

Awareness of the light field: the case of deformation

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Abstract. Human observers group local shading patterns into global super-patterns that appear to be illuminated in some unitary fashion. Many years ago, this was noticed for the case of uniform, unidirectional illumination. Recently, we found that it also applies to convergent and divergent illumination flows, but that human observers are blind to rotational light flow patterns (in the sense of being unable to group the local shading patterns). We now report that human observers are also blind to deformation patterns. This is perhaps interesting because convergent, divergent, rotational, and deformation patterns all occur in natural light fields. This is an idiosyncrasy of the human visual system, on par with the fact that visual awareness fails to present the observer with saddle shapes.

Keywords: shading, light fields, ambiguity, grouping, depth, shape.

1 Introduction

The conventional stimulus⁽¹⁾ in shape from shading research for about a century has been a circular disk filled with a linear intensity gradient (Metzger, 1975; Ramachandran, 1988a, 1988b). It represents most likely an attempt to catch the essential information in shading patterns, which is supposed to be the first-order intensity variation in a small neighborhood about a point (Turhan, 1935). The result is somewhat troublesome, because of the effect of supposedly irrelevant factors. For instance, not just the intensity gradient determines the resulting perception, the circular outline is just as important (a square outline yields a very different perception; Wagemans, van Doorn, & Koenderink, 2010). However, this basic pattern has been adopted in a large volume of research, which is the reason why we used it in the present study (Figure 1).

Most observers report that the basic stimulus appears to them as a pictorial relief, modulated in depth. A small group of observers sees a flat intensity gradient (we are not in a position to put a number on this, from our experience, we would guess at least one out of ten). Observers who report a depth articulation declare to see either a “cap” (convexity, like the outside of an eggshell) or a “cup”

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⁽¹⁾ We say “conventional stimulus,” because this is the stimulus that pervades the literature on “shape from shading” from the earliest times on. There is no doubt it was (and is) used because conceived of as “the minimal stimulus”. However, this it is not. The conventional stimulus is made up of a localized linear gradient (which indeed might count as a “minimal stimulus”) and a circular disk with a sharp edge. The edge typically accounts for much of the shape impressions evoked by the stimulus. For instance, changing it to a square evokes a cylindrical, instead of spherical shape, and changing it to triangular induces a conical shape. The true “minimal” stimulus would be a localized gradient in the appropriate formal sense (scale space; Koenderink, 1984), thus an intensity distribution modulated with the first derivative of a symmetric Gaussian function. It is hardly ever used although because the edge is so important in inducing a phenomenal three-dimensional shape impression (Erens, Kappers, & Koenderink, 1993), that is to say, because of the mixed nature of the conventional stimulus.

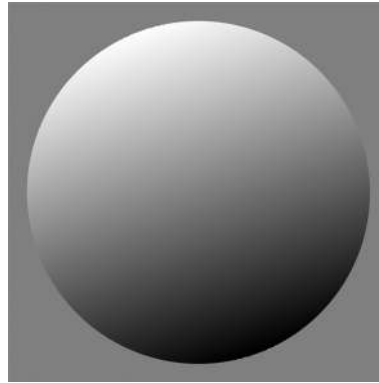


Figure 1. The conventional stimulus pattern used in psychophysical shape from shading research. It is a circular disk, filled with a linear intensity gradient, placed on a background of the average intensity of the disk.

(concavity, like the inside of an eggshell). The depth ambiguity has been familiar for a long time (Rittenhouse, 1786). As we have argued elsewhere, the fact that only caps or cups are reported is perhaps surprising, since inverse optics shows that cylinders (either convex or concave) and saddle shapes are equally valid interpretations. Here “valid” refers to “Shape From Shading” (SFS) theory, based on standard radiometry (Boyd, 1983; Lambert, 1760; Moon & Spencer, 1981). SFS has a large literature; we mention only Belhumeur, Kriegman, & Yuille (1999), Forsyth & Ponce (2002), Horn & Brooks (1989), van Diggelen (1959). From a Bayesian viewpoint, saddles should be the preferred interpretation (Koenderink & van Doorn, 2003b). Whereas cylinders are indeed reported for a square outline, saddle shapes are (in our experience) never spontaneously reported. The “cap or cup” decision is often determined by the direction of the gradient, which is possibly interpreted as the direction of illumination in the “scene”.

It is well known that a group of such basic stimulus patterns tends to “synchronize” (Kleffner & Ramachandran 1992). That is to say, when you view a group of such stimuli with gradient directions all lined up, one tends to be aware of a group of all caps or all cups (Figure 2, left). If there is an outlier in a flock of caps, it is often reported as a cup and vice versa (Figure 2, second from the left). This finding is also related to the work on the perception of light flow direction (Koenderink & Pont 2003; Koenderink and van Doorn, 1996; Koenderink, van Doorn, & Pont, 2004; Koenderink, van Doorn, Kappers, te Pas, & Pont, 2003; Koenderink, Pont, van Doorn, Kappers, & Todd, 2007; Pont & Koenderink, 2003, 2004). In the Appendix, we explain the concept of “light field” and “surface illuminance flow” in some detail. Roughly speaking, the “light field” describes the transport of radiant power, whereas the “surface illuminance flow” describes the aspect of the illumination that photometrically reveals the roughness of the illuminated surface.

Recently, we have found (van Doorn, Koenderink, & Wagemans, 2011) that this “synchronization” (a kind of “grouping”) also occurs for divergent and convergent illuminance flows (Figure 2, third from left) but not for rotational illuminance flows (Figure 2, right). This is interesting, because all first-order singularities of the illuminance flow field (i.e., divergence, rotational, and deformation fields) occur in the daily environment (Mury, Pont, & Koenderink, 2007, 2009a, 2009b). Thus, the apparent selective sensitivity appears perhaps surprising. In any case, it is a nontrivial property of the

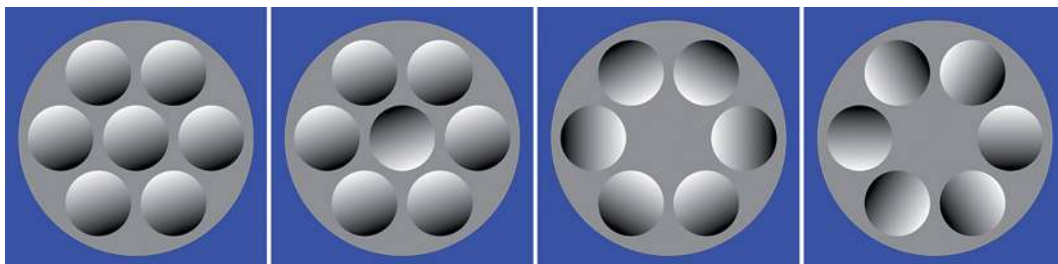


Figure 2. Left, a group of basic stimuli with the same gradient direction (uniform illuminance flow). Second from left has an outlier at the center. Third from left has concurrent gradient directions (divergent illuminance flow). Right, a group of basic stimuli with a rotational illuminance flow.

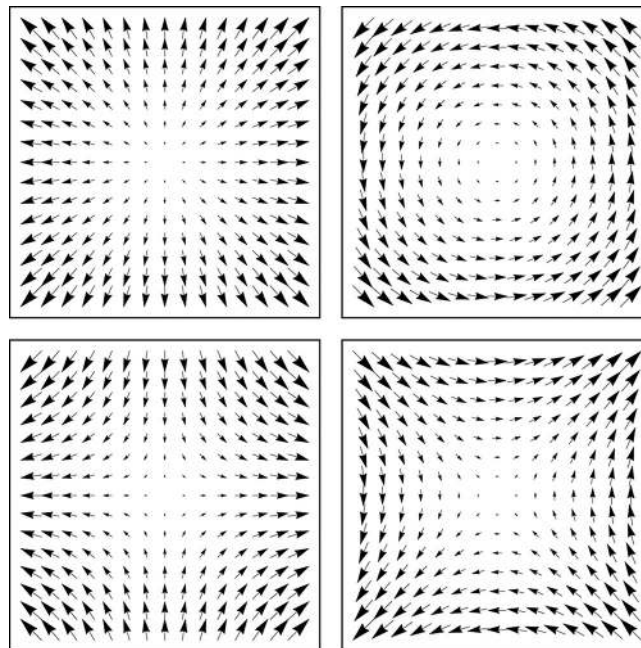


Figure 3. A basis for the first-order structure of the illuminance flow. Any first-order structure can be expressed as a linear combination of these components. At top left one has a divergence, at top right a rotation, in the bottom row, two mutually independent deformation patterns. The divergence and rotation are specified by magnitude alone, the deformations also have an orientation.

structures in human visual awareness. The stimuli shown in [Figure 2](#) may be understood as abstractions from naturally occurring patterns, rendered suitable for a structured empirical investigation.

In [Figure 3](#), we show the basis for the linear (local) approximation of illuminance flow fields. The zeroth-order, unidirectional, uniform field can be locally deformed by linear combinations of these four patterns. When the uniform flow vanishes, at singular points of the illuminance flow field, one generically has a purely linear flow pattern. All these components occur in the natural environment. Thus, it makes sense to find out whether deformation patterns can group (like divergent patterns) or not (like rotational patterns).

In this paper, we test whether human observers are able to group deformation patterns as they do uniform or divergent patterns or whether they are blind to these as they are to cyclic (rotational) patterns.

2 Methods

2.1 Observers

Observers AD, JK, JT, and JW were the authors, whereas EG, MS, RK, and SC were naive to the issue. We do not consider the addition of a group of naive observers to be essential, because we are interested in a possible form of selective blindness. If a group of experienced observers turns out to be unable to group deformation patterns, it is hard to imagine why this would be different for naive observers. Moreover, the advantage of the experienced observers is that they are very well documented as observers in various subdomains of visual perception. Three (AD, JK, and JW) were also observers in a previous, related study (van Doorn et al., [2011](#)). One (JT) has a known tendency to report only “caps” (never “cups”) when confronted with the conventional stimulus pattern. As it turned out to be the case (see Results), both groups of observers yielded equivalent results.

2.2 Stimulus design

The stimulus design was similar to that used in an experiment upon which we have reported before (van Doorn et al., [2011](#)). The only difference is the number of basic stimulus patterns placed on the annulus ([Figure 4](#)), which is eight in the present and six in the previous experiment. The number is important, because the deformation stimuli cannot be represented very well using only six samples, whereas eight samples suffice.

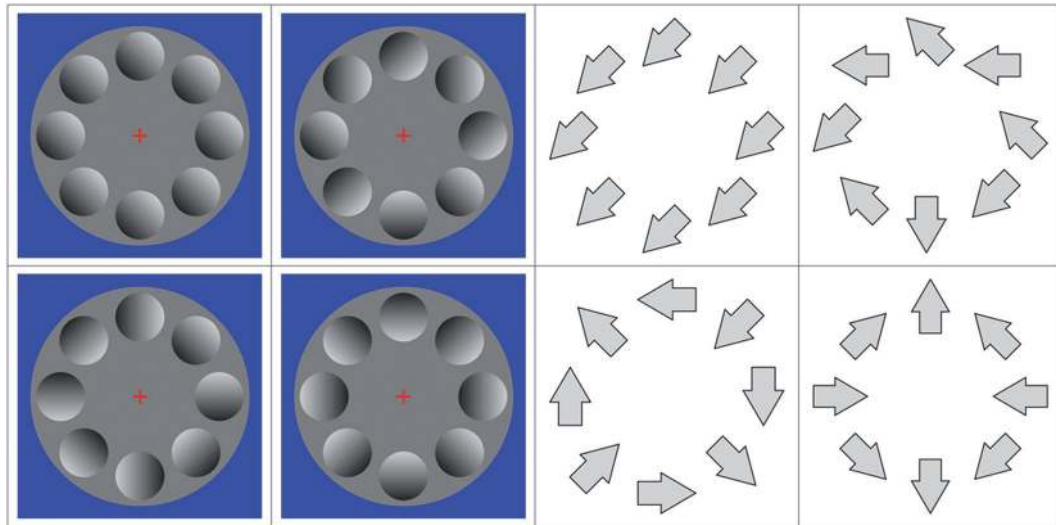


Figure 4. Left, The stimulus patterns used in the experiment. At top left a uniform field, at top right a random field (of course this is just one instance, all instances are different). The bottom row shows two deformation fields with different orientations. The red cross is used for fixation. Right, the same patterns, but shading gradient indicated with arrows.

3 Experiments

We report on two related experiments. Both potentially yield an answer to our research question, although the second experiment perhaps allows some additional conclusions.

3.1 Simultaneous task

In the “simultaneous task,” we presented stimulus patterns like those shown in Figure 4, randomly rotated over some multiple of 45° . The observer fixated the red cross, then was presented with the pattern for 2 s, after which the observer had to indicate whether the eight patterns were either

- all caps,
- all cups,
- not all the same.

This is a simple, overall decision. When all subpatterns are either cup or cap, the whole pattern looks like it was made of one piece, it becomes a single Gestalt.

The patterns were presented on a computer screen. The background gray disk was 15 cm in

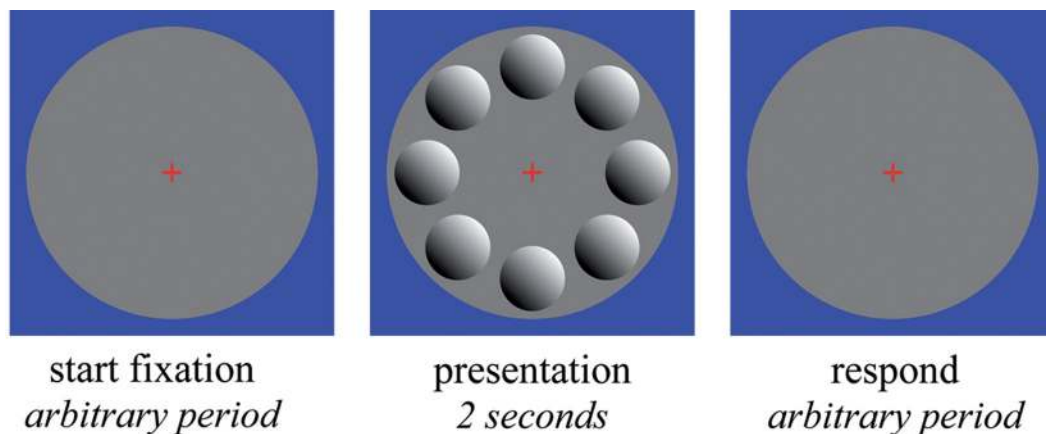


Figure 5. The presentation sequence for the simultaneous task. Each trial starts with a blank plus fixation mark. This enables the observer to fixate the red mark. The observer initiates the next presentation, which presents the stimulus pattern for 2 s. In the next phase, the blank pattern is on until the observer has responded. The response indicates a triadic judgment: all caps, all cups, or mixed.

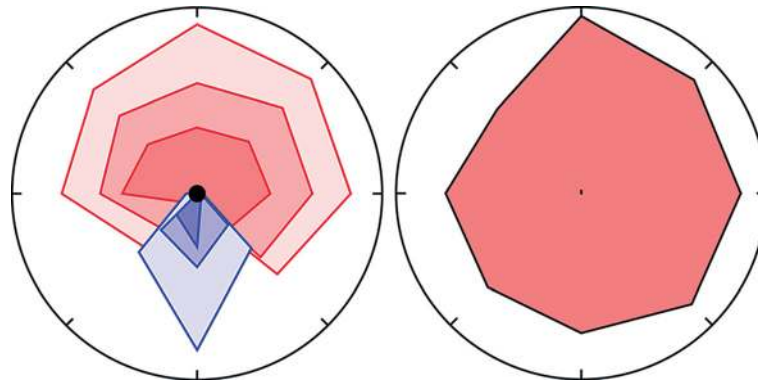


Figure 6. At left, the polar frequency plot for reporting “all caps” (red) or “all cups” (blue) as a function of the direction of the gradient. The concentric areas show the quartiles over all observers. Notice the decided preference for cap over cup. At right, the plot for observer JT. This observer is special in that he never reports awareness of all cups.

diameter and binocularly viewed from 50 cm. Room lights were dimmed. The trials were self-paced by the observers (See [Figure 5](#).)

Observers were tested with 500 presentations in all (1,000 AD, JK, JT, and JW), equally distributed over uniform, random, and deformation fields, in random order. In the case, the observer felt fully uncertain, the trial could be dumped, and another one initiated. This happened in not more than about one in 100 cases. Overall, all observers judged the task as “easy but dull”. All but one observers completed the task in one session, observer JW needed two sessions.

For the uniform patterns, we checked how the all cap/cup report correlated with the direction of the gradient ([Figure 6](#)). We find the expected pattern for the observers. Outlier is observer JT, who (as we knew) will never report all cups. The remaining observers have a strong but idiosyncratic preference for cap over cup.

3.1.1 Conclusions simultaneous task

The results of responding “caps”, “cups”, or “different” are shown as a triangle plot in [Figure 7](#). One has to judge whether the deformation results are more like the random or the uniform results. The answer is immediately obvious. The result for the deformation patterns is like that for the random patterns, and very different from that for the uniform patterns.

The conclusion is that human observers fail to group deformation into a single Gestalt. The

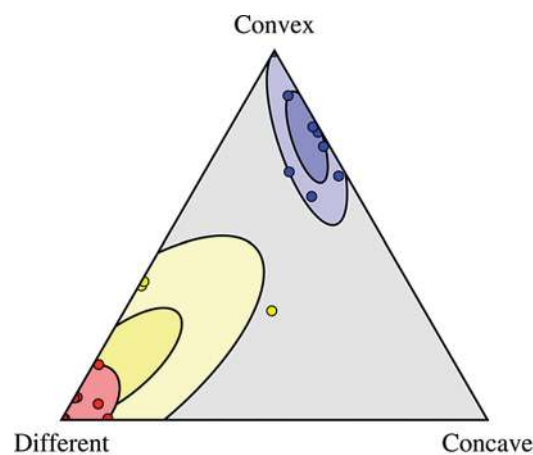


Figure 7. Triangle plot of responding convex (“all caps”), concave (“all cups”), or “different”. The elliptical areas show the 95% and 50% probability areas, the dots show individual cases. The bluish area corresponds to the uniform pattern, the reddish area to the random patterns, whereas the yellowish area represents the deformation patterns. Conclusion is that the results for the deformation patterns resemble those for the random, rather than the uniform patterns.

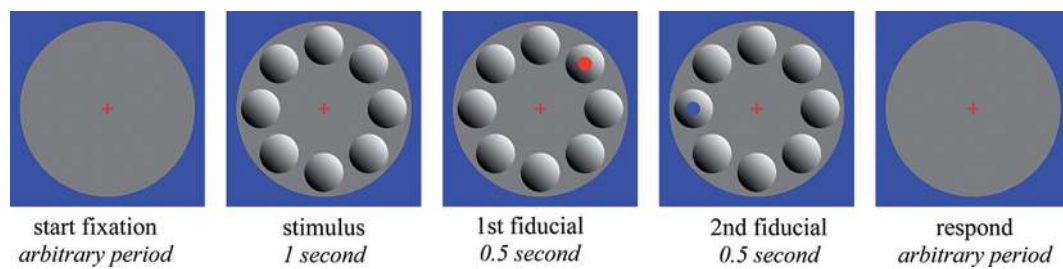


Figure 8. The sequence of presentations in a trial of the successive task. The trial starts with a blank plus fixation mark, enabling the observer to fixate. The observer initiates the following three presentations. First the stimulus pattern is presented for 1 s. Then, follow two one-half-second presentations in which the fiducial patterns are identified: the first through a red, the second through a blue dot. The fourth presentation is blank again. It ends when the observer has responded. The response involves a cap-cup decision for the first (red) pattern and one for the second (blue pattern). Thus, possible responses are cap-cap, cap-cup, cup-cap, and cup-cup.

deformation patterns look qualitatively different from the uniform patterns (and, to the three observers who participated in an earlier experiment, to the convergence or divergence patterns). They look more like the random patterns (and again, to the three observers who participated in an earlier experiment, to the rotation patterns).

3.2 Successive task

In the “successive task,” we again presented stimulus patterns like those shown in [Figure 4](#). The observer fixated the red cross and was then presented with the pattern for 1 s. After this, there came two successive periods of half a second each. In each period, one subpattern was indicated with an easily visible dot. For the first period, the dot was red; for the second, it was blue. In this task, the observer has to indicate, for both the first and the second interval, whether the indicated subpattern was judged to be a cap or a cup. Thus, there are four possible responses, namely *cap-cap*, *cup-cup*, *cap-cup* or *cup-cap*. In the former two cases, the two partial responses are correlated; in the latter two cases, they are anti-correlated. (See [Figure 8](#).)

The timing of the presentations is important. Naive observers initially tend to complain that the presentation is fast, although they evidently are able to handle it. Experienced observers sometimes complain of the slowness of the presentations because they notice occasional cap-cup or cup-cap flips which interfere with the task. The timing that we eventually settled on is somewhat of a compromise. The temporal intervals were chosen such that it is unlikely that the second fiducial would have flipped with respect to its appearance during the indication of the first fiducial. We could draw on our previous experience here. The probability of a flip is less than a percent or so.

The patterns were presented on a computer screen. They were again 15 cm in diameter and binocularly viewed from 50 cm. Room lights were dimmed. The trials were self-paced by the observers. Again, random, uniform, and deformation patterns were generated with equal probability. They were randomly rotated over multiples of 45°. We used a total of 500 presentations (1,000 for AD, JK, JT, and JW). This task is rather more difficult than the simultaneous task. In a few percent of cases, the observer failed to see the first or second indication. Such cases could be discarded, and an additional trial generated.

In the case of observer JT, who never reports to be aware of cups, the “cup” judgment was replaced with a “not cap” judgment. Judging from the results, this at least partly succeeded. At least the random and deformation cases are evidently distinct from the uniform case. Observer JT reports to be aware of pictorial relief; he apparently does not merely respond to the gradient directions. In the case of caps, these appear as convincingly three-dimensional; in the case of cups, these are “less convincing” and might as well be interpreted as planar gradients.

What is interesting from a phenomenological viewpoint is that the observers apparently used the relief appearance, rather than the (two-dimensional) gradient representation. It happens occasionally that one is aware of a cap-cap event, whereas (in reflective afterthought) one notices that the gradients were opposite. In such (rare) cases, the microgenetic process possibly assumed different local illumination directions for the first and second fiducial subpattern.

From the results, we may compute autocorrelation functions (see [Figure 9](#)). These specify precisely how seeing cap or cup at the second presentation depends upon cap or cup at the first presenta-

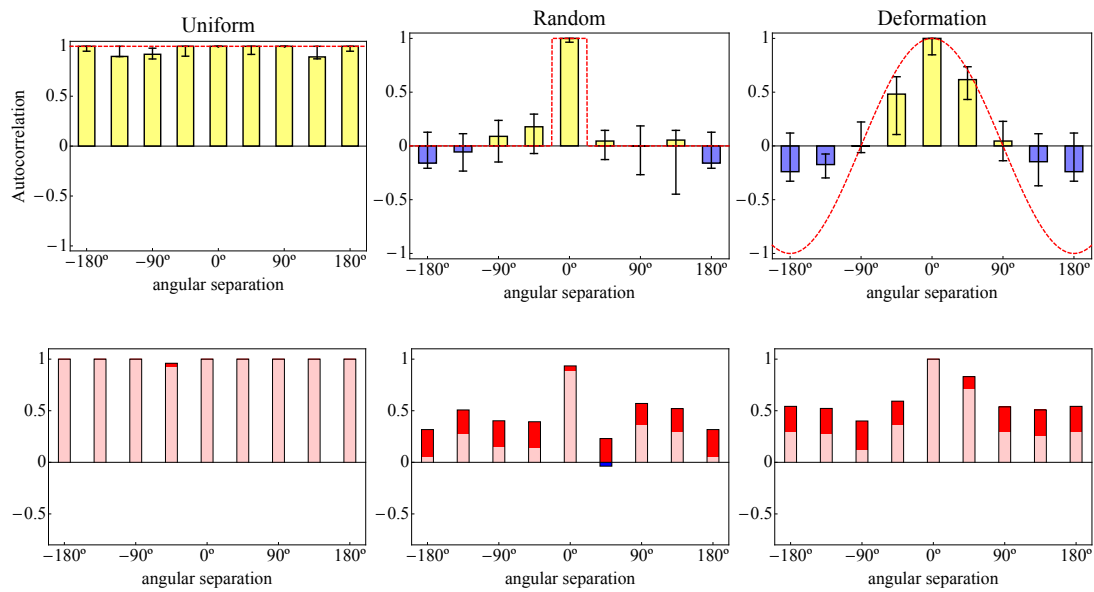


Figure 9. At top, the median autocorrelation functions for all observers except JT and the three stimuli (from left to right: uniform, random, and deformation). The error bars indicate the interquartile ranges. The correlations specify how the judgment at the second location depends on that at the first location, as a function of the angular distance between the locations. (notice that the outermost positions are repeats of the opposite location.) Yellow indicates positive, blue negative (anti-) correlation. The dashed red lines indicate naive predictions. At bottom, the autocorrelation functions for observer JT (the 95% significance intervals are indicated in red). This observer evidently shows a different pattern. JT's results for the uniform patterns are not different from those for the other observers. His results for the random and deformation patterns are different but do not contradict our overall conclusions.

tion, as a function of the angular separation of the subpatterns.

3.2.1 Conclusions successive task

The results from the successive task—at the expense of some hard labor—much more detailed than those of the simultaneous task. A cursory glance shows that the deformation case is much like the random and very much unlike the uniform case. Thus, the conclusion that human observers appear to be blind to the deformation patterns is immediate, without the need for any significance testing.

Of course, one has certain predictions. In the case of the uniform pattern, we expect the appearance to be either all caps or all cups. Thus, one expects the autocorrelation function to be equal to one (perfect correlation) for all angular separations. This expectation is borne out to a surprising extent.

For the random case, one expects (of course) the correlation to be perfect if the angular separation is zero. Anything less would imply cap-cup flips within a half-second interval. This is pretty much borne out. One also expects correlations for non-zero angular intervals to be essentially (within the random scatter) zero. Perhaps surprisingly, this is not borne out. Except for observer JT, who fails to report cups in any case, observers tend to (minor) anti-correlation, except for the smallest angular separations, for which there is a (minor) tendency to positive correlation. We do not understand these (minor) trends, which are evidently systematic.

For the deformation case, one expects (in case grouping would fail to pertain) antipodes (around 180° angular separation) to anti-correlate separations of plus or minus ninety degrees not to correlate at all. Except for the (special) case of observer JT, this appears to be borne out by the data. However, the difference with the random case (which we fail to fully understand) is only slight, so we can hardly conclude otherwise than that the deformation case is very much like the random one.

4 Overall conclusions

The overall conclusion from these experiments is clear-cut: *human observers do not group deformation patterns like they do uniform, convergent or divergent ones*. We have not been able to robustly differentiate the results for the deformation patterns from the results of the random patterns. They are categorically different from the results for the uniform patterns though. Thus, the research question posed at the initiation of the experiments has been definitively answered.

There remain interesting unanswered questions. For instance, it is unclear how the anti-correlations for non-zero angular separations in the random case arise. We guess that this has to do with local grouping processes (for angular separations of less than half the annulus) that mutually compete to settle the global grouping. The “grouping” seems to be accompanied by an “anti-grouping”. We have met similar effects in a previous experiment for the case of rotations. This appears to be a worthy target for experiments yielding more detailed data. Such experiments are not likely to be easy though in view of the fact that it took several decades to move from preliminary qualitative observations to detailed quantitative data. There is clearly more detail to be probed. For instance, our observers occasionally spontaneously noticed a dissociation between pictorial relief (cap-cup distinction) and visual field (gradient direction) properties. It might be possible to probe this.

A side issue, which is nevertheless of interest, is the difference between observer JT and the others. All observers are aware of compellingly three-dimensional *caps*, but whereas all but one observer are also aware of equally compelling *cups*, observer JT is not. Whereas the “cups” are obviously “non-caps” (which is why JT was able to complete the experiment), they are not compelling as three-dimensional objects but might as well be described as planar figures. In the past, we have encountered similar observers, as well as observers who failed to see any conventional stimulus as convincingly three-dimensional. In two cases, we met with reviewers who declared that they failed to “get the illusion”, meaning that they were not visually aware of caps or cups. Apparently, one may distinguish among various groups of distinct observers. There appears to be no literature on the issue; it seems a potentially rewarding research topic.

In conclusion, the human visual system appears to be idiosyncratic in that it easily groups unidirectional, convergent, and divergent illuminance flow patterns but fully ignores rotational and deformation patterns. This is the case even though the first-order patterns all occur in the natural environment (Mury et al., 2007, 2009b). This “selective blindness” is reminiscent of the inability to be visually aware of the conventional stimulus pattern as anything else but cap or cup (Wagemans et al., 2010). In the case of surface illuminance flow (Pont & Koenderink 2003, 2004), we actually know that the saddles which are ignored are actually more abundant in the natural environment than the caps and cups taken together (Koenderink & van Doorn, 2003b). To understand such idiosyncrasies, one will need to study the affordances of these patterns in further detail than has been attempted so far. Perhaps insights from the arts, rather than from classical physics will prove useful there (Baxandall, 2005; Jacobs, 1988; Luckiesh, 1916; Koenderink & van Doorn, 2003a).

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References

- Adams, A. (1952). *Natural light photography*. Hastings-on-Hudson, NY: Morgan and Morgan.
[doi:10.2307/362172](https://doi.org/10.2307/362172) ◀
- Baxandall, M. (2005). *Shadows and enlightenment* (2nd ed.). London, England: Yale University Press. ◀
- Belhumeur, P. N., Kriegman, D. J., & Yuille, A. L. (1999). The bas-relief ambiguity. *International Journal of Computer Vision*, 35, 33–44. [doi:10.1023/A:1008154927611](https://doi.org/10.1023/A:1008154927611) ◀
- Boyd, R. W. (1983). *Radiometry and the detection of optical radiation*. New York, NY: Wiley. ◀
- Erens, R. G. F., Kappers, A. M. L., & Koenderink, J. J. (1993). Perception of local shape from shading. *Perception & Psychophysics*, 54, 145–156. [doi:10.3758/BF03211750](https://doi.org/10.3758/BF03211750) ◀
- Euler, L. (1736). Solutio problematis ad geometriam situs pertinentis. *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, 8, 128–140. ◀
- Foley, J. D., van Dam, A., Feiner, S. K., & Hughes, J. F. (1990). *Computer graphics: Principles and practice*. Reading, MA: Addison-Wesley. ◀
- Forsyth, D. A., & Ponce, J. (2002). *Computer vision: A modern approach*. Upper Saddle River, NJ: Prentice-Hall. ◀
- Gershun, A. (1936). The light field (translated by P. Moon and G. Timoshenko, 1939). *Journal of Mathematical Physics*, 18, 51–151. ◀

- Horn, B. K. P., & Brooks, M. J. (Eds.). (1989). *Shape from shading*. Cambridge, MA: MIT Press. ◀
- Hunter, F., & Fuqua, P. (1990). *Light, Science & Magic*. Boston, MA: Focal Press. ◀
- Jacobs, T. S. (1988). *Light for the artist*. New York, NY: Watson-Guption. ◀
- Kleffner, D. A., & Ramachandran, V. S. (1992). On the perception of shape from shading. *Perception & Psychophysics*, *52*, 18–36. doi:10.3758/BF03206757 ◀
- Koenderink, J. J. (1984). The structure of images. *Biological Cybernetics*, *50*, 363–370. doi:10.1007/BF00336961 ◀
- Koenderink, J. J., & Pont, S. C. (2002). Texture at the terminator. In G. M. Cortelazzo, & C. Guerra (Eds.). *Proceedings of the First International Symposium on 3D Data Processing Visualization and Transmission*. (pp. 406–415). Los Alamitos, CA: IEEE Computer Society. doi:10.1109/TDPVT.2002.1024096 ◀
- Koenderink, J. J., & Pont, S. C. (2003). Irradiation direction from texture. *Journal of the Optical Society of America A*, *20*, 1875–1882. doi:10.1364/JOSAA.20.001875 ◀
- Koenderink, J. J., & van Doorn, A. J. (1983). Geometrical modes as a general method to treat diffuse interreflections in radiometry. *Journal of the Optical Society of America A*, *73*, 843–850. doi:10.1364/JOSA.73.000843 ◀
- Koenderink, J. J., & van Doorn, A. J. (1996). Illuminance texture due to surface mesostructure. *Journal of the Optical Society of America A*, *13*, 452–463. doi:10.1364/JOSAA.13.000452 ◀
- Koenderink, J. J., & van Doorn, A. J., (2003a). Shape and shading. In L. M. Chalupa, & J. S. Werner (Eds.). *The visual neurosciences* (pp. 1090–1105). Cambridge, MA: MIT Press. ◀
- Koenderink, J. J., & van Doorn, A. J., (2003b). Local structure of Gaussian texture. *IEEE Transactions on Information and Systems*, *86*, 1165–1171. ◀
- Koenderink, J. J., Pont, S. C., van Doorn, A. J., Kappers, A. M. L., & Todd, J. T. (2007). The visual light field. *Perception*, *36*, 1595–1610. doi:10.1167/7.9.288 ◀
- Koenderink, J. J., van Doorn, A. J., Kappers, A. M. L., te Pas, S. F., & Pont, S. C. (2003). Illumination direction from texture shading. *Journal of the Optical Society of America A*, *20*, 987–995. doi:10.1364/JOSAA.20.000987 ◀
- Koenderink, J. J., van Doorn, A. J., & Pont, S. C. (2004). Light direction from shad(ow)ed random Gaussian surfaces. *Perception*, *33*, 1405–1420. doi:10.1068/p5287 ◀
- Lambert, J. H. (1760). *Photometria, sive, De mensura et gradibus luminis, colorum et umbrae*. Augsburg, Germany: V. E. Klett. ◀
- Luckiesh, M. (1916). *Light and shade and their applications*. New York, NY: Van Nostrand. ◀
- Metzger, W. (1975). *Gesetze des Sehens*. Frankfurt AM, Germany: Verlag Waldemar Kramer. ◀
- Moon, P., & Spencer, D. E. (1981). *The photic field*. Cambridge, MA: MIT Press. doi:10.1177/109114218100900110 ◀
- Morse, M. (1934). *The calculus of variations in the large* (Vol 18). New York, NY: American Mathematical Society Colloquium Publication. ◀
- Mury, A. A., Pont, S. C., & Koenderink, J. J. (2007). Spatial properties of light fields in natural scenes. In S. N. Spencer (Ed.). *Proceedings of the 4th Applied Perception in Graphics and Visualization (APGV 2007), ACM SIGGRAPH* (pp. 140). New York, NY: ACM. doi:10.1364/AO.46.007308 ◀
- Mury, A. A., Pont, S. C., & Koenderink, J. J. (2009a). Representing the light field in finite three-dimensional spaces from sparse discrete samples. *Applied Optics*, *48*, 450–457. doi:10.1364/AO.48.000450 ◀
- Mury, A. A., Pont, S. C., & Koenderink, J. J., (2009b). Structure of light fields in natural scenes. *Applied Optics*, *48*, 5386–5395. doi:10.1364/AO.48.005386 ◀
- Pont, S. C., & Koenderink, J. J. (2003). Illuminance flow. In N. Petkov, & M. A. Wetsenberg (Eds.). *Computer analysis of images and patterns* (LNCS 2756). (pp. 90–97). Berlin/Heidelberg, Germany: Springer-Verlag. ◀
- Pont, S. C., & Koenderink, J. J. (2004). Surface illuminance flow. In Y. Aloimonos, & G. Taubin (Eds.). *Proceedings of the Second International Symposium on 3D Data Processing Visualization and Transmission* (pp. 2–9). Los Alamitos, CA: IEEE Computer Society. ◀
- Ramachandran, V. S. (1988a). Perceiving shape from shading. *Scientific American*, *258*, 76–83. doi:10.1038/scientificamerican0888-76 ◀
- Ramachandran, V. S. (1988b). Perception of shape from shading. *Nature*, *331*, 163–166. doi:10.1038/331163a0 ◀
- Rittenhouse, D. (1786). Explanation of an optical deception. *Transactions of the American Philosophical Society*, *2*, 37–42. doi:10.2307/1005164 ◀
- Turhan, M. (1935). Über räumliche Wirkungen von Helligkeitsgefällen. *Psychologische Forschung*, *21*, 1–49. ◀
- van Diggelen, J. (1959). Photometric properties of lunar crater floors. *Recherches Astronomiques de l'Observatoire d'Utrecht*, *14*, 1–114. ◀
- van Doorn, A. J., Koenderink, J. J., & Wagemans, J. (2011). Light fields and shape from shading. *Journal of Vision*, *11*, 1–21. doi:10.1167/11.3.21 ◀
- Wagemans, J., van Doorn, A. J., & Koenderink, J. J. (2010). The shading cue in context. *i-Perception*, *1*, 159–177. doi:10.1068/i0401 ◀

Appendix: “Surface Irradiance Flow”

Chiaroscuro is mainly due to the radiation scattered by surfaces in the scene. In an “ideal” scene, all objects have been painted a matte white (“Lambertian BRDF”; Lambert, 1760). Such scenes have been used by photographers (Hunter & Fuqua, 1990), to learn lighting, and by visual artists (using plaster models), to learn “shading” (Jacobs, 1988). In an ideal image of such a scene, the sources of radiation are not included. Such cases virtually never occur in real life, but they are a good approximation *modulo* a variety of “special effects” like object color, specularities (highlights), translucencies, reflexes, vignetting, air light, and so forth (Jacobs, 1988).

The ideal case will do fine for this appendix. We mention only basic facts. These are, as a matter of course, thoroughly understood by anyone working on the psychophysics of shading and light fields, which is why we put them in an appendix. We do not pretend to discuss anything novel.

For a collimated source, the radiance scattered to the eye is proportional to the irradiance of the scattering surface element, which is again proportional to the normal irradiance caused by the incident beam, and the obliqueness of the surface element with respect to the source (“Lambert’s cosine law”; Lambert, 1760). The normal irradiance caused by the incident beam is proportional to the radiance of the source and (for a smallish source) the solid angle subtended by the source as seen from the surface element (Horn & Brooks 1989; Moon & Spencer 1981). These facts are sufficient to determine the “shading”, for instance, to compute a computer graphics rendering (Foley, van Dam, Feiner, & Hughes, 1990).

Shading proper is the radiance scattered to the eye by surface elements in the scene. The notion of “surface element” needs to be made more precise though. For instance, a roughly plastered wall has surface elements that all have the spatial orientation of the wall itself, but on a finer scale, one becomes aware of a random distribution of spatial attitudes that we will denote “microfacets” (Koenderink & van Doorn, 1996). These microfacets are variously shaded, according to their spatial attitudes, and give rise to shading texture (Koenderink & Pont 2003; Koenderink & van Doorn 1996). Such texture is categorically different from the “wall paper texture” due to spatial albedo variations. The shading proper is a local average over a sample of many texture elements. The structure of the texture depends upon the direction of the irradiating beam, that is, on the tangential (surface) component of the irradiation direction, not on its normal component (which determines the shading proper; Koenderink & Pont 2003). The field of tangential components depends both on the source distribution and on the shape of the surface. A planar wall in sunlight will receive a uniform, unidirectional “surface irradiance flow”, whereas a spherical surface under the same illumination will receive a diverging flow. (The center of divergence being the point where the sphere is normally irradiated, the tangential component vanishing at that point).

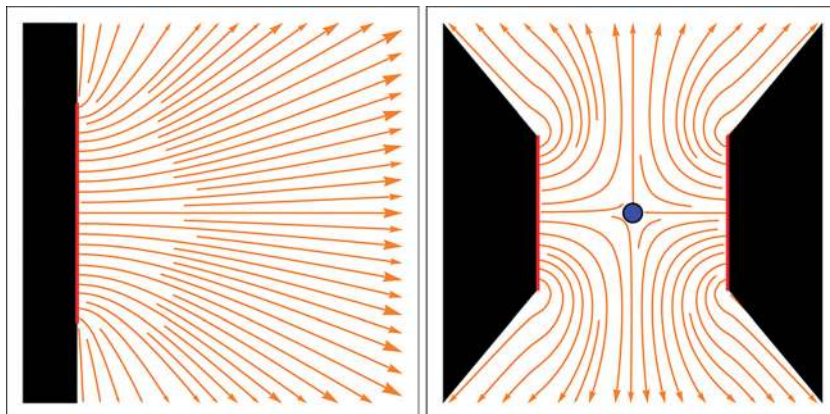


Figure A1. Examples of light fields. At left, the field due to a diffusely radiating plane (e.g., a window with diffusing screen), indicated with the red bar, at right two opposite, otherwise identical radiating planes. The black regions are in shadow (we assume that the surfaces radiate only into a single half-space). Notice that—unlike “light rays”—the field lines of the light field are generically curved. (They can even form closed loops.) They indicate the average direction of transport of radiant power. In the right figure, a deformation singularity is formed at the center (indicated with the blue point), where the two irradiation directions “collide head on”. The light vector vanishes at that point, although the volume density of radiation is high. It lacks direction though.

The texture due to the surface irradiance flow will be of high contrast where the tangential component is large as compared to the normal component, that is, at the terminator of the attached shadow, and low where the tangential component is small as compared to the normal component, that is, near the points that are normally irradiated, that is, face the source (Koenderink & Pont, 2002). The texture due to the surface irradiance flow is crucial for the perception of the nature of material surfaces, a fact that is highly appreciated by photographers (Adams, 1952) For instance, in portrait photography, skin texture is minimized by irradiation from the camera position and maximized by having the radiant source at right angles to the camera direction (Hunter & Fuqua, 1990).

The “light field” (because “light” is a *quale*, this historical term is perhaps an unfortunate one) in space is a formal notion used in radiometric calculations, for instance, in interior architecture (Gershun, 1936; Moon & Spencer 1981). The light field is a field of vectors that specify the average flow of radiant power through arbitrary virtual surface elements. (Here “virtual” indicates the absence of a material surface.) The light field is itself invisible (as all radiation that does not arrive in the eye is invisible) but specifies what would happen if you were to put an object at a certain location. Human observers are visually aware of the light field on the bases of irradiated objects in the scene and have expectations regarding the way a novel object introduced at some location would appear to them. In that sense, they are indeed aware of the light field *in empty space* (Koenderink et

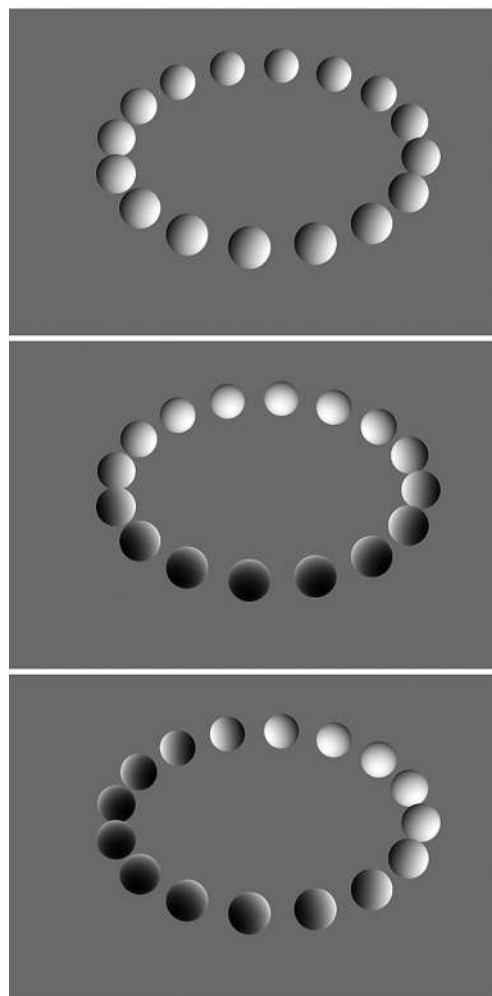


Figure A2. Examples of light fields as revealed by a circular necklace of spherical beads. At top a uniform, unidirectional field, at center a diverging field, at bottom a deformation field (it may take some scrutiny to parse the latter). In the experiment, we offer a “view from the top”.

al, 2007).

The surface irradiance flow exists only on the surfaces of objects in the scene. It can be seen if a surface is rough on the microscale, at least if this scale is coarser than the human visual resolution. On the basis of such texture, human observers are sensitive to the direction of irradiance flow, being able to estimate it to within 10° or so (Koenderink & Pont 2003). If the surfaces are smooth (e.g., plastics, many artificial objects, perhaps most objects used in vision research using screen presentations), the surface irradiance flow is not detectable.

The structure of the light field is determined by the distribution of radiant sources, including “secondary radiators,” that are illuminated surfaces that scatter radiation into all directions available to them. (Of course, this implies “tertiary”, etc., sources. The overall light field involves infinite sums over such contributions; Koenderink & van Doorn, 1983). The light field in a realistic scene has numerous “singular points”, which are of various qualitatively different natures. In planar sections, one has cases of convergent or divergent, whirl (rotational) and deformation fields (Mury et al., 2007). An example is shown in Figure A1. In three dimensions, one has combinations of these. The singularities can be seen when strategically located test objects are placed in the scene. This is illustrated in Figure A2.



Figure A3. Examples of surface irradiance flow. The direction of the uniform, unidirectional irradiating beam is indicated with red arrows. At left, a convexity on which one obtains a divergent flow. At right, a saddle on which one obtains a deformation flow. Along the red curve, the flow approaches the center, along the blue curve, it moves away from the center.

The structure of the surface irradiance flow depends both on the structure of the light field and on the shape of the surface. In the simplest case of uniform, unidirectional irradiation, the flow depends only on the shape of the surface. A sphere in sunlight will yield a divergent field, a spherical cup a convergent field, a saddle surface a deformation field (See Figure A3.) An intuitive way to grasp the structure of the surface irradiance flow is as the flow of water that runs downward along the surface.

The texture will also depend on the nature of the corrugated surface. When the surface is statistically identical to its depth-reversed copy (as with a Gaussian random surface), only the orientation (as opposed to the direction) of the flow is revealed (Koenderink & Pont, 2003; Koenderink et al., 2004). In such a case, it is impossible to make out the difference between a convergent or divergent field. This makes it also very hard to glean a good impression of a deformation field (Figure A4 top row). When the surface is less symmetrical, the direction of surface irradiance flow is revealed (See Figure A5.) In the center column of Figure A4, one sees a divergent field. This image is very similar to the view of a plastered ceiling around a light source mounted on the ceiling. If the surface corrugations are more regular, the texture approaches the case used in the experiment. In the bottom row of Figure A4, one easily spots the uniform, unidirectional field and the divergent field, whereas it is difficult to gain a coherent impression of the deformation field. People’s impressions appear to vary. One common observation is that the convexities tend to invert in parts of the image, depending on the momentary fixation point.

The stimulus in our experiment is a highly simplified configuration that schematizes either a distribution of objects in space or a corrugated surface. Thus, it may be understood as either showing a planar section of the light field or a sample of surface irradiance flow. It does not really matter which, as we are only interested in establishing whether human observers are perceptually aware of

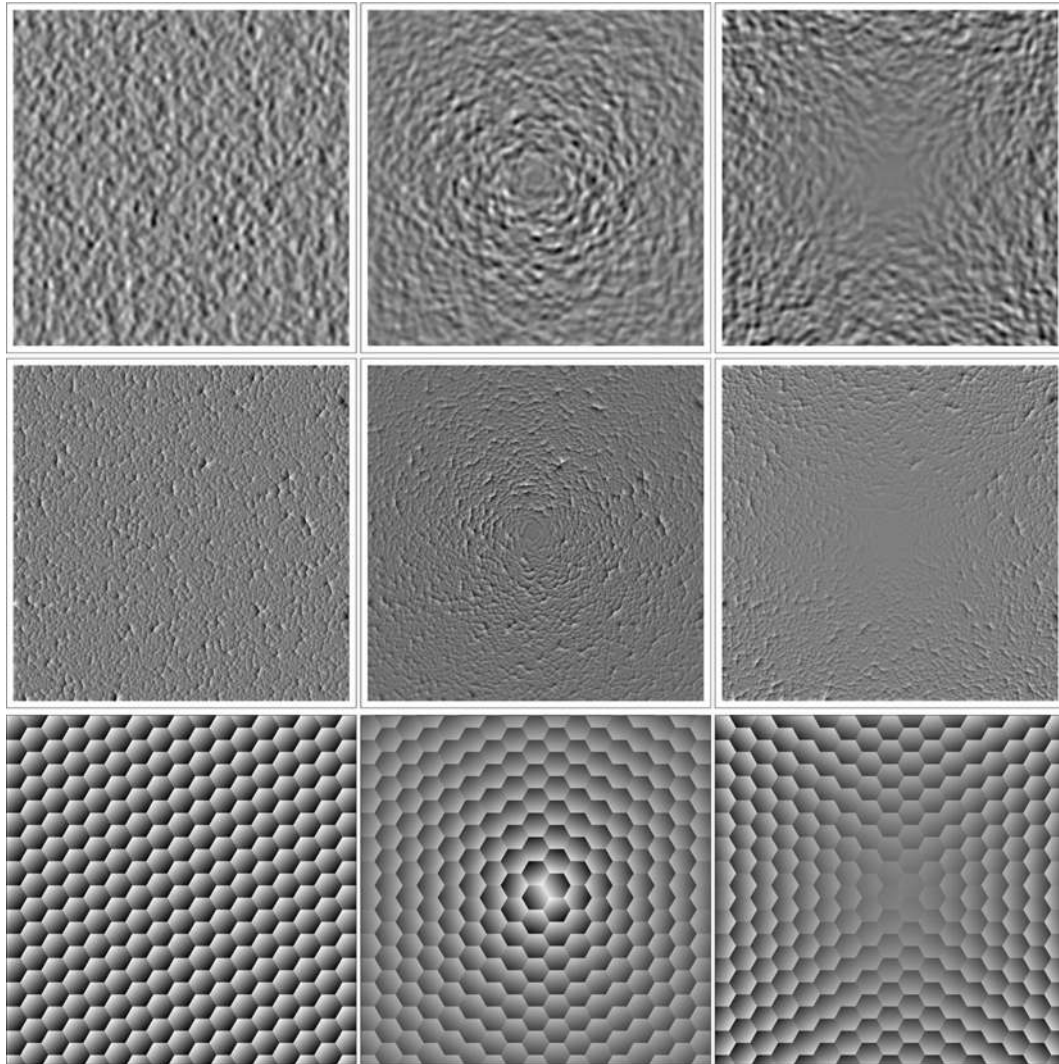


Figure A4. Examples of surface irradiance flows. Left column, a uniform, unidirectional flow; center column, a diverging flow; right column, a deformation flow. The textures are due to a random Gaussian surface (top row), a spherical hills surface (middle row), and a honeycomb pattern of spherical hills.



Figure A5. Cross-section of heights for the surface in the middle row of Figure A4. The top profile shows a sample of the surface itself, the bottom row a sample of a depth-inverted version. The two types are easy enough to distinguish; thus, you can easily notice whether you see the surface (or perhaps only part of the surface) in the images of the middle row of Figure A4 as intended, or reversed. You may want to practice on the leftmost image. You should be able to “get” (in visual awareness) both the correct and the inverted version. In the rightmost image, you may notice correct and inverted patches, shifting as you change your fixation.

certain shading patterns.

How frequent do deformation singularities occur in nature? For the case of surface irradiance flow, one may use a theorem by Morse (1934), which relates the numbers of vergent points (either convergent or divergent) and the number of saddle points (the centers of deformation patterns) to the Euler (1736) characteristic of a surface. For a piece of surface that is topologically equivalent to a disk (no holes of any kind), this characteristic equals one. Then, the number of deformation singularities is one less than the number of vergent patterns. (In this case, whirl patterns do not occur.) Thus, the upshot is that the vergent and deformation patterns are roughly equally frequent. If you consider the patterns on the ground in an open forest, this implies that you will probably have hundreds of deformation patterns in your visual field.



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