Averaging Spacetime: Where do we go from here?

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The Idealized Universe

- Universe is assumed to be homogeneous and isotropic on very large scales.
- Some observational data to support these assumptions [CMB, Galaxy Surveys, etc].
- GR results in a Universe described by a single function of time $R(t)$.
- Mathematically elegant.

but ...

- There is structure on smaller scales.
- The smaller the scale, the larger the inhomogeneity.
- What are the effects of these inhomogeneities on our smoothed out idealized model?
- Can we ignore these effects?
- How do we model these effects?

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The Big Problem

- Einstein's Field Equations (EFEs);

$$G_{\alpha\beta}(g) = \kappa T_{\alpha\beta}$$

  considered a success on solar system scales
- for larger scales, not quite so
- for cosmology, RHS commonly modeled as a fluid on very large scales.
- **The Big Problem:** an averaging/smoothing procedure has been employed without a corresponding averaging/smoothing procedure on the LHS.

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The Big Solution?

- Shirokov and Fisher (63): early recognition of the problem
- Ellis (84): Detailed description of the issues related to the problem
- Both suggested modified gravitational equations for cosmology,

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Further Issues: Problem A

How does one average tensor fields on a manifold?

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- How can one relate $\overline{G}_{\alpha\beta}[g]$ with $G_{\alpha\beta}[\overline{g}]$?
- Can we simply assume $\overline{G}_{\alpha\beta}[g] = G_{\alpha\beta}[\overline{g}]$?
- **NO**, due to non-linearity of the EFEs.
- **Solution** Both S+F and E, introduce a Gravitational Correlation Tensor $C_{\alpha\beta}$

$$G_{\alpha\beta}[\overline{g}] + C_{\alpha\beta} = \kappa T^{\rm fluid}_{\alpha\beta}$$

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Further Issues: Problem C

What is the nature of the gravitational correlation $C_{\alpha\beta}$?

Shirokov and Fisher(63)

- Appears to be the first to propose a solution
- Employed spacetime averaging procedure

$$\overline{T}^{\alpha}_{\beta}(x) = \frac{\int_{\xi \in \Sigma_x} T^{\alpha}_{\beta}(x+\xi)\sqrt{-g(x+\xi)}\, d^4\xi}{\int_{\xi \in \Sigma_x} \sqrt{-g(x+\xi)}\, d^4\xi}$$

- x is the location of the macro-observer (center of averaging region)
- ξ is the location of the micro-observer with respect to x
- Perturbatively determined the nature of the gravitational correlation
- Weakness: non-covariant and perturbative

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Isaacson(68)

- Main interest in Gravitational Radiation, not cosmology
- Appears to be the first to use a covariant averaging procedure

$$\overline{T}^{\alpha}_{\beta}(x) = \int_{all\ space} g^{\alpha'}_{\alpha}(x,x')g^{\beta}_{\beta'}(x,x')T^{\alpha'}_{\beta'}(x')f(x,x')\,d^4x'$$

- $f(x,x')$ is a weighting function
- $\int_{all\ space} f(x,x')\,d^4x' = 1$
- $g^{\alpha'}_{\alpha}(x,x')$ is the parallel propagator along geodesics.
- Perturbatively determined the nature of the gravitational correlation
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- Claims to extends Isaacson's result by averaging the RHS of EFEs, but
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- Gauge invariant proposal for averaging
- Uses a Window Function : Similar to Isaacsons $f(x, x')$ function
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Zalaletdinov(92)

- Similar to Isaacson
- Assumes bi-local transport operators $\mathcal{A}_{\alpha}^{\alpha'}(x, x')$
- Defines the spacetime averaging operation

$$\overline{T}_{\alpha}^{\beta}(x) = \frac{\int_{x' \in \Sigma_x} \mathcal{A}_{\alpha}^{\alpha'}(x, x') \mathcal{A}_{\beta'}^{\beta}(x, x') T_{\beta'}^{\alpha'}(x') \sqrt{-g(x')}\, d^4x'}{\int_{x' \in \Sigma_x} \sqrt{-g(x')}\, d^4x'}$$

- Apply averaging procedure to Cartan structure equations,
- Determines an averaged spacetime by defining $\Gamma^{\alpha}_{\beta\gamma} = \left\langle \mathcal{F}^{\alpha}_{\beta\gamma} \right\rangle$ to be LC connection for this space
- $\mathcal{F}^{\alpha}_{\beta\gamma}$ a bi-local extension of the LC connection of the original manifold.

Zalaletdinov(92) cont.

- Defines a 2nd order Connection Correlation tensor,
- Few more assumptions to obtain splitting rules for products of Riemann and metric
- Apply averaging procedure to EFE's
- Complete set of field equations including a new field, with its own set of equations
- Gravitational Correlation determined exactly (non-perturbative)
- Weakness: existence of bi-local transport operators

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Futamase(88,89,96)

- Futamase(88,89),
  - noncovariant averaging procedure,
  - perturbative determination of the gravitational correlation
- Futamase(96),
  - used geodesic parallel propagator on 3-surface
  - perturbative determination of the gravitational correlation

Kasai(93)

- Foliates spacetime by flow orthogonal hypersurfaces with 3-metric g_{ij}
- Assumes inhomogeneous dust model,
- Averages the energy density only
- Defines 3 correlations [Extrinsic Curvature, Ricci 3-Curvature, Density Contrast]
- Determines conditions for the EFE's of inhomogeneous models to have the form of a dust FRW on average.
- If met, some of the correlations are determined
- Weakness: Part of gravitational correlation is assumed zero

Buchert(00,01)

- Foliate spacetime by flow orthogonal hypersurfaces with 3-metric g_{ij}
- Define a spatial averaging operation suitable for scalars

$$\overline{T}(X^i, t) = \frac{1}{V_D} \int_D T(X^i, t)\sqrt{det(g_{ij})}d^3X$$

- No fixed background
- Define volume scale factor $a_D(t) = \left(\frac{V_D(t)}{V_{D_0}}\right)^{1/3}$
- Apply averaging procedure to scalar parts of the EFEs
- Yields 2 scalar equations for three unknowns
- Weakness: Not Closed: Ignores the tensorial parts of the EFE's

- Boersma (98)
  - Defines a general averaging operator $\hat{A}$
  - Assumes FRW is a stable fixed point of the averaging operator
  - Shows that linearized averaging operation for metric perturbations, can be defined as a spatial averaging operation for scalars applied to δg_{00} and δg^i_i in synchronous coordinates
- Paranjape and Singh (07)
  - Spatial averaging limit of Zalaletdinov averaging
  - More general but agrees with Buchert averaging

Other Promising Approaches

- Debasch (04), ensemble averaging, no gravitational correlation
- Sussman (08), defines quasi-local scalars, and averages scalar EFE's, similar to Buchert
- Behrend (08), spacetime averaging of maximally smooth tetrad field to determine average metric
- Hehl and Mashoon (08), Non-local gravity, $GR_{||}$, a causal spacetime averaged theory of gravity
- Khosravi, Mansouri and Kourkchi (08), Preliminary ideas of “on” and “in” Light Cone Averaging
- Coley (09) Discusses the need for Lightcone Averaging: Averages the Raychaudhuri Equation on the Null Cone

Averaging and parallel transport

- Averaging involves integration/summation of tensor fields
- Not straightforward on an arbitrary affinely connected and curved manifold.
- How to add tensor fields which are located at finitely separated points?
- Requires a notion of parallel transport of a tensor at a point x' along some curve C to a base point x in a unique and well defined manner.

Well defined transportation procedure

A well defined a unique transportation will require either

1. transportation along well defined curves: e.g. geodesics or
2. the transportation should be independent of the path

Selection of Unique Curve: Geodesic

- select unique curve, in this case, the geodesic,
- appears “natural”, as there are no other “natural” curves that connect x' and x.
- in Riemannian space, the geodesic is the shortest and straightest path connecting points x' and x.
- (we assume a unique geodesic exists connecting x' and x)
- the **elementary parallel propagators** no longer depend on an arbitrary curve and are functions of the endpoints x' and x.
- these special parallel propagators are denoted with a lower case g, i.e, $g_w(x, x'), g^{\alpha}_{\alpha'}(x, x'), g^{\alpha'}_{\alpha}(x, x')$

Path Independent Parallel Transport

- Parallel Transport is independent of path iff curvature of the connection is zero.
- cannot use the Levi-Cevita connection
- employ a different connection, one in particular that has zero curvature
- Let $e_i{}^{\alpha}$ $(i = 1, \ldots, n)$ be n linearly independent vector fields
- Assume covariantly constant with respect to some unknown connection
- This requirement uniquely defines an affine connection $W^{\alpha}{}_{\beta\gamma} = e_i{}^{\alpha} e^i{}_{\beta,\gamma}$
- The result is a Weitzenbock connection.

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Form of the parallel propagators in a T_n

In this case, the **elementary parallel propagators** are factorable and can be shown to have the form

$$P_w(x,x') = \left(\frac{e(x)}{e(x')}\right)^w \qquad e = det(e_i{}^\alpha) \tag{1}$$

$$P^\alpha_{\alpha'}(x,x') = e_i{}^\alpha(x) e^i{}_{\alpha'}(x') \tag{2}$$

$$P^{\alpha'}_\alpha(x,x') = e^i{}_\alpha(x) e_i{}^{\alpha'}(x') \tag{3}$$

Basically the frame components of any tensor are invariant under parallel transport with respect to $W^\alpha_{\beta\gamma}$.

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Our Options

Two options in developing a well defined covariant averaging procedure. Parallel transport along geodesic,

- Curve uniquely chosen
- Parallel transported along geodesic with respect to Levi-Cevita connection
- Use $g^{\alpha}_{\alpha'}$ as the transporter
- Approach used by Isaacson

or ...

Our Options

Path Independent transportation

- Parallel transported with respect to the Weitzenbock connection
- Use $P^{\alpha}_{\alpha'}$ as the transporter
- closely resembles the approach of Zalaletdinov

One can now integrate vector and/or tensor fields over compact regions of the manifold, and consequently, can define an averaging procedure.

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Averaging/Smoothing Procedure (Path Independent)

Definition (Averaging/Smoothing Procedure)

Let $\mathcal{M}$ be a simply connected metric manifold. Let $T^{\alpha}_{\beta}(x)$ be a continuous tensor field defined on some simply connected region $\mathcal{R} \subset \mathcal{M}$. Let Σ_x be a compact subset of $\mathcal{R}$ at supporting point x. We define the average of the tensor field $T^{\alpha}_{\beta}(x)$, denoted $\overline{T}^{\alpha}_{\beta}(x)$, as the definite integral at supporting point x,

$$\overline{T}^{\alpha}_{\beta}(x) \equiv \frac{1}{V_{\Sigma_x}} \int_{x' \in \Sigma_x} P^{\alpha}_{\alpha'}(x,x') P^{\beta'}_{\beta}(x,x') T^{\alpha'}_{\beta'}(x') \sqrt{-g(x')} d^4x'$$

where

$$V_{\Sigma_x} = \int_{x' \in \Sigma_x} \sqrt{-g(x')} d^4x'$$

Averaging/Smoothing Procedure (Geodesic)

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$$V_{\Sigma_x} = \int_{x' \in \Sigma_x} \sqrt{-g(x')} d^4 x'$$

Comments

- In either procedure, $\overline{g_{\alpha\beta}} = g_{\alpha\beta}$
- Bonus: Constant Curvature spacetimes are fixed points of either procedure
- Does it make sense to average the metric?
- Which geometrical object of the micro geometry when averaged, yields information about the macro geometry?
- Levi-Cevita connection? Possibly.
- Perhaps it is $R^{\alpha}_{\beta\gamma\delta}(g)$? Better possibility?
- Perhaps it is the Kontosion tensor?
- Illustrated an averaging procedure for tensor fields (**Problem A), have not averaged the EFE's (Problem C), so more work to do**

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- If GR is appropriate for the solar system, then what is the effective change to the Einstein Field Equations upon averaging?
- Should result be GR plus bits, or a new theory?
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- The gravitational correlation (polarization) should it be determined
  - through perturbative techniques, or
  - assigned via geometrical assumptions a la Zalaletdinov? or
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- Symmetries and Averaging?
- Is the average of a bundle of null geodesics, a null geodesic? What about Causality?
- Should we use a fully covariant spacetime averaging procedure, or one better suited to cosmology (1+3 split).

Unanswered Questions and Other Issues

- Can the inhomogeneities in the un-averaged geometry manifest an effective acceleration in the averaged geometry?
- Cosmology is tested with observations, and observations take place down the Null Cone: Should we not be averaging down the Null Cone?
- The Fitting Problem.

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DISCUSSION