threshold, the value assigned to the link is *zero*, indicating it is not available.

At each simulation iteration the state of all the links is set randomly and independently as described above. Then the connections are scanned in a sequential order which varies randomly at each iteration. Each connection is available if the working path is available (all its links have state one). Else, all the associated spare paths are scanned (with a priority sequence dependent on the protection technique). The £rst ”all-one” free path to the destination found is used for recovery. The links crossed by such a path are assigned to the connection. If no free ”all-one” path is found, the connection fails. All the subsequent connections can use only unassigned ”one” links for a possible backup. At the end of the simulation

All the favorable events, i.e. connections which could be established successfully are counted. MC simulation is particularly useful when many connections compete for the shared protection resources. While the analytical method of section III, based on a time-invariant approach, assumed all these cases as unfavorable, they are instead adequately measured and counted by the MC simulation.

Table IV compares analysis and MC simulation results concerning the networks represented in Fig. 13. A_i is 0.9 for all the links and 50 million iterations are generated to gather MC statistics. The table proves a good convergence of analysis to simulation even for high values of link unavailability.

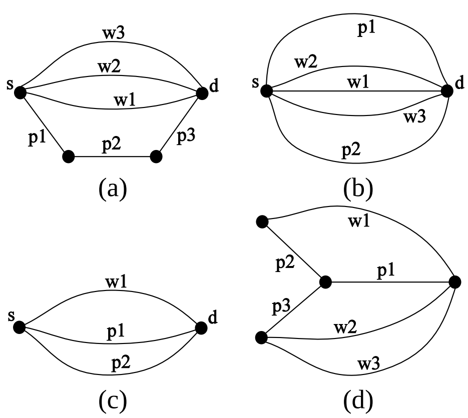

Fig. 13. Topologies used for the Monte Carlo simulations

TABLE IV

NUMERICAL COMPARISON BETWEEN MONTE CARLO APPROACH AND ANALYTICAL METHOD

Topologies	MC	Analysis	% error
Fig. 13.a, w_1	0.9813	0.9729	0.865
Fig. 13.b, w_1	0.99693	0.99477	0.217
Fig. 13.c, w_1	0.99899	0.999	0.0012
Fig. 13.d, w_1	0.97399	0.96715	0.707
Fig. 13.d, w_2	0.97351	0.96634	0.742

VI. CONCLUSION

In this paper a rigorous methodology is proposed which can be used to quantify the connection availability under several protection schemes. Different protection strategies provide a different availability degree in a set of simple topologies.

The proposed method is capable of estimating both connection and system availability of many protection techniques. In treating shared protection we have introduced an approximation that allows us to analyze complex topologies.

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