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AXISYMMETRIC SCRAPE-OFF PLASMA TRANSPORT

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ABSTRACT

The two-dimensional flow of a collision dominated hydrogen scrape-off plasma in an axisymmetric tokamak is examined. This flow is described by a set of equations which contain the dominant terms in a maximal ordering appropriate to high density experimental divertors and reactor scrape-off plasmas. Comparison of the theory to estimates of scrape-off parameters in the Doublet III expanded boundary plasmas suggests that analysis of classical and neoclassical processes alone may be sufficient to predict plasma transport in high density scrape-off plasmas of practical importance.

I. INTRODUCTION

Tokamak divertors remove impurities and power by scraping off the outer layer of the plasma and channeling it along open magnetic flux surfaces to a plate or wall away from the main plasma. Some of the purposes of a divertor are to prevent sputtering of impurities and erosion of the vessel wall. In a reactorlike device, the problems of sputtering and erosion may then be transferred to the divertor plate. Fortunately, tokamak divertors have recently been able to operate in a high density, low temperature mode. This mode is important for tokamak fusion plasmas for several reasons: (1) sputtering of impurities and wall erosion are reduced due to the low edge temperatures;[1] (2) energy can be lost by plasma-neutral interactions (e.g., ionization and excitation) rather than by collisions with the divertor plate; and (3) the overall plasma confinement may be increased.[2] Also, outflux of helium and fusion power is facilitated by operating with the highest possible scrape-off density.[3]

It has been shown that the low temperature, high density mode can be achieved in divertors by establishing a large particle recycling (i.e., the repeated neutralization of plasma on the plate, followed by ionization, and neutralization).[3-6] Under conditions of low temperature and high density, the Coulomb collision frequency is large. There is, therefore, a need for a theory of collision-dominated transport in the scrape-off region. Such a theory can be used to model existing divertor experiments as well as predict the scrape-off thickness of reactorlike configurations. In this paper, we analyze the fluid equations appropriate to such high collisionality divertors, determine the order of all the terms, and solve the ordering equations to find an estimate for the scrape-off thickness.

We use the velocity moments of the Fokker-Planck equations to obtain a steady-state description of the transport for a plasma with a single hydrogen isotope in an axisymmetric magnetic field. The magnetic field is assumed to be determined by currents external to the scrape-off region. We examine only the part of the scrapeoff where the Debye length is small compared to characteristic scale lengths, so the ion and electron densities are related by quasineutrality, and the electrostatic potential can be determined from the moment equations.

First, we define a specific ordering which is "maximal," in the sense that all of the important physical processes which may contribute to collision-dominated scrape-off transport appear in the same order. The moment equations are written as tensor invariants, which can be readily transformed to any given coordinate system. The dominant terms are then displayed in orthogonal toroidal flux coordinates.

We find that neoclassical theory alone should be adequate to predict the scrape-off thickness in sufficiently dense reactor scrape-off plasmas because neoclassical transport in the radial direction across magnetic flux surfaces increases rapidly with increasing collisionality. Also, the radial scrape-off thickness is estimated for Doublet-III (D-III) by extending the model to conditions found in the D-III expanded boundary divertor configuration. We find reasonable agreement with the scrape-off thickness measured in D-III during operation in the high density, low temperature mode.

II. TRANSPORT WITH MAXIMAL ORDERING

The steady-state Fokker-Planck equation for ions is

$$\vec{v}\cdot\nabla f + \frac{e}{m}\left(-\nabla\phi + \vec{v}\times\vec{B}/c\right)\cdot\nabla_v f = C + \Sigma \,, \qquad (1)$$

where e and m are the ion charge and mass, ∇_v is the velocity space derivative, C is a sum over all Coulomb collisions, and Σ is a sum over all collisions of ions with neutrals. A similar equation holds for electrons with charge −e and mass m_e. The relevant velocity moments are[7]

$$\left(nu^{\alpha}\right)_{,\alpha} = S \ , \tag{2}$$

$$P^{\alpha\beta}{}_{,\beta} + en\left(g^{\alpha\beta}\phi_{,\beta} - \varepsilon^{\alpha\beta\gamma}u_{\beta}B_{\gamma}/c\right) = F^{\alpha} + X^{\alpha}, \tag{3}$$

$$Q^{\alpha}{}_{,\alpha} + enu^{\alpha}\phi_{,\alpha} = Q_{\Delta} + u_{\alpha}F^{\alpha} + Y, \tag{4}$$

$$R^{\alpha\beta}{}_{,\beta} + \frac{e}{m}\left(\phi_{,\beta}P^{\beta\alpha} + \frac{1}{2}g^{\alpha\beta}\phi_{,\beta}P^{\gamma}_{\gamma} - \varepsilon^{\alpha\beta\gamma}Q_{\beta}B_{\gamma}/c\right) = G^{\alpha} + Z^{\alpha} . \tag{5}$$

Here n is the quasineutral plasma density, u is the ion flow velocity, $S = \int d^3\vec{v}\Sigma$ is the ion particle source, $P^{\alpha\beta} = nTg^{\alpha\beta} + \Pi^{\alpha\beta} + mnu^{\alpha}u^{\beta}$ is the stress tensor, $nT = m\int d^3\vec{v}\ [(v^{\alpha} - u^{\alpha})(v_{\alpha} - u_{\alpha})/3]f$ is the ion pressure, $\Pi^{\alpha\beta} = m\int d^3\vec{v}\ [(v^{\alpha} - u^{\alpha})(v^{\beta} - u^{\beta}) - g^{\alpha\beta}(v^{\gamma} - u^{\gamma})(v_{\gamma} - u_{\gamma})/3]f$ is the viscosity tensor, $g^{\alpha\beta}$ is the metric tensor, $\varepsilon^{\alpha\beta\gamma}$ is the Levi-Civita permutation tensor, $F^{\alpha} = \int d^3\vec{v}\ v^{\alpha}C$ is the force on ions due to Coulomb friction, $X^{\alpha} = m\int d^3\vec{v}\ v^{\alpha}\Sigma$ is the neutral friction on the ions, $Q^{\alpha} = q^{\alpha} + (5/2)nTu^{\alpha} + u_{\beta}\Pi^{\beta\alpha} + mnu^{\beta}u_{\beta}u^{\alpha}/2$ is the ion energy flux, $q^{\alpha} = \int d^3\vec{v}\ (m/2)(v^{\beta} - u^{\beta})(v_{\beta} - u_{\beta})(v^{\alpha} - u^{\alpha})f$ is the ion heat flux, $Q_{\Delta} = \int d^3\vec{v}\ (m/2)(v^{\alpha} - u^{\alpha})(v_{\alpha} - u_{\alpha})C$ is the Coulomb energy interchange, $Y = \int d^3\vec{v}\ (m/2)v^{\alpha}v_{\alpha}\Sigma$ is the ion energy source due to collisions with neutrals, $R^{\alpha\beta} = \int d^3\vec{v}(mv^{\gamma}v_{\gamma}/2)v^{\alpha}v^{\beta}f$ is the energy-weighted stress tensor, $G^{\alpha} = \int d^3\vec{v}\ (mv^{\beta}v_{\beta}/2)v^{\alpha}C$ is the change of ion energy flux due to Couloumb collisions, and $Z^{\alpha} = \int d^3\vec{v}\ (mv^{\beta}v_{\beta}/2)v^{\alpha}\Sigma$ is the change in ion energy flux due to collisions with neutrals. Subscripts denoting ions are suppressed here and throughout this paper. The covariant derivative is represented by a comma, and repeated indices imply summation. Similar equations can be obtained for electrons.

The ion continuity relation, Eq. (2), provides a starting point for the derivation of the maximally ordered transport equations. In a right-handed orthogonal toroidal coordinate system (ψ,θ,ξ), where ψ labels magnetic flux surfaces, θ is a poloidal angle, and ξ is the toroidal symmetry angle, Eq. (2) is

$$g^{-1/2}\left[\frac{\partial}{\partial\psi}\left(nu_\psi|\nabla\psi|g^{1/2}\right)+\frac{\partial}{\partial\theta}\left(nu_\theta|\nabla\theta|g^{1/2}\right)\right]=S, \tag{6}$$

where $g^{1/2} = 1/\nabla\psi \times \nabla\theta\cdot\nabla\xi$ is the Jacobian. The radial ion flux, nu_ψ, is to be found from the toroidal component of the ion momentum balance, Eq. (3). The poloidal ion flow, u_θ, can be resolved into a parallel component, $u_\parallel$, along the magnetic field and a diamagnetic component, u_d, in the direction $\vec{B} \times \nabla\psi$:

$$u_\theta = (B_\xi/B)u_d + (B_\theta/B)u_\parallel. \tag{7}$$

u_d will be found from the radial component of the ion momentum balance. $u_\parallel$ will be determined from the component of the total momentum balance parallel to the magnetic field. We shall assume that sources in Eq. (6) are not so large as to allow one of the terms in the flux divergence to dominate the others. (Cases where sources are balanced by one of the fluxes in Eq. (6) are comparatively simple and have been treated elsewhere.)[3-6,8]

The expression for the radial ion particle flux, which can be derived from the toroidal component of Eq. (3), is considerably simplified by a set of approximations appropriate to the high-collisionality scrapeoff. It is convenient to express the ordering in terms of the geometric parameter, ϵ, the inverse aspect ratio. We order $B_\theta/B \sim \epsilon^2$, since this relation is generally valid (except for a small region near the poloidal field null in divertor tokamaks) for the highest MHD stable pressure and current in a reactor plasma. We choose an electron/ion mass ratio $m_e/m \sim \epsilon^8$ (which is approximately correct for practical reactor parameters) since this gives the largest number of terms remaining to lowest order in ϵ. (In our maximal ordering; i.e., an ordering where the largest possible number of terms occur together at lowest order, it is desirable to have as many terms as possible of order ϵ.) In most of the spatial extent of the scrapeoff, it suffices to order the parallel mach number $M = u_\parallel/v_{th} \ll 1$, where $v_{th} = (2T/m)^{1/2}$. Post et al.[3] have described why it is appropriate to assume $M \ll 1$ in the case of interest here, where a high scrape-off density is maintained by a high rate of recycling near the divertor or limiter plate. Briefly, the recycling source near the plate leads to a larger particle flux along the magnetic field near the plate than farther upstream in the flow. Sonic flow throughout would imply a large gradient in density and in the dynamic head, $mnu_\parallel^2/2$. A large force would be required to drive this dynamic head against the plate, and no such force is available. Therefore, the flow is subsonic except in the region of intense recycling near the plate. We exclude this intense recycling region from consideration in the present paper. For the maximal ordering, we take $M \sim \epsilon$; results for smaller values of M are easily obtained once the maximal ordering results are in hand. We also assume that the electrostatic potential, ϕ, due to voltage applied across the region of interest is given by $e\phi/T \lesssim 1$. Finally, the restriction to high collisionality must be reflected in the ordering assumptions. We anticipate the large scrape-off thickness obtained in a cool, dense plasma by assuming a small ratio $\delta_p = \rho_p/L_\phi$ of the poloidal gyroradius to the radial scale height, L_ϕ. (Here $\rho_p = v_{th}/\Omega_p$, where

$\Omega_p = eB_\theta/(mc)$ is the poloidal gyrofrequency.) For the maximal ordering, we choose $\delta_p \sim M$. The generalization to higher collisionality cases with $\delta_p \ll M$ is straightforward.

A. Radial ion flux

With the ordering just described, the dominant contribution to the radial ion flux can be shown to be the neoclassical flux, which results from the component of the Coulomb friction parallel to the magnetic field. As in the Pfirsch-Schlüter theory for closed magnetic flux surfaces,[7] this gives a neoclassical contribution

$$nu_\psi = -\frac{1}{m\Omega_p}\frac{B_\xi}{B}F_\parallel , \tag{8}$$

$$F_\parallel = -\frac{\alpha_o m_e \nu_e J_\parallel}{e} + \beta_o n \frac{B_\theta}{B}\frac{\partial T}{\partial \theta}|\nabla\theta|, \tag{9}$$

where $\alpha_o = 0.5129$, $\beta_o = 0.711$, ν_e is the electron Coulomb collision frequency, and $J_\parallel \equiv en\left(u_\parallel - u_\parallel^e\right)$ is the current parallel to the magnetic field.[9] As shown in the Appendix, other toroidal forces drive fluxes only of smaller order and are neglected. By investigating the total toroidal momentum balance, using a simple analysis of the ordering described elsewhere,[10] it can be shown *a posteriori* that, to lowest order in ε,

$$\frac{\partial}{\partial\theta}\left(\frac{B_\theta}{B}J_\parallel|\nabla\theta|g^{1/2}\right) = -\frac{\partial}{\partial\theta}\left(\frac{B_\xi}{B}J_d|\nabla\theta|g^{1/2}\right), \tag{10}$$

(i.e., the radial flux is ambipolar to this order, as shown in the Appendix.) The diamagnetic current J_d is determined from the total radial momentum balance to be

$$J_d = \frac{c}{B} \frac{\partial(2nT)}{\partial\psi} |\nabla\psi|. \tag{11}$$

Note that the total pressure is twice the ion pressure, as the high collisionality assumed here can be shown to force the ion and electron temperatures to be approximately equal,[10] as shown in the Appendix.

The temperature gradient required to compute the thermal force in Eq. (8) is described by the total heat balance. For flows which are subsonic and also have $\delta_p << 1$, the solution for the energy flux using Eq. (5) proceeds as described in the review by Hinton and Hazeltine.[9] Here we present the results obtained after separating classical and neoclassical contributions and determining the dominant contributions assuming maximal ordering described above.

The total heat balance is obtained by summing the ion and electron versions of Eq. (4) and subtracting the mechanical energy.[9] Classical viscous heating is negligible for the subsonic flows under consideration, and we also neglect any neoclassical viscous heating (cf. Section IIC below). In orthogonal toroidal coordinates, the heat balance is

$$3nu_\psi \frac{\partial T}{\partial\psi} |\nabla\psi| + \frac{3}{2} n\left(u_\theta + u_\theta^e\right) \frac{\partial T}{\partial\theta} |\nabla\theta| + nTg^{-1/2}\{2 \frac{\partial}{\partial\phi} [u_\psi |\nabla\psi| g^{1/2}]$$

$$+ \frac{\partial}{\partial\theta} [(u_\theta + u_\theta^e)|\nabla\theta| g^{1/2}]\}$$

$$= g^{-1/2}\{\frac{\partial}{\partial\theta} [\frac{B_\theta}{B} (\kappa_\parallel^e \frac{\partial T}{\partial\theta} |\nabla\theta| + \beta_o TJ_\parallel/e)|\nabla\theta| g^{1/2}] - \frac{\partial}{\partial\phi} [\frac{2nT}{m\nu} (\frac{\nu}{\Omega})^2 \frac{\partial T}{\partial\phi} |\nabla\phi|^2 g^{1/2}]\}$$

$$- F_\parallel J_\parallel/(en) + W_{neutrals} , \tag{12}$$

where $\kappa_\parallel^e = 3.1616\ nT/(m_e \nu_e)$, $\Omega = eB/(mc)$, and ν is the ion-ion collision frequency.[9] Here ambipolarity has been used to combine contributions from radial ion and electron flows. The poloidal electron flow velocity, $u_\theta^e = u_\theta - (J_d B_\xi/B + J_\parallel B_\theta/B)/(en)$, is related to the ion flows by Eqs. (7), (10), and (11). The dominant contributions to the heat conduction term are due to classical electron conduction along the magnetic field and classical ion heat conduction across magnetic flux surfaces. Ion and electron heat fluxes in the diamagnetic direction cancel for equal ion and electron temperatures. Similarly, neoclassical radial ion and electron heat fluxes cancel (in contrast to the case for closed flux surfaces).[7] The neutral source term can be simplified by noting that electron-neutral friction produces negligible force and that $u_\psi \ll u_d \ll u_\parallel$, as described in Section IIB below. Then, assuming the neutral flow velocity and mean energy per particle are not of greater order than that of the ions,

$$W_{\text{neutrals}} = Y + Y^e - 3TS \ , \tag{13}$$

where Y^e is the energy loss rate due to interaction of electrons with neutrals (including radiation losses). For subsonic flows, the work due to neutral friction is negligible (as is dilution of the "ram" energy in the directed flow) and does not appear in the energy balance.

B. Poloidal ion flow

Flows along and across the magnetic field within a flux surface are now described. The diamagnetic ion flow velocity is given by the dominant terms in the radial ion momentum balance

$$nu_d = \frac{|\nabla\psi|}{m\Omega}\left(\frac{\partial(nT)}{\partial\psi} + en\frac{\partial\phi}{\partial\psi}\right). \tag{14}$$

The potential, ϕ, can be integrated in poloidal angle from any specified boundary condition using the dominant contributions to the parallel Ohm's law,

$$en\frac{B_\theta}{B}\frac{\partial\phi}{\partial\theta}|\nabla\theta| = -F_\parallel , \tag{15}$$

where $F_\parallel$ is the Coulomb friction given in Eq. (9).

The parallel ion flow velocity, $u_\parallel$, is determined by the dominant contributions to the total plasma momentum balance parallel to the magnetic field,

$$mn\left(u_\psi\frac{\partial u_\parallel}{\partial\psi}|\nabla\psi| + u_\theta\frac{\partial u_\parallel}{\partial\theta}|\nabla\theta|\right) = -\frac{B_\theta}{B}\frac{\partial(2nT)}{\partial\theta}|\nabla\theta| + F_{vis} + X_\parallel - mu_\parallel S. \tag{16}$$

The lowest order classical contribution to the parallel viscosity can be written

$$F_{vis} = \frac{\partial}{\partial\psi}\left[\left(-\eta_2\frac{\partial u_\parallel}{\partial\psi}|\nabla\psi| + \eta_4\frac{\partial u_\parallel}{\partial\theta}|\nabla\theta|\right)|\nabla\psi|\right] - \frac{\partial}{\partial\theta}\left[\eta_4\frac{\partial u_\parallel}{\partial\psi}|\nabla\psi||\nabla\theta|\right], \tag{17}$$

where $\eta_4 = nT/\Omega$ and $\eta_2 = 6\nu\eta_4/(5\Omega)$ are viscosity coefficients.[9] No comparable neoclassical contributions[11] to the parallel viscous force are known, but the relevant kinetic theory has not been thoroughly investigated for a collision dominated scrape-off plasma.

III. DISCUSSION

Equations (6) through (17) are a complete set of partial differential equations which include only the lowest order terms in the maximal ordering described above. Here we discuss suitable boundary conditions for these equations, summarize the maximal ordering assumptions, comment on results for other orderings, and estimate the scrape-off thickness for a high density Doublet III scrapeoff.

Equations (6) through (11) contain six derivatives in the radial coordinate, ψ. Three boundary conditions can be provided by extracting the radial particle and energy flux, and by analyzing the plasma rotation with the usual flux-surface-averaged neoclassical theory[7,11,12] applied on a flux surface far from the separatrix or edge of the limiter. Three additional radial boundary conditions are provided by the requirement that the density, temperature, and rotation be negligible many radial scale heights outside the separatrix or innermost edge of a toroidally symmetric limiter. Equations (6) through (17) also contain six derivatives in the poloidal angle, θ. A complete set of poloidal boundary conditions requires specification of: the potential difference along a flux tube applied at the plates at each end of the tube, the net poloidal current emitted at the plates, and the energy and ion particle fluxes flowing poloidally to each end. While an exact treatment requires a kinetic analysis of the region near the plates, rough estimates of end losses are available from simple plasma sheath theory.[13,14] On closed flux surfaces near the scrapeoff, these poloidal boundary conditions are, of course, replaced by the appropriate periodicity relations.

The maximal ordering assumptions described above can be summarized as follows: $\delta_p \sim M \sim \epsilon$, $B_\theta/B \sim \epsilon^2$, and $(m_e/m)^{1/2} \sim \epsilon^4$. The parallel momentum balance, Eq. (17) (see Appendix) restricts the poloidal pressure drop to be at most of order M^2. That is, $(nT - \langle nT\rangle)/\langle nT\rangle \lesssim M^2$ where $\langle\rangle$ denotes the flux surface average. However, the density may increase and the temperature decrease by a factor of order 1 when approaching the poloidal boundaries of the regions described by Eqs. (6) through (17). This explains the ordering of Ohm's law given in Eq. (15) where the pressure gradient is negligible when compared to the parallel thermal force and the resistance to the parallel current (cf. Eq. 9).

The radial scale height, L_ψ, for the plasma density can be estimated from Eqs. (8) through (12). In terms of dimensionless quantities, the result is

$$\delta_p^2 \sim M\Delta\,(m_e/m)^{1/2}, \tag{18}$$

where the inverse collisionality, $\Delta = \lambda_{mfp}/L_\parallel$, is the ratio of the Coulomb mean-free path, $\lambda_{mfp} = v_{th}/\nu$, to the parallel scale height, $L_\parallel = (B/B_\theta)/|\nabla\theta|$. Equation (18) can be solved for the radial scale height, L_ψ, using the relation $\delta_p = \rho_p/L_\psi$. The result, expressed in convenient units, is

$$L_\psi(\mathrm{cm}) \sim 1.1\ \left(\frac{B}{10B_\theta}\right)\left[\frac{(\Lambda/10)A^{1/2}L_{\parallel m}n_{14}}{T_{eV}B_T^2\,M}\right]^{1/2}, \tag{19}$$

where Λ is the Coulomb logarithm,[9] A is the hydrogen isotope mass in amu, $L_{\parallel m}$ is the parallel scale height in meters, $n_{14} = n/(10^{14}\mathrm{cm}^{-3})$, T_{eV} is the temperature in eV, and B_T is the toroidal magnetic field in Tesla. (It should be noted that we have assumed $L_\psi \ll 1/|\nabla\theta|$ in deriving the results in Section II. From Eq. (9), it can be seen that this is not a significant limitation for a reasonable tokamak scrape-off plasma.)

For practical applications, it is necessary to estimate the radial scale height for plasma parameters which do not satisfy the maximal ordering assumptions described above. It is especially important to relax the restrictions $\delta_p \sim M \sim \epsilon$ in order to treat cases with various recycling rates and plasma densities and temperatures. In particular, a high density scrapeoff plasma should have a very short Coulomb mean-free path, and small inverse collisionality, $\Delta = \lambda_{mfp}/L_\parallel$. Equation (18) suggests examining cases with $\delta_p \lesssim M \lesssim \epsilon$ for such plasmas. It has been found that Eqs. (18) and (19) also apply to cases with this relaxed ordering.[10] Equations (6) through (17) remain an accurate description of the plasma transport for this relaxed ordering.

In the opposite case where $\delta_p \gg M$, diamagnetic flows dominate the poloidal plasma transport. One complication for large values of δ is that loss cone instabilities may compromise the validity of simple kinetic[15] and fluid theories when the neoclassical radial particle flux is insufficient to guarantee $\delta \ll 1$.

Advection of energy by neoclassical radial particle fluxes becomes relatively less significant if the maximal ordering is relaxed to allow $(m_e/m)^{1/2} \ll \epsilon^4$ or $B_\theta/B \ll \epsilon^2$. The first of these conditions never applies to practical reactor geometries, and the second occurs only in a small region near the separatrix of a poloidal divertor, where $B_\theta/B \ll \epsilon^2$.

Not all of the physical effects contained in Eqs. (6) through (17) have been included previously in fluid models of the tokamak scrapeoff. Several computational fluid models have contributions of parallel flows to conservation of ions, momentum, and energy.[4-6,13,16] However, diamagnetic flows and the related momentum transport have not been considered in these models. These treatments have also omitted the radial particle flux, nu_ϕ, and the frictional heating, $-F_\parallel J_\parallel/(en)$, due the Pfirsch-Schlüter return currents described by Eq. (10). Viscous forces have also been omitted, although the first term in Eq. (17) was discussed by Tokar and Nedospasov.[16] Thus, there is considerable scope for improving fluid models of the axisymmetric tokamak scrapeoff at high collisionality.

We have estimated a density scale height of 3 cm from Eq. (20) for the D-III expanded boundary plasma at a point midway between the poloidal field null and the vacuum vessel. The plasma parameters were taken at the inflection point of the radial temperature profiles reported by Petravic et al.[13] (Their results were obtained by integrating one-dimensional conservation equations along field lines from a set of initial radial profiles chosen to match estimates of the scrape-off thickness.) To within the accuracy of our ordering estimates and of the experimental data, the neoclassical theory gives good agreement with the scrape-off thickness measured for the expanded boundary configuration in D-III. It would be desirable to make more detailed numerical computations using Eqs. (6) through (17) and to compare results with experimental data on the high collisionality axisymmetric plasmas which have recently become available.

IV. SUMMARY

We analyzed the applicability of fluid theory for a collision dominated (low-T, high-n) scrape-off plasma, such as is observed in a number of tokamak divertors. The fluid equations were cast in a form suitable for numerical solution, and an estimate was made of the relative importance of the classical and neoclassical terms for subsonic flow. An estimate of the radial scrape-off width was derived from an ordering analysis.

Radial particle flow is primarily due to thermal gradients or resistance to Pfirsch-Schlüter return currents. Poloidal flows result from two competing processes. Poloidal rotation of the ion flow is driven by radial gradients of the pressure and electric potential. Poloidal pressure gradients drive a flow along magnetic field lines, and this flow has a poloidal component.

Advection of energy and work done by the flow are modified by heat conduction parallel to the magnetic field, classical radial ion heat conduction, frictional dissipation of the Pfirsch-Schlüter return current, and interaction of the plasma with neutral gas. Many of these processes have been omitted from previous treatments of scrape-off flows in tokamaks.

An estimate of the radial scrape-off width was derived from an ordering analysis. This procedure is useful for interpreting experiments and for reactor designs. In the case of D-III expanded boundary configurations we find good agreement between the measurements of the scrapeoff and our theoretical estimates.

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APPENDIX: MAXIMAL ORDERING

Here we describe an ordering consistent with the equations presented in Section II. We demonstrate why the radial particle flow is ambipolar in this ordering and explain why this ordering leads to the relations $e\phi \sim T_e \approx T$ (where T_e is the electron temperature and T is the ion temperature). The ordering of interest here is "maximal" in the sense that the largest possible number of terms occur together at lowest order; e.g., in the ion continuity equation:

$$\frac{nu_\psi}{L_\psi} \sim \frac{nu_\parallel}{L_\parallel} \sim \frac{nu_d}{L_d} \sim S \,, \tag{A1}$$

where $L_\psi = n/((\partial n/\partial\psi)|\nabla\psi|)$, $L_d = L_\theta = 1/|\nabla\theta|$, and $L_\parallel = L_\theta B/B_\theta$. We also order the neutral-ion friction $X \sim mu_\parallel S$ and the energy sources $Y \sim Y^e \sim TS$, which assumes a momentum transfer $\sim mu_\parallel$ and an energy transfer $\sim T$ per ionization event.

The ordering is defined by parallel Mach number $M \sim \epsilon$, poloidal gyroradius parameter $\delta_p \sim \epsilon$, $B_\theta/B \sim \epsilon^2$, and hence $\delta = \delta_p B_\theta/B \sim \epsilon^3$. The restriction on mass ratio $(m_e/m)^{1/2} \sim \epsilon^4$, then implies an inverse parallel collisionality, $\Delta \sim \epsilon^5$, which is sufficient to make the ordering maximal. Ambipolar radial flow results from the fact that the dominant term in the toroidal ion moment balance is $enu_\psi B_\theta/c = F_\parallel$ and that the electron charge and Coulomb friction are $e_e = -e$ and $F_\parallel = -F_\parallel^e$. This relation yields the same result for u_ψ for both species. It can be readily shown that other terms in the toroidal ion momentum balance are of higher order in ϵ than $F_\parallel \sim nT/L_\parallel$. (The ordering $F_\parallel \sim nT/L_\parallel$ is immediately evident for the second term in Eq. (9); for the first term in Eq. (9), $F_\parallel \sim nT/L_\parallel$ follows from the discussion of $J_\parallel$ given below.) The ordering of the neutral friction is $X_\parallel L_\parallel/(nT) \sim$

$u_\parallel SL_\parallel/(nT) \sim mu_\parallel (nu_\parallel/L_\parallel)L_\parallel/(nT) \sim M^2 \sim \varepsilon^2 \ll 1$. It can also be shown that the largest contribution of the toroidal viscous force on the ions is of order ε^2. For example, the toroidal projection of the first term in Eq. (17) is of order $(nT/\Omega)(\nu/\Omega)(u_\parallel/L_\phi^2)L_\parallel/(nT) \sim [v_{th}/(\Omega L_\phi)]^2 (u_\parallel/v_{th})(L_\parallel \nu/v_{th}) \sim \delta^2 M/\Delta \sim \varepsilon^6\varepsilon/\varepsilon^5 \sim \varepsilon^2$. While these forces act primarily on the ions rather than the electrons, they nevertheless do not destroy ambipolarity of the lowest order radial flows.

The result just derived has been used to reduce the current continuity equation to the simple form given in Eq. (10). Equation (10) is an expression of the result $J_\phi/L_\phi \ll J_\parallel/L_\parallel \sim J_d/L_d$. This relation follows from the result of the previous paragraph, since $J_\phi/L_\phi \sim e(nu_\phi - nu_\phi^e) \sim M^2 enu_\phi/L_\phi \sim M^2 enu_d/L_d \sim M^2 J_d/L_d \ll J_d/L_d$. (Here we have used the maximal ordering in Eq. (A1), and noted that $S - S_e \sim \delta S \ll S$ to neglect sources of current resulting from the small difference between the ion source, S, and the electron source, S_e.)

That the maximal ordering assumptions are consistent with Eq. (A1) can now be confirmed. We start by noting that $J_d \sim env_{th}\delta$ (from Eq. 11) and $J_\parallel \sim env_{th}\delta_p \sim env_{th}\varepsilon$ (from Eq. 10, taking $\delta_p \sim \varepsilon$). The ordering $(m_e/m)^{1/2} \sim \varepsilon^4$ then leads to $(\alpha_o m_e \nu_e J_\parallel/e)L_\parallel/(nT) \sim F_\parallel L_\parallel/(nT) \sim 1$ in Eq. (9) and $\Delta \sim \varepsilon^5$. Equation (8) then gives $u_\phi/L_\phi \sim F_\parallel/(nm\Omega_p) \sim (nT/L_\parallel)/(nm\Omega_p) \sim \delta_p v_{th}/L_\parallel \sim Mv_{th}/L_\parallel \sim u_\parallel/L_\parallel$, since $M \sim \delta_p \sim \varepsilon$ for the maximal ordering, thus confirming the first relation in Eq. (A1).

The ordering of the diamagnetic flow given in Eq. (A1) is readily seen to be consistent with Eqs. (14) and (15). Ohm's law, Eq. (15), yields $e\Phi \sim T$. The radial ion momentum balance then provides the ordering $u_d/L_d \sim (nT/L_\phi)/(nm\Omega) \sim \delta v_{th}/L_d \sim \delta_p v_{th}/L_\parallel \sim Mv_{th}/L_\parallel \sim u_\parallel/L_\parallel$.

The absence of a pressure gradient in the Ohm's law given by Eq. (15) follows from applying the maximal ordering assumptions to the total parallel momentum balance given in Eq. (16). Like the other terms discussed above, the inertial terms on the right-hand side of Eq. (16) are readily seen to be of order $M^2nT/L_\parallel$. Therefore, Eq. (16) restricts the validity of the maximal ordering to regions bounded by a small parallel pressure gradient of order $(B_\theta/B)(\partial(2nT)/\partial\theta)|\nabla\theta| \lesssim M^2nT/L_\parallel$. Thus we must avoid a small region of intense recycling immediately in front of the material boundary of a high density scrapeoff, as discussed in Section I.

Finally, we justify the use of equal ion and electron temperatures. This is done by comparing the Coulomb energy interchange,[9] $Q_\Delta = 3n\nu_e(T_e - T)m_e/m \sim (nTv_{th}/L_\parallel)\ (m_e/m)^{1/2}\ \Delta^{-1}(T_e - T)/T \sim (nTv_{th}/L_\parallel)\ \varepsilon^{-1}(T_e - T)/T$, to the terms in the heat balance, Eq. (12), which are of order $\varepsilon(nTv_{th}/L_\parallel)$. Equating $(nTv_{th}/L_\parallel)\ \varepsilon^{-1}(T_e - T)/T \sim \varepsilon(nTv_{th}/L_\parallel)$ implies that there is at most enough differential heating to produce a temperature difference of order $(T_e - T)/T \sim \varepsilon^2$. This has the important consequence that equal ion and electron temperature gradients along the magnetic field produce equal and opposite neoclassical radial heat conduction to lowest order in ε, as can be shown from an analysis[7] of Eq. (5). Since these neoclassical heat flux divergences are separately only of order comparable to the other terms in Eq. (12), no net neoclassical effect appears in the total heat balance to lowest order under the maximal ordering.

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