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AZIMUTHAL TWO-PARTICLE CORRELATIONS FROM $M=0$ LORENTZ POLES

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A B S T R A C T

We give a general expression for the first non-leading asymptotic term of the contribution of a Lorentz pole with $M = 0$ to the two particle inclusive distribution, by means of the derivatives of the leading term. This expression, containing the cosine of the relative azimuthal angle, provides correlations between the observed particles. We suggest to test this result in reactions in which the Pomeron dominates at relatively low energies, the other Lorentz pole contributions being cancelled by exchange degeneracy.

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It is well known that inclusive distributions [1] can be expanded [2)-7)] in certain asymptotic limits in terms of Reggeon contributions, or, more exactly, in terms of Lorentz pole contributions.

In previous papers [7),8)], we have remarked that azimuthal correlations in two-particle inclusive distributions can arise from the exchange of Lorentz poles (or cuts) with $M \geqslant 1$ [9)] or also from the non-leading part of the contribution of Lorentz poles with $M = 0$. As possible Lorentz poles with $M \geqslant 1$ are thought to have a rather low intercept, the non-leading Pomeron contributions could well be, in certain kinematical conditions, the most relevant source of azimuthal correlations.

In Ref. 8) we considered the special case in which both the observed particles are in the pionization region. The first non-leading Pomeron contribution, which is proportional to the cosine of the relative azimuthal angle $\varphi_{\beta\alpha}$, was expressed in a simple way in terms of a derivative of the leading term, which is directly connected with the rather well-known transverse momentum distribution.

The aim of the present paper is to extend these ideas, releasing the condition that the observed particles are in the pionization region, in order to permit the treatment of processes at a lower total energy.

Our investigation is based on the assumption that the Pomeron is essentially a Lorentz pole with factorized residue. The effect of cuts near the Pomeron pole has been studied in Ref. 10), where it was suggested that they could give rise to azimuthal correlations. These effects should decrease much more slowly than the ones we are considering for increasing relative rapidity of the observed particles. In Ref. 8) we have shown, on the basis of a transverse momentum sum rule, that these effects are probably small in the pionization region. The same argument cannot be applied to the fragmentation regions in which Ref. 10) is explicitly concerned.

In order to write the Lorentz pole expansion, we use the formalism developed in Refs. 11),12), which is in agreement with the general results of multiperipheral dynamics. We introduce two standard frames of reference, S_A and S_B. We indicate by P_A and P_B the four momenta of the incoming particles and by P_α and P_β the four momenta of the two observed particles. In the frame S_A we have

$$\begin{cases} P_A = (M_A, 0, 0, 0) , \\ P_\alpha = \left(\sqrt{q_\alpha^2 + M_\alpha^2}\, , 0, 0, q_\alpha\right) , \end{cases} \tag{1}$$

and in the frame S_B

$$\begin{cases} P_B = (M_B, 0, 0, 0) , \\ P_\beta = \left(\sqrt{q_\beta^2 + M_\beta^2}\, , 0, 0, -q_\beta\right) . \end{cases} \tag{2}$$

These two frames are connected to the laboratory system S_L by the two Lorentz transformations

$$\begin{cases} a_{AL} = u_z(\varphi_\alpha)\, u_y(\theta_\alpha)\,, \\ a_{BL} = u_z(\varphi_\beta)\, a_z(\xi)\, u_y(-\theta_\beta)\,. \end{cases} \tag{3}$$

We have indicated by $u_z(\varphi)$ a rotation around the z axis of an angle φ and by $a_z(\xi)$ a boost along the z axis with rapidity ξ. As a consequence, in the laboratory system we have

$$\begin{cases} P_A = (M_A, 0, 0, 0)\,, \\ P_\alpha = \left(\sqrt{q_\alpha^2+M_\alpha^2},\, q_\alpha \sin\theta_\alpha \cos\varphi_\alpha,\, q_\alpha \sin\theta_\alpha \sin\varphi_\alpha,\, q_\alpha \cos\theta_\alpha\right), \\ P_B = (M_B \cosh\xi, 0, 0, M_B \sinh\xi)\,, \\ P_\beta = \left(\sqrt{q_\beta^2+M_\beta^2}\cosh\xi - q_\beta\cos\theta_\beta \sinh\xi,\, q_\beta \sin\theta_\beta \cos\varphi_\beta\,, \right. \\ \quad \left. q_\beta \sin\theta_\beta \sin\varphi_\beta,\, -q_\beta \cos\theta_\beta \cosh\xi + \sqrt{q_\beta^2+M_\beta^2}\sinh\xi\right). \end{cases} \tag{4}$$

We see that p_α, θ_α and φ_α define in the usual way the modulus and the direction of the momentum of the particle α in the laboratory frame. The quantities p_β, θ_β and φ_β have a similar meaning in the projectile system. In particular, we have

$$s = (P_A + P_B)^2 = M_A^2 + M_B^2 + 2 M_A M_B \cosh\xi\,. \tag{5}$$

The two particle inclusive distribution is given by

$$\begin{aligned} F^{\alpha\beta}(P_\alpha, P_\beta, s) &= \frac{d^6\sigma}{d^3\underline{P}_\alpha\, d^3\underline{P}_\beta}\, 4 P_\alpha^0 P_\beta^0 = \\ &= (4 M_A M_B \sinh\xi)^{-1}\, G^{\alpha\beta}(P_\alpha, P_\beta, s)\,. \end{aligned} \tag{6}$$

The relevant group-theoretical variable is

$$a_{BA} = (a_{AL})^{-1} a_{BL} = u_y(-\theta_\alpha)\, u_z(\varphi_{\beta\alpha})\, a_z(\xi)\, u_y(-\theta_\beta), \tag{7}$$

so that the contribution, averaged over the spins, of a Lorentz pole with $M = 0$ and intercept a is

$$G^{\alpha\beta}(P_\alpha, P_\beta, s) \simeq \sum_{j\,j'} r_j^A(q_\alpha)\, \mathcal{Q}^{0,-a-1}_{j0j'0}(a_{BA})\, r_{j'}^B(q_\beta), \tag{8}$$

where $\mathcal{Q}^{0,-a-1}_{j0j'0}(a)$ are the function of the second kind on $O(3,1)$ defined in Refs. 9) and 13).

Using the asymptotic expansion 9),13) of these functions for large ξ, we get

$$\begin{aligned} G^{\alpha\beta}(P_\alpha, P_\beta, s) \simeq & \sum_{j\,j'} r_j^A(q_\alpha)\, P_j(\cos\theta_\alpha)\, c_j^a \exp(a\xi)\, \cdot \\ & \cdot c_{j'}^a\, P_{j'}(\cos\theta_\beta)\, r_{j'}^B(q_\beta) - \sum_{j\,j'} r_j^A(q_\alpha) \sin\theta_\alpha \,\cdot \\ & \cdot P_j'(\cos\theta_\alpha)\, 2a^{-1} c_j^a \exp[(a-1)\xi]\, c_{j'}^a \sin\theta_\beta \,\cdot \\ & \cdot P_{j'}'(\cos\theta_\beta)\, r_{j'}^B(q_\beta) \cos\varphi_{\beta\alpha} + O(\exp[(a-2)\xi]), \end{aligned} \tag{9}$$

where c_j^a are known coefficients.

If we put

$$\mathcal{F}^A(q_\alpha, \theta_\alpha) = 2^{-\frac{1}{2}} M_A^{-a} \sum_j r_j^A(q_\alpha)\, c_j^a\, P_j(\cos\theta_\alpha), \tag{10}$$

and we define in a similar way $\mathcal{F}^B(q_\beta, \theta_\beta)$, we get

$$\begin{aligned} F^{\alpha\beta}(P_\alpha, P_\beta, s) \simeq & \mathcal{F}^A(q_\alpha, \theta_\alpha)\, \mathcal{F}^B(q_\beta, \theta_\beta)\, s^{a-1} - \\ & - 2a^{-1} M_A \frac{\partial}{\partial\theta_\alpha} \mathcal{F}^A(q_\alpha, \theta_\alpha)\, M_B \frac{\partial}{\partial\theta_\beta} \mathcal{F}^B(q_\beta, \theta_\beta)\, \cdot \\ & \cdot s^{a-2} \cos\varphi_{\beta\alpha} + O(s^{a-3}). \end{aligned} \tag{11}$$

If we are considering the Pomeron we have to put $a = 1$. We perform this substitution only in the final result, since in the intermediate steps it gives rise to difficulties which will be discussed in detail in a forthcoming paper. Of course, if one wants to take into account the contribution of several Lorentz poles, one has to add several expressions of the kind (11).

The result of Ref. 8) can be obtained as a special case in the limit of large q_α and q_β with finite transverse momenta $q_\alpha \sin\theta_\alpha$ and $q_\beta \sin\theta_\beta$. As the functions $\mathfrak{F}^A$ and $\mathfrak{F}^B$ depend on θ_α and θ_β mainly through the transverse momenta, we see that their derivatives contain the factors q_α and q_β. It follows that the ratio between the second and the first term in Eq. (11) contains the factor $q_\alpha q_\beta s^{-1}$, which is the relevant parameter of the asymptotic expansion we are considering.

In order to check Eq. (11) in the simplest way, one has to choose a situation in which only the Pomeron is important. This cannot be achieved increasing s (for fixed values of the other parameters), because the correlation effect decreases as s^{-1}. This limit would be suitable instead, for studying the cut effects suggested in Ref. 10).

A more favourable possibility is to consider processes in which the contributions of the non-dominant poles cancel each other, due to the exchange degeneracy required by duality [14)-16)]. It has been shown that, in these situations, the one particle inclusive distributions reach their asymptotic limit already for an incoming momentum of the order of 10 GeV/c. Then the functions $\mathfrak{F}^A$ and $\mathfrak{F}^B$ are proportional to the one-particle inclusive distributions and a considerable amount of experimental information about them is already available. Therefore, the second term in Eq. (11) can be computed numerically in many cases, in order to compare it with experimental data on two particle correlations as soon as they will be available. The factor $q_\alpha q_\beta s^{-1}$ should be chosen not too small in order to have a sizable effect, but not too large in order to avoid a failure of the asymptotic expansion (11).

The agreement of Eq. (11) with experiments would support the assumption that the Pomeron is essentially a well-behaved Lorentz pole. A discrepancy would suggest the existence of singularities with $M \gg 1$ in the Lorentz complex plane. Their nature could be investigated by a detailed analysis of the energy dependence of the correlation effects.

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