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AVERAGE BODY-WAVE RADIATION COEFFICIENTS

BY DAVID M. BOORE AND JOHN BOATWRIGHT

ABSTRACT

Averages of P- and S-wave radiation patterns over all azimuths and various ranges of takeoff angles (corresponding to observations at teleseismic, regional, and near distances) have been computed for use in seismological applications requiring average radiation coefficients. Various fault orientations and averages of the squared, absolute, and logarithmic radiation patterns have been considered. Effective radiation patterns combining high-frequency direct and surface-reflected waves from shallow faults have also been derived and used in the computation of average radiation coefficients at teleseismic distances.

In most cases, the radiation coefficients are within a factor of 1.6 of the commonly used values of 0.52 and 0.63 for the rms of P- and S-wave radiation patterns, respectively, averaged over the whole focal sphere. The main exceptions to this conclusion are the coefficients for P waves at teleseismic distances from vertical strike-slip faults, which are at least a factor of 2.8 smaller than the commonly used value.

INTRODUCTION

Average P- and S-wave radiation coefficients are used both in theoretical predictions of ground motions and radiated energy and in observational inversions for source parameters. In most cases, the rms averages over the whole focal sphere are used (Aki and Richards, 1980, prob. 4.6)

$$\sqrt{\langle (F^P)^2 \rangle} = \sqrt{4/15} = 0.52 \tag{1}$$

$$\sqrt{\langle (F^S)^2 \rangle} = \sqrt{2/5} = 0.63. \tag{2}$$

These averages are independent of fault orientation. The S represents the total S motion ($= \sqrt{SH^2 + SV^2}$).

Although commonly used, the radiation coefficients given by equations (1) and (2) have a number of theoretical shortcomings. In particular

1. The rms average is inappropriate for those applications where the absolute value or the logarithm of a seismological quantity is being computed. In fact, about the only application where the rms average is appropriate is in energy computations (Boatwright, 1980; Boatwright and Fletcher, 1984).
2. In many applications, a specific portion of the focal sphere is being sampled, rather than the whole local sphere.
3. The type of faulting, not required in deriving equations (1) and (2), is necessary if either SH or SV motion is considered or if the average over only a portion of the focal sphere is desired.

One purpose of this paper is to assess the practical importance of these factors. To do this, we present radiation coefficients for averages of squared, absolute-value, and logarithmic radiation patterns for ranges of take-off angles appropriate for teleseismic, regional, and close distances. In so doing, we will also accomplish our goal of providing tables of radiation coefficients that may be useful in future applications. Averages of the absolute value are useful, for example, if seismic moment is derived from an arithmetic average of moments from individual stations.

A geometric average of moments, however, demands the average of the logarithm of the radiation pattern, as does prediction of earthquake magnitude. We consider P, S, SV and SH radiation. For teleseismic distances and shallow faults, we approximate the effective high-frequency radiation pattern obtained from the interaction of the direct and surface-reflected phases. von Seggern (1970) also discussed the effect of fault orientation and a limited range of takeoff angles on the average radiation pattern. He considered averages of the logarithm of the radiation pattern for teleseismic distances. He was concerned with effects on M_S, m_b systematics, however, and did not present the effective radiation patterns in a form useful for theoretical predictions or inversion of data for source parameters.

METHOD

The following integral defining the average must be computed

$$\langle G \rangle = \frac{\int_0^{\pi} \int_0^{2\pi} [W(\theta, \phi) G(\theta, \phi)] \sin \theta d\phi \, d\theta}{\int_0^{\pi} \int_0^{2\pi} [W(\theta, \phi)] \sin \theta d\phi \, d\theta} \tag{3}$$

where G is taken to be F^2, $|F|$, or $\log |F|$, and F is the radiation pattern for the appropriate type of wave. ϕ is the azimuth and θ is the takeoff angle (measured from the downward vertical; see Fig. 4.14 in Aki and Richards, 1980, where their i_ξ is our θ). We assume that the weighting function $W(\theta, \phi)$ is independent of azimuth and is nonzero (and equal to one) only for takeoff angles θ in the range $\theta_1 < \theta < \theta_2$

$$W(\theta, \phi) = H(\theta - \theta_1) - H(\theta - \theta_2) \tag{4}$$

where $H(\theta)$ is the Heaviside step function. The radiation patterns for P, SV, SH, and $S = \sqrt{SV^2 + SH^2}$ are given, e.g., by Aki and Richards [1980, equations (4.84), (4.85), and (4.86)]. Following the example of Boatwright and Choy (1984), effective radiation coefficients for the P- and S-wave groups from a shallow earthquake have been derived under the assumption that at high frequencies the energy density spectra of the direct and surface-reflected phases add. Assuming a simple ω^{-2} spectral behavior at high frequencies, this leads to the following generalized P-wave radiation pattern

$$F^{g^P} = \left[(F^P)^2 + (\acute{P}\grave{P} F^{p^P})^2 + \left(\left(\frac{\alpha}{\beta}\right)^3 \frac{1}{\delta^2} \acute{S}\grave{P} F^{s^P} \right)^2 \right]^{1/2} \tag{5}$$

where δ is the ratio of P to S corner frequencies and α, β are compressional and shear velocities. $\acute{P}\grave{P}$ and $\acute{S}\grave{P}$ are the free surface reflection coefficients [Aki and Richards, 1980, equations (5.26) and (5.30)], the latter multiplied by the ratio of the P- to S-vertical wavenumbers to correct for the spherical wavefront (Bache, written communication, 1975). The superscripts of F denote the type of wave considered, with the appropriate takeoff angle implied [$\pi - \theta$ for pP and $\pi - \sin^{-1}\left(\frac{\beta}{\alpha} \sin \theta\right)$ for sP]. Generalized radiation patterns for S waves can be derived in a similar way.

The integrals in equation (3) were evaluated using a Monte Carlo scheme in which N random numbers η_i, ξ_i uniformly distributed between 0 and 1 were converted to azimuths and takeoff angles according to the following mapping

$$\theta_i = \cos^{-1}[(1 - \xi_i)\cos\theta_1 + \xi_i \cos\theta_2] \tag{6a}$$

$$\phi_i = 2\pi\eta_i. \tag{6b}$$

With this mapping, θ has a probability density distribution equal to

$$\frac{W \sin\theta}{\displaystyle\int_0^{\pi}\int_0^{2\pi} W \sin\theta d\phi\, d\theta}.$$

The integral in equation (3) can then be identified as the expected value of G, which can be approximated by the average of G evaluated at the N pairs of θ_i, ϕ_i

$$\langle G \rangle \approx \frac{1}{N}\sum_{i=1}^{N} G(\theta_i, \phi_i). \tag{7}$$

This is not an optimum method for computing the integral, but it is convenient and easy to program. Equation (3) is a deterministic integral with a precise numerical value for a given fault orientation and range of takeoff angles; the use of random numbers in a Monte Carlo scheme is only one of many ways of evaluating the integral. The integral should, of course, be independent of the particular set of random numbers used, at least to within the desired accuracy of the answer. This in turn requires a large set of random numbers in order to guarantee a sufficiently small standard error of $\langle G \rangle$. Choosing $N = 1000$ gives adequate answers; to derive results for this paper, however, $N = 10{,}000$ was used for integrations over the whole focal sphere and $N = 5000$ for more limited ranges of takeoff angle. Further discussion of the Monte Carlo method, including ways of speeding up the computations, is given by Hammersley and Handscomb (1964). For some cases, the integrals in equation (3) are easily done analytically (e.g., $\langle F^2 \rangle$ and $\langle |F| \rangle$ for a vertical strike-slip fault) and provide useful checks for the numerical equations.

Results

Besides the averages for the whole focal sphere, three ranges of takeoff angles were considered

1. $17° \leqq \theta \leqq 25°$ for use at teleseismic distances
2. $60° \leqq \theta \leqq 120°$ for use at regional distances (tens to hundreds of kilometers)
3. $120° \leqq \theta \leqq 180°$ for use at close distances (within a source depth).

For each range of takeoff angles, computations were done for P, SV, SH, and total S motion corresponding to three fault types

1. strike-slip on a vertical fault
2. dip-slip on a 30° dipping fault
3. oblique-slip (rake angle = 45°) and a 45° dipping fault.

The results for case (2) are identical to those from dip-slip motion on a 60° dipping fault. Averages over a distribution of fault orientations would perhaps have been better, although not as convenient as assuming representative fault geometries. If a distribution of fault orientations had been used, it would be appropriate to assign a standard error to the average radiation coefficients. In a sense, the range of takeoff angles compensates for the lack of a range in fault orientations.

Nodes were avoided by assigning an absolute lower bound (a water level) of 0.1 to the radiation pattern. Numerical tests showed that most of the results are not sensitive to the value of this lower bound. [Spottiswoode and McGarr (1975, p. 103) avoided the problem of nodes by computing median values of the P- and S-radiation patterns over the whole focal sphere. They found values of 0.39 and 0.57 for the P- and S-radiation patterns, respectively.]

TABLE 1

EFFECTIVE RADIATION COEFFICIENTS, AVERAGED OVER WHOLE FOCAL SPHERE

	$\sqrt{\langle F^2 \rangle}$	$\langle \lvert F \rvert \rangle$	$10^{\langle \log \lvert F \rvert \rangle}$
P	0.52	0.44	0.33
S	0.63	0.60	0.55
Vertical, strike-slip			
SV	0.26	0.23	0.20
SH	0.58	0.50	0.40
30° dip, dip-slip			
SV	0.54	0.48	0.40
SH	0.32	0.28	0.24
45° dip, oblique slip			
SV	0.48	0.43	0.36
SH	0.41	0.36	0.30

TABLE 2

EFFECTIVE P-WAVE RADIATION COEFFICIENTS, AVERAGED OVER $17° \leqq \theta \leqq 25°$ (TELESEISMIC DISTANCES)

		$\sqrt{\langle F^2 \rangle}$	$\langle \lvert F \rvert \rangle$	$10^{\langle \log \lvert F \rvert \rangle}$
Vertical, strike-slip				
Direct	$\{ P$	0.11	0.11	0.11
Direct and reflected	$\{ P$	0.18	0.18	0.17
30° dip, dip-slip				
Direct	$\{ P$	0.74	0.70	0.64
Direct and reflected	$\{ P$	0.99	0.98	0.97
45° Dip, Oblique-Slip				
Direct	$\{ P$	0.62	0.57	0.51
Direct and reflected	$\{ P$	0.84	0.82	0.81

The results of these calculations are presented in a series of tables. For reference and comparison with later results, Table 1 contains the P- and S-radiation patterns averaged over the whole focal sphere. The effective radiation coefficients appropriate for teleseismic distances are contained in Tables 2 and 3, and Tables 4 and 5 contain results for regional and teleseismic distances, respectively.

DISCUSSION

The primary purpose for the tables is to provide seismologists with radiation coefficients for various uses. Second, the tables can be used to judge whether our earlier point regarding the inapplicability of equations (1) and (2) is a practical concern.

Although extracting general inferences from the mass of numbers in the tables is difficult (and beside the point of this paper), several general conclusions can be drawn.

1. As expected, the effective radiation coefficients from averages of the squared patterns are greater than from the absolute values, which in turn are greater than obtained using logarithms of the radiation patterns.
2. The types of faulting and averages and the range of takeoff angles clearly lead to differences in the effective radiation coefficients, but the coefficients are generally within a factor of 1.6 of each other. The primary exception is P-wave radiation at teleseismic distances, which for strike-slip motion on a vertical fault is at least 5 times smaller than for dip-slip or oblique-slip motion on dipping faults. P-wave radiation coefficients for strike-slip faults at teleseismic distances are more sensitive to the water level than in the other cases considered here (see below).
3. The effect of surface reflections is to increase the effective P-wave radiation coefficient by a factor of about 1.5 at teleseismic distances, relative to direct P waves.
4. The commonly used rms radiation coefficients given by equations (1) and (2) generally overestimate the radiation coefficients. This is particularly true for the coefficients based on averages of the logarithms of the radiation patterns. The main exceptions to this conclusion are for P waves and total S waves, including surface reflections, at teleseismic distances radiated from dipping faults.

TABLE 3

EFFECTIVE S-WAVE RADIATION COEFFICIENTS, AVERAGED OVER $17° \leqq \theta \leqq 25°$ (TELESEISMIC DISTANCES)

		$\sqrt{\langle F^2 \rangle}$	$\langle \lvert F \rvert \rangle$	$10^{\langle \log \lvert F \rvert \rangle}$
Vertical, strike-slip				
Direct	S	0.36	0.35	0.35
	SV	0.24	0.23	0.20
	SH	0.26	0.24	0.21
Direct and reflected	S	0.52	0.51	0.51
	SV	0.37	0.34	0.30
	SH	0.36	0.34	0.30
30° Dip, Dip-Slip				
Direct	S	0.63	0.56	0.48
	SV	0.52	0.44	0.35
	SH	0.35	0.31	0.26
Direct and reflected	S	0.87	0.85	0.82
	SV	0.72	0.67	0.61
	SH	0.49	0.46	0.42
45° Dip, Oblique-Slip				
Direct	S	0.59	0.57	0.53
	SV	0.47	0.41	0.34
	SH	0.36	0.32	0.28
Direct and reflected	S	0.84	0.81	0.77
	SV	0.66	0.61	0.55
	SH	0.52	0.49	0.46

All of the results shown in the tables have been recomputed using water levels of 0.05 and 0.20. As expected, the smallest radiation coefficients computed from the logarithms of the radiation patterns are most sensitive to the water level. For the vertical strike-slip fault, the direct P-wave radiation coefficient increases from 0.08

TABLE 4

EFFECTIVE RADIATION COEFFICIENTS, AVERAGED OVER $60° \leqq \theta \leqq 120°$ (REGIONAL DISTANCES)

	$\sqrt{\langle F^2 \rangle}$	$\langle \lvert F \rvert \rangle$	$10^{\langle \log \lvert F \rvert \rangle}$
Vertical, strike-slip			
P	0.65	0.59	0.50
S	0.70	0.66	0.60
SV	0.20	0.18	0.16
SH	0.67	0.61	0.50
30° dip, dip-slip			
P	0.48	0.38	0.28
S	0.56	0.52	0.48
SV	0.46	0.39	0.30
SH	0.32	0.28	0.25
45° dip, oblique-slip			
P	0.53	0.44	0.34
S	0.60	0.57	0.53
SV	0.43	0.38	0.32
SH	0.43	0.38	0.32

TABLE 5

EFFECTIVE RADIATION PATTERNS, AVERAGED OVER $120° \leqq \theta \leqq 180°$ (CLOSE DISTANCES)

	$\sqrt{\langle F^2 \rangle}$	$\langle \lvert F \rvert \rangle$	$10^{\langle \log \lvert F \rvert \rangle}$
Vertical, strike-slip			
P	0.34	0.28	0.22
S	0.55	0.53	0.50
SV	0.32	0.28	0.25
SH	0.45	0.39	0.32
30° dip, dip-slip			
P	0.55	0.49	0.40
S	0.70	0.67	0.64
SV	0.62	0.57	0.52
SH	0.34	0.29	0.23
45° dip, oblique-slip			
P	0.51	0.43	0.32
S	0.66	0.62	0.57
SV	0.54	0.48	0.41
SH	0.39	0.34	0.28

to 0.20 as the water level increases from 0.05 to 0.20. For dipping faults, the largest effect is for *P*- and *S*-wave radiation coefficients at close distances, with increases of up to a factor of 1.5 as the water level is increased.

Although the averaged radiation coefficients presented here differ from the commonly used ones by amounts comparable to the scatter in many seismological measurements, their use can avoid systematic errors in derived quantities that might result from the use of the coefficients given in equations (1) and (2). Not only should the effect of fault type and portion of the focal sphere being sampled be taken into consideration, but also for most seismological applications the average of the logarithm of the radiation patterns, rather than the square, should be used.

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U.S. Geological Survey
345 Middlefield Road, Mail Stop 977
Menlo Park, California 94025

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