Axions and the Strong CP Problem

Jihn E. Kim
Seoul National University

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1. Symmetries (weak CP,
strong CP,
a SUSY example)

2. The strong CP problem

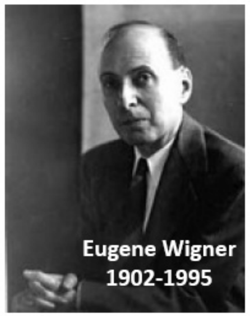
3. Axion physics

(4. Axino CDM)

1. Symmetries

The charge conjugation C and parity P have been known as exact symmetries in atomic physics, i.e. in electromagnetic interactions.

1924: Atomic wave functions are either symmetric or antisymmetric:
Laporte rule



1927: Nature is parity symmetric, Wigner:
Laporte rule = parity symmetric

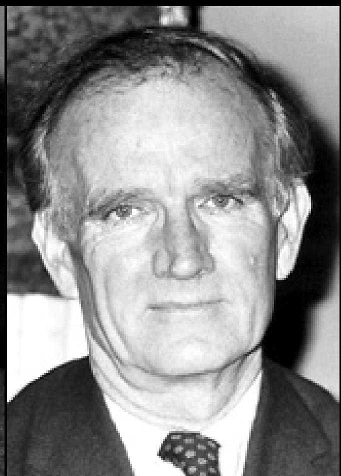
But Parity has been known to be broken, as shown in the tau-theta puzzle. This led to

"P violation in weak interactions"

For the chiral model, we must mention the "V-A" theory of Marshak-Sudarshan(1957); Feynman-Gell-Mann(1958).

In the SM, the P violation in weak interactions is ultimately given at low energy perspective by the Glashow-Salam-Weinberg chiral model of weak interactions.

The charge conjugation C is also broken in the GSW model, but the product CP or T is usually unbroken. T is an anti-unitary operator needing complex conjugation in QFT. So, CP violation observed in the neutral K-meson system needed to introduce a CP violation model with a phase in weak interactions. It is given by the Kobayashi-Maskawa model.

CP violation in weak interactions in the SM, four quark model is not enough but six quarks are needed.

In 1972, u, d, s quarks were known. With four quarks of u,d,s, c, CP violation was attempted by Mohapatra. Sub. to PRD in April, 1972. As far as I know, it was the first try

Because of spin, we can think of LH and RH quarks independently. Only LH quarks participate in the weak CP violation.
This was known in 1977.

$$Q_{em} = \frac{2}{3} \qquad Q_{em} = \frac{-1}{3} \qquad \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix}$$

Not known in 1972

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \qquad \begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \begin{pmatrix} t \\ b' \end{pmatrix}$$

3x3 matrix Has a phase.

Compare KM's submission to Prog.Theor. Phys. on 01.09.1972.

In addition, the quark mixing involves only the LH quarks. It was footnoted by Gell-Mann and Levy in 1961 and suggested as a mixing model by Cabibbo in 1963..

Weak CP Violation

SM: SU(2) x U(1) x SU(3)
chiral model vector model

Seems to have CP violation in weak interactions, but not in strong interactions:

Weak CP violation: good and needed in the K phenomenology and baryogenesis or leptogenesis

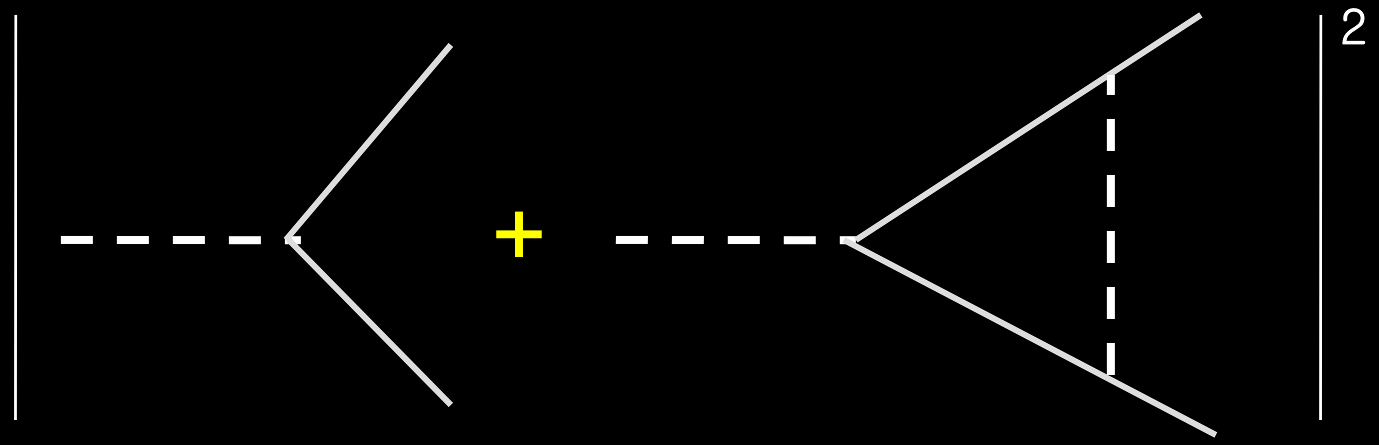
Strong CP violation: not allowed phenomenologically

The observable CP symmetry with a complex field involved is an interference phenomenon, due to the freedom in the definition of the CP phase. Always, we have to look at this freedom of redefinition of phases of complex fields.

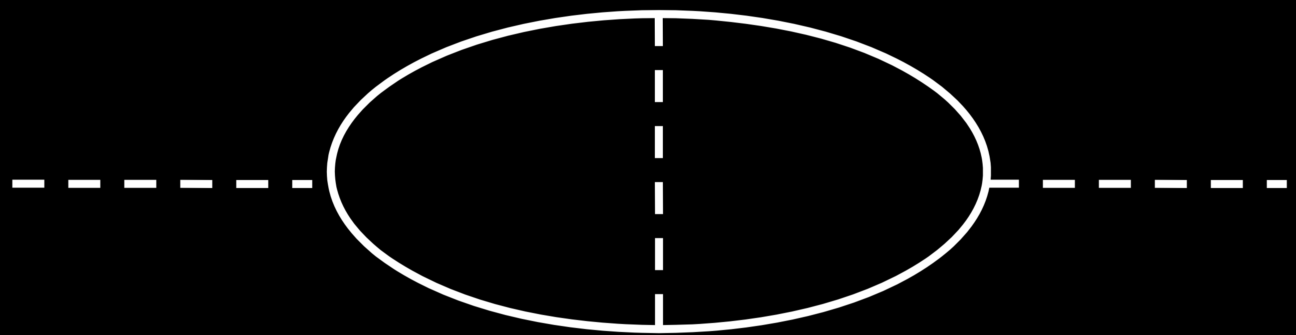
$$(CP)L(CP)^{-1} \to L$$

If there are appropriate CP phases for this to hold, then CP is conserved.

For example, in a heavy particle decay, it is like

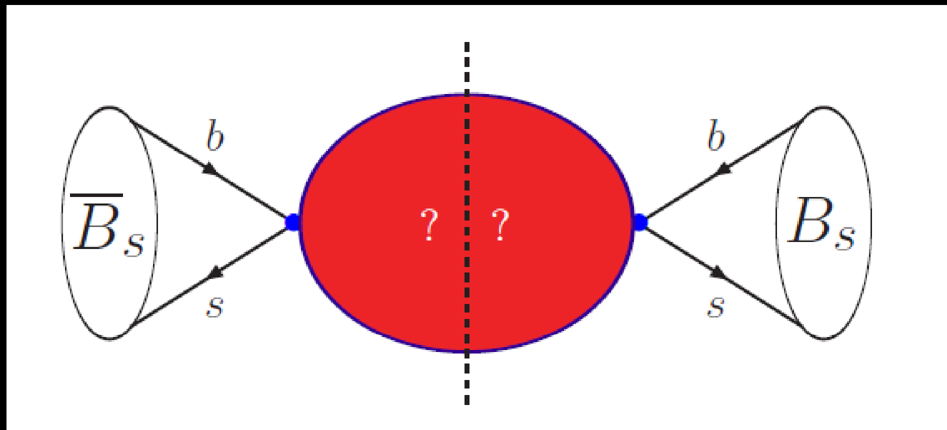


The interference of these two introduces an impossibility of redefining the phases such that the whole thing becomes real.



In the KM model, we need at least three families for this to happen.

For the K and B meson system, this kind of original CP violation is encoded in the effective Lagrangians.



But when we consider only real fields, this interference is not looked for, as considered in

$K_S^0, \quad K_L^0 \quad Decays$

to well defined CP eigenstates

But neutral K mesons are not fundamental, i.e. composite in the SM, and we must consider the interference terms as presented above.

Current Weak CP Issue from D0

In the proton-antiproton machine (Tevatron), if CP is good, we do not expect a particle-antiparticle asymmetry. Considered observables are the same sign di-leptons (*A* parameter) and wrong sign lepton from B or B-bar decay (*a* parameter).

$$A_{sl}^{b} = \frac{N^{++} - N^{--}}{N^{++} + N^{--}}$$

The same sign di-lepton asymmetry

$$a_{sl}^{s} = \frac{\Gamma(\overline{B}_s \to \mu^+ X) - \Gamma(B_s \to \mu^- X)}{\Gamma(\overline{B}_s \to \mu^+ X) + \Gamma(B_s \to \mu^- X)}$$

The wrong sign leptons

These are related, and D0 gives at 1.96 TeV as

$$A_{sl}^{b} = (0.506 \pm 0.043)a_{sl}^{d} + (0.494 \pm 0.043)a_{sl}^{s}$$

If we neglect the wrong sign leptons from B^d, we have

$$a_{sl}^{s} \approx -(19.37 \pm 5.08 \pm 2.96) \times 10^{-3}$$

which is known to be outside the SM prediction. So if we try to interpret it with a new physics, we can try a phenomenological neutral B-meson mass matrix as

$$M_{12} - \frac{i}{2}\Gamma_{12} \to M_{12}^{SM} + M_{12}^{NP} - \frac{i}{2}\left(\Gamma_{12}^{SM} + \Gamma_{12}^{NP}\right)$$

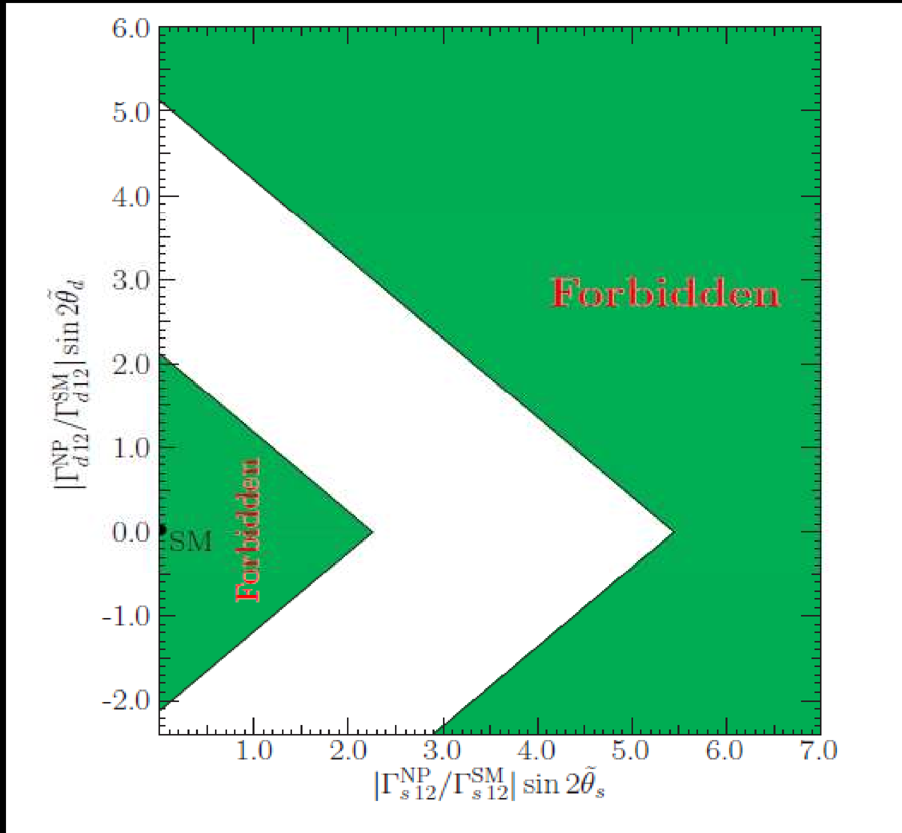
Assuming that M_{q12} has a small NP contribution, i.e. ignoring $|M_{q12}^{NP} / M_{q12}^{SM}|$ and ϕ_{sm}^{q}, [note ϕ_{sm}^{s}= (0.0042+-0.0014)]

$$2\tilde{\theta}_q = relative \quad phase \quad comp. \quad to \quad 2\tilde{\phi}$$
$$2\tilde{\phi}_q = SM \quad phases$$

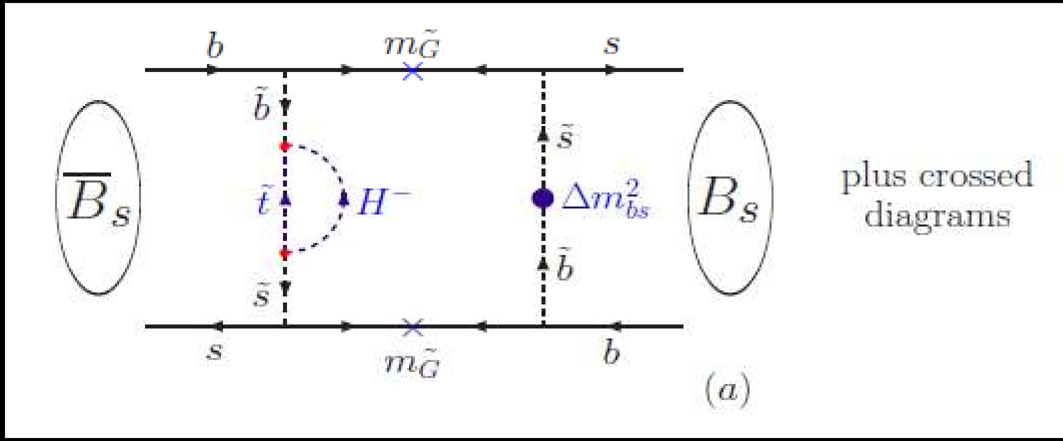
Then,

$$A^b_{sl} \cong -\frac{1}{2}\left|\frac{\Gamma^{NP}_{d12}}{\Gamma^{SM}_{d12}}\right|\sin(2\tilde{\theta}_d)(0.526\times10^{-2}) - \frac{1}{2}\left|\frac{\Gamma^{NP}_{s12}}{\Gamma^{SM}_{s12}}\right|\sin(2\tilde{\theta}_s)(0.497\times10^{-2})$$

The allowed parameter space is given by [K-Seo-Shin]

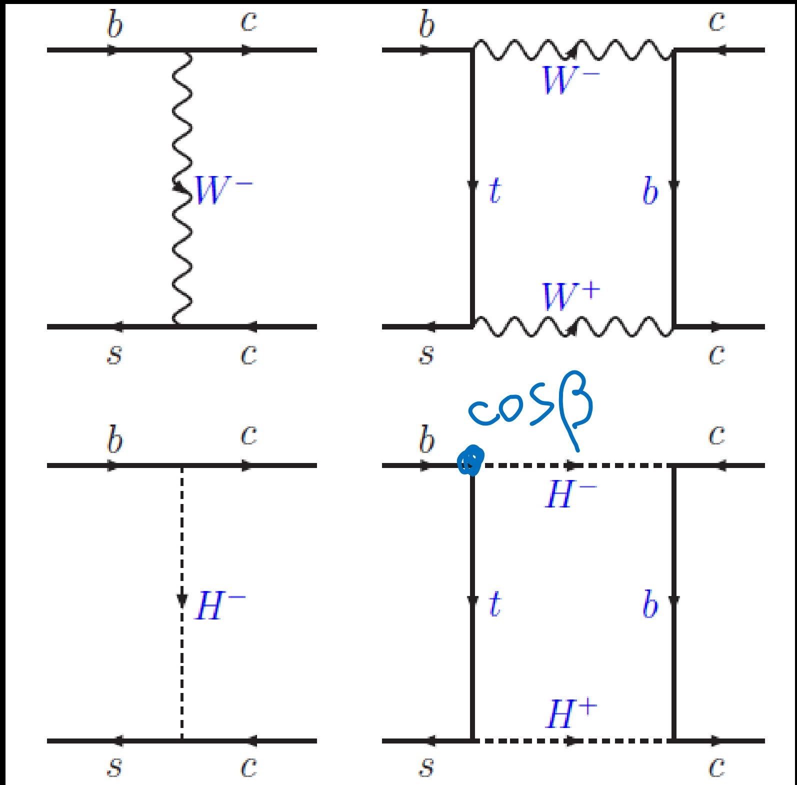


So, both NP Γ and Θ must be appreciable. This is usually hard to achieve. Because CP violation is an interference phenomenon. We show two examples.



In the MSSM, this is small, because it is a two-three loop effect while the SM is one loop effect.

For a two Higgs doublet model, we consider



$$-\cos\beta \quad f_{it}^{(u)*}\,\overline{t}_R\, d_L^i H^+$$
$$-\cos\beta \quad f_{it}^{(u)}\,\overline{d}_L^i\, t_R H^-$$

In the region where tan β is large, cos β is small. Again, THD model is not in a good shape introducing a large imaginary mass.

Just by showing these interferences, let's stop the weak CP discussion.

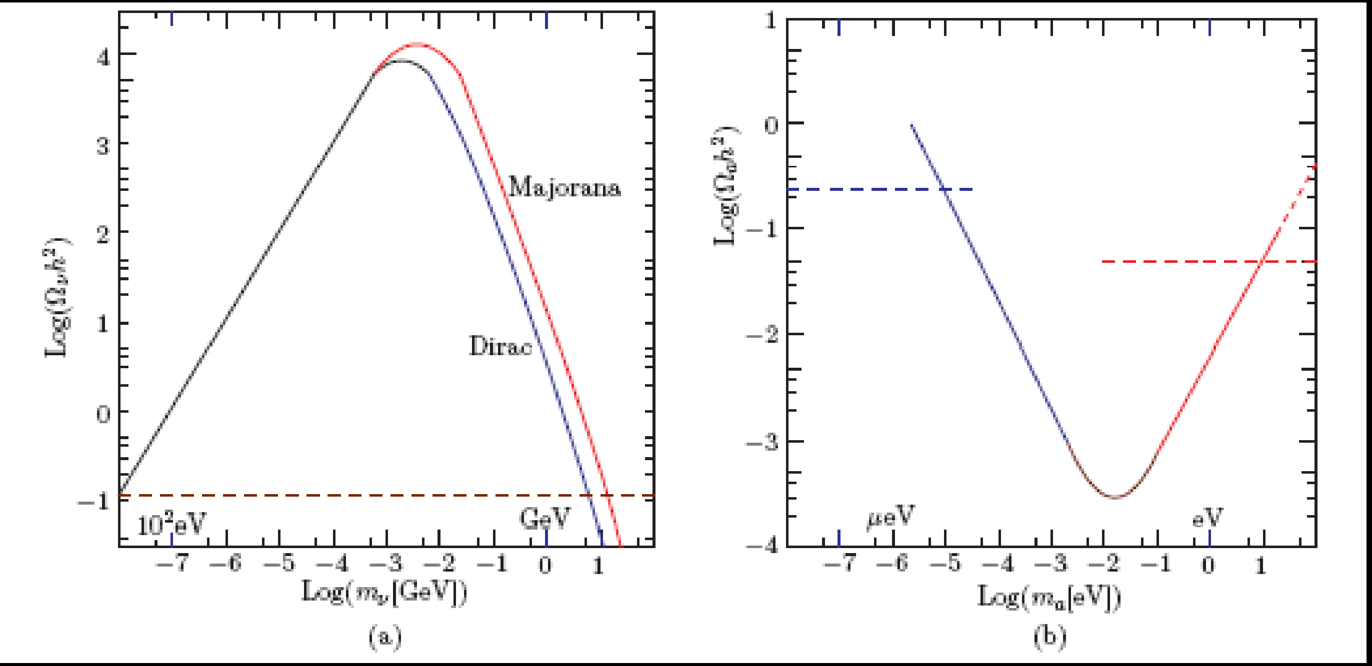
Strong CP

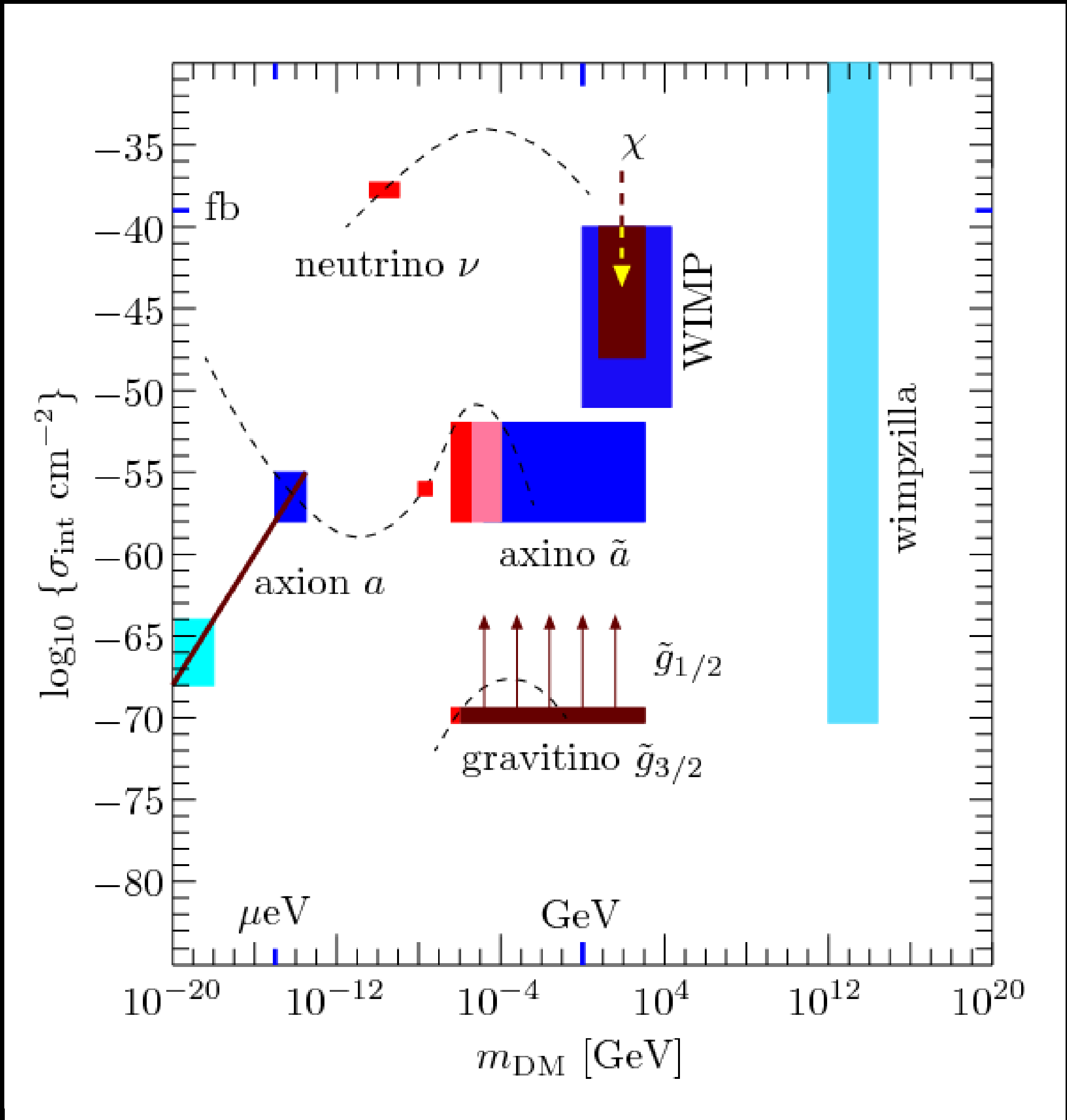
In this Introduction, let me mention just the attractive feature of axion related to DM. Axion is a Goldstone boson arising when the PQ global symmetry is spontaneously broken. The simple form dictates that its interaction is only through the anomaly term(hadronic axion), etc. The axion models have the spontaneous symmetry breaking scale F and the axion decay constant F_a which are related by $F=N_{DW}\,F_a$.

Here, I present the the general idea on axions and then focus on the phenomenology of axion and axino.

The axion cosmic energy density has the opposite behavior from that of WIMP. It is because it is the bosonic collective motion.

Kim-Carosi, "axions and the strong CP problem" RMP 82, 557 (2010) [arXiv:0807.3125]





A rough sketch of masses and cross sections. Bosonic DM with collective motion is always CDM.

[Kim-Carosi with Roszkowski modified]

A recent calculation of the cosmic axion density is,

$$10^9 \text{ GeV} < F_a < \{10^{12} \text{ GeV ?}\}$$

Turner (86), Grin et al (07), Giudice-Kolb-Riotto (08), Bae-Huh-K (JCAP 08, [arXiv:0806.0497]): recalculated including the anharmonic term carefully with the new data on light quark masses.

It is the basis of using the anthropic argument for a large F_a.

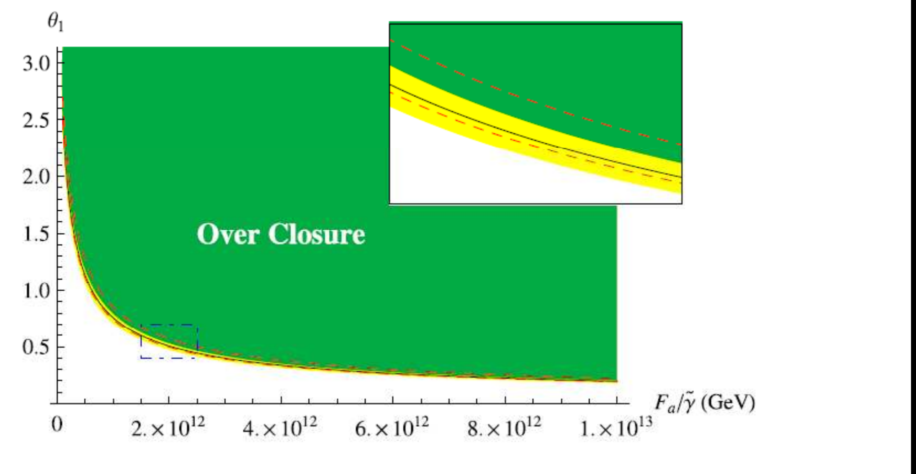
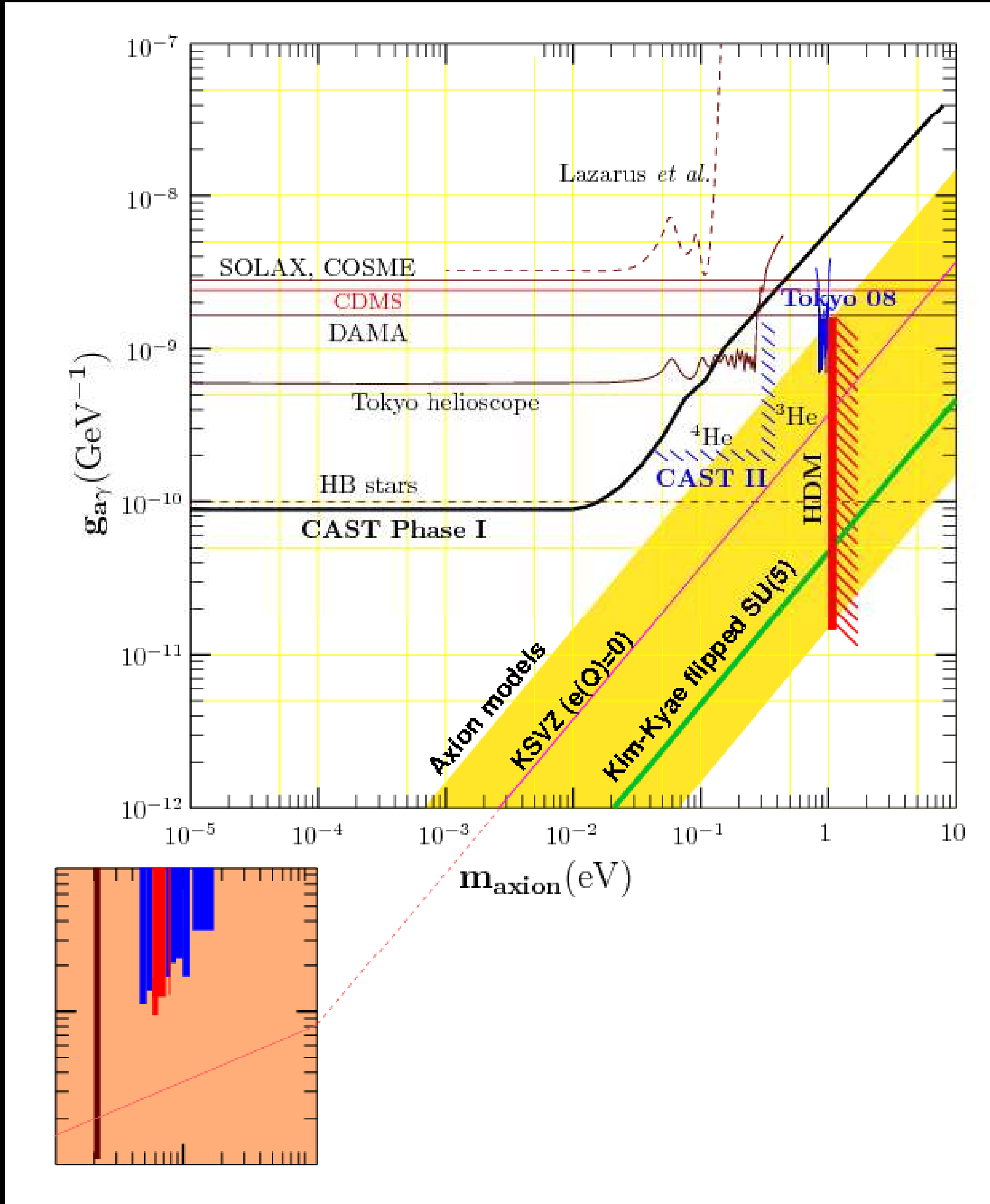


Figure 9. The bound from overclosure of the universe. The yellow band shows the error bars of Λ and two red dashed lines are the limits of the allowed current quark masses. The anharmonic effect is taken into account, including the initial correction factor of equation (15). Here, the entropy production ratio γ is absorbed into the bracket of F_a: $\tilde{\gamma} = \gamma^{(n+4)/(n+6)} \simeq \gamma^{0.84}$.

Many lab. searches were made, and we hope the axion be discovered .

The current status is



A SUSY Example

In this school, we encountered the NMSSM many times.

$$W = H_u U^c Q + H_d D^c Q + H_u H_d S + S^3$$

But this model seems to have an R symmetry. Look! However, it is broken by the gravitational effects, and there appear the A-terms, violating U(1)-R, and no problem!

$$V \supset m_{3/2}(H_u U^c Q + H_d D^c Q + H_u H_d S + S^3)$$

Then, we ask "how $m_{3/2}$ arises?" Maybe by the process of SUSY breaking? However, if it arises from spontaneous breaking when $m_{3/2}$ is generated, then there must be a Goldstone boson: R-axion.

So, the NMSSM introduced to solve the mu-problem without any dangerous light pseudoscalar has another light pseudoscalar. How do we resolve this dilemma? Most probably, in a complete theory like in a string model. String models do not have global symmetries, except MI axion.

So, approximate global symmetries are the only methods.

(1) This was explicitly studied in Z12-I orbifold model [K-Kyae, NPB 770, 47 (hep-ph/0608086)] first for the QCD axion [K.-S. Choi- I W Kim- JEK, (hep-ph/0612107)]

(2) For U(1)-R, this statement also applies. [Nilles et al., PRL 102, 121602 (2009) (arXiv:0812.2120)]
Even, a power law gauge hierarchy suggested.

In string compactification, the Yukawa couplings or superpotntial terms, including higher dimensional ones, are allowed if string superselection rules allow them. So, at the string compactification scale, there must appear U(1)-R breaking superpotential terms. These must give the R-axion a mass.

In this way, we may achieve the NMSSM objective. However, this must be stated in a specific model. Then, there are many sources contributing to the generation of mu. Introduction of S^3 as propaganded does not have a deep root at that level.

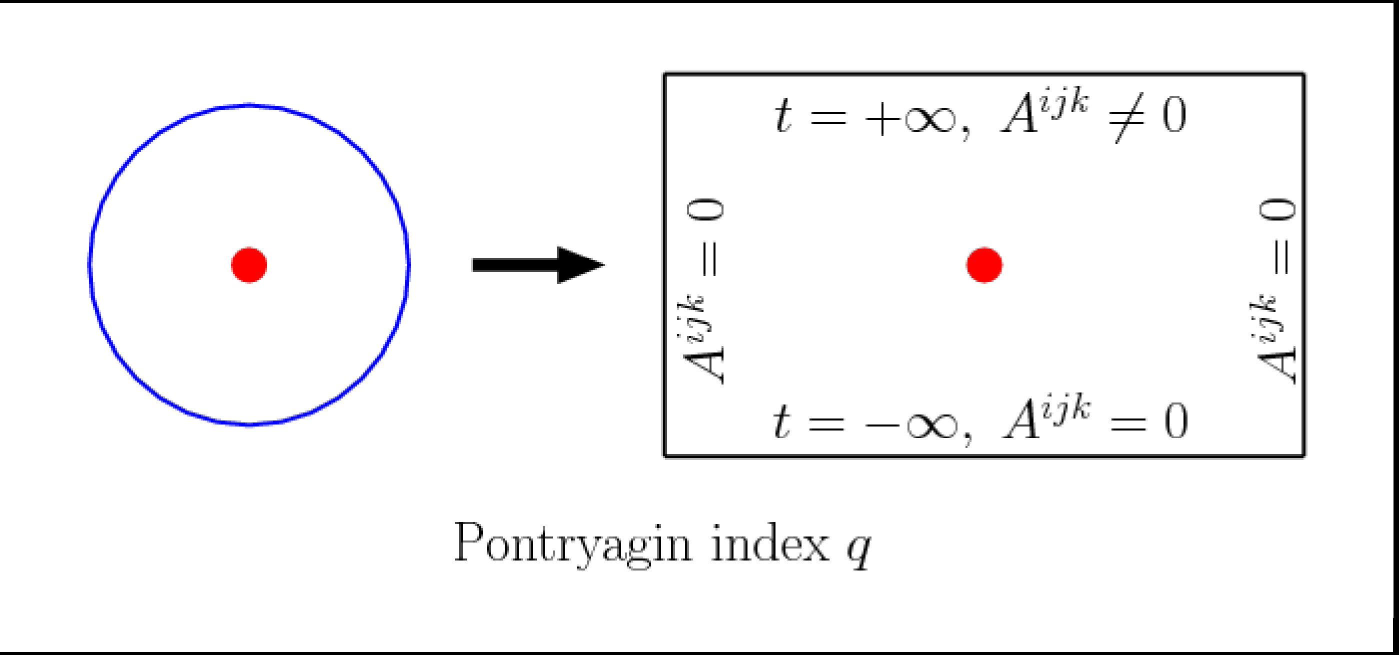
This closes an example of considering symmetries, and we move on to the discussion of axions.

2. Strong CP problem

Many considers the axion ‘attractive’ because it is a DM candidate.

But, axion’s strong CP solution is the bottom line in every past and future axion search experiments. So, let us start with the strong CP problem.

The instanton solution introduces the so-called θ term, and the resulting NEDM.



The arbitrary field configurations can be distinguished by the topological property, depending on its Pontryagin index. Thus, the classical vacuum can be a superposition of vacua of different Pontryagin indices. The criterion of superposition is that the vacuum is invariant under the gauge transformation. That vacuum is the so-called theta vacuum,

$$|\theta\rangle \propto \sum_{n=-\infty}^{+\infty} e^{in\theta} |n\rangle$$

The existence of instanton solution in nonabelian gauge theories needs θ vacuum [CDG, JR]. It introduces the θ term,

$$\frac{1}{32\pi^2}\bar{\theta}\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}G^{\mu\nu}G^{\rho\sigma}=\bar{\theta}\{G\tilde{G}\}$$

$$\bar{\theta}=\theta_{QCD}+\theta_{weak},\qquad \theta_{weak}=\arg.Det.M_q$$

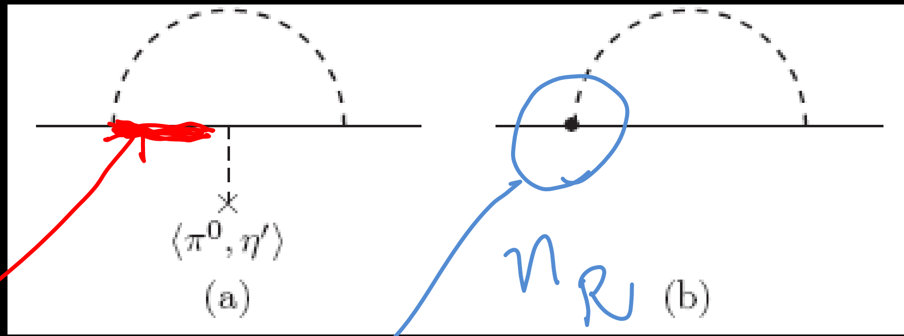
$$P:\quad G\tilde{G}\to -G\tilde{G}$$

$$T:\quad G\tilde{G}\to -G\tilde{G}$$

Here theta-bar is the final value taking into account the electroweak CP violation. For QCD to become a correct theory, this CP violation must be sufficiently suppressed.

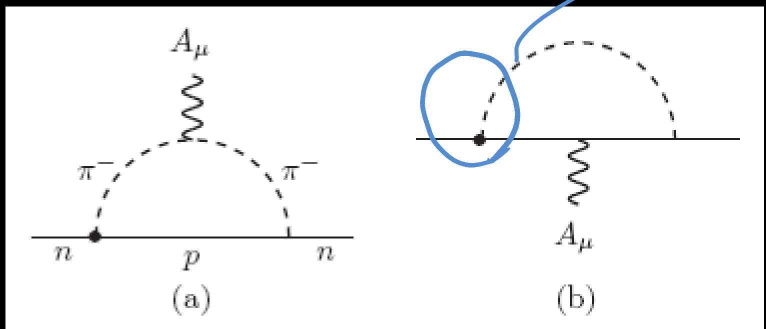
Look for the neutron mass term by CPV meson VEVs

$$g_{\pi NN}\frac{i}{-m_n}\left\langle m_n e^{-i\pi/f_\pi}\right\rangle$$



CP|π^0> = - |π^0>

The NMDM and NEDM terms



Neutrom mass is real.

The mass term and the NMDM term have the same chiral transformation property. So, (b)s are simultaneously removed.

(a) So, d(proton)= - d(neutron). is the NEDM contribution.

In our study, so the VEV of pi-zero determine the size of NEDM.

As stated in Sec. I, the real field pi0 having a VEV is AN OBSERVABLE phenomenon. No phase kind of thing.

$$\overline{g_{\pi NN}} = -\bar{\theta}\frac{Z}{(1+Z)} \simeq -\frac{\bar{\theta}}{3}$$

$$d_n = \frac{g_{\pi NN}\overline{g_{\pi NN}}}{4\pi^2 m_N} \ln\left(\frac{m_N}{m_\pi}\right) e\mathrm{cm}$$

We used C A Baker et al, PRL 97 (2006) 131801, to obtain

$$|\bar{\theta}| < 0.7 \times 10^{-11}.$$

It is an order of magnitude stronger than Crewther et al bound.

Why is this so small? : Strong CP problem.

1. Calculable θ (???), 2. Massless up quark (X)
3. Axion

1. Calculable θ

The Nelson-Barr CP violation is done by introducing vectorlike heavy quarks at high energy. This model produces the KM type weak CP violation at low energy. Still, at one loop the appearance of θ must be forbidden, and a two-loop generation is acceptable (???).

Earlier attempts: Beg-Tsao, Mohapatra-Senjanovic, Georgi, Segre-Weldon, Barr-Langacker

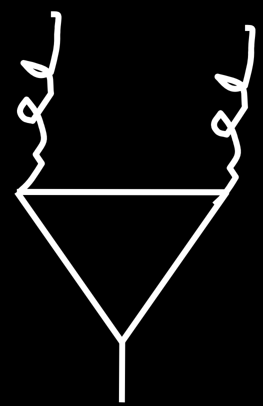
The weak CP violation must be spontaneous so that θ_0 must be 0.

2. Massless up quark

Suppose that we chiral-transform a quark,

$$q \to e^{i\gamma_5 \alpha} q: \quad \int (-m\bar{q}q + \frac{\theta}{32\pi^2} G\tilde{G})$$

$$\to \int (-m\bar{q} e^{2i\gamma_5\alpha} q + \frac{\theta - 2\alpha}{32\pi^2} G\tilde{G})$$



If m=0, it is equivalent to changing θ → θ -2α. Thus, there exists a shift symmetry θ → θ -2α. Here, θ is not physical, and there is no strong CP problem. The problem is, "Is massless up quark phenomenologically viable?"

The famous up/down quark mass ratio from chiral pert. calculation is originally given as 5/9 [Weinberg, Leutwyler] which is very similar to the recent compilation,

$$\frac{m_u}{m_d}=0.5,$$

$$m_u=2.5\mp 1MeV,$$

$$m_d=5.1\pm 1.5MeV$$

(Manohar-Sachrajda)

Excluding the lattice cal., this is convincing that m_u=0 is not a solution now.

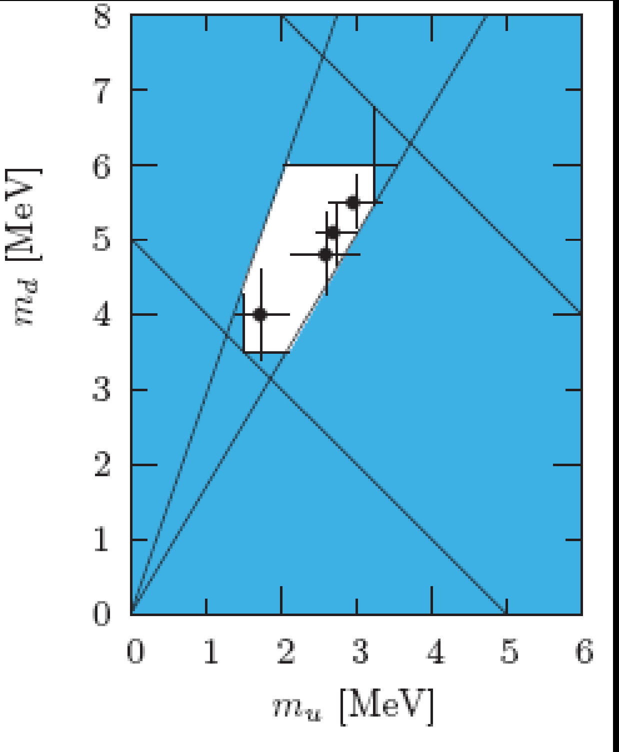


FIG. 6: The allowed $m_u - m_d$ region [265].

Particle Data (2008)

3. Axions

Kim-Carosi, RMP 82, 557 (2010) arXiv:0807.3125

Historically, Peccei-Quinn tried to mimick the symmetry θ → θ -2α, by the full electroweak theory. They found such a symmetry if H_u is coupled to up-type quarks and H_d couples to down-type quarks,

$$L = \bar{q}_L u_R H_u + \bar{q}_L d_R H_d - V(H_u, H_d) + \cdots$$

$$q \to e^{i\gamma_5\alpha} q, \{H_u, H_d\} \to e^{i\beta}\{H_u, H_d\}$$

$$\to \int (-H_u e^{i\beta} \bar{u} e^{i\gamma_5\alpha} u - H_d e^{i\beta} \bar{d} e^{i\gamma_5\alpha} d + \frac{\theta - 2\alpha}{32\pi^2} G\tilde{G})$$

Eq. β=α achieves the same thing as the m=0 case.

The Lagrangian is invariant under changing $\theta \to \theta - 2\boldsymbol{\alpha}$. Thus, it seems that θ is not physical, since it is a phase of the PQ transformation. But, θ is physical. At the Lagrangian level, there seems to be no strong CP problem. But $\langle H_u \rangle$ and $\langle H_d \rangle$ breaks the PQ global symmetry and there results a Goldstone boson, axion *a* [Weinberg,Wilczek] . Since θ is made field, the original cosθ dependence becomes the potential of the axion *a.*

If its potential is of the cosθ form, always $\theta = a/F_a$ can be chosen at 0 [Instanton physics,PQ,Vafa-Witten]. So the PQ solution of the strong CP problem is that the vacuum chooses

$$\theta = 0$$

History: The Peccei-Quinn-Weinberg-Wilczek axion is ruled out early in one year [Peccei, 1978]. The PQ symmetry can be incorporated by heavy quarks, using a singlet Higgs field [KSVZ axion]

$$L = \overline{Q}_L Q_R S - V(S, H_u, H_d) + \cdots$$

Here, Higgs doublets are neutral under PQ. If they are not neutral, then it is not necessary to introduce heavy quarks [DFSZ]]. In any case, the axion is the phase of the SM singlet *S,* if the VEV of *S* is much above the electroweak scale.

Now the couplings of *S* determines the axion interaction. Because it is a Goldstone boson, the couplings are of the derivative form except the anomaly term.

In most studies, a specific example is discussed. Here, we consider an effective theory just above the QCD scale. All heavy fields are integrated out.

In axion physics, heavy fermions carrying color charges are special. So consider the following Lagrangian

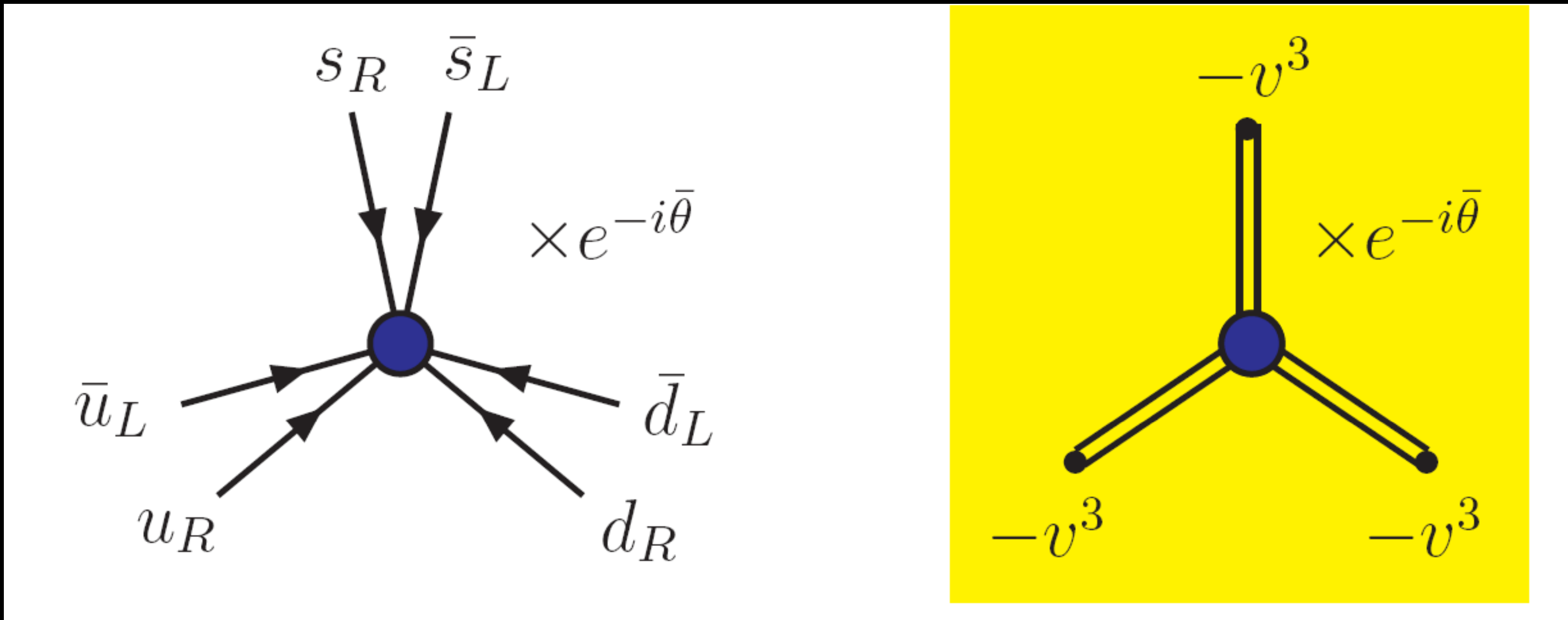
Heavy Q's are Integrated out

$$\mathcal{L}_\theta = \frac{1}{2} f_S^2 \partial^\mu \theta \partial_\mu \theta - \frac{1}{4g_c^2} G^a_{\mu\nu} G^{a\mu\nu} + (\bar{q}_L i \not{D}_{q_L} + \bar{q}_R i \not{D}_{q_R})$$
$$+ c_1 (\partial_\mu \theta) \bar{q} \gamma^\mu \gamma_5 q - (\bar{q}_L m q_R e^{ic_2\theta} + \text{H.c.})$$
$$+ c_3 \frac{\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} (\text{or } \mathcal{L}_{\text{det}}) + c_{\theta\gamma\gamma} \frac{\theta}{32\pi^2} F^i_{\text{em}\ \mu\nu} \tilde{F}^{i\mu\nu}_{\text{em}}$$
$$+ \mathcal{L}_{\text{leptons},\theta}, \qquad (19)$$

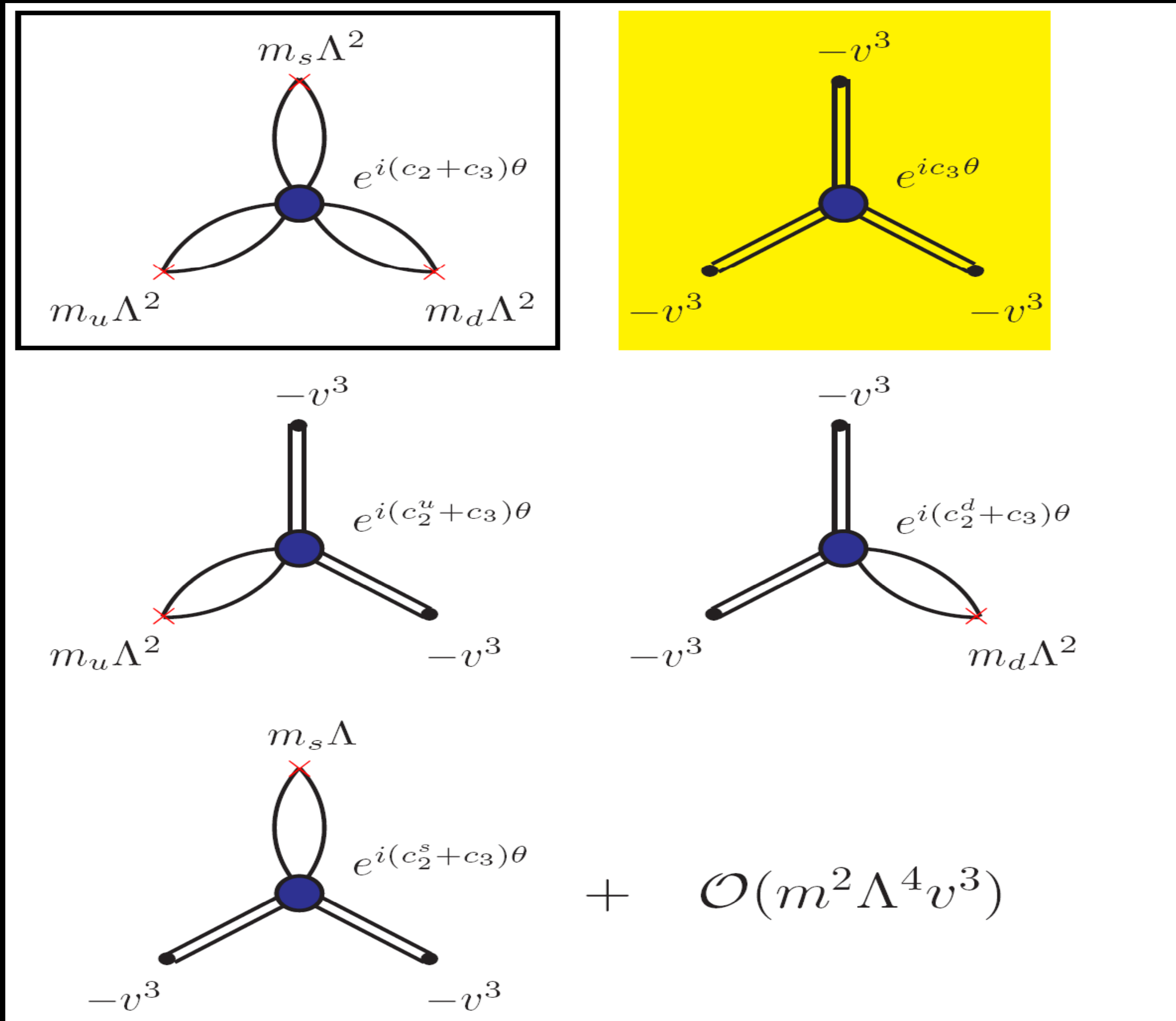
$$\mathcal{L}_{\text{det}} = -2^{-1} i c_3 \theta (-1)^{N_f} \frac{e^{-ic_3\theta}}{K^{3N_f-4}} \text{Det}(q_R \bar{q}_L) + \text{H.c.},$$

$$\Gamma_{1PI}[a(x), A^a_\mu(x); c_1, c_2, c_3, m, \Lambda_{\text{QCD}}]$$
$$= \Gamma_{1PI}[a(x), A^a_\mu(x); c_1 - \alpha, c_2 - 2\alpha, c_3 + 2\alpha, m, \Lambda_{\text{QCD}}].$$

The axion mass depends only on the combination of (c_2+c_3). The 'hadronic axion' usually means c_1=0, c_2=0, $c_3 \neq 0$.



't Hooft determinental interaction and the solution of the U(1) problem. If the story ends here, the axion is exactly massless. But,....



$m_s\Lambda^2$
$e^{i(c_2+c_3)\theta}$
$m_u\Lambda^2$
$m_d\Lambda^2$
$-v^3$
$e^{ic_3\theta}$
$-v^3$
$-v^3$
$-v^3$
$e^{i(c_2^u+c_3)\theta}$
$m_u\Lambda^2$
$-v^3$
$-v^3$
$e^{i(c_2^d+c_3)\theta}$
$-v^3$
$m_d\Lambda^2$
$m_s\Lambda$
$e^{i(c_2^s+c_3)\theta}$
$-v^3$
$-v^3$
$+ \quad \mathcal{O}(m^2\Lambda^4 v^3)$

$$\mathcal{L} = -m_u \langle \bar{u}_L u_R \rangle e^{i[(\theta_\pi + \theta_{\eta'}) + c_2^u \theta]} - m_d \langle \bar{d}_L d_R \rangle e^{i[(-\theta_\pi + \theta_{\eta'}) + c_2^d \theta]} + \text{h.c.} + \mathcal{L}_{\rm det}$$

$$-V = m_u v^3 \cos(\theta_\pi + \theta_{\eta'}) + m_d v^3 \cos(-\theta_\pi + \theta_{\eta'}) + \frac{v^9}{K^5} \cos\left(2\theta_{\eta'} - (c_2^u + c_2^d + c_3)\theta\right)$$
$$+ m_u \frac{\Lambda_u^2 v^6}{K^5} \cos\left(-\theta_\pi + \theta_{\eta'} - (c_2^u + c_2^d + c_3)\theta\right) + m_d \frac{\Lambda_d^2 v^6}{K^5} \cos\left(\theta_\pi + \theta_{\eta'} - (c_2^u + c_2^d + c_3)\theta\right)$$

$$M^2_{a,\eta',\pi^0} = \begin{pmatrix} c^2[\Lambda^4_{\eta'} + 2\mu\Lambda^3_{\rm inst}]/F^2 & -2c[\Lambda^4_{\eta'} + \mu\Lambda^3_{\rm inst}]/f'F & 0 \\ -2c[\Lambda^4_{\eta'} + \mu\Lambda^3_{\rm inst}]/f'F & [4\Lambda^4_{\eta'} + 2\mu\Lambda^3_{\rm inst} + m_+ v^3]/f'^2 & -m_- v^3/ff' \\ 0 & -m_- v^3/ff' & (m_+ v^3 + 2\mu\Lambda^3_{\rm inst})/f^2 \end{pmatrix}$$

$$m^2_{\pi^0} \simeq \frac{m_+ v^3 + 2\mu\Lambda^3_{\rm inst}}{f_\pi^2}$$

$$m^2_{\eta'} \simeq \frac{4\Lambda^4_{\eta'} + m_+ v^3 + 2\mu\Lambda^3_{\rm inst}}{f_{\eta'}^2}$$

$$m_a^2 \simeq \frac{c^2}{F^2} \frac{Z}{(1+Z)^2} f_\pi^2 m^2_{\pi^0} (1 + \Delta)$$

$$\Delta = \frac{m_-^2}{m_+} \frac{\Lambda^3_{\rm inst}(m_+ v^3 + \mu\Lambda^3_{\rm inst})}{m^4_{\pi^0} f_\pi^4}$$

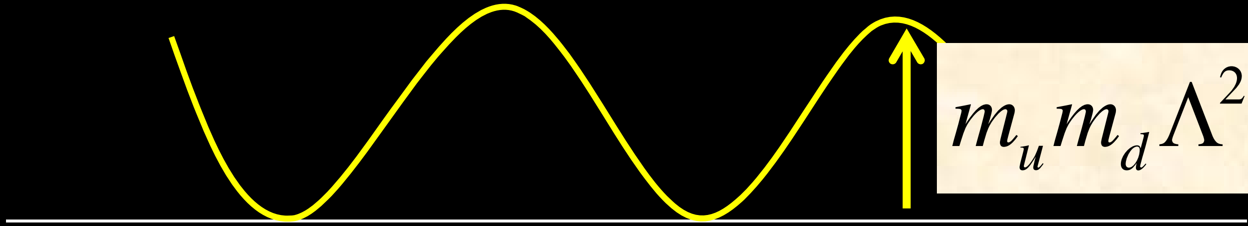
Leading to the cos form determines the axion mass

$$m_a = \frac{\sqrt{Z}}{1+Z}\frac{f_\pi m_{\pi^0}}{F_a}(1+\Delta)$$

The instanton contribution is included by Δ.

Numerically, we use

$$-m_u\Lambda^3\cos\frac{a}{F_a} \Rightarrow m_a = \frac{\sqrt{Z}}{1+Z}\frac{f_\pi m_\pi}{F_a} = 0.6[eV]\frac{10^7 GeV}{F_a}$$



$$m_u m_d \Lambda^2 \leftarrow \Lambda^4$$

$\uparrow$

$\bar{\theta} = 0$

$$V[a] = \frac{Z}{(1+Z)^2} f_\pi^2 m_\pi^2 (1 - \cos\frac{a}{F_a})$$

The essence of the axion solution is that <a> seeks

θ=0

whatever happened before. In this sense it is a cosmological solution. The height of the potential is the scale Λ of the nonabelian gauge interaction.

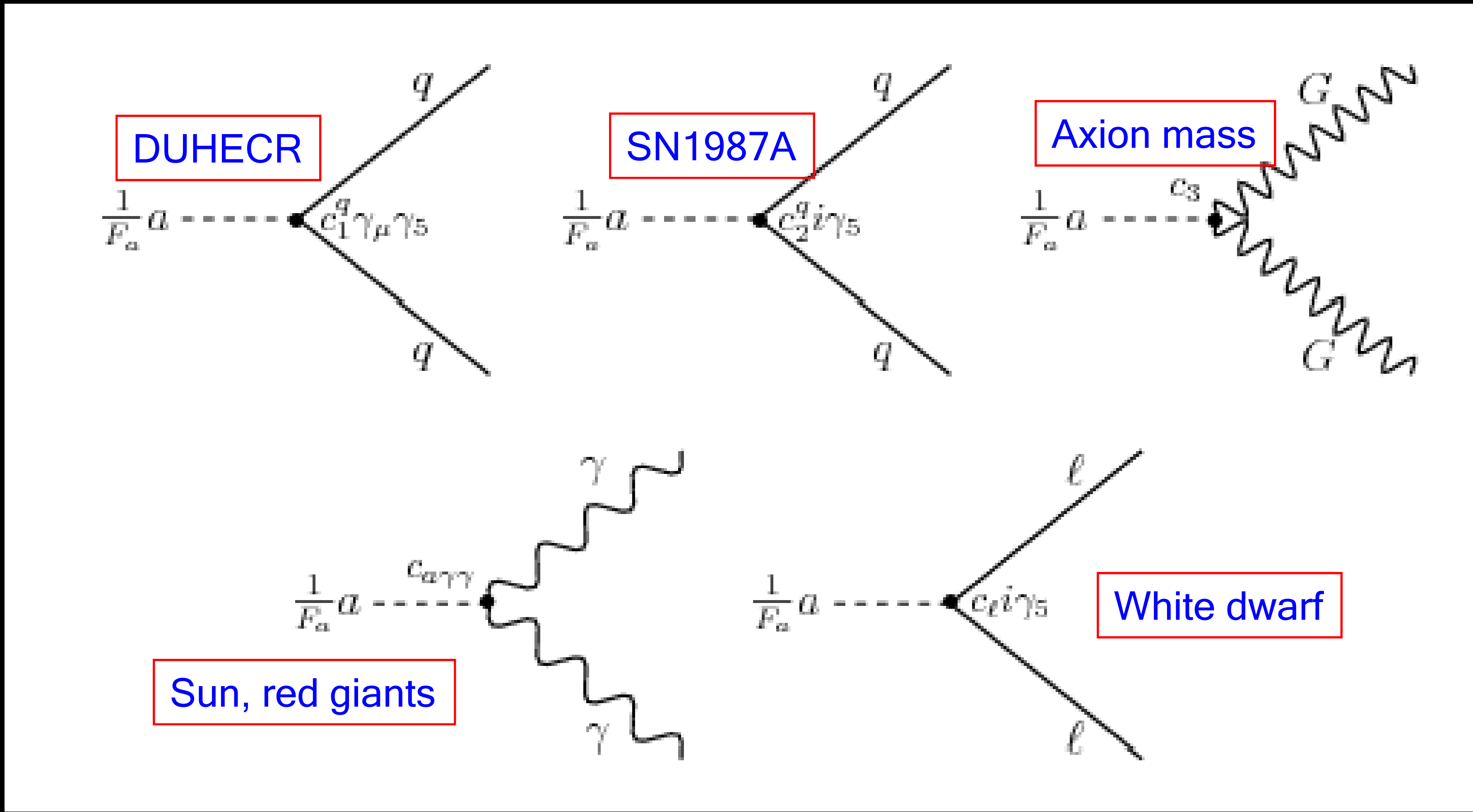
Axion couplings

Above the electroweak scale, we integrate out heavy fields. If colored quarks are integrated out, its effect is appearing as the coefficient of the gluon anomaly. If only bosons are integrated out, there is no anomaly term. Thus, we have

KSVZ: $c_1=0$, $c_2=0$, $c_3=$nonzero

DFSZ: $c_1=0$, $c_2=$nonzero, $c_3=0$

PQWW: similar to DFSZ


DUHECR
SN1987A
Axion mass
Sun, red giants
White dwarf
$\frac{1}{F_a}a$
$c_1^q\gamma_\mu\gamma_5$
$c_2^q i\gamma_5$
c_3
$c_{a\gamma\gamma}$
$c_\ell i\gamma_5$
q
G
γ
ℓ

Hadronic axion coupling is important for the study of supernovae:
The chiral symmetry breaking is properly taken into account, using the reparametrization invariance so that c3'=0.

KSVZ:

$$\bar{c}_1^{u,d} = \frac{1}{2}\bar{c}_2^{u,d}$$
$$\bar{c}_2^{u} = \frac{1}{1+Z}, \quad \bar{c}_2^{d} = \frac{Z}{1+Z}$$

DFSZ:

$$\bar{c}_1^{u} = -\frac{|v_d|^2}{2v_{EW}^2} + \frac{1}{2}\bar{c}_2^{u}, \quad \bar{c}_1^{d} = -\frac{|v_u|^2}{2v_{EW}^2} + \frac{1}{2}\bar{c}_2^{d},$$
$$\bar{c}_2^{u} = \frac{1}{1+Z}, \quad \bar{c}_2^{d} = \frac{Z}{1+Z},$$

The KSVZ axion has been extensively studied. Now the DFSZ axion can be studied, too.

General very light axion:

$$\bar{c}_2^u = \frac{1}{1+Z}$$
$$\bar{c}_2^d = \frac{Z}{1+Z}$$
$$\bar{c}_1^u = \frac{1}{2}\frac{1}{1+Z} \mp \frac{|v_d|^2}{2v_{EW}^2}\delta_{H_u}$$
$$\bar{c}_1^d = \frac{1}{2}\frac{Z}{1+Z} \mp \frac{|v_u|^2}{2v_{EW}^2}\delta_{H_d}$$

Axial vector couplings:

$$(\bar{c}_{1,2}^u - \bar{c}_{1,2}^d)F_3 + \frac{\bar{c}_{1,2}^u + \bar{c}_{1,2}^d}{\sqrt{3}}F_8 + \frac{\bar{c}_{1,2}^u + \bar{c}_{1,2}^d}{6}\mathbf{1}$$

Axion mixing in view of hidden sector

Even if we lowered some F_a, we must consider hidden sector also. In this case, axion mixing must be considered. There is an important theorem.

Cross theorem on decay constant and condensation scales

[Kim, hep-ph/9811509, hep-ph/9907528]:

Suppose two axions a_1 with F_1 and a_2 with F_2 ($F_1 << F_2$) couples to two nonabelian groups whose scales have a hierarchy, $\Lambda_1 << \Lambda_2$.

Then, diagonalization process chooses the

larger condensation scale Λ_2 chooses smaller decay constant F_1,

smaller condensation scale Λ_1 chooses larger decay constant F_2.

So, just obtaining a small decay constant is not enough. Hidden sector may steal the smaller decay constant. It is likely that the QCD axion chooses the larger decay constant. [See also, I.-W. Kim-K, PLB639 (2006) 342]

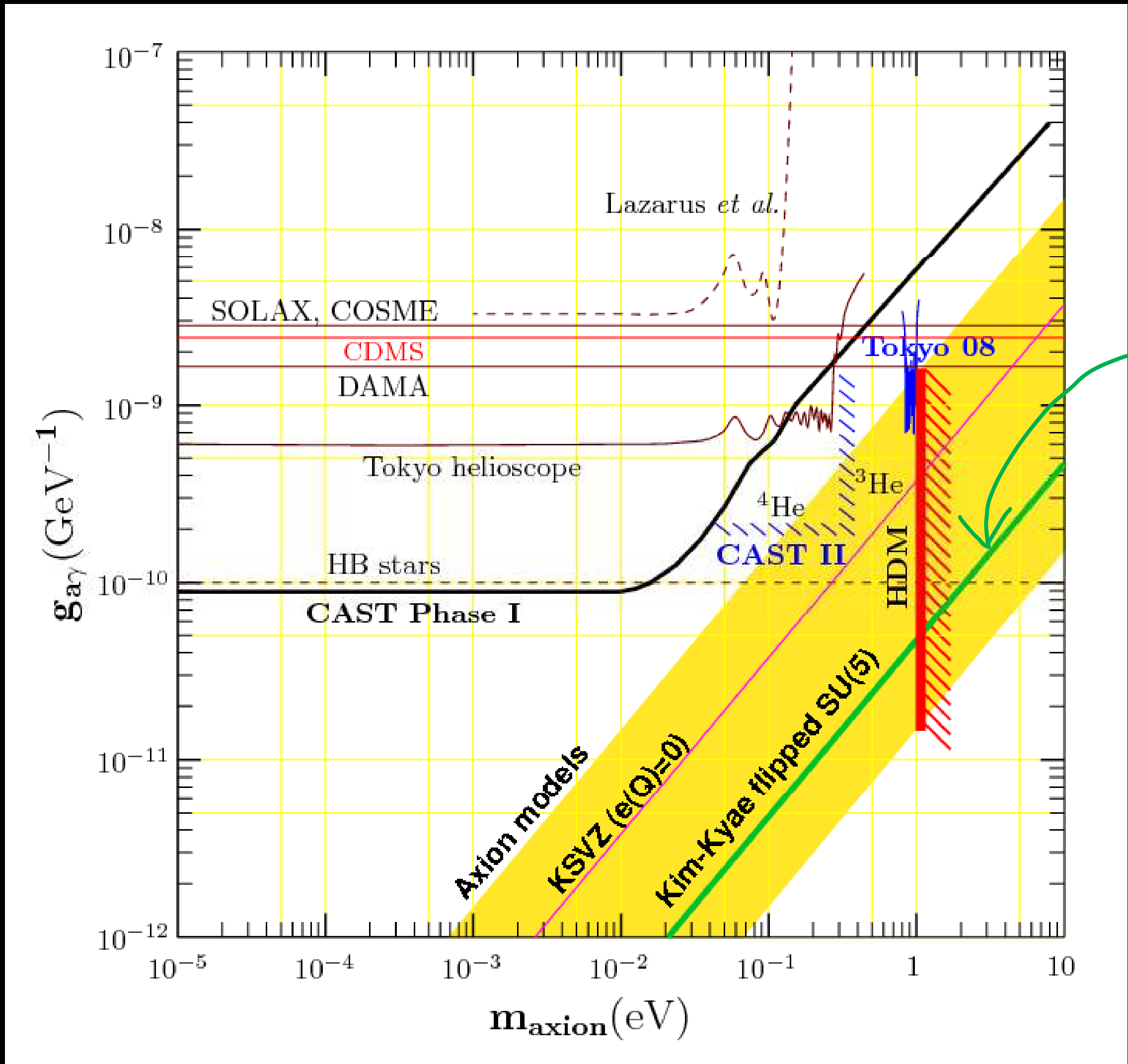
In this regard, we point out that the MI-axion with anomalous U(1) always has a large decay constant since all fields are charged under this anomalous U(1). Phenomenologically successful axion must need the approximate PQ.

An approximate PQ global symmetry with discrete symmetry in SUGRA was pointed out long time ago: for Z_9 given by [L-P-Shafi]. Z_9 is not possible in orbifold compactification. May need $Z_3 x Z_3$ orbifold.

Approx. PQ symmetry from heterotic string:

Choi-Kim-Kim, JHEP 03 (2007) 116 [hep-ph/0612107]

Choi-Nilles-RamosSanches-Vaudrevange, arXiv:0902.3070.



String models give definite numbers. [I-W Kim-K]

There exist only one calculation in string compactification, In a model explaining all MSSM phenomenology.

Axions in the universe

The axion potential is of the form

The vacuum stays there for a long time, and oscillates when the Hubble time(1/H) is larger than the oscillation period($1/m_a$)

$$3H < m_a$$

This occurs when the temperature is about 0.92 GeV.

The axion is created at $T=F_a$, but the universe • (<*a*>)does not roll until $3H=m_a$ ($T=0.92$ GeV [Bae-Huh-Kim]). From then, the classical

field <*a*> starts to oscillate. Harmonic oscillator

$m_a^2\, F_a^2$ = energy density = m_a x *number density* = like CDM.

See, Bae-Huh-Kim, arXiv:0806.0497 [JCAP09 (2009) 005]

$$\rho_a(T_\gamma = 2.73\mathrm{K}) = m_a(T_\gamma) n_a(T_\gamma) f_1(\theta_2) = \frac{\sqrt{Z}}{1+Z} m_\pi f_\pi \frac{3\cdot 1.66 g_{*s}(T_\gamma)T_\gamma^3}{2\sqrt{g_*(T_1)}M_\mathrm{P}} \frac{F_a}{T_1} \frac{\theta_2^2 f_1(\theta_2)}{\gamma} \left(\frac{T_2}{T_1}\right)^{-3-n/2}$$

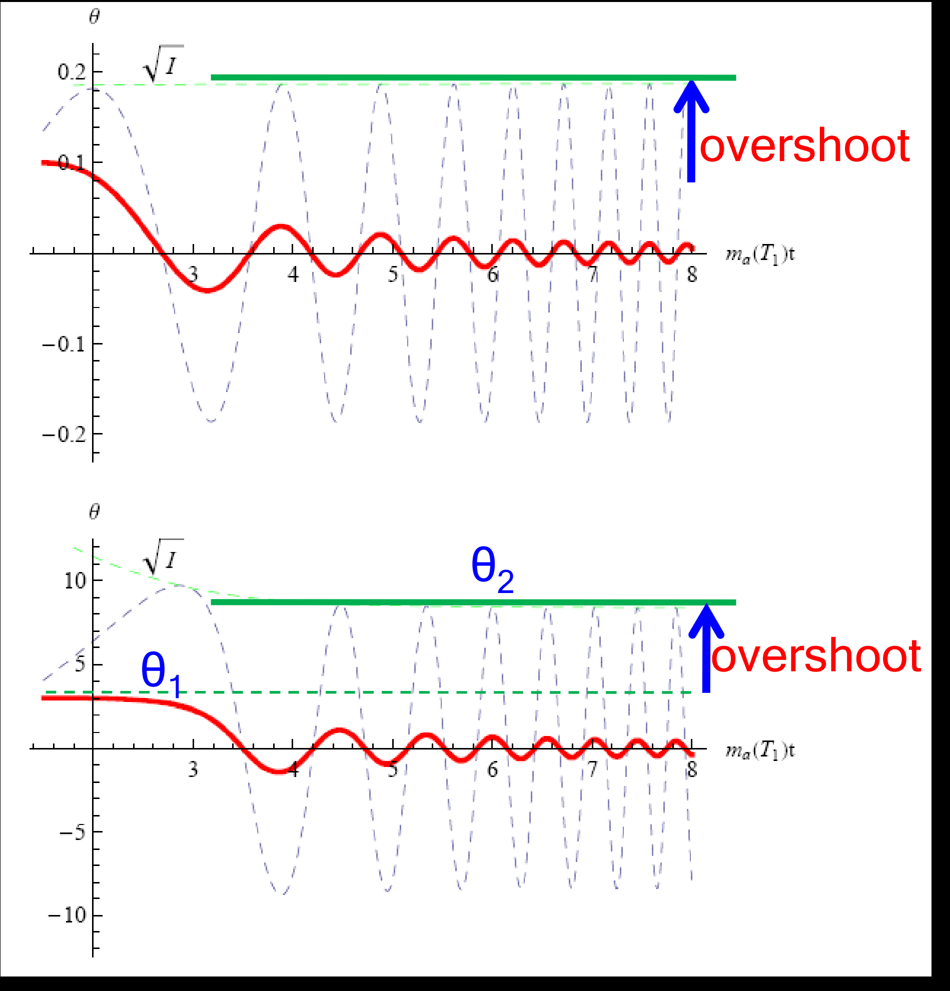
There is an overshoot factor of 1.8. So we use theta2, rather than theta1. If F_a is large(> 10^{12} GeV), then the axion energy density dominates. Since the ener gy density is proportional to the number density, it behaves like a CDM, but

$$10^9 \text{ GeV} < F_a < 10^{12} \text{ GeV},$$

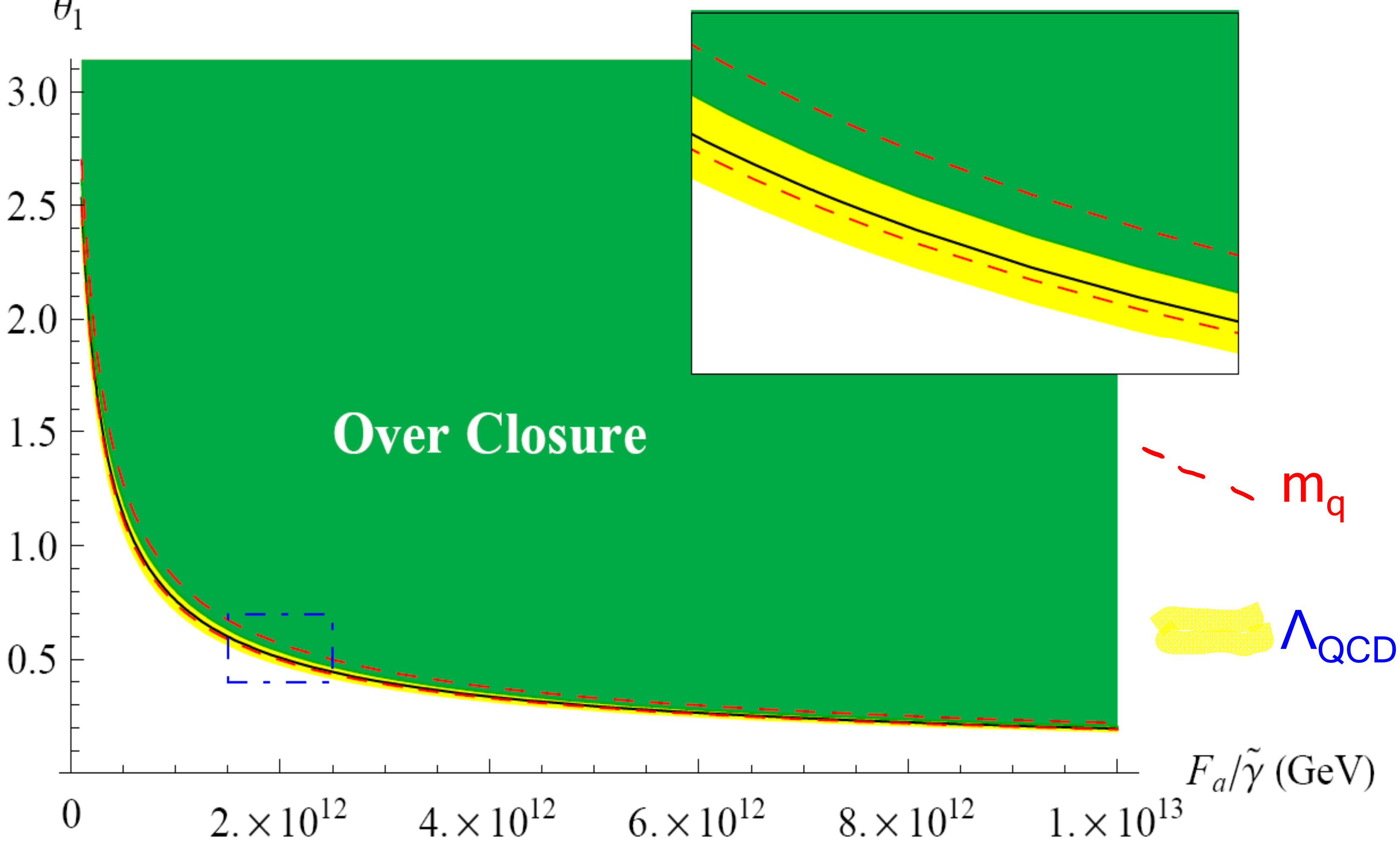
The axion field evolution eq. and time-varying Lagrangian

The adiabatic condition:

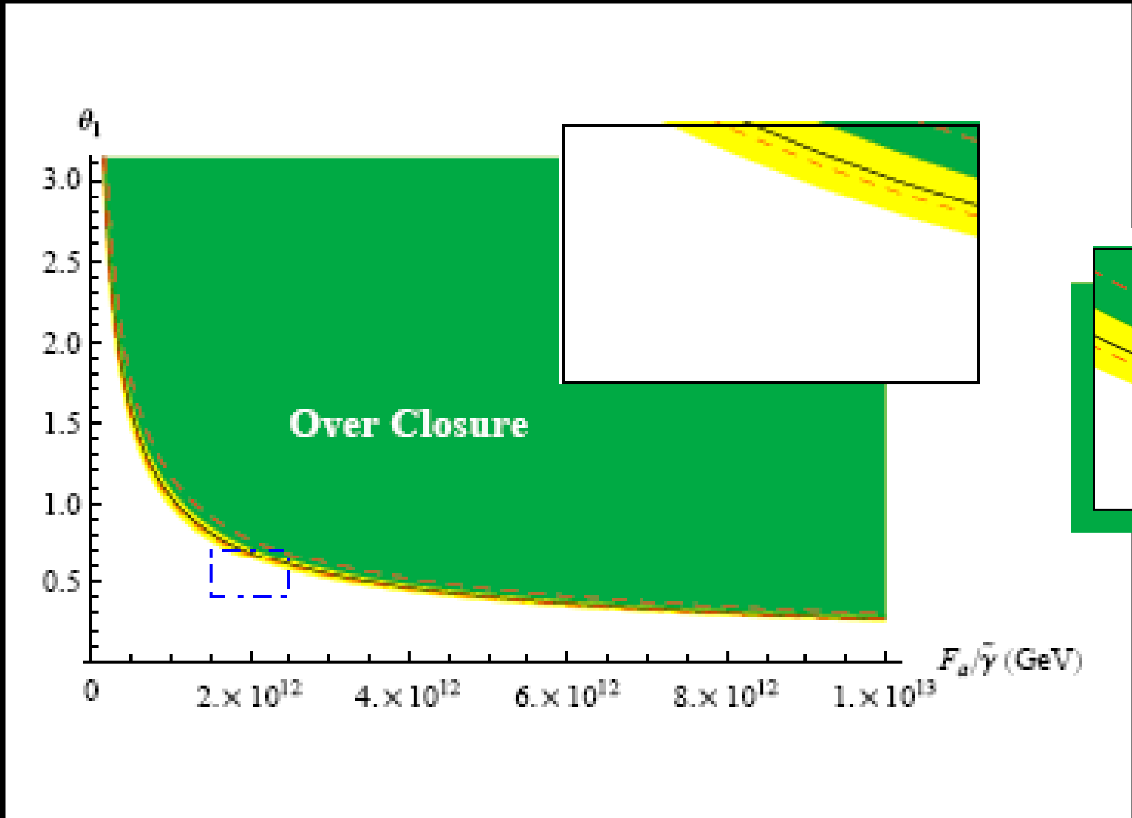
The adiabatic invariant quantity:


θ
$\sqrt{I}$
overshoot
$m_a(T_1)t$
θ
$\sqrt{I}$
θ_2
θ_1
overshoot
$m_a(T_1)t$

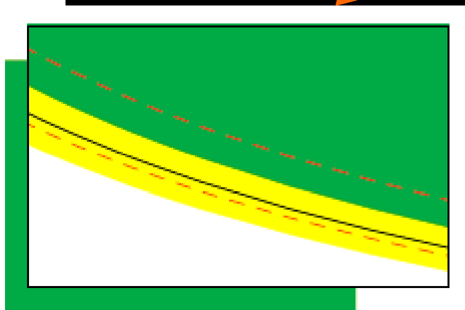
Bae-Huh-Kim, JCAP0809, 005



If we do not take into account the overshoot factor and the anharmonic correction,



Inclusion of these showed the region, prev. figure



The anharmonic effect and the overshoot of roughly a factor of 1.8 (realized after a half cycle) are taken into account. Then, the axion energy fraction is given by

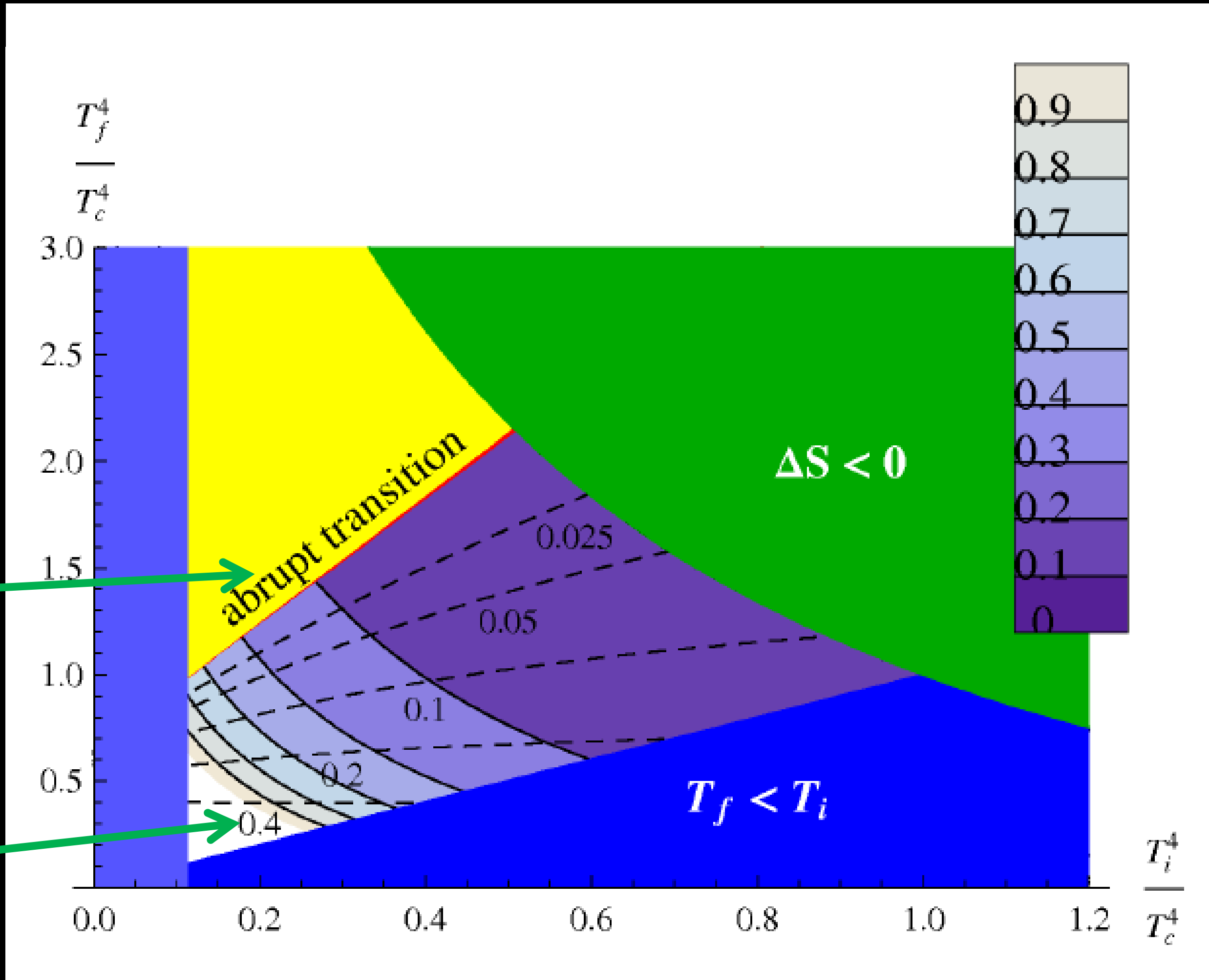
$$\Omega_a \cong 0.379 \times \left(\frac{m_u m_d m_s}{3 \cdot 6 \cdot 103 MeV^3} \right)^{-0.092} \left(\frac{\Lambda_{QCD}}{380 MeV} \right)^{-0.733} \left(\frac{0.701}{h} \right)^2 \times \left(\frac{\theta_1^2 F(\theta_1)}{\gamma} \right) \times \left(\frac{F_a}{10^{12} GeV} \right)^{1.184 - 0.010x}$$

$$x = \frac{\Lambda_{QCD}}{380 MeV} - 1$$

QCD phase transition effect does not change
The current axion density calculated above.

$t = 3.7\times10^{-9}\,s$

$t = 3.7\times10^{-8}\,s$



Cosmic axion search

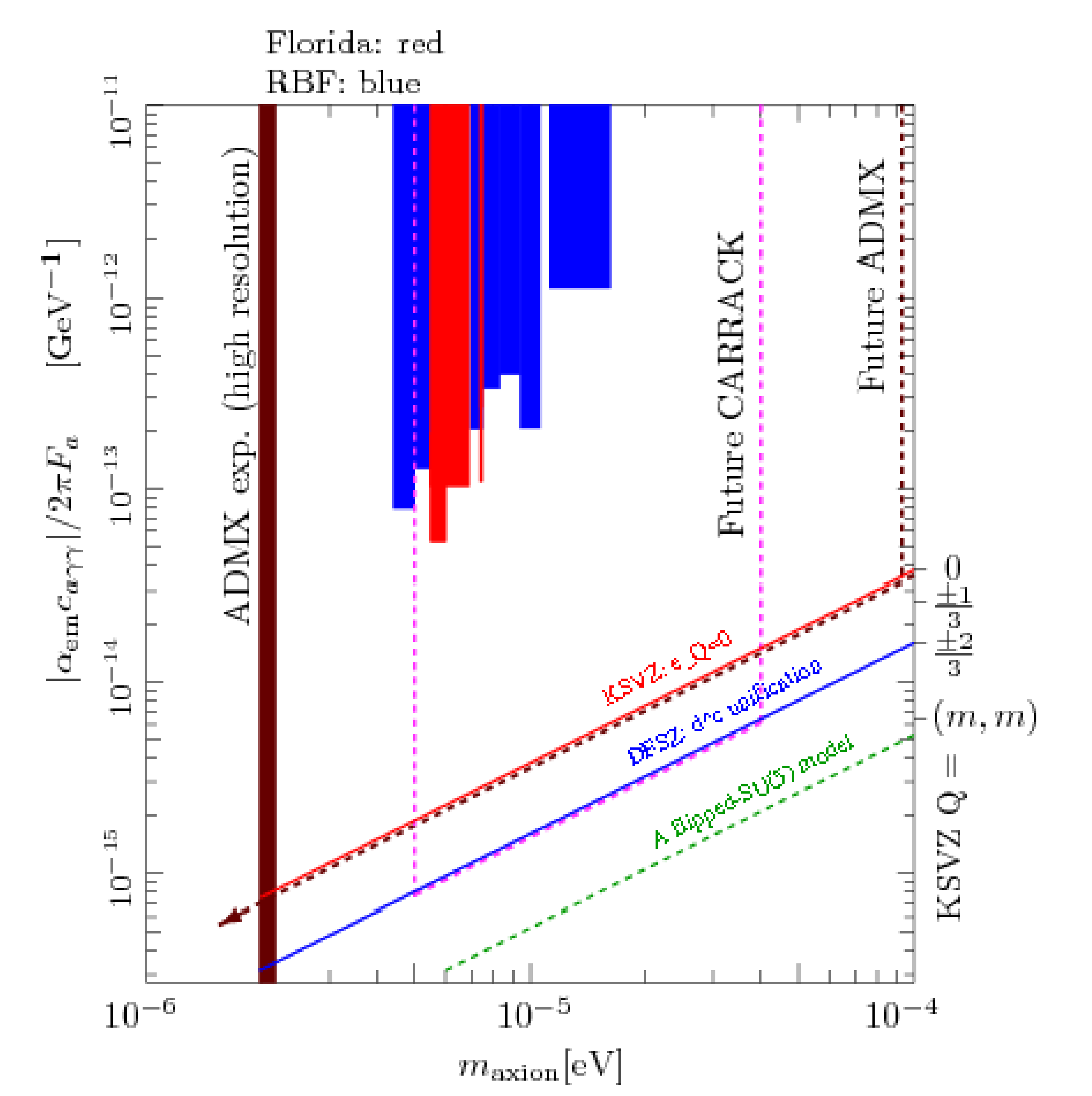
If axion is the CDM component of the universe, then they can be detected [Sikivie]. K. van Bibber's efforts. The feeble coupling can be compensated by a huge number of axions. The number density ~ F_a^2, and the cross section ~ $1/F_a^2$, and there is a hope to detect [10^{-5} eV range].

$$L = -\frac{a}{F_a} c_{a\gamma\gamma} \frac{e^2}{16\pi^2} F_{em}\tilde{F}_{em} \quad \Rightarrow \quad E\cdot B$$

$$c_{a\gamma\gamma} = \bar{c}_{a\gamma\gamma} + 6 \sum_{i=light\ quarks} \tilde{\alpha}_i (Q_i^{em})^2 = \bar{c}_{a\gamma\gamma} - 1.95$$

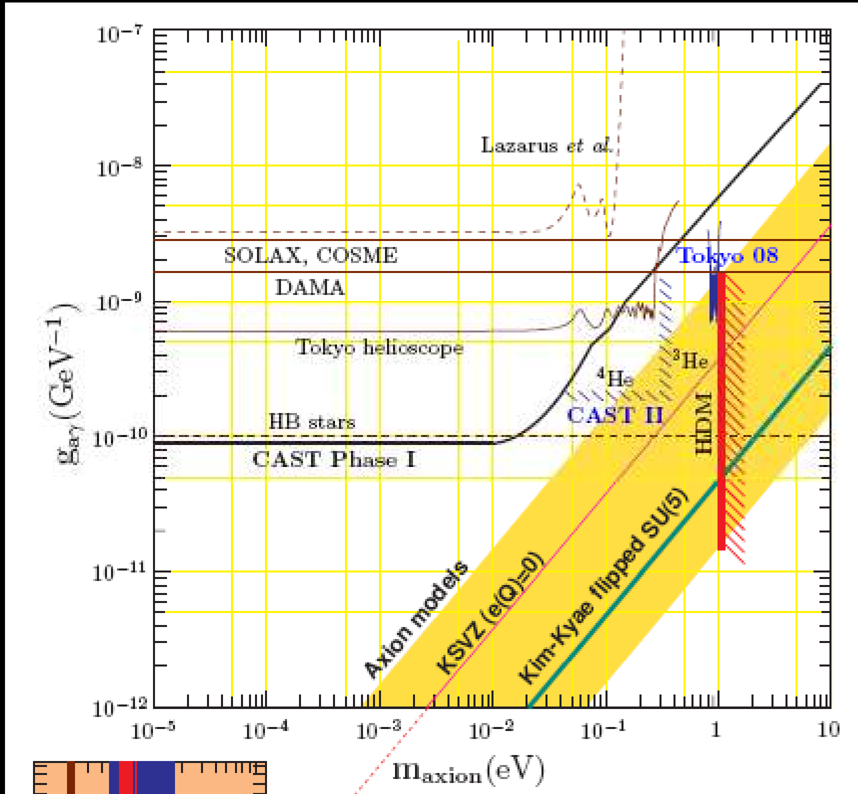
$$\bar{c}_{a\gamma\gamma} = Tr(Q_{em}^2)|_{E>>M_Z} = 0, \ \frac{8}{3}$$

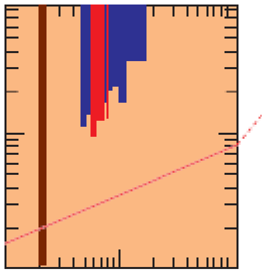
Positive for 1 HQ



From local density with $f_{a\gamma\gamma}E\cdot B$

Future ADMX and CARRACK will cover the interesting region.





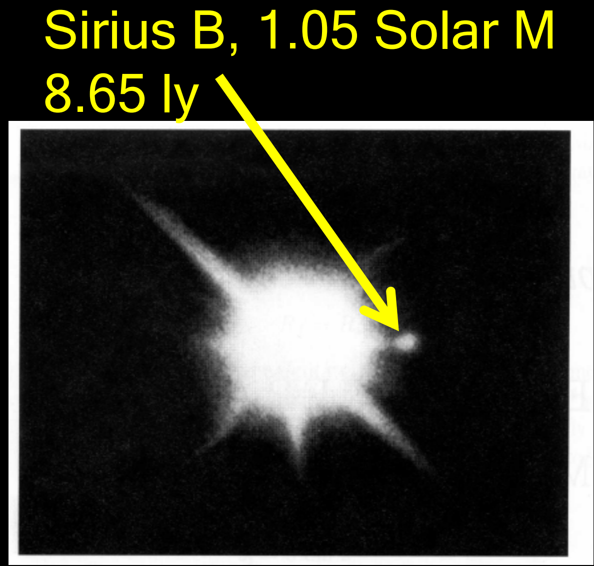
KSVZ		DFSZ		
Q_{em}	$c_{a\gamma\gamma}$	$x = \tan\beta = v_u/v_d$,	same Higgs for (q^c, e) masses,	$c_{a\gamma\gamma}$
0	-1.95	any x,	(d^c, e)	0.72
$\pm\frac{1}{3}$	-1.28	any x,	(u^c, e)	-1.28
$\pm\frac{2}{3}$	0.72			
± 1	4.05			
(m, m)	-0.28			

Two outer space examples

Low energy example: White dwarf energy loss

Very high energy example: Ultra High Energy
Cosmic Rays exceeding GZK bound

White dwarf axion possibility



White dwarfs can give us useful information about their last stage evolution. Main sequence stars will evolve after consuming all their nuclear fuel to WDs if their mass is less than 1.08 M_{Sol}. WDs of Sun's mass have the size of Earth, and DA WDs are studied most.

The exceptionally strong pull of WD's gravity is the reason for the thin hydrogen surface of DA white dwarfs. In fact, the core of WDs follows simple physics, the degenerate fermion gas.

The Fermi energy at T=0 K is

$$\varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$
$$= \frac{\hbar^2}{2m}\left(3\pi^2 \frac{Z}{A}\frac{\rho}{m_H}\right)^{2/3}$$

The condition for a degenerate electron gas is

$$\frac{T}{\rho^{2/3}} < 1.3\times 10^5\, Kcm^2 gr^{-2/3}$$

Sirius B: $3.6\text{x}10^3$

The pressure of the degenerate electron gas is

$$P = \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e} \left[\left(\frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{5/3}$$

The Chandrasekhar limit

The astronomers are able to recover the history of star formation in our Galaxy by studying the statistics of WD temperatures.

For this, the energy transport mechanism from the core is essential. Unlike in Sun, it is transported by neutrinos at high T since most electron are filling the Degenerate energy levels. So, the transport mechanism is very simple. And the resulting luminosity at the surface is calculable and reliable.

$$L_{wd} = CT^{7/2}$$

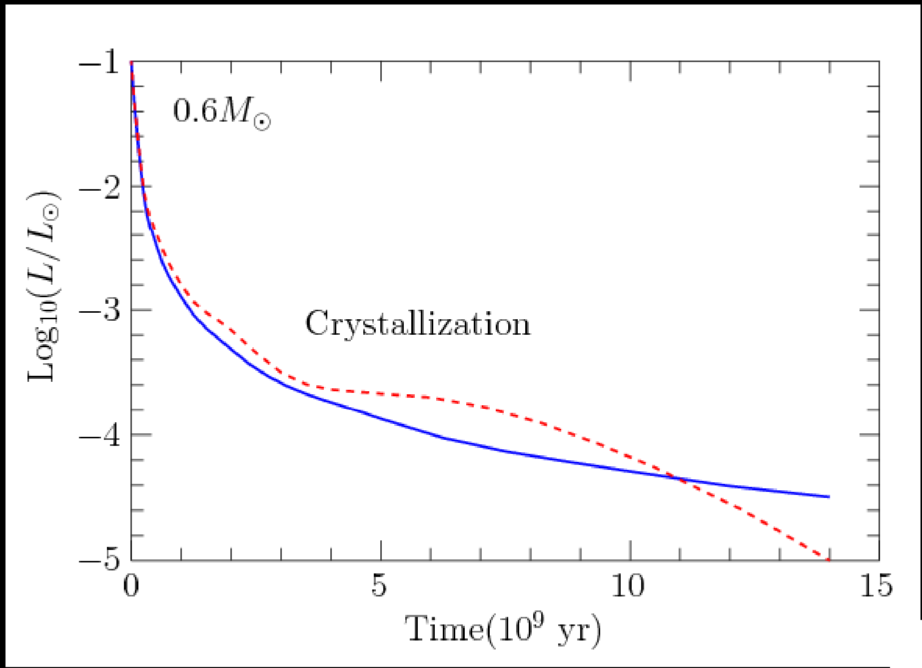
$$C = 7.3\times10^{5}\left(\frac{M_{wd}}{M_{Sun}}\right)\frac{\mu}{Z(1+X)}, \mu = av. \quad molar \quad wt.$$

The later stage of evolution is cristalization from the core. As time goes on, the luminosity drops. In terms of t,

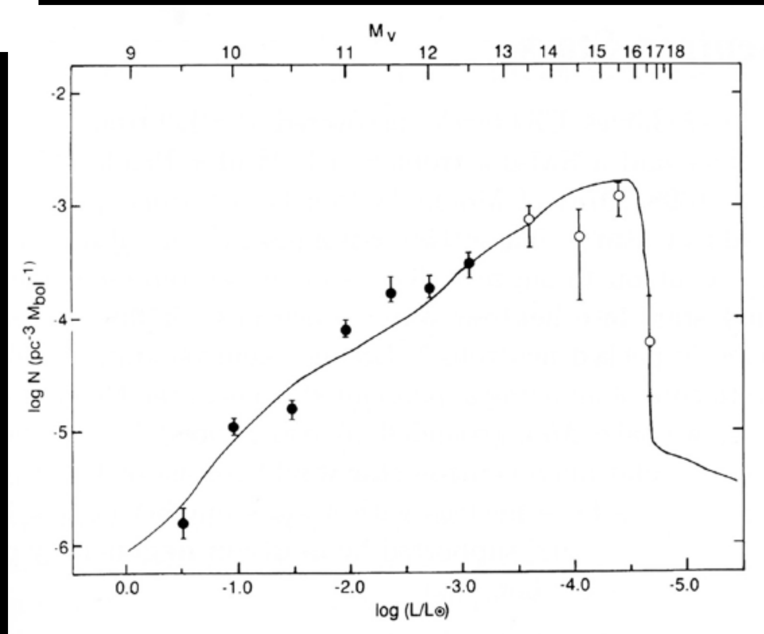
$$L_{wd} = L_0\left(1+\frac{5}{2}\frac{t}{\tau_0}\right)^{-7/5}, \qquad \tau_0 \cong 2.16\times10^{7}\, yrs$$

characteristic time of WD

A more complete treatment changes this simple behavior little bit (red dash line). With more data, Isern et al. gives a very impressive figure.



Winget et al., Ap. J. Lett. 315 (1987) L77.



The energy loss in the early stage is through the photon conversion to neutrino pairs in the electron plasma.
This calculation of the photon decay was initiated in 1960s, but the accurate number was available after 1972 when the NC interaction was taken into account.

D. A. Dicus, PRD6 (1972) 941;
E. Braaten, PRL66 (1991) 1655;
N. Itoh et al., Ap. J. 395 (1992) 622;
Braaten-Segel, PRD48 (1993)1478;
Y. Kohyama et al., Ap. J. 431 (1994) 761

Isern et al., [Ap. J. Lett. 682 (2008) 109] gives a very impressive figure from the recent calculation, including this early stage and the crystalization period.

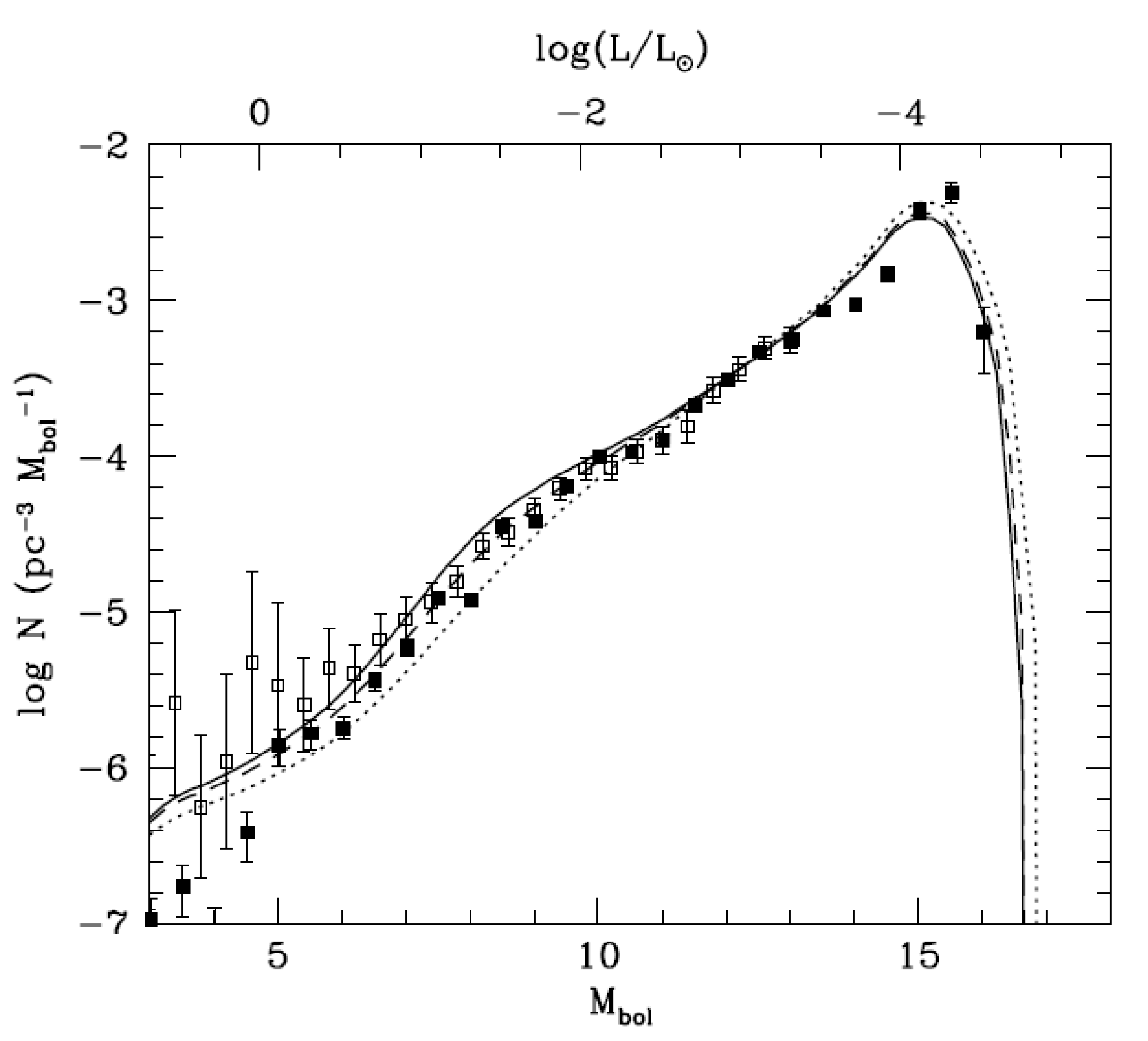


FIG. 3.— White dwarf luminosity functions for different values of the axion mass. The luminosity functions have been computed assuming $m_a \cos^2 \beta = 0$ (solid line), 5 (dashed line) and 10 (dotted line) meV.

Isern et al., Ap. J. Lett. 682 (2008) 109

Here, the luminocity is smaller than the above calculation.

One obvious possibility is the contribution from neutrino transition magnetic moments, and their plasmon decay leads to:

$$\frac{1}{2}\mu_{ij}\nu^{iT}C\gamma^{\mu\nu}\nu^{j}F_{\mu\nu}\to \quad \Gamma=\frac{|\mu|^2}{24\pi}Z_{T,L}\frac{\left(\omega_{T,L}^2-\vec{p}_{plasmon}^{\,2}\right)^2}{\omega_{T,L}}$$

which can be compared to the SM decay to neutrinos in the plasma,

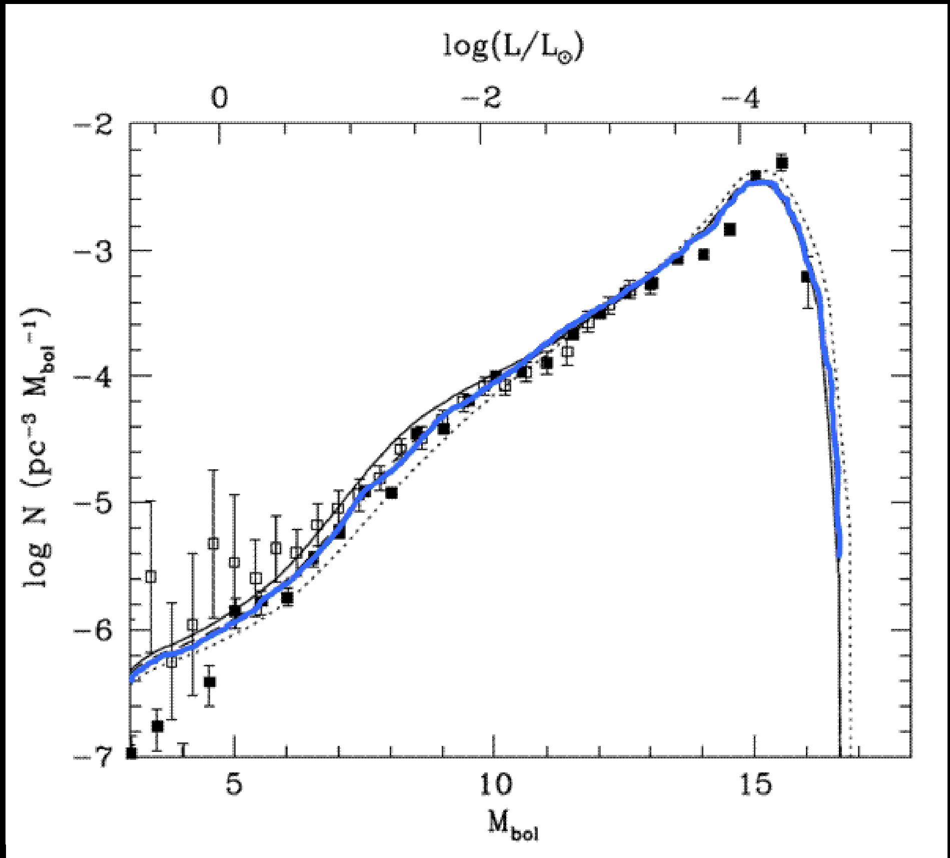
$$C_V=(e\,\nu)\ vector\ NC\ coupling\to \quad \Gamma=\frac{G_F^2\,C_V^2}{48\pi^2\alpha_{em}}Z_{T,L}\frac{\left(\omega_{T,L}^2-\vec{p}_{plasmon}^{\,2}\right)^3}{\omega_{T,L}}$$

So, the radiation rate ratio is [Raffelt's book]

$$\frac{Q_{mag.\ mom.}}{Q_{SM}}=\ 6.01\left(\frac{\mu}{10^{-11}\mu_{Bohr}}\right)^2\left(\frac{23\ keV}{\omega_P}\right)^2\frac{Q_3}{Q_2},\quad \frac{Q_3}{Q_2}=O(1)$$

The neutrino magnetic moment possibility is out in the SM.

Isern et al. varied the star burst rates which is the only important uncertainty, and found that in the middle the predicted WD number stays almost the same. So, they used this almost burst rate independent region to estimate the WD luminocity.



So, they conclude that there must be another mechanism for the energy loss, and considered the axion possibility.

We translate their number to the axion-electron coupling

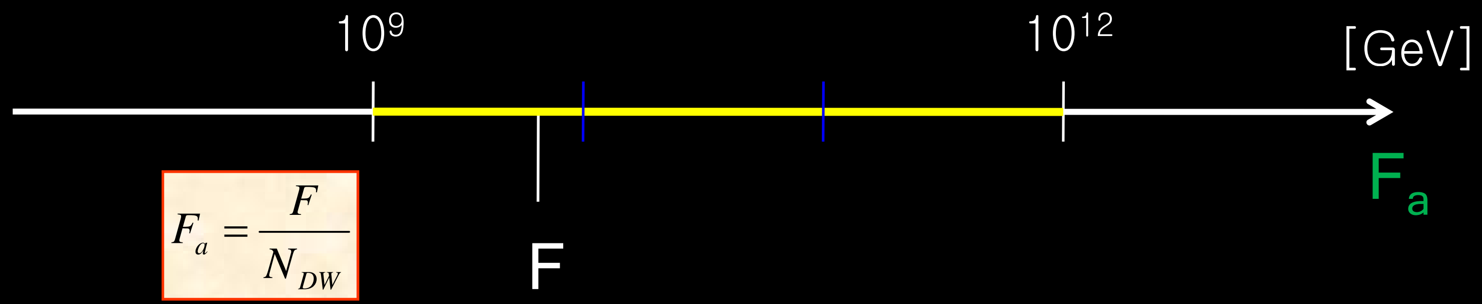
$$\left|\frac{m_e\Gamma(e)}{F}\right| = \frac{m_e}{0.72\times 10^{10}\,GeV} \cong 0.7\times 10^{-13} : any \quad axion \quad model$$

$$axion-electron \quad coupling: \quad \frac{m_e\Gamma(e)}{F}\bar{e}i\gamma_5 ea, \quad F = N_{DW}F_a$$

So, the axion-electron coupling has the form,

$$\frac{m_e\Gamma(e)/N_{DW}}{F_a}\bar{e}i\gamma_5 ea, \qquad F = N_{DW}F_a, \qquad \Gamma(e) = PQ \quad charge$$

To have a QCD axion at the intermediate scale, 10^9 - 10^{12} GeV, we need some PQ charge carrying scalar develop VEV(s) at that scale. But the domain wall number relates $F=N_{DW}F_a$ with $N_{DW}=1/2$.



$$F_a = \frac{F}{N_{DW}}$$

If we anticipate the axion decay constant at the middle of the axion window, N_{DW} must be smaller than 1 since the needed axion-electron coupling is quite large.

If it is done by the phase of a singlet scalar S, presumably the PQ charges of the SM quark fields must be odd such that sum of the PQ charges of all the quarks(including heavy ones) be 1. But sum of the PQ charges of e_{2L} and e_R is 2. Then we obtain N_{DW} =1/2. Because our objective is the quark-lepton unification, this choice is the simplest.

An enhanced electron coupling compared to the axion lower bound is possible by,

(i) Assign a large PQ charge to e.
The quark-lepton unification makes this idea not very promising, especially in GUTs.

(ii) Assign 1 PQ charge to e, but let the DW number be fractional. In this case, only ½ is possible.
For the quark sector, effectively only one chirality of one quark carries PQ charge, but both e_L and e_R carries PQ charges.

Bae-Huh-Kim-Kyae-Viollier, NPB817 (2009) 58
used only u_R for an effective PQ charged quark. It is
Possible in the flipped SU(5) since (u, nu, e)L appear
and e_R can be a singlet.

Ultra High Energy Cosmic Ray

GZK bound: 0.6×10^{11} GeV $\rightarrow$ the range of F_a

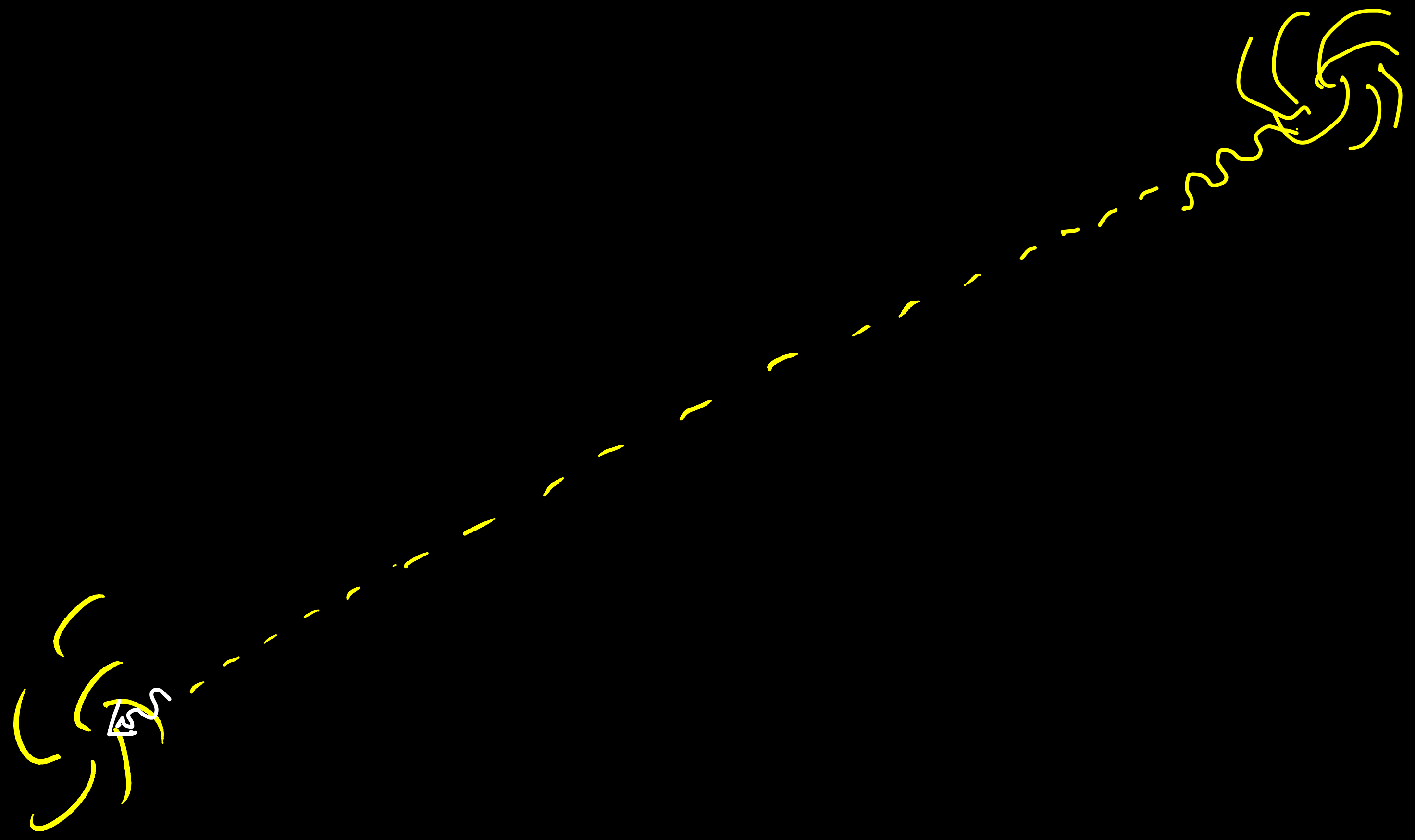
Pierre Auger, Flyer's Eye, AGASA observed UHECR
with E> GZK bound (sphere 100 Mpc)
(Albuquerque and Chou summary, arXiv:1001.0972)

1.48×10^{11} GeV direction PKS1245-19 3.8 Gpc away
3.2×10^{11} GeV direction QSO 3C147 2 Gpc away

These seem to point radio galaxies, AGNs, quasars.
These are known to have high Faraday rotation, and capable of accerating charged particles to ultra high energy.

DUHECR

Auger: 1.48x10^{11} GeV direction PKS1245-19   3.8 Gpc away
F. Eye: 3.2x10^{11} GeV direction QSO 3C147   2 Gpc away



In our Galaxy the axion-photon oscillation is described by

$$M^2 = \begin{pmatrix} m_a^2 & X \\ -X & m_\gamma^2 \end{pmatrix}$$

$$\sin\theta = \frac{m_2^2}{\sqrt{X^2 + m_2^2}} = \frac{1}{\sqrt{2}}\sqrt{1 - \sqrt{1 - 4X^2/m_a^4}}\ .$$

In the limit $X^2 \ll m_a^4$, we obtain

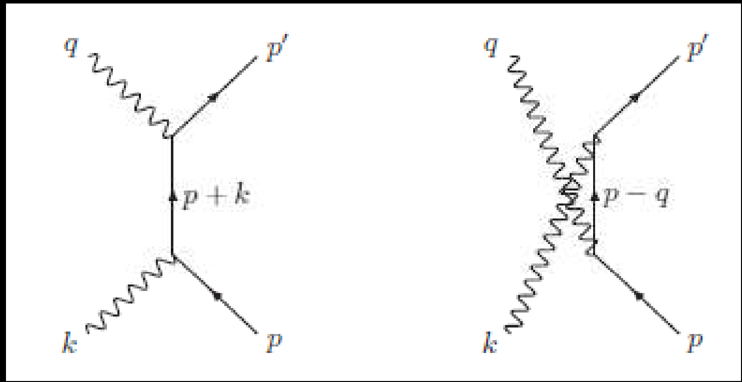
$$\sin\theta \simeq \frac{X}{m_a^2} = \frac{\alpha_{\rm em} c_{a\gamma\gamma} EB}{2\pi F_a m_a^2}$$

$$\approx 0.65 \times 10^{-2} c_{a\gamma\gamma} \left(\frac{F_a E}{10^{22}\ {\rm GeV}^2}\right)\left(\frac{B}{\mu{\rm G}}\right).$$

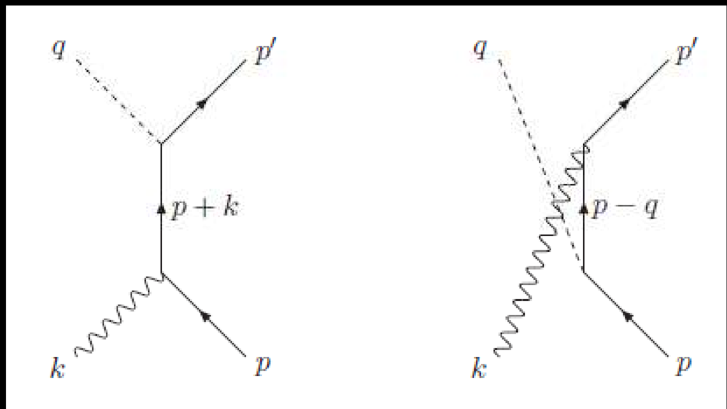
Gorbunov-Raffelt-Semikoz(arXiv:hep-ph/0103175)
Albuquerque-Chou(arXiv:1001.0972)

concluded that the axion transportation does not work, because the B field needed at AGN is not matched in our Galaxy (AC), and also in our Galaxy conversion rate of axion to photon is less than 10^{-8} (GRS) and considered ALP.

But we can also consider the bremsstrahlung processes, in addition to the oscillation.



$$\sigma \propto \frac{\alpha_{em}^2}{s}$$

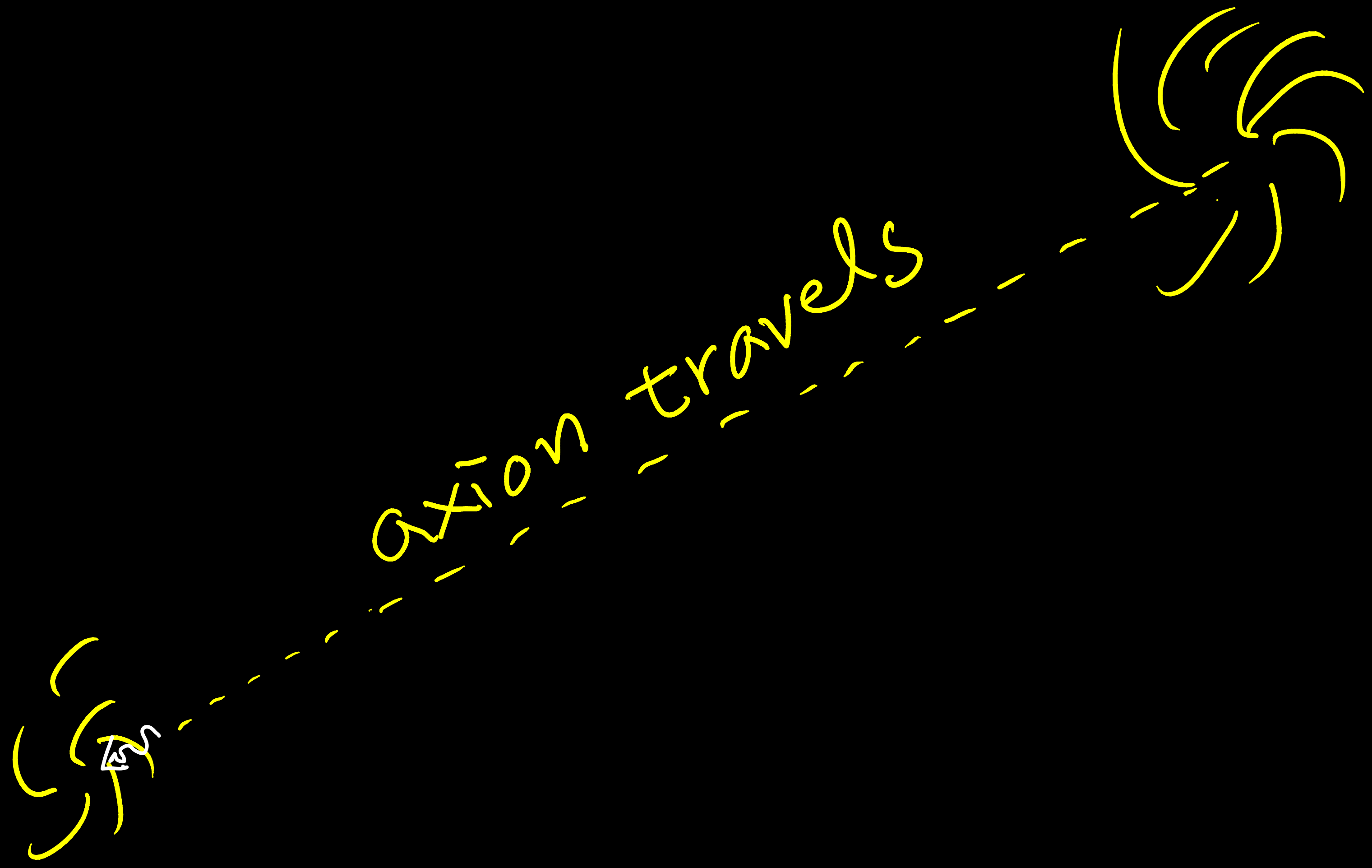


$$\sigma \propto \frac{\alpha_{em}}{F_a^2}$$

This ratio can be of order 1 if energy of DUHECR and F_a and m are comparable: KSVZ

$$Ratio = \frac{c_1^2}{\pi \alpha_{em}} \left(\frac{(m/F_a)^2}{1 + 4\ln(\sqrt{s}/m)} \right),$$

$$m = quark \quad mass$$


axion travels

This important comparable cross section is possible only when the total CM energy and quark mass are comparable to the axion decay constant.

In our Galaxy, total CM energy is much smaller than DUHECR energy E, since the target is at rest.
But in an AGN, the total CM energy can be of order E itself.
So, this axion bremsstrahlung process becomes important.
[Huh-Kim-Munoz (to appear)]

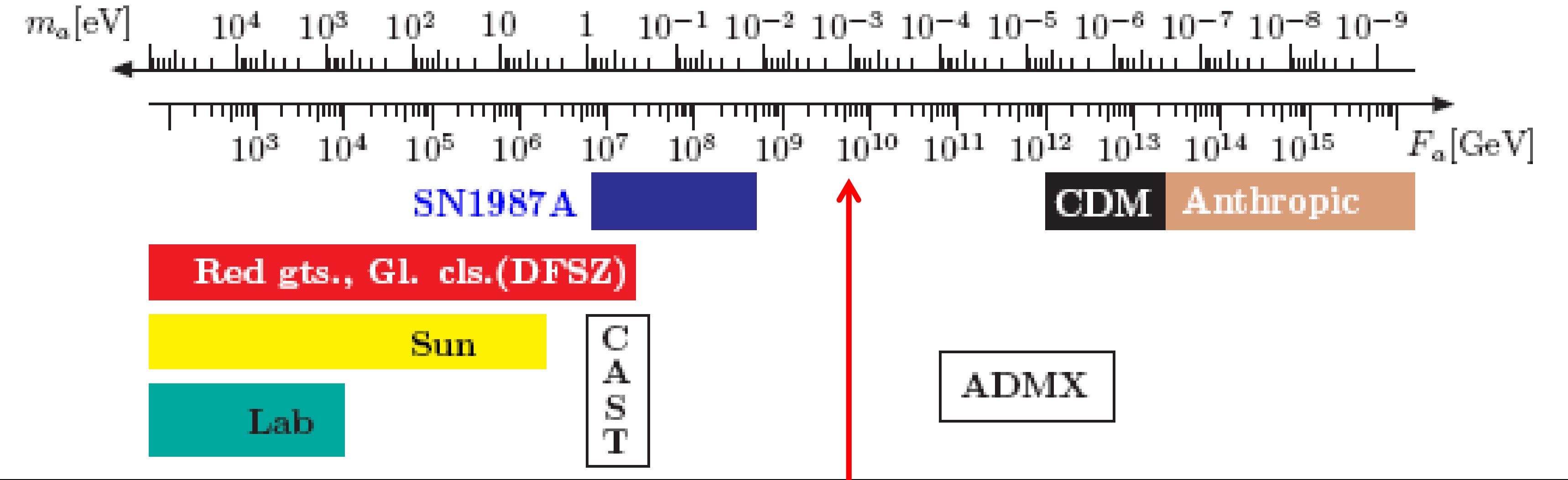
In AGN or quasar, in the strong B and E fields, charged particles are supposed to be accelerated to 10^{11} GeV. There are differently charged quarks and leptons which have different masses. Even, heavy quarks of mass below F_a can be accelerated. These synchtron-radiate photons and gluons. The directions of these radiated bosons may not be aligned.

So, the incident momenta **p** and **k** are not necessarily aligned. Let us suppose that they are 20 degrees off.

$$s=(p+k)^2=(E+E)^2-(\vec{p}+\vec{k})^2$$
$$\cong 2E_1E_2-2E_1E_2\cos 20^o$$
$$=0.12E_1E_2\approx(0.35E)^2\approx O(E^2)$$

So, we expect that the CM energies of the scattering is of order 10^{11} GeV above the heavy quark mass.

And, the axion production in AGN is comparable to the photon production. We will report the feasibility of this idea soon [Huh-K-Munoz].



White dwarf bound
(1st hint at the center of the axion window)

Is the window of hadronic axion still open?

$0.06 \text{ eV} < m_a < 0.6 \text{ eV}$
[Raffelt-Deabon, PRD 36 (1987) 2211]
$3\text{x}10^5 \text{ GeV} < F_a < 3\text{x}10^6 \text{ GeV}$, or
$0.02 \text{ eV} < m_a < 0.2 \text{ eV}$
[Chang-Choi, PLB 316 (1993) 51]

The hadronic axion in the 0.1 eV range has been allowed.

4. SUSY extension and axino

Strong CP solution and SUSY:

axion : implies a superpartner axino

The gravitino constraint: gravitinos produced thermally after inflation decays very late in cosmic time scale (>10^3 sec) and can dissociate the light nuclei by its decay products. Not to have too many gravitinos, the reheating temperature must be bounded,

$$T_R < 10^9 \text{ GeV(old), or } 10^7 \text{ GeV(recent)}$$

Ellis-Kim-Nanopoulos   Khori-Kawasaki-Moroi

Thus, in SUSY theories we must consider the relatively small reheating temperature.

The LSP seems the most attractive candidate for DM simply because the TeV order SUSY breaking scale introduces the LSP as a WIMP. This scenario needs an exact or effective R-parity for it to be sufficiently long lived.

For axino to be LSP, it must be lighter than the lightest neutralino. The axino mass is of prime importance. The conclusion is that there is no theoretical upper bound on the axino mass. For axino to be CDM, it must be stable or practically stable. Thus, we require the practical

R-parity or effective R-parity

KeV axinos can be warm DM (90s) [Rajagopal-Turner-Wilczek]
GeV axinos can be CDM (00s) [Covi-H. B. Kim-K-Roszkowski]

TeV axino (decaying) to DM [Huh- Kim, PRD 80, 075012 (2009)]

CDM axino comes into two categories:

(1) GeV scale LSP: The LSP χ decays to axino. There can be thermal axino density [Covi-K-Roszkowski] and non- thermal axino density arising from

χ → axino + photon [Covi-Kim-K-Roszkowski]

(2) TeV scale decaying axino:

(a) Around several hundred GeV, producing nonthermal neutralinos. [Choi-K-Lee-Seto]

(b) Much above TeV [Huh-K] in view of PAMELA/Fermi data

$$\tilde{a} \to N + \bullet\bullet\bullet \quad and \quad \tilde{G} + \bullet\bullet\bullet$$
$$\downarrow$$
$$\chi + \bullet\bullet\bullet$$

Concentrate on this possibility

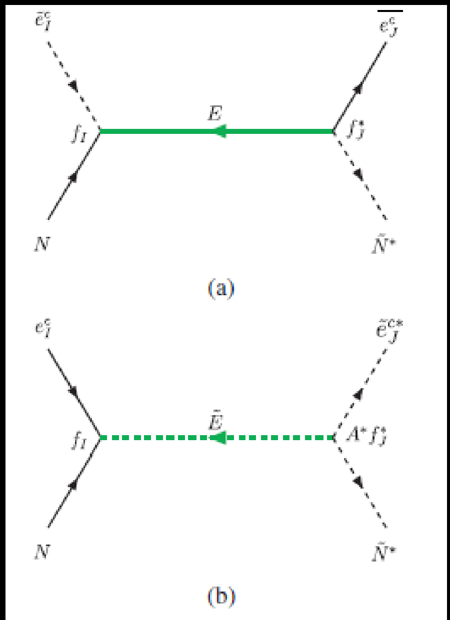
Let us comment on (2) first.
N is the decaying DM.

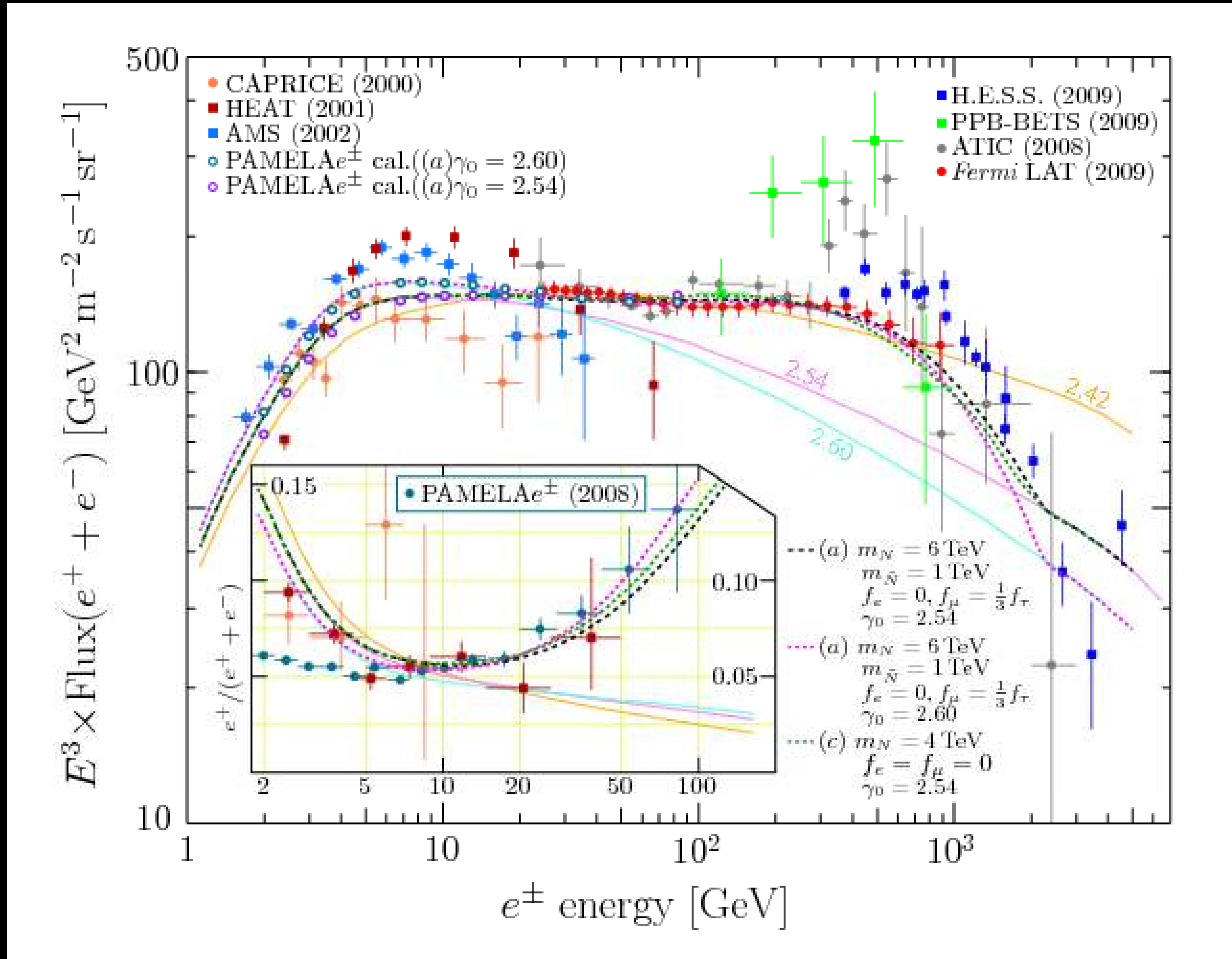
$$\int d^2\vartheta\left(\frac{1}{4M'}NNXX-\frac{c_g\alpha_g}{4\sqrt{2}\pi}\theta_g W_g W_g\right),$$

$$c_g\theta_g=\frac{A}{F_a}$$

$$\frac{No.\quad of\quad N}{No.\quad of\quad \chi}=2\left(\frac{32\pi^2}{\alpha_3^2}\right)\left(\frac{\langle X\rangle}{M'}\right)^2$$

We need $m_{N\chi}/m_N \sim 10^{-2}$ and $F_a \sim 4\text{x}10^{11}$ GeV $M' \sim 2\text{x}10^{15}$ GeV





Huh-K, PRD 80 (2009) 075012 [arXiv:0908.0152]

Gravitino problem is resolved if gravitino is NLSP, since the TP gravitinos would decay to axino and axion which do not affect BBN produced light elements. [Ellis et al, Moroi et al]

$$m_{\tilde{a}} < M_{3/2} < m_{\chi}$$

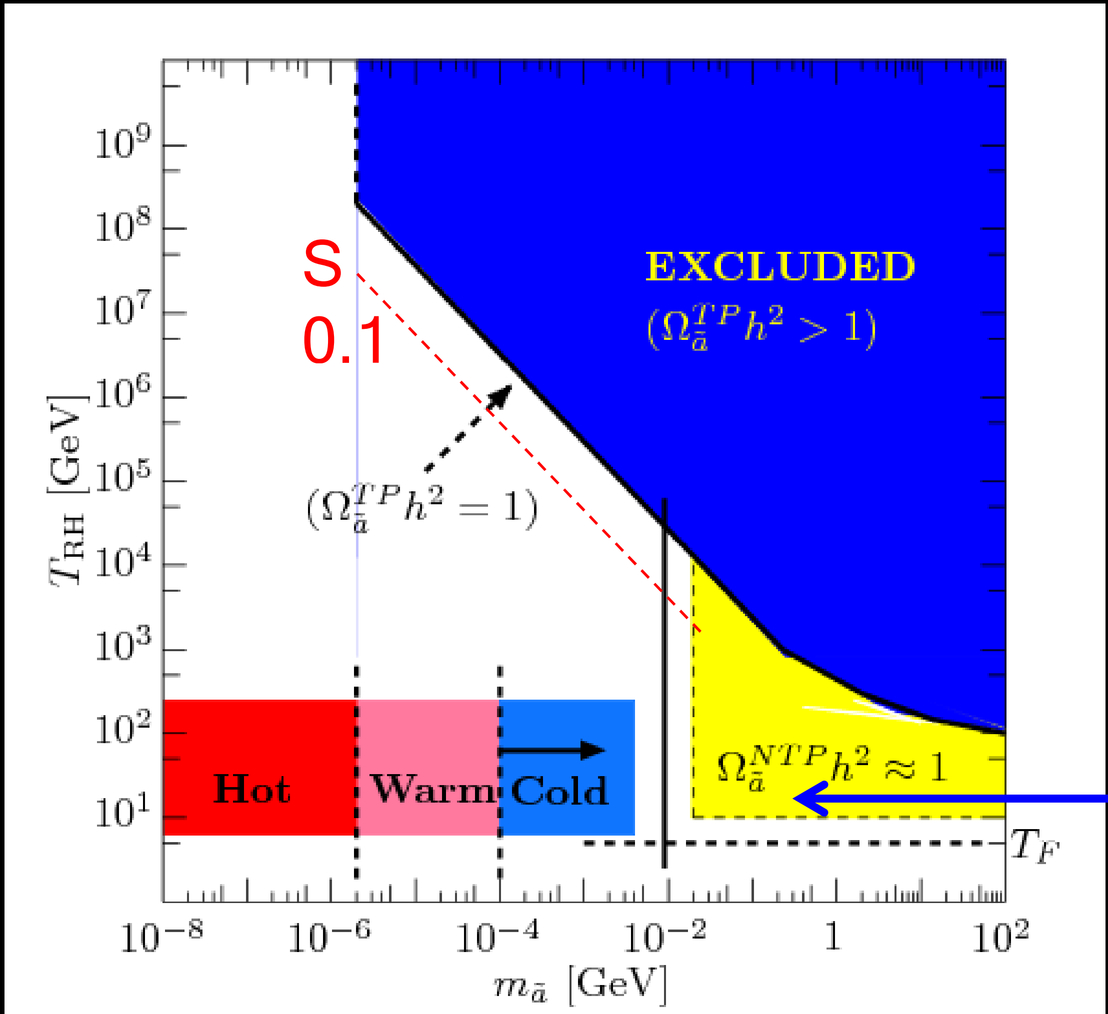
On the other hand, if χ is NLSP(=LOSP), the TP mechanism restricts the reheating temperature after inflation. At high reheating temperature, TP contributes dominantly in the axino production.

If the reheating temperature is below the c. energy density line, there still exists the CDM possibility by the NTP axinos. [Covi et al]

NTP: $$\Omega_{\tilde{a}} h^2 = \frac{m_{\tilde{a}}}{m_\chi} \Omega_\chi h^2$$ for

Even though the axino density itself is not estimateded thermally, its mother WIMP density is estimated by the thermal equilibrium.

It is a close cousin of the WIMP scenario.



$m_\chi = 100 GeV$

$F_a = 10^{11} GeV$

Covi-K-H B Kim-Roszkowski

Low re-heating T

Strumia, arXiv:1003.5847

$$\Omega_{\tilde a} h^2 = 1.24 g_3^4 F(g_3)\left(\frac{m_{\tilde a}}{GeV}\right)\left(\frac{T_{RH}}{10^4 GeV}\right)\left(\frac{10^{11} GeV}{F_a}\right)^2, \quad F(g_3) \approx 20 g_3^2 \ln\frac{3}{g_3}$$

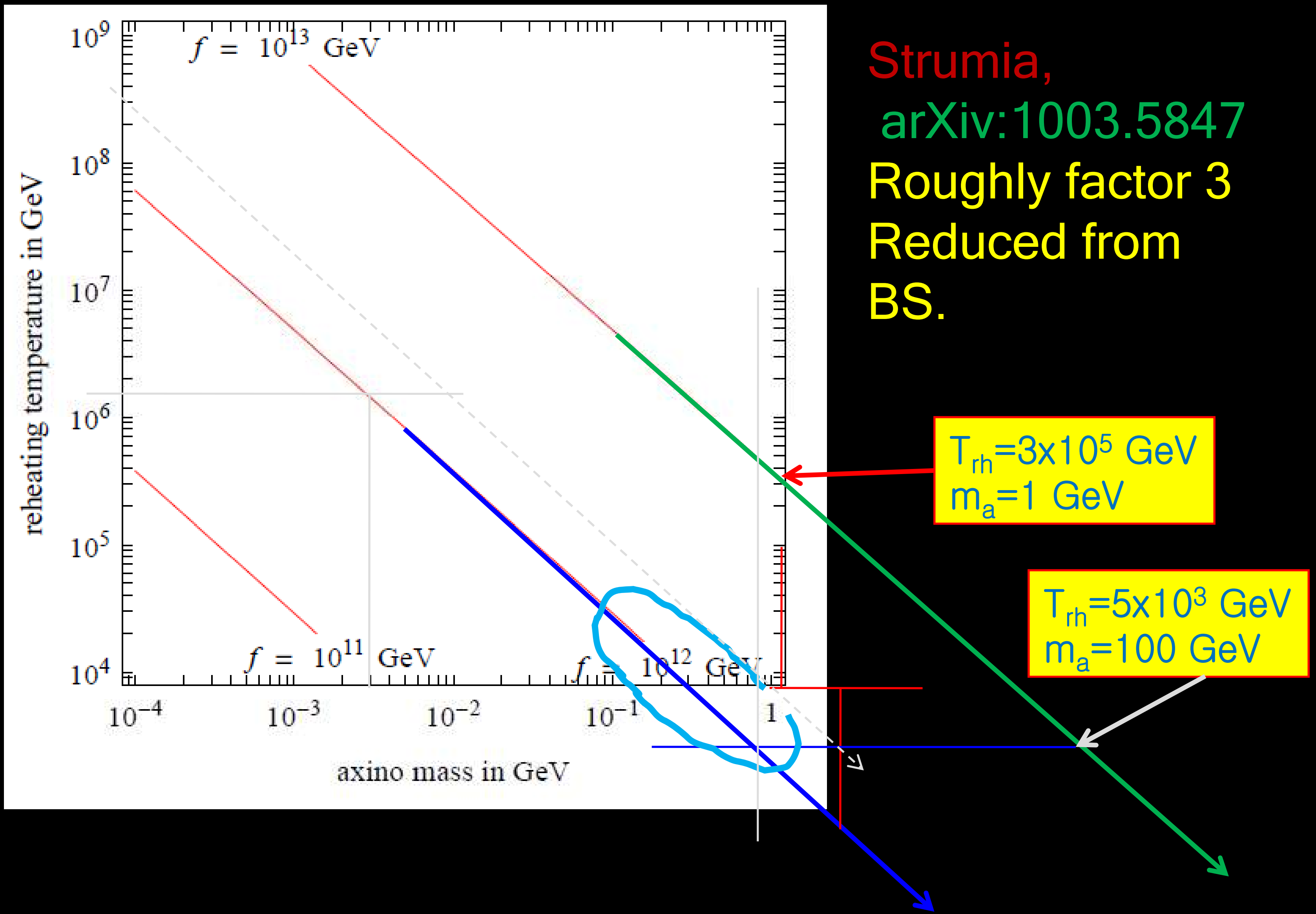
Strumia's number almost the same with CKKR.

CKKR drew for $\Omega h^2 = 1$ while Strumia for $\Omega h^2 = 0.1$.

BS gives a few times larger than the number from the effective mass of CKKR.

Strumia is a 2-3 times smaller than BS.

BS and Strumia compensate compared to CKKR.



Strumia,
arXiv:1003.5847
Roughly factor 3
Reduced from
BS.

If the reheating temperature is greater than 500 GeV, the axino needs F_a larger than 10^{12} GeV to close the Universe by GeV thermal axinos. Then, the axion density dominates that of axino.

High reheating temperature with SUSY with O(GeV) axino implies the axion domination of the Universe.

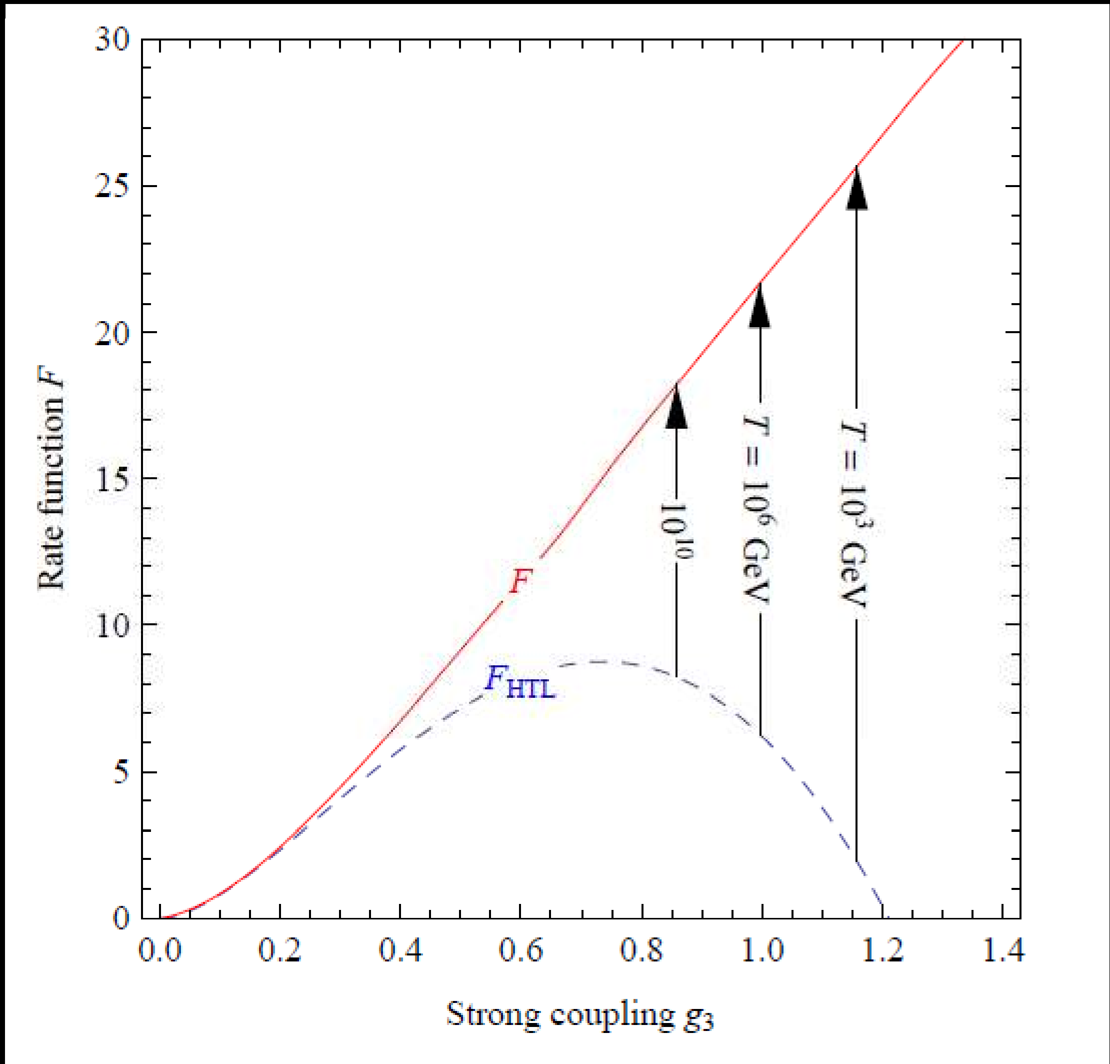
Strumia, arXiv:1003.5847

Considered the previously ignored term.

It is the axino-gluino-squark-squark coupling

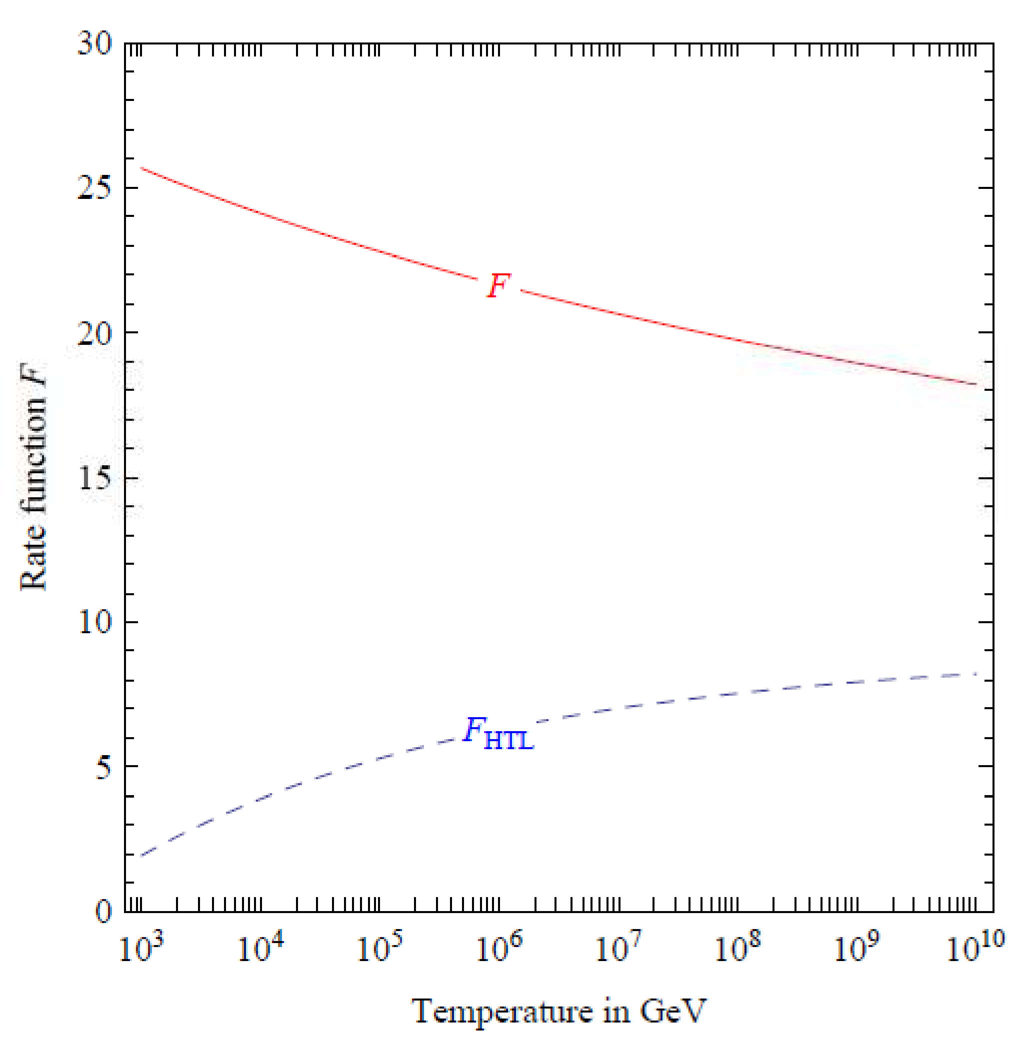
$$\frac{\alpha_3}{8\pi F_a}\left[i\overline{\psi}_a G^{\alpha}_{\mu\nu}\frac{[\gamma^\mu,\gamma^\nu]}{2}\gamma_5\tilde{g}^\alpha - 2\overline{\psi}_a\tilde{g}^\alpha D^\alpha \right], \quad D^\alpha = -g_3\sum_{\tilde{q}}\tilde{q}^* T^\alpha \tilde{q}$$

Term not considered before,
but considered in the calculation
as arising from the gluon exchange.
But HTL calculation seems giving
a dominant change.



Strumia,
arXiv: 1003.5847

Hard thermal loop
(Brandenbur+Steffen)
vs. Strumia on the
rate function F.
(Coupling fn)



Strumia,
arXiv: 1003.5847

Hard thermal loop (Brandenbur+Steffen) vs. Strumia on the rate function F. (Temperature fn)

In this figure, NTP axinos can be CDM for relatively low reheating temperature < 0.5 TeV, in the region

$$10 MeV < m_{\tilde{a}} < m_{\chi}$$

NTP axino as CDM possibility

The shaded region corresponds to the MSSM models with $\Omega_\chi h^2 < 10^4$, but a small axino mass renders the possibility of axino closing the universe or just 30 % of the energy density. If all SUSY mass parameters are below 1 TeV, then $\Omega\chi\ h^2 < 100$ but a sufficient axino energy density may not result for

$$m_{\tilde{a}} > 1 GeV \quad and \quad T_{RH} > TeV$$

Conclusion

I discussed CP, weak and strong, and axion with related issues.

1. Solutions of the strong CP problem :

   Nelson-Barr, *m_u=0 ruled out now*, axion.

2. Axions can be detected by cavity experiments. Most exciting is, its discovery confirms instanton physics of QCD by experiments.

3. Cosmology and astrophysics give bounds on the axion parameters. Maybe, axions are coming out from WD cooling process and DUHECRs. It is the first hint, in the middle of

   the axion window. A specific variant very light axion

   model has been constructed for N_{DW}=1/2 for WD E loss mech.

3. With SUSY extension, O(GeV) axino can be CDM or decaying axino to CDM [Choi-K-Lee-Seto(08)] can produce the needed number of nonthermal neutralinos. In any case, to understand the strong CP with axions in SUSY framework, the axino must be considered in the discussion.