Avoiding closed timelike curves with a collapsing rotating null dust shell

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Introduction

- Several solutions to EFE display CTCs: van Stockum, Gödel, NUT, Gott (usually associated to rotation).

- Cosmic censorship implies that for reasonable matter one cannot evolve CTCs from generic initial data, since maximal development is globally hyperbolic and generically inextendible.

- Example: what happens if one tries to create a spinning cosmic string from an incoming rotating cylindrical null shell?

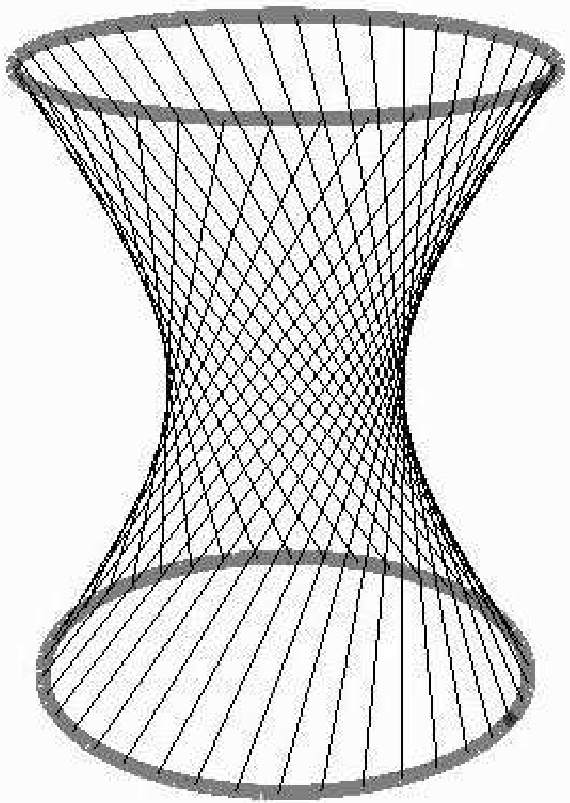
Hyperboloid in Minkowski space

- Parameterize hyperboloid $-\tau^2 + \xi^2 + \eta^2 = a^2$ in Minkowski space $g^- = -d\tau^2 + d\xi^2 + d\eta^2$ by

$$\begin{cases} \tau = u \\ \xi = a\cos\psi - u\sin\psi \\ \eta = a\sin\psi + u\cos\psi \end{cases}$$

- Induced metric is

$$h^- = 2a\,du\,d\psi + (u^2 + a^2)\,d\psi^2$$



Rotating cosmic string

- Rotating cosmic string is given by the metric

$$g^{+} = -(dt + m d\varphi)^2 + C^2 dr^2 + r^2 d\varphi^2$$

It is just flat space with unusual identifications.

- CTCs for $r^2 < m^2$.

- Mass and angular momentum per unit length are

$$\mu = \frac{C-1}{4C} \qquad\qquad J = \frac{m}{4}$$

Hyperboloid in rotating cosmic string metric

- Null geodesics are parameterized by

$$\begin{cases} t = bC\tan\lambda - mC\lambda \\ r = b\sec\lambda \\ \varphi = C\lambda + \psi \end{cases}$$

- In the new set coordinates $\{\lambda, b, \psi\}$ the metric becomes

$$g^{+} = 2bC(b-m)\sec^2\lambda\, d\lambda\, d\psi + C^2 db^2 - 2mC\tan\lambda\, db\, d\psi + (b^2\sec^2\lambda - m^2)\, d\psi^2$$

- The metric induced on a hypersurface $\{b = \text{constant}\}$ is

$$h^{+} = 2bC(b-m)\sec^2\lambda\, d\lambda\, d\psi + (b^2\sec^2\lambda - m^2)\, d\psi^2$$

or setting $u = b\tan\lambda$

$$h^{+} = 2C(b-m)\, du\, d\psi + (u^2 + b^2 - m^2)\, d\psi^2$$

- Matching conditions are

$$\begin{cases} a = C(b-m) \\ a^2 = b^2 - m^2 \end{cases}$$

In particular that the matching requires $b > m$, so that the shell bounces before CTCs are revealed in the exterior.

Shell matter

- Computing the jump on the second fundamental form one arrives at

$$T^{\alpha\beta}\partial_\alpha \otimes \partial_\beta = \frac{m}{8\pi C a\rho}\,\delta\left(\rho - \sqrt{\tau^2 + a^2}\right)\frac{\partial}{\partial u} \otimes \frac{\partial}{\partial u}$$

where $\rho^2 = \xi^2 + \eta^2$.

- Therefore matter is a null dust with surface density

$$\sigma = \frac{m}{8\pi a C\rho} = \frac{C^2 - 1}{16\pi C^2\rho} = \frac{(C+1)}{2C}\frac{\mu}{2\pi\rho}$$

- Similarly

$$-T^{\alpha\beta}\left(\frac{\partial}{\partial\tau}\right)_{\alpha}\left(\frac{\partial}{\partial\varphi}\right)_{\beta} = a\sigma\delta\left(\rho - \sqrt{\tau^2 + a^2}\right)$$

corresponding to a surface angular momentum density

$$j = a\sigma = \frac{m}{8\pi C\rho} = \frac{1}{C}\frac{J}{2\pi\rho}$$

- Densities slightly puzzling. Notice that solution is not stationary (so one cannot use Komar integrals).