

$B \rightarrow P$ and $B \rightarrow V$ form factors from B -meson light-cone sum rules beyond leading twist

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ABSTRACT: We provide results for the full set of form factors describing semileptonic B -meson transitions to pseudoscalar mesons π , K , \bar{D} and vector mesons ρ , K^* , \bar{D}^* . Our results are obtained within the framework of QCD Light-Cone Sum Rules with B -meson distribution amplitudes. We recalculate and confirm the results for the leading-twist two-particle contributions in the literature. Furthermore, we calculate and provide new expressions for the two-particle contributions up to twist four. Following new developments for the three-particle distribution amplitudes, we calculate and provide new results for the complete set of three-particle contributions up to twist four. The form factors are computed numerically at several phase space points using up-to-date input parameters, including correlations across phase space points and form factors. We use a model ansatz for all contributing B -meson distribution amplitudes that is self-consistent up to twist-four accuracy. We find that the higher-twist two-particle contributions have a substantial impact on the results, and dominate over the three-particle contributions. Our numerical results, including correlations, are provided as machine-readable files in the supplementary material. We discuss the qualitative phenomenological impact of our results on the present b anomalies.

KEYWORDS: QCD Phenomenology

ARXIV EPRINT: [1811.00983](https://arxiv.org/abs/1811.00983)

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1 Introduction

Form factors for B -meson decays to either pseudoscalar (P) or vector hadrons (V) arise in the hadronic $B \rightarrow P$ and $B \rightarrow V$ matrix elements of local flavour-changing currents $\bar{q}_1 \Gamma b$, where Γ denotes some spin structure. The hadronic form factors are a crucial input for accurate and precise predictions of observables (e.g. branching ratios and angular coefficients) in various semileptonic B -meson decays. In light of the recent b anomalies,¹ the form factors for $B \rightarrow K^{(*)}$ and $B \rightarrow \bar{D}^{(*)}$ transitions in particular have moved into the focus of theoretical and phenomenological interest. In this article we study the full set of form factors that arise in $B \rightarrow P, V$ transitions. Results for the tensor form factors in $B \rightarrow D^{(*)}$ transitions are provided at small momentum transfer for the first time.

¹See references [1] and [2] for recent reviews on the topic.

The hadronic matrix elements for the transitions under discussion are genuinely non-perturbative quantities. Presently, the only ab-initio method for their determination is Lattice QCD (LQCD), which uses discretized spacetime as a UV regulator [3]. In the long run, lattice determinations are expected to dominantly contribute to our understanding of the hadronic matrix elements discussed here. At present, lattice results for the form factors are restricted to the phase space region in which the final state exhibits only small recoil momentum in the B rest frame, corresponding to high dilepton invariant mass square q^2 . However, for the anomalies in $B \rightarrow K^* \mu^+ \mu^-$ decays, the phase space region containing the largest deviations from the SM predictions corresponds to large recoil of the K^* (or equivalently to low q^2). For semileptonic $B \rightarrow \bar{D}^{(*)}$ decays, it has been demonstrated [4, 5] that inputs at maximal hadronic recoil can influence the extraction of $|V_{cb}|$ and the SM prediction for the Lepton-Flavour Universality ratios $R_{D^{(*)}}$. The present lattice results for the relevant form factor therefore require an extrapolation from the phase space region in which they are obtained to the phase space region in which they are required.

An alternative method, which we apply in this work, is to determine the form factors from Light-Cone QCD Sum Rules (LCSRs). In the framework of B -meson LCSRs [6–10]² the time-ordered product of two local quark currents $\bar{q}_2(x)\Gamma_2 q_1(x)$ and $\bar{q}_1(0)\Gamma_1 b(0)$ is expanded in a series of non-local operators $\bar{q}_2(x)\Gamma h_v(0)$, where h_v denotes a b -flavoured HQET field, v_μ is the four-velocity of the B -meson and x is a light-like separations: $x^2 \simeq 0$. The operators' hadronic matrix elements between an on-shell B meson state and the hadronic vacuum can then be expressed as convolutions of hard scattering kernels with Light-Cone Distribution Amplitudes (LCDAs) of the B meson. The LCDAs are organized in terms of their twist, i.e., the difference between an operator's mass dimension and the canonical spin of the operator. Here the canonical spin of the h_v field is chosen such that the leading-twist contributions enter at twist two, similar to LCDAs of light mesons [23]. While the hard scattering kernels are perturbatively calculable, the LCDAs are genuine but universal non-perturbative input to the sum rules. The non-local B to vacuum matrix element is then related to the sought-after form factors via dispersion relations and semi-local quark-hadron duality. In this process, the relevant duality threshold parameters can be determined from moments of the correlations function. LCSRs therefore constitute a systematic approach to determine hadronic matrix elements relevant for exclusive B decays. The light-cone dominance $x^2 \simeq 0$ of the expansion of the correlator is only fulfilled for small values of the momentum transfer q^2 [6]. As a consequence LQCD and LCSRs provide complementary inputs toward the determination of the B -meson form factors as functions of the momentum transfer q^2 , albeit with with starkly different levels of uncertainties.

Our work improves upon the results in the literature in several places. First, we include matrix elements of two-particle operators at twist-four level. Second, we calculate the contributions arising from the full set of eight independent Lorentz structures [24] of three-particle matrix elements. A set of models for the three-particle LCDAs, consistent with the relevant equations of motion [25] and theoretical constraints, has been worked out for

²This setup is complementary to more commonly used LCSRs in which the B meson is interpolated and the final state meson is taken on-shell; see e.g. [11–18]. Our B -LCDA-based setup is similar to the framework of SCET sum rules [19–22].

the first time in ref. [23]. Our results therefore go beyond the three-particle contributions previously discussed [6–8]. Third, we use a rigorous statistical framework to estimate the theoretical parametric uncertainties of the form factors. In this, we follow closely a previous work that proved the concept at the hand of $B \rightarrow \pi$ form factors in LCSRs with π LCDAs. The $B \rightarrow V$ transitions discussed here all involve vector states $V = \rho, K^*, \bar{D}^*$ that are unstable and decay strongly to pairs of pseudoscalar mesons. Throughout this article we work within the narrow-width approximation, i.e., we assume the vector mesons to be stable under QCD. To go beyond this approximation requires further studies. A dedicated programme has emerged in recent years to study $\pi\pi$ [10, 26–31] and $K\pi$ [32–35] final states beyond a simple resonance ansatz. LCSR analyses of $B \rightarrow P_1 P_2$ form factors require ongoing updates [36, 37] by other authors to improve our understanding of the semileptonic processes as well as purely hadronic multi-body B decays [38–44].

The structure of our article is as follows. We provide details on the derivation of the sum rules at next-to-next-to-leading twist and the master formulas for the analytical results in section 2. Our numerical results in section 3 are comprised of the actual LCSR results (in section 3.1) and a fit to LCSR and LQCD results (in section 3.2). We continue to discuss some selected phenomenological implications of our results in section 4, before concluding in section 5. In a series of appendices we discuss the details of the B -meson LCDAs (in appendix A), provide the lengthy formulas of our analytical LCSR results in form of scalar coefficient functions (in appendix B), and illustrate our numerical results at the hand of plots of all form factors considered here (in appendix C).

2 Details on the computation and analytical results

We construct the LCSRs for the full set of form factors in the hadronic matrix elements of local flavour-changing $b \rightarrow q_1$ currents. For $B \rightarrow P$ transitions, with $P = \pi, K, \bar{D}$, we discuss the three independent non-vanishing form factors $f_+^{B \rightarrow P}$, $f_0^{B \rightarrow P}$ and $f_T^{B \rightarrow P}$, which are defined via

$$\langle P(k) | \bar{q}_1 \gamma^\mu b | B(p) \rangle = \left[(p+k)^\mu - \frac{m_B^2 - m_P^2}{q^2} q^\mu \right] f_+^{B \rightarrow P} + \frac{m_B^2 - m_P^2}{q^2} q^\mu f_0^{B \rightarrow P}, \quad (2.1)$$

$$\langle P(k) | \bar{q}_1 \sigma^{\mu\nu} q_\nu b | B(p) \rangle = \frac{i f_T^{B \rightarrow P}}{m_B + m_P} \left[q^2 (p+k)^\mu - (m_B^2 - m_P^2) q^\mu \right]. \quad (2.2)$$

Here and throughout this article the form factors are functions of the momentum transfer $q^2 \equiv (p-k)^2$, and p and k denote the B -meson's and the final-state meson's momentum, respectively. For $B \rightarrow V$ transitions, with $V = \rho, K^*, \bar{D}^*$, we discuss the seven non-vanishing form factors $A_0^{B \rightarrow V}$, $A_1^{B \rightarrow V}$, $A_2^{B \rightarrow V}$, $V^{B \rightarrow V}$, $T_1^{B \rightarrow V}$, $T_2^{B \rightarrow V}$, and $T_3^{B \rightarrow V}$, which are defined via:

$$\langle V(k, \eta) | \bar{q}_1 \gamma^\mu b | B(p) \rangle = \epsilon^{\mu\nu\rho\sigma} \eta_\nu^* p_\rho k_\sigma \frac{2V^{B \rightarrow V}}{m_B + m_V}, \quad (2.3)$$

$$\langle V(k, \eta) | \bar{q}_1 \gamma^\mu \gamma_5 b | B(p) \rangle = i \eta_\nu^* \left[g^{\mu\nu} (m_B + m_V) A_1^{B \rightarrow V} - \frac{(p+k)^\mu q^\nu}{m_B + m_V} A_2^{B \rightarrow V} - q^\mu q^\nu \frac{2m_V}{q^2} (A_3 - A_0) \right], \quad (2.4)$$

$$\langle V(k, \eta) | \bar{q}_1 i\sigma^{\mu\nu} q_\nu b | B(p) \rangle = \epsilon^{\mu\nu\rho\sigma} \eta_\nu^* p_\rho k_\sigma 2T_1^{B \rightarrow V}, \quad (2.5)$$

$$\begin{aligned} \langle V(k, \eta) | \bar{q}_1 i\sigma^{\mu\nu} q_\nu \gamma_5 b | B(p) \rangle = i\eta_\nu^* \left[(g^{\mu\nu}(m_B^2 - m_V^2) - (p+k)^\mu q^\nu) T_2^{B \rightarrow V} \right. \\ \left. + q^\nu \left(q^\mu - \frac{q^2}{m_B^2 - m_V^2} (p+k)^\mu \right) T_3^{B \rightarrow V} \right], \end{aligned} \quad (2.6)$$

with η representing the polarization of the vector meson, and we use the Bjorken-Drell convention with $\epsilon_{0123} = +1$. If the final state is either a π^0 or a ρ^0 , the l.h.s. of eqs. (2.1)–(2.6) have to be multiplied with a factor of $\sqrt{2}$. Note that $A_3^{B \rightarrow V}$ is redundant, since it is a linear combination of the two other axial form factors

$$A_3^{B \rightarrow V} \equiv \frac{m_B + m_V}{2m_V} A_1^{B \rightarrow V} - \frac{m_B - m_V}{2m_V} A_2^{B \rightarrow V}. \quad (2.7)$$

However, the decomposition of the matrix elements including the $A_3^{B \rightarrow V}$ form factor is convenient for the extraction of the form factors within a sum rule approach, as discussed below.

The matrix elements defined in eq. (2.1) and eq. (2.4) exhibit apparently unphysical singularities at $q^2 = 0$. These are removed by the identities

$$f_+^{B \rightarrow P}(q^2 = 0) = f_0^{B \rightarrow P}(q^2 = 0), \quad A_0^{B \rightarrow V}(q^2 = 0) = A_3^{B \rightarrow V}(q^2 = 0). \quad (2.8)$$

In addition, the algebraic relations between $\sigma^{\mu\nu}$ and $\sigma^{\mu\nu}\gamma_5$ give rise to the identity

$$T_1^{B \rightarrow V}(q^2 = 0) = T_2^{B \rightarrow V}(q^2 = 0). \quad (2.9)$$

It is common to replace the form factors $A_2^{B \rightarrow V}$ and $T_3^{B \rightarrow V}$ with the linear combinations

$$A_{12}^{B \rightarrow V} \equiv \frac{(m_B + m_V)^2(m_B^2 - m_V^2 - q^2)A_1 - \lambda(q^2)A_2}{16m_B m_V^2(m_B + m_V)}, \quad (2.10)$$

$$T_{23}^{B \rightarrow V} \equiv \frac{(m_B^2 - m_V^2)(m_B^2 + 3m_V^2 - q^2)T_2 - \lambda(q^2)T_3}{8m_B m_V^2(m_B - m_V)}, \quad (2.11)$$

where $\lambda(q^2) \equiv [(m_B + m_V)^2 - q^2][(m_B - m_V)^2 - q^2]$ is the Källén function. The linear combinations $A_{12}^{B \rightarrow V}$ and $T_{23}^{B \rightarrow V}$ correspond to form factors for the transition into a longitudinal vector state and therefore simplify the structure of angular coefficients in the differential decay rate of the semileptonic B -meson decays.

The starting point for the construction of the B -LCSRs is the correlation function

$$\Pi^{\mu\nu}(q, k) \equiv i \int d^4x e^{ik \cdot x} \langle 0 | \mathcal{T} \{ J_{\text{int}}^\nu(x), J_{\text{weak}}^\mu(0) \} | \bar{B}_{q_2}(q+k) \rangle \quad (2.12)$$

of two quark currents $J_{\text{int}}^\nu \equiv \bar{q}_2(x)\Gamma_2^\nu q_1(x)$ and $J_{\text{weak}}^\mu(0) \equiv \bar{q}_1(0)\Gamma_1^\mu h_\nu(0)$. The various choices of spin structures $\Gamma_{1,2}$ and quark flavours q_1 and q_2 for the form factors extracted in this article are shown in table 1. The correlator (2.12) is calculated in the framework of heavy quark effective theory (HQET), i.e. the b -quark field is replaced by the HQET field h_ν . In the kinematic regime $q^2 \leq m_b^2 + m_b k^2/\Lambda_{\text{had.}}$ and $k^2 \ll -\Lambda_{\text{had.}}^2$, the dominant contributions

process	J_{int}^ν	J_{weak}^μ	form factor
$\bar{B}^0 \rightarrow \pi^+$	$\bar{d}\gamma^\nu\gamma_5 u$	$\bar{u}\gamma^\mu h_\nu$ $\bar{u}\sigma^{\mu\{q\}}h_\nu$	$f_+^{B\rightarrow\pi}, f_{+/-}^{B\rightarrow\pi}$ $f_T^{B\rightarrow\pi}$
$\bar{B}^0 \rightarrow \bar{K}^0$	$\bar{d}\gamma^\nu\gamma_5 s$	$\bar{s}\gamma^\mu h_\nu$ $\bar{s}\sigma^{\mu\{q\}}h_\nu$	$f_+^{B\rightarrow K}, f_{+/-}^{B\rightarrow K}$ $f_T^{B\rightarrow K}$
$\bar{B}^0 \rightarrow D^+$	$\bar{d}\gamma^\nu\gamma_5 c$	$\bar{c}\gamma^\mu h_\nu$ $\bar{c}\sigma^{\mu\{q\}}h_\nu$	$f_+^{B\rightarrow D}, f_{+/-}^{B\rightarrow D}$ $f_T^{B\rightarrow D}$
$\bar{B}^0 \rightarrow \rho^+$	$\bar{d}\gamma^\nu u$	$\bar{u}\gamma^\mu h_\nu$ $\bar{u}\gamma^\mu\gamma_5 h_\nu$ $\bar{u}\sigma^{\mu\{q\}}h_\nu$ $\bar{u}\sigma^{\mu\{q\}}\gamma_5 h_\nu$	$V^{B\rightarrow\rho}$ $A_0^{B\rightarrow\rho}, A_1^{B\rightarrow\rho}, A_2^{B\rightarrow\rho}$ $T_1^{B\rightarrow\rho}$ $T_2^{B\rightarrow\rho}, T_3^{B\rightarrow\rho}$
$\bar{B}^0 \rightarrow \bar{K}^{*0}$	$\bar{d}\gamma^\nu s$	$\bar{s}\gamma^\mu h_\nu$ $\bar{s}\gamma^\mu\gamma_5 h_\nu$ $\bar{s}\sigma^{\mu\{q\}}h_\nu$ $\bar{s}\sigma^{\mu\{q\}}\gamma_5 h_\nu$	$V^{B\rightarrow K^*}$ $A_0^{B\rightarrow K^*}, A_1^{B\rightarrow K^*}, A_2^{B\rightarrow K^*}$ $T_1^{B\rightarrow K^*}$ $T_2^{B\rightarrow K^*}, T_3^{B\rightarrow K^*}$
$\bar{B}^0 \rightarrow D^{*+}$	$\bar{d}\gamma^\nu c$	$\bar{c}\gamma^\mu h_\nu$ $\bar{c}\gamma^\mu\gamma_5 h_\nu$ $\bar{c}\sigma^{\mu\{q\}}h_\nu$ $\bar{c}\sigma^{\mu\{q\}}\gamma_5 h_\nu$	$V^{B\rightarrow D^*}$ $A_0^{B\rightarrow D^*}, A_1^{B\rightarrow D^*}, A_2^{B\rightarrow D^*}$ $T_1^{B\rightarrow D^*}$ $T_2^{B\rightarrow D^*}, T_3^{B\rightarrow D^*}$

Table 1. Summary of the various combinations of weak and interpolating currents used to extract the form factors. We abbreviate $\sigma^{\mu\{q\}} \equiv \sigma^{\mu\nu}q_\nu$.

to the correlator eq. (2.12) arise at light-like distances $x^2 \simeq 0$ [7]. This motivates a systematic expansion of the time-ordered product in terms of bi-local operators with light-like separation $\bar{q}_2(x)\Gamma[x,0]h_\nu(0)$, where the $[x,0]$ denotes a gauge link that renders the bi-local operators gauge invariant. The expansion of the q_1 propagator up to next-to-leading power in x^2 near the light-cone $x^2 \simeq 0$ gives rise to two-particle and three-particle contributions to the correlator. Four-particle contributions are not taken in account in this work. The two-particle contributions can be summarized as

$$\Pi^{\mu\nu}(q, k) \Big|_{2\text{p}} \equiv \int d^4x \int d^4p' e^{i(k-p')\cdot x} \left[\Gamma_2^\nu \frac{\not{p}' + m_1}{m_1^2 - p'^2} \Gamma_1^\mu \right]_{\alpha\beta} \langle 0 | \bar{q}_1^\alpha(x) h_\nu^\beta(0) | \bar{B}_{q_2}(v) \rangle, \quad (2.13)$$

where α, β are spinor indices. The three-particle contributions involve a further gluon field:

$$\Pi^{\mu\nu}(q, k) \Big|_{3\text{p}} \equiv \int d^4x \int d^4p' \int_0^1 du e^{i(k-p')\cdot x} \left[\Gamma_2^\nu S_{3\text{p}}^{\lambda\rho}(u, p') \Gamma_1^\mu \right]_{\alpha\beta} \langle 0 | \bar{q}_1^\alpha(x) G_{\lambda\rho}(ux) h_\nu^\beta(0) | \bar{B}_{q_2}(v) \rangle. \quad (2.14)$$

In the above $G_{\lambda\rho} \equiv g_s(\lambda^a/2)G_{\lambda\rho}^a(x)$ denotes the gluon field strength tensor, and

$$S_{3p}^{\lambda\rho}(u, p') \equiv \frac{\bar{u}(p' + m_1)\sigma^{\lambda\rho} + u\sigma^{\lambda\rho}(p' + m_1)}{2(p'^2 - m_1^2)^2}, \quad \bar{u} \equiv 1 - u, \quad (2.15)$$

is the momentum-space representation of the next-to-leading-power term in the light-cone expansion of the quark propagator [45], in which u is the position of the gluon field as a fraction of the light-like distance $[0, x]$. The B -meson to vacuum matrix elements of the non-local heavy-light currents, appearing in eq. (2.13) and eq. (2.14), are parametrized in terms of B -meson Light-Cone Distribution Amplitudes (LCDAs). The full expressions of these non-local matrix elements for the B -meson LCDAs used in this work are collected in appendix A. Previous works [6–8] calculate the correlation functions up to twist three for the two-particle LCDAs, and use an incomplete set of three-particle LCDAs. In our work we go beyond the accuracy of the previous calculations by including all contributions to the correlator from two- and three-particle Fock states up to twist four [23].

In order to construct the sum rule, one has to insert a complete set of hadronic states in eq. (2.12), thereby obtaining a hadronic dispersion relation for $\Pi^{\mu\nu}$:

$$\Pi^{\mu\nu}(q, k) = \frac{\langle 0 | J_{\text{int}}^\nu(x) | M(k) \rangle \langle M(k) | J_{\text{weak}}^\mu(0) | \bar{B}_{q_2}(q+k) \rangle}{m_M^2 - k^2} + \frac{1}{2\pi} \int_{s_0^h}^{\infty} ds \frac{\rho^{\mu\nu}(s)}{s - k^2}, \quad (2.16)$$

with $M = P, V$. The last term involving the spectral density $\rho(s)$ on the r.h.s. of eq. (2.16) captures the contributions arising from excited and continuum states, s_0^h is the corresponding threshold for the lowest-mass excited or continuum state. The local M to vacuum matrix elements are proportional to decay constants f_P and f_V :

$$\begin{aligned} \langle 0 | \bar{q}_2 \gamma^\nu \gamma_5 q_1 | P(k) \rangle &= ik^\nu f_P, \\ \langle 0 | \bar{q}_2 \gamma^\nu q_1 | V(k, \eta) \rangle &= i\eta^\nu m_V f_V. \end{aligned} \quad (2.17)$$

The $B \rightarrow P$ and $B \rightarrow V$ matrix elements have been already introduced in eqs. (2.1)–(2.6). Again, if the initial state of the equation above is a π^0 or a ρ^0 , the l.h.s. receives an additional factor of $\sqrt{2}$.

Using the formulas given in eq. (A.1) and eq. (A.2), we can cast the two- and three-particle terms in eq. (2.13) and eq. (2.14) into an integral form similar to eq. (2.16). Using semi-local quark hadron duality to subtract the continuum contributions we obtain the sum rule. In the process, we apply a Borel transformation from k^2 to M^2 , which removes surfaces terms in the integrals and improves the numerical stability of the sum rule. The latter is achieved by accelerating the convergence of the twist expansion, and by reducing the sensitivity to the duality approximation. The sum rule can then be written in the following form for all the form factors F and final states $M = P, V$ considered here:

$$\begin{aligned} F &= \frac{f_B M_B}{K^{(F)}} \sum_{n=1}^{\infty} \left\{ (-1)^n \int_0^{\sigma_0} d\sigma e^{(-s(\sigma, q^2) + m_{P,V}^2)/M^2} \frac{1}{(n-1)!(M^2)^{n-1}} I_n^{(F)} \right. \\ &\quad \left. - \left[\frac{(-1)^{n-1}}{(n-1)!} e^{(-s(\sigma, q^2) + m_{P,V}^2)/M^2} \sum_{j=1}^{n-1} \frac{1}{(M^2)^{n-j-1}} \frac{1}{s'} \left(\frac{d}{d\sigma} \frac{1}{s'} \right)^{j-1} I_n^{(F)} \right]_{\sigma=\sigma_0} \right\}, \quad (2.18) \end{aligned}$$

where we use the auxiliary variable s and its derivative

$$s(\sigma, q^2) = \sigma m_B^2 + \frac{m_1^2 - \sigma q^2}{\bar{\sigma}}, \quad s'(\sigma, q^2) = \frac{ds(\sigma, q^2)}{d\sigma}. \quad (2.19)$$

In eq. (2.18), the expressions involving powers of differential operators should always be read as

$$\left(\frac{d}{d\sigma} \frac{1}{s'} \right)^n I(\sigma) \rightarrow \left(\frac{d}{d\sigma} \frac{1}{s'} \left(\frac{d}{d\sigma} \frac{1}{s'} \dots I(\sigma) \right) \right).$$

We further abbreviate $\bar{\sigma} \equiv 1 - \sigma$ and $\sigma_0 \equiv \sigma(s_0, q^2)$, where s_0 is an effective threshold parameter not to be confused with s_0^h , from which it differs in general. The functions $I_n^{(F)}$ can be represented as integrals involving the two-particle and three-particle LCDAs:

$$I_n^{(F, 2p)}(\sigma, q^2) = \frac{1}{\bar{\sigma}^n} \sum_{\psi_{2p}} C_n^{(F, \psi_{2p})}(\sigma, q^2) \psi_{2p}(\sigma m_B), \quad \psi_{2p} = \phi_+, \bar{\phi}, g_+, \bar{g}; \quad (2.20)$$

$$I_n^{(F, 3p)}(\sigma, q^2) = \frac{1}{\bar{\sigma}^n} \int_0^{\sigma m_B} d\omega_1 \int_{\sigma m_B - \omega_1}^{\infty} \frac{d\omega_2}{\omega_2} \sum_{\psi_{3p}} C_n^{(F, \psi_{3p})}(\sigma, u, q^2) \psi_{3p}(\omega_1, \omega_2) \Big|_{u=(\sigma m_B - \omega_1)/\omega_2}, \quad \psi_{3p} = \phi_3, \phi_4, \psi_4, \chi_4; \quad (2.21)$$

with $\sigma = \omega/m_B$ in eq. (2.20) and $\sigma = (\omega_1 + u\omega_2)/m_B$ in eq. (2.21), respectively. The coefficients $C^{(F, \psi)}$, as well as the normalization factors $K^{(F)}$ of eq. (2.18) are listed in the appendix B. In the cases $F = f_+^{B \rightarrow P}$, $f_-^{B \rightarrow P}$, $A_1^{B \rightarrow V}$, $A_2^{B \rightarrow V}$, $V^{B \rightarrow V}$, and $T_1^{B \rightarrow V}$ we can construct the sum rule for the form factor directly, whereas for the remainder of the cases F denotes one of the following linear combinations of form factors:

$$f_{+/-}^{B \rightarrow P} \equiv f_+^{B \rightarrow P} + f_-^{B \rightarrow P}, \quad (2.22)$$

$$A_{30}^{B \rightarrow V} \equiv A_3^{B \rightarrow V} - A_0^{B \rightarrow V}, \quad (2.23)$$

$$T_{23A}^{B \rightarrow V} \equiv T_2^{B \rightarrow V} + \frac{q^2}{m_B^2 - m_V^2} T_3^{B \rightarrow V}, \quad (2.24)$$

$$T_{23B}^{B \rightarrow V} \equiv \frac{1}{2} T_2^{B \rightarrow V} + \frac{1}{2} \left(\frac{q^2}{m_B^2 - m_V^2} - 1 \right) T_3^{B \rightarrow V}. \quad (2.25)$$

Here $f_-^{B \rightarrow P}$ is given by

$$f_0^{B \rightarrow P} = f_+^{B \rightarrow P} + \frac{q^2}{m_B^2 - m_P^2} f_-^{B \rightarrow P}. \quad (2.26)$$

Our results for the analytical expressions are always provided for a generic final state meson P, V with valence quark content $(q_1 \bar{q}_2)$. To the precision we work at, only the mass m_1 of the quark field q_1 enters the expressions.

We fully reproduce the two-particle leading-twist contributions proportional to ϕ_+ and ϕ_- given in ref. [7]. Furthermore, we extend the two-particle results adding the terms containing g_+ and g_- , that take in account corrections up to twist five. The results for three-particle contributions in refs. [7, 8] are obtained for only a subset of the three-particle

LCDAs: $\psi_{3p} = \psi_V, \psi_A, X_A,$ and Y_A . When artificially restricting the LCDAs to the same subset, we reproduce the results of refs. [7, 8]. Our results for the coefficients $C^{(F,\psi)}$ provide for the first time the complete results for the two- and three-particle contributions up to and including twist four.

3 Numerical results

3.1 LCSR results

We implement the sum rules for the full set of $B \rightarrow P$ and $B \rightarrow V$ form factors as part of the EOS software [46], which is an open source project for the evaluation of flavour observables [47]. Our implementation is agnostic of the concrete parametrization of the various LCDAs entering the sum rules. This is achieved by computing all contributing integrals numerically. For this work, the LCDAs implemented in EOS conform to the exponential model put forward in ref. [23]. However, further LCDA models can readily be added to EOS, in order to challenge the (implicit) dependency of the sum rules on the LCDA model. Realistically, this can only be done after measurements of the photo-leptonic decay $B^- \rightarrow \gamma \ell^- \bar{\nu}$; see [48] for a recent update of the theoretical framework for the extraction of the LCDA model parameters.

In order to obtain numerical predictions for the form factors and to estimate the theory uncertainties due to the input parameters, we follow the statistical procedure used in ref. [16]. Within a Bayesian framework we first define an a-priori Probability Density Function (PDF) for the input parameters. A summary of the process-specific elements of this PDF is given in table 2. The universal elements of this PDF can be summarized as the following independent Gaussian PDFs for the B -meson to vacuum matrix elements:

$$\begin{aligned}
 f_B &= (189.4 \pm 1.4) \text{ MeV}, & 1/\lambda_{B,+} &= (2.2 \pm 0.6) \text{ GeV}^{-1}, \\
 \lambda_E^2 &= (0.03 \pm 0.02) \text{ GeV}^2, & \lambda_H^2 &= (0.06 \pm 0.03) \text{ GeV}^2.
 \end{aligned}
 \tag{3.1}$$

We use the f_B value from the most precise LQCD analysis available [49], our own estimate of $1/\lambda_{B,+}$ and the $\lambda_{E,H}^2$ from ref. [50]. Two classes of the process-specific parameters deserve a more detailed discussion: the Borel parameters M^2 and the duality thresholds s_0 .

Borel parameters. The Borel parameters M_P^2 and M_V^2 for the pseudoscalar and vector final states are taken from previous studies [6–8]. As usual in QCD sum rules, a window should be chosen for the Borel parameter M^2 such that:

- a) M^2 is not too large, to ensure that excited and continuum state contributions to the correlation function are exponentially suppressed; and
- b) M^2 is not too small, to ensure that the impact of higher-twist contributions are suppressed by powers of $1/M^2$.

We explicitly confirm that the central values of the form factors for $B \rightarrow K^*, \bar{D}^{(*)}$ transitions exhibit a plateau in their M^2 dependence. For the remaining form factors we find no such plateau, which, however, does not preclude us from applying the sum rules with

meson	decay constant $f_{P,V}$ [MeV]	s_0 [GeV ²]	M^2 [GeV ²]
π	130.2 ± 1.4	$0.7 \pm 0.014^\times$	1.0 ± 0.5
K	155.6 ± 0.4	$1.05 \pm 0.021^\times$	1.0 ± 0.5
D	212.6 ± 0.5	$[5.8, 7.8]^\dagger$	4.5 ± 1.5
ρ	213 ± 5	$1.6 \pm 0.032^\times$	1.0 ± 0.5
K^*	204 ± 7	$[1.4, 1.7]^\dagger$	1.0 ± 0.5
D^*	249 ± 21	$[6.9, 8.0]^\dagger$	4.5 ± 1.5

Table 2. Overview of transition and form-factor specific numerical inputs used in our calculations. Values marked with \times are taken from refs. [51–53], with the uncertainties estimated from the uncertainty of the corresponding decay constant. Intervals marked with \dagger represent the union of intervals for the individual form factors obtained in our analyses. For the decay constants we use values given in refs. [17, 49, 54–61]. The Borel parameters are the same as the ones used in [7, 8].

some increased systematic uncertainties. Based on the variations of the form factors under change of the Borel parameters, we assign a systematic uncertainty as a percentage of the central value as follows:

$$\begin{aligned}
 B \rightarrow \pi : 15\%, \quad B \rightarrow \rho : 12\%, \\
 B \rightarrow K : 8\%, \quad B \rightarrow K^* : 5\%, \\
 B \rightarrow \bar{D}^{(*)} : 3\%.
 \end{aligned}
 \tag{3.2}$$

For π , K and ρ final states the systematic uncertainties can be further reduced through a simultaneous analyses of the form factors and the light-meson decay constants within the framework of QCD sum rules, since both analyses have the Borel parameters and the thresholds in common. This effect has been previously shown in the case of LCSRs with π LCDAs [16]. For K^* and $\bar{D}^{(*)}$ final states the uncertainty arising from the variation of the Borel parameter can be included in the statistical procedure. Given the present knowledge of the B -meson LCDA parameter(s), these uncertainties are presently subleading to the parametric uncertainties due to thresholds and LCDA parameters. We leave both of these improvements to future work.

Power corrections. Using the full set of LCDAs up to twist-four accuracy, the authors of ref. [23] expect to account for the contributions of HQET operators up to and including $1/m_b$ corrections. This expectation is based on the observation that an increase by two units of collinear twist corresponds to a suppression by a factor of $1/m_b$ [62]. Moreover, four-particle LCDAs also start to contribute at the twist-four level and are presently unknown. Given the small size of the three-particle contributions to the sum rules we do not expect sizeable contributions from the four-particle terms, which we ignore throughout. The corrections at order $1/m_b^2$ are presently unknown, and we estimate them based on naive dimensional arguments at $\sim 5\%$. We add this uncertainty in quadrature to the systematic uncertainty incurred by the Borel parameters.

Duality threshold parameters. The threshold parameters $s_0^{(F)}$ can in principle be determined by closely following the procedure carried out in ref. [16]. First, one defines a prior interval with uniform probability for the threshold parameters. In this step one also varies the LCDA parameters $1/\lambda_{B,+}$, λ_E^2 and λ_H^2 to determine the correlations between thresholds and LCDA model parameters. In a next step, the a-priori PDF is challenged with a theoretical likelihood. The pseudo-observables that are constrained through the likelihood are the “first moments” of the form factors’ correlation function. These moments warrant a more careful definition: for any form factor F we differentiate its scalar-valued correlator $\Pi^F(q^2; M^2)$ with respect to $-1/M^2$ and normalize it to Π^F . The resulting ratio is a pseudo-observable that is expected to yield the final state’s mass square m_P^2 or m_V^2 , respectively, within the accuracy of the light-cone OPE for the correlation function.

We carry out this procedure for the K^* and $\bar{D}^{(*)}$ final states. Within the likelihood, we impose that the theory prediction for the first moments match the square of the respective final state hadron mass. We impose relative uncertainties of 5% on these predictions, in order to account for the impact of $1/m_b^2$ corrections to the correlators. The added uncertainties are considerably larger than in the $B \rightarrow \pi$ analysis [16]. We think our more conservative treatment is warranted as we expect the “first moments” to exhibit a substantial but difficult-to-quantify dependence on the B -meson LCDA model. For some of the threshold parameters we find a marked non-gaussianity for the two-dimensional joint posterior PDF of a single threshold parameter and $1/\lambda_{B,+}$.

For pseudo-Goldstone bosons such as the π and K , and for the ρ meson with its substantial decay width, the first moments are not expected to reproduce the meson mass squares. As an exercise, we attempt anyway to apply the procedure described above and find it to be too unstable to determine the duality threshold for any of these states. We therefore adopt the thresholds used in ref. [7], which are determined ($\lambda_{B,+}$ independently) from two-point QCD sum rules of the π , K and ρ decay constants.

In our analysis of form factors to K^* and $\bar{D}^{(*)}$ final states a further complication arises from the fact that the first moments of the correlation functions exhibit a noticeable but mild q^2 dependence. We choose to study this effect as follows: for each form factor, the theory likelihood includes the form factor’s first moment for seven values of q^2 in the range -15 GeV^2 to 0 GeV^2 , with increments of 2.5 GeV^2 . We make a linear ansatz for the q^2 dependence of the threshold parameters $s_0^{(F)}$:

$$s_0^{(F)}(q^2) = s_0^{(F)} + q^2 s_0^{\prime(F)}. \tag{3.3}$$

We then determine the two parameters $s_0^{(F)}$ and $s_0^{\prime(F)}$ for each form factor from the theory likelihood. Subsequently we repeat the fit while fixing the slope parameters $s_0^{\prime(F)}$ to zero. For most of the form factors we find a negligible difference in the constant parts $s_0^{(F)}$. The only exception is the form factor $f_T^{B \rightarrow D}$, for which the two parameters s_0 and s_0' are very strongly linearly correlated. We can therefore not reliably obtain the threshold parameter for this form factor, and choose to use the same threshold as for $f_+^{B \rightarrow D}$, which is a good approximation for other vector/tensor pairs of form factors and holds at the 3% level for e.g. the pair $V^{B \rightarrow D^*}$, $T_1^{B \rightarrow D^*}$. Nevertheless, we increase the systematic uncertainty on $f_T^{B \rightarrow D}$ by 5% due to this treatment. Considering the full set of form factors and final

form factor	2 pt.			3-pt. [10^{-2}]
	ϕ_{\pm}	g_+	g_-^{WW}	
$f_+^{B \rightarrow \pi}$	0.28	+0.00	-0.06	-0.00
$f_T^{B \rightarrow \pi}$	0.25	+0.01	-0.07	-0.29
$f_+^{B \rightarrow K}$	0.35	+0.00	-0.08	-0.01
$f_T^{B \rightarrow K}$	0.33	+0.02	-0.09	-0.37
$f_+^{B \rightarrow D}$	0.84	+0.02	-0.21	-0.03
$f_T^{B \rightarrow D}$	0.65	+0.33	-0.41	-0.52
$A_1^{B \rightarrow \rho}$	0.28	-0.08	+0.01	-0.19
$A_{12}^{B \rightarrow \rho}$	0.31	+0.01	-0.07	-0.10
$V^{B \rightarrow \rho}$	0.37	-0.11	-0.00	-0.34
$T_1^{B \rightarrow \rho}$	0.32	-0.09	+0.01	-0.25
$T_{23}^{B \rightarrow \rho}$	0.69	+0.07	-0.18	-0.96
$A_1^{B \rightarrow K^*}$	0.33	-0.08	+0.01	-0.21
$A_{12}^{B \rightarrow K^*}$	0.26	+0.01	-0.05	-0.06
$V^{B \rightarrow K^*}$	0.44	-0.12	-0.00	-0.38
$T_1^{B \rightarrow K^*}$	0.37	-0.10	+0.01	-0.28
$T_{23}^{B \rightarrow K^*}$	0.68	+0.04	-0.14	-0.84
$A_1^{B \rightarrow D^*}$	0.73	-0.17	+0.04	-0.10
$A_{12}^{B \rightarrow D^*}$	0.21	+0.01	-0.03	-0.01
$V^{B \rightarrow D^*}$	1.02	-0.29	-0.04	-0.38
$T_1^{B \rightarrow D^*}$	0.83	-0.21	+0.01	-0.19
$T_{23}^{B \rightarrow D^*}$	0.88	+0.08	-0.15	-0.37

Table 3. Detailed budget of the ϕ_{\pm} , g_+ , g_-^{WW} and three-particle contributions to our LCSR results for the form factors at $q^2 = 0$.

state hadrons, we find only negligible impact due to our treatment of q^2 dependence of the threshold parameters when comparing to the dominant uncertainties incurred by the B -meson LCDA parameters. We therefore proceed with the assumption of q^2 -independent duality thresholds. However, we remark that this problem needs to be revisited once the parametric uncertainties due to the LCDA model-dependence are under better control.

For the \bar{D} and \bar{D}^* final states, we find that increasing q^2 to positive values increases the uncertainty in the prediction of the first moments substantially. In fact, for $q^2 \simeq 5 \text{ GeV}^2$ we find very broad intervals that include $s_0 = 0$ at 68% probability. This increase in uncertainty is accompanied by a substantial growth of relative contributions (to $\sim 50\%$ and beyond) due to the higher-twist two-particle terms. This clearly poses a problem for the calculation of the $B \rightarrow \bar{D}^{(*)}$ form factors at positive q^2 . It remains to be seen if this effect is due to the modelling of the LCDAs, or indicates an earlier-than-expected breakdown of the Light-Cone OPE at positive q^2 .

Predictions. Based on the procedure discussed above, we obtain threshold parameters for the individual form factors. A summary of these parameters and their uncertainties are listed in table 2. We then proceed to produce posterior-predictive distributions for the form factors at five different q^2 points: $q^2 = \{-15, -10, -5, 0, +5\} \text{ GeV}^2$. Note that the form factors $A_0^{B \rightarrow V}$ and $T_2^{B \rightarrow V}$ are linearly dependent on the remaining form factors at $q^2 = 0$, and therefore this particular point is dropped from the predictions for these two quantities. For heavy final states $M = D, D^*$ we remarked previously that the threshold computation becomes unstable for $q^2 > 0$. We therefore drop the point $q^2 = +5 \text{ GeV}^2$ for these two final states. The resulting Probability Density Functions (PDFs) of the form factors at the various q^2 points are most readily communicated in form of machine readable files, containing the mean values and covariance matrices of a multivariate Gaussian density. The results are included in the EOS software [46] as of version v0.2.3 as YAML files, defining the following named constraints:

```

B->pi::FormFactors[f_+,f_0,f_T]@GKvD2018
B->rho::FormFactors[V,A_0,A_1,A_2,T_1,T_2,T_23]@GKvD2018
B->K::FormFactors[f_+,f_0,f_T]@GKvD2018
B->K^*::FormFactors[V,A_0,A_1,A_2,T_1,T_2,T_23]@GKvD2018
B->D^(*)::FormFactors[f_+,f_0,f_T,V,A_0,A_1,A_2,T_1,T_2,T_23]@GKvD2018
    
```

We provide a detailed budget of the individual contributions to the form factors at $q^2 = 0$ in table 3. We also compare our results and their uncertainties, including all sources of systematic uncertainties, with results in the literature in table 4.

Our numerical results can subsequently be used to fit concrete parametrizations of the respective form factors. We carry out such fits for the BSZ parametrization [17] in the next subsection.

3.2 Parametrization and fits to LCSR and lattice QCD constraints

With our LCSR results in hand at selected q^2 values $\leq 5 \text{ GeV}^2$, we proceed to extrapolate the form factors to large positive q^2 values. This is most readily achieved using a z expansion of the form factors. We adopt the same parametrization as used in ref. [17], which also facilitates comparisons between the results therein and ours. The parametrization of any form factor F reads

$$F(q^2) \equiv \frac{1}{1 - q^2/m_{R,F}^2} \sum_{k=0}^2 \alpha_k^{(F)} [z(q^2) - z(0)]^k. \quad (3.4)$$

Here $m_{R,F}$ denotes the mass of sub-threshold resonances compatible with the quantum numbers of the form factor F , as listed in table 5. We also apply the conformal map from q^2 to z :

$$z(t) \equiv \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}, \quad (3.5)$$

where $t_{\pm} = (m_B \pm m_{P,V})^2$ and t_0 is a free parameter that governs the size of z in the semileptonic phase space. As in ref. [17] we use $t_0 \equiv t_+ \left(1 - \sqrt{1 - t_-/t_+}\right)$.

form factor	our result	literature	reference
$f_+^{B \rightarrow \pi}$	0.21 ± 0.07	0.258 ± 0.031	[11]
		0.25 ± 0.05	[7]
		0.28 ± 0.05	[14]
		0.31 ± 0.02	[16]
		0.281 ± 0.038	[21]
$f_T^{B \rightarrow \pi}$	0.19 ± 0.06	0.253 ± 0.028	[11]
		0.21 ± 0.04	[7]
		0.273 ± 0.021	[18]
		0.26 ± 0.06	[63]
$f_+^{B \rightarrow K}$	0.27 ± 0.08	0.331 ± 0.041	[11]
		0.31 ± 0.04	[7]
		0.395 ± 0.033	[18]
		0.364 ± 0.05	[63]
$f_T^{B \rightarrow K}$	0.25 ± 0.07	0.358 ± 0.037	[11]
		0.27 ± 0.04	[7]
		0.381 ± 0.027	[18]
		0.363 ± 0.08	[63]
$f_+^{B \rightarrow D}$	0.65 ± 0.08	0.69 ± 0.2	[8]
$f_T^{B \rightarrow D}$	0.57 ± 0.05	—	—
$A_1^{B \rightarrow \rho}$	0.22 ± 0.10	0.24 ± 0.08	[7]
$A_2^{B \rightarrow \rho}$	0.19 ± 0.11	0.262 ± 0.026	[17]
$V^{B \rightarrow \rho}$	0.27 ± 0.14	0.21 ± 0.09	[7]
		0.32 ± 0.10	[7]
$T_1^{B \rightarrow \rho}$	0.24 ± 0.12	0.327 ± 0.031	[17]
		0.28 ± 0.09	[7]
$T_{23}^{B \rightarrow \rho}$	0.56 ± 0.15	0.272 ± 0.026	[17]
		0.747 ± 0.076	[17]
$A_1^{B \rightarrow K^*}$	0.26 ± 0.08	0.30 ± 0.08	[7]
		0.269 ± 0.029	[17]
$A_2^{B \rightarrow K^*}$	0.24 ± 0.09	0.26 ± 0.08	[7]
		0.39 ± 0.11	[7]
$V^{B \rightarrow K^*}$	0.33 ± 0.11	0.341 ± 0.036	[17]
		0.33 ± 0.10	[7]
$T_1^{B \rightarrow K^*}$	0.29 ± 0.10	0.282 ± 0.031	[17]
		0.668 ± 0.083	[17]
$T_{23}^{B \rightarrow K^*}$	0.58 ± 0.13	0.668 ± 0.083	[17]
$A_1^{B \rightarrow D^*}$	0.60 ± 0.09	0.73 ± 0.19	[8]
$A_2^{B \rightarrow D^*}$	0.51 ± 0.09	0.66 ± 0.30	[8]
$V^{B \rightarrow D^*}$	0.69 ± 0.13	0.96 ± 0.29	[8]
$T_1^{B \rightarrow D^*}$	0.63 ± 0.10	—	—
$T_{23}^{B \rightarrow D^*}$	0.81 ± 0.11	—	—

Table 4. Comparison of our LCSR results for the form factors at $q^2 = 0$ with previous results in the literature. Note that the $B \rightarrow D$ form factors from ref. [8] have been obtained from a different interpolating current J_{int} than our results. Note that the $f_T^{B \rightarrow P}$, $T_1^{B \rightarrow V}$, $T_2^{B \rightarrow V}$ and $T_{23}^{B \rightarrow V}$ are scale-dependent quantities, evaluated at $\mu^2 = 1 \text{ GeV}^2$ for $P = \pi, K$ and $V = \rho, K^*$, and at $\mu^2 = 4.5 \text{ GeV}^2$ for $P = \bar{D}$ and $V = \bar{D}^*$.

For each final state we perform two fits. The first fit includes only the information at small q^2 values, obtained from the LCSRs, within the likelihood. Within all plots in appendix C, the results of this fit are displayed as a dark gray band. For the second fit, we add further information from lattice QCD analyses of the form factors at large values of q^2 as available [64–70]. Due to the absence of lattice QCD analyses of the $B \rightarrow \rho$ transitions there is no combined fit for the respective form factors. Results arising from the second fit are displayed as blue bands, throughout this work.

For the LCSR-only fits we have four data points and three parameters per form factor, equivalent to one degree of freedom (three data points and two parameters in the case of $f_0^{B \rightarrow P}$, $A_0^{B \rightarrow V}$ and $T_2^{B \rightarrow V}$). Given the large uncertainties and small number of degrees of freedom, it is not surprising that we find a p value $\gg 3\%$, our a-priori threshold, in each of these fits. For the combined fits to LCSR and LQCD inputs, we find p values very close to one, indicating an excellent fit in each of these analyses.

As for the LCSRs, the posterior PDFs of our fits are most readily provided as machine readable files containing the mean values and covariance matrices of a multivariate Gaussian density. The results are included in the EOS software [46] as of version v0.2.3 as YAML files, defining the following named constraints:

```

B->pi::FormFactors[parametric,LCSR]@GKvD2018
B->pi::FormFactors[parametric,LCSRLattice]@GKvD2018
B->rho::FormFactors[parametric,LCSR]@GKvD2018
B->K::FormFactors[parametric,LCSR]@GKvD2018
B->K::FormFactors[parametric,LCSRLattice]@GKvD2018
B->K^*::FormFactors[parametric,LCSR]@GKvD2018
B->K^*::FormFactors[parametric,LCSRLattice]@GKvD2018
B->D^(*)::FormFactors[parametric,LCSR]@GKvD2018
B->D^(*)::FormFactors[parametric,LCSRLattice]@GKvD2018

```

Moreover, we provide our results also through machine-readable JSON files in the same format as used in ref. [17]. These files are part of the supplementary material of this article. Moreover, our results will be available by default to users of the `flavio` software [71] from the next release on.

4 Selected phenomenological implications

We will briefly discuss the impact of our results for the form factors on the present b anomalies.

4.1 The $B \rightarrow K^* \mu^+ \mu^-$ anomaly and P'_5

Rare semileptonic b decays presently exhibit a number of measurements that deviate individually by about 2σ from their respective Standard Model (SM) predictions. These include all exclusive $b \rightarrow s \mu^+ \mu^-$ branching ratios [73–75] (with the exception of $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ [76]); the full set of angular observables in $B \rightarrow K^* \mu^+ \mu^-$ [77–79]; and most notably the Lepton Flavour Universality (LFU) ratios R_K [80] and R_{K^*} [81].

J^P	form factors	resonance masses [GeV]		
		$B_{u,d}(J^P)$	$B_s(J^P)$	$B_c(J^P)$
0^-	$A_0^{B \rightarrow V}$	5.279	5.336	6.275
0^+	$f_0^{B \rightarrow P}$	5.540	5.630	6.420
1^-	$f_+^{B \rightarrow P}, f_T^{B \rightarrow P}, V^{B \rightarrow V}, T_1^{B \rightarrow V}$	5.325	5.412	6.330
1^+	$A_1^{B \rightarrow V}, A_{12}^{B \rightarrow V}, T_2^{B \rightarrow V}, T_{23}^{B \rightarrow V}$	5.724	5.829	6.767

Table 5. Overview of the lowest-lying resonances in the individual $b \rightarrow \{u, d\}$, $b \rightarrow s$ and $b \rightarrow c$ transitions, and the association to the respective form factors. The masses above enter the parametrization of the form factors eq. (3.4) as the resonance mass parameter $m_{R,F}$. The $B_{u,d,s}$ masses have been taken from ref. [17], to ensure interoperability of their and our results. The B_c resonance masses have been taken from ref. [72].

Several studies [82–92] come to the conclusion that a negative shift to the short-distance coupling C_9^μ , and potentially to some couplings that vanish in the SM, can explain simultaneously the deviations in all anomalous $b \rightarrow s\ell^+\ell^-$ measurements; see [2] for a recent review and the definition of the low-energy Lagrangian. In the case of e vs μ universality with a lower dilepton mass cut $q^2 \leq 1$ GeV, the SM predictions of the LFU ratios are insensitive to the hadronic form factors [93–95]. We will therefore not discuss them here any further. Instead, we will discuss the qualitative impact of our results on fits of the $b \rightarrow s\ell^+\ell^-$ short-distance couplings to the available data on exclusive $B \rightarrow K^*\mu^+\mu^-$ decays, which have presently the biggest impact in global $b \rightarrow s\mu^+\mu^-$ fits.

Assuming the global fits to correctly account for non-local effects arising from four-quark operators in the $B \rightarrow K^*\mu^+\mu^-$ amplitudes [98], the data leads to two possible conclusions [84, 86]:

1. the ratio of form factors $V^{B \rightarrow K^*}/A_1^{B \rightarrow K^*}$ deviates from the ratio predicted by symmetry relations at large kaon energies [96, 97] as well as a-priori predictions from extrapolations of lattice QCD result [99] and light-meson LCSR [17], leading to global fits with border-line goodness of fit; or
2. there is a New Physics (NP) shift to the short-distance coefficient C_9 corresponding to $\sim 25\%$ of its SM value.

This interpretation has been strengthened recently by a proof-of-concept analysis in which the non-local matrix elements are further constrained in shape due to their properties following from analyticity and unitarity [98]. The particular solution to obtaining a good fit in the absence of NP effects requires the ratio $V^{B \rightarrow K^*}/A_1^{B \rightarrow K^*}$ to not only deviate in value from the large-energy limit prediction, but also in shape. We show explicitly in figure 1 that our predictions are compatible with the symmetry limit at large energies; with extrapolations of lattice QCD results within their large uncertainties; and with the rather precise results obtained from LCSRs with K^* LCDAs.

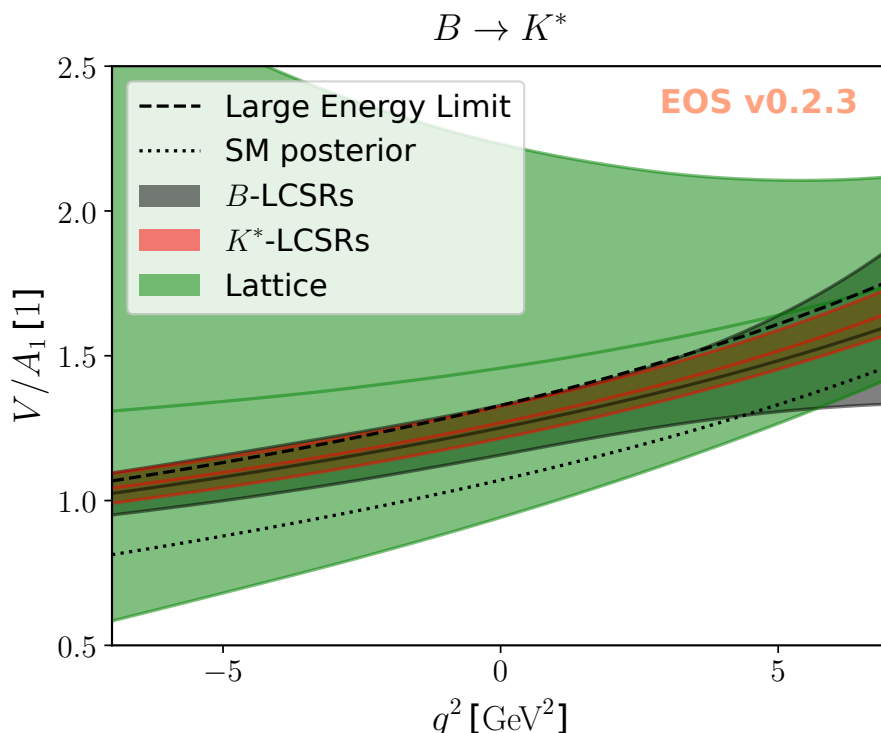


Figure 1. The ratio $V(q^2)/A_1(q^2)$ for $B \rightarrow K^*$ transitions. The green, red and gray lines and shaded areas correspond to the central values and the 68% probability envelope of the form factors obtained from fits to only LQCD results, fits to LCSR results from ref. [17], and fits to our LCSR results, respectively. The dashed line correspond to the large-energy limit for this ratio [96, 97]. The dotted line corresponds to the central value of the SM fit to $B \rightarrow K^* \mu^+ \mu^-$ data from ref. [98].

4.2 Standard model predictions for $B \rightarrow D^{(*)} \ell \bar{\nu}$ and $R(D^{(*)})$

The exclusive semileptonic decays $B \rightarrow \bar{D}^{(*)} \ell \bar{\nu}$ are of great phenomenological interest. On the one hand, they can be used to extract the magnitude of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{cb} . Its determinations from exclusive and inclusive B decays has been famously in tension with each other for the last decade. On the other hand, the exclusive decays allow to test the SM through LFU ratios $R(D)$ and $R(D^*)$

$$R(D^{(*)}) \equiv \frac{\mathcal{B}(B \rightarrow \bar{D}^{(*)} \tau^- \bar{\nu})}{\mathcal{B}(B \rightarrow \bar{D}^{(*)} \ell^- \bar{\nu})}, \quad \text{with } \ell = e, \mu. \quad (4.1)$$

Both the extraction of $|V_{cb}|$ and testing the SM through LFU violation require accurate predictions of the relevant form factors. The Heavy-Quark-Expansion, in combination with data, can help in this particular case of heavy-to-heavy flavour-changing quark transitions; see refs. [100, 101] and references therein for dispersive bounds, and ref. [102] for the SM prediction of $R(D^*)$. It has been recently argued that strict adherence to the so-called CLN parametrization [101] is, at least partially, responsible for the exclusive-vs-inclusive tension [4, 5, 103] when determining V_{cb} from semileptonic $B \rightarrow \bar{D}^*$ transitions. In the case of $B \rightarrow \bar{D}$, recent lattice QCD analyses yield V_{cb} values that are compatible with both the inclusive and the $B \rightarrow \bar{D}^*$ determinations.

LCSR determinations of $B \rightarrow \bar{D}^*$ form factors [8] play an important role in some of the phenomenological analyses [4, 5], e.g. through form factor ratios at $q^2 = 0$. With our updated results for the form factors, we are in the position to also update these ratios and also to provide parametric correlations between them. The ratios under discussion are labelled R_0 , R_1 and R_2 (see e.g. ref. [101] for their definitions), which are functions of the recoil parameter w , with $m_B m_D^* w = p \cdot k$. At maximal recoil w_{\max} , corresponding to $q^2 = 0$, one has:

$$R_0(q^2=0) = \frac{A_0(q^2=0)}{A_1(q^2=0)}, \quad R_1(q^2=0) = \frac{V(q^2=0)}{A_1(q^2=0)}, \quad R_2(q^2=0) = \frac{A_2(q^2=0)}{A_1(q^2=0)}. \quad (4.2)$$

Moreover, at $q^2 = 0$ the equation of motion implies that only two of these ratios are linearly independent. Based on our correlated results for the form factors we obtain

$$\begin{aligned} R_0(q^2=0) &= 1.117 \pm 0.061, & R_1(q^2=0) &= 1.151 \pm 0.114, \\ R_2(q^2=0) &= 0.856 \pm 0.076. \end{aligned} \quad (4.3)$$

The correlation coefficients ρ between R_1 and R_2 reads

$$\rho_{12} = 0.5154. \quad (4.4)$$

Our correlated results are compatible with the previous LCSR determinations of $R_1(q^2 = 0)$ and $R_2(q^2 = 0)$ [8] at less than one standard deviation.

We can also use our results to calculate the values of the LFU observables $R(D)$ and $R(D^*)$ in the SM and beyond. Using the correlated results for the form factor parameters obtained in section 3.2 from the fit to only our LCSR results, we obtain:

$$\text{LCSR only} \quad R(D) \Big|_{\text{SM}} = 0.269 \pm 0.100. \quad (4.5)$$

$$\text{LCSR only} \quad R(D^*) \Big|_{\text{SM}} = 0.242 \pm 0.048. \quad (4.6)$$

Our prediction for $R(D^*)$ is the first theory prediction that does not use either symmetry arguments based on the simultaneous expansion in $1/m_c$ and $1/m_b$ or experimental data from $\bar{B} \rightarrow D^*(e, \mu)\bar{\nu}$ decays. Given the substantial uncertainties of our prediction, these values are in good agreement with the predictions obtained from heavy quark symmetry relations, lattice inputs and $B \rightarrow \bar{D}^{(*)}\ell\bar{\nu}$ data [4, 5, 103]. Using the form factors parameters obtained in section 3.2 from a fit to both our LCSR results and two LQCD inputs for the $B \rightarrow D^*$ form factor $A_1^{B \rightarrow D^*}$ [66, 68, 70], we obtain:

$$\text{LCSR + Lattice} \quad R(D) \Big|_{\text{SM}} = 0.296 \pm 0.006, \quad (4.7)$$

$$\text{LCSR + Lattice} \quad R(D^*) \Big|_{\text{SM}} = 0.256 \pm 0.020. \quad (4.8)$$

Our result for $R(D)$ is dominated by the precise LQCD inputs [68] beyond zero recoil, and the agreement with the LQCD prediction $R(D) = 0.300 \pm 0.008$ is therefore not surprising.

Our result for $R(D^*)$, on the other hand, is supported by two LQCD inputs for the $A_1^{B \rightarrow D^*}$ form factor at only the zero recoil point. We find excellent agreement with the values obtained using heavy quark symmetry relations, i.e.:

$$R(D^*) \Big|_{\text{SM, [103]}} = 0.257 \pm 0.003, \tag{4.9}$$

$$R(D^*) \Big|_{\text{SM, [4]}} = 0.260 \pm 0.008. \tag{4.10}$$

As a closing remark, we wish to emphasize that the predictions in the framework of heavy quark symmetry relations are complicated by the proliferation of matrix elements associated with $1/m_c$ corrections that are not present in our LCSR-derived results.

5 Summary and outlook

We have presented a comprehensive update of Light-Cone Sum Rules (LCSRs) results for the full set of form factors relevant to semileptonic B decays. Our update includes, for the first time, a consistent treatment of *all* two-particle and three-particle Light-Cone Distribution Amplitudes (LCDAs) up to twist four. Moreover, our work also updates the numerical inputs across the board.

We have implemented our analytical results agnostic of the concrete expressions for the distribution amplitudes, thereby ensuring that our analysis can be readily repeated once our knowledge of either the properties of the amplitudes or their parameters improves in the future. The relevant computer code is publicly available [46] under an open source license as part of the EOS software [47]. Moreover, all of our numerical results are available as machine-readable files. For form factors that are common to our and a previous LCSR analysis using light-meson LCDAs [17] we have ensured interoperability of the data files with the supplementary material of this article.

Within our analyses we find sizable contributions from two-particle states at the twist-four level, which exceed the twist-three and twist-four three-particle contributions by one order of magnitude. Our analysis has been carried out strictly in the framework of Heavy-Quark Effective Theory, which enables us to be agnostic of the final state quark flavour, thereby facilitating the analysis. However, it also precludes us from using the $\mathcal{O}(\alpha_s)$ corrections to the leading-twist two-particle results obtained in the framework of SCET Sum Rules for massless [21] and massive [22] pseudoscalar final states. The logical next step is therefore to extend our present framework with these radiative corrections, and to check if the combined twist and α_s expansion of the non-local operators in the light-cone OPE is well behaved. Particularly, we wonder if the instability of inferring the duality thresholds for π , K and ρ final states can be overcome by including the radiative corrections, or by including contributions at the twist-five and twist-six levels; see ref. [63] for recent efforts in the latter direction.

Finally, we have selected two phenomenological applications connected to the present B anomalies to highlight the usefulness of our results. Our findings weaken the interpretation of the $B \rightarrow K^* \mu^+ \mu^-$ angular anomalies as effects of our lack of knowledge of hadronic form

factors. Furthermore, we have updated the form factor ratios R_1 and R_2 , relevant for V_{cb} extractions and predictions of the LFU ratio $R(D^*)$. Our results permit for the first time to account for correlations among the relevant $B \rightarrow \bar{D}^*$ form factors, and are in agreement with previous results at less than one standard deviation.

Acknowledgments

This work is supported by the DFG within the Emmy Noether Programme under grant DY-130/1-1 and the DFG Collaborative Research Center 110 ‘‘Symmetries and the Emergence of Structure in QCD’’. We are very grateful to S ebastien Descotes-Genon, Paolo Gambino and Javier Virto for helpful discussions, and to Martin Jung, David Straub and Javier Virto for comments on the manuscript. We are also grateful to the authors of [36, 37] for discussing their results prior to publication.

A B -meson distribution amplitudes

In this appendix we collect formulas relevant to the parametrization in terms of momentum-space of B -LCDAs of the non-local matrix elements in eqs. (2.13) and (2.14). The two-particle B -LCDAs are defined via

$$\begin{aligned} \langle 0 | \bar{q}_1^\alpha(x) h_v^\beta(0) | \bar{B}_{q_2}(v) \rangle = & -\frac{if_B m_B}{4} \int_0^\infty d\omega e^{-i\omega v \cdot x} \left\{ (1 + \not{v}) \left[\phi_+(\omega) - g_+(\omega) \partial_\sigma \partial^\sigma \right. \right. \\ & \left. \left. + \left(\frac{\bar{\phi}(\omega)}{2} - \frac{\bar{g}(\omega)}{2} \partial_\sigma \partial^\sigma \right) \gamma^\mu \partial_\mu \right] \gamma_5 \right\}^{\beta\alpha}, \end{aligned} \quad (\text{A.1})$$

while, for the three-particle B -LCDAs, we have

$$\begin{aligned} \langle 0 | \bar{q}_1^\alpha(x) G_{\mu\nu}(ux) h_v^\beta(0) | \bar{B}_{q_2}(v) \rangle = & \frac{f_B m_B}{4} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 e^{-i(\omega_1 + u\omega_2) v \cdot x} \\ & \times \left\{ (1 + \not{v}) \left[(v_\mu \gamma_\nu - v_\nu \gamma_\mu) [\psi_A - \psi_V] - i\sigma_{\mu\nu} \psi_V \right. \right. \\ & + (\partial_\mu v_\nu - \partial_\nu v_\mu) \bar{X}_A - (\partial_\mu \gamma_\nu - \partial_\nu \gamma_\mu) [\bar{W} + \bar{Y}_A] + i\epsilon_{\mu\nu\alpha\beta} \partial^\alpha v^\beta \gamma_5 \bar{X}_A \\ & \left. \left. - i\epsilon_{\mu\nu\alpha\beta} \partial^\alpha \gamma^\beta \gamma_5 \bar{Y}_A - u(\partial_\mu v_\nu - \partial_\nu v_\mu) \not{\partial} \bar{W} + u(\partial_\mu \gamma_\nu - \partial_\nu \gamma_\mu) \not{\partial} \bar{Z} \right] \gamma_5 \right\}^{\beta\alpha} (\omega_1, \omega_2), \end{aligned} \quad (\text{A.2})$$

where a gauge link is implied in the above, and the derivatives are abbreviated as $\partial_\mu \equiv \partial/\partial l^\mu$, is the momentum-space representation where $l^\mu = \omega v^\mu$ in the two-particle case and $l^\mu = (\omega_1 + u\omega_2)v^\mu$ in the three-particle case. Throughout, these derivatives are understood to act on the hard-scattering kernel. In addition, we define the following shorthand notation:

$$\begin{aligned} \bar{\phi}(\omega) & \equiv \int_0^\omega d\eta (\phi_+(\eta) - \phi_-(\eta)), \\ \bar{g}(\omega) & \equiv \int_0^\omega d\eta (g_+(\eta) - g_-(\eta)), \end{aligned}$$

$$\begin{aligned}\bar{\psi}_{3p}(\omega_1, \omega_2) &\equiv \int_0^{\omega_1} d\eta_1 \psi_{3p}(\eta_1, \omega_2), \\ \bar{\bar{\psi}}_{3p}(\omega_1, \omega_2) &\equiv \int_0^{\omega_1} d\eta_1 \int_0^{\omega_2} d\eta_2 \psi_{3p}(\eta_1, \eta_2),\end{aligned}\tag{A.3}$$

where ψ_{3p} represents any of the three-particle LCDAs. We use $\epsilon_{0123} = +1$ in both the definition of the form factors and in eq. (A.2), which matches the conventions of refs. [12, 17]. Accounting for the different convention, we reproduce the results of refs. [7, 8]. The “traditional” basis of three-particle LCDAs can be related to a basis of LCDAs with definite twist as follows [23]:

$$\begin{aligned}\phi_3(\omega_1, \omega_2) &= [\psi_A - \psi_V](\omega_1, \omega_2), \\ \phi_4(\omega_1, \omega_2) &= [\psi_A + \psi_V](\omega_1, \omega_2), \\ \psi_4(\omega_1, \omega_2) &= [\psi_A + X_A](\omega_1, \omega_2), \\ \chi_4(\omega_1, \omega_2) &= [\psi_V - \tilde{X}_A](\omega_1, \omega_2), \\ \tilde{\phi}_5(\omega_1, \omega_2) &= [\psi_A + \psi_V + 2Y_A - 2\tilde{Y}_A + 2W](\omega_1, \omega_2), \\ \psi_5(\omega_1, \omega_2) &= [-\psi_A + X_A - 2Y_A](\omega_1, \omega_2), \\ \chi_5(\omega_1, \omega_2) &= [-\psi_V - \tilde{X}_A + 2\tilde{Y}_A](\omega_1, \omega_2), \\ \phi_6(\omega_1, \omega_2) &= [\psi_A - \psi_V + 2Y_A + 2\tilde{Y}_A + 2W - 4Z](\omega_1, \omega_2).\end{aligned}\tag{A.4}$$

Note that we adopt the same nomenclature for the LCDAs as in ref. [23], except for renaming $\tilde{\psi}_{4,5} \rightarrow \chi_{4,5}$ such that our notation involving barred LCDAs (see eq. (A.3)) becomes more legible. It is possible to invert these relations. We obtain:

$$\begin{aligned}\psi_A &= \frac{1}{2} [\phi_3 + \phi_4](\omega_1, \omega_2), \\ \psi_V &= \frac{1}{2} [-\phi_3 + \phi_4](\omega_1, \omega_2), \\ X_A &= \frac{1}{2} [-\phi_3 - \phi_4 + 2\psi_4](\omega_1, \omega_2), \\ Y_A &= \frac{1}{2} [-\phi_3 - \phi_4 + \psi_4 - \psi_5](\omega_1, \omega_2), \\ \tilde{X}_A &= \frac{1}{2} [-\phi_3 + \phi_4 - 2\chi_4](\omega_1, \omega_2), \\ \tilde{Y}_A &= \frac{1}{2} [-\phi_3 + \phi_4 - \chi_4 + \chi_5](\omega_1, \omega_2), \\ W &= \frac{1}{2} [\phi_4 - \psi_4 - \chi_4 + \tilde{\phi}_5 + \psi_5 + \chi_5](\omega_1, \omega_2), \\ Z &= \frac{1}{4} [-\phi_3 + \phi_4 - 2\chi_4 + \tilde{\phi}_5 + 2\chi_5 - \phi_6](\omega_1, \omega_2).\end{aligned}\tag{A.5}$$

A parametrization of the set of three-particle LCDAs at the twist-five and twist-six level has been recently suggested [63]. This set includes three twist-five LCDAs and one twist-six LCDA. However, to obtain the full set of three-particle LCDAs one has to expand the position-space non-local matrix elements around the light-cone $x^2 \simeq 0$ in a consistent manner. Including the terms $\propto x^2$ for the structures multiplied by ϕ_3 , ϕ_4 , ψ_4 and χ_4 ,

the full set of momentum-space matrix elements at the twist-six level can be obtained from eq. (A.2) by using eq. (A.5) in combination with the replacements

$$\begin{aligned}
 \phi_3 &\mapsto \phi_3 - g_{\phi_3} \partial_\sigma \partial^\sigma, \\
 \phi_4 &\mapsto \phi_4 - g_{\phi_4} \partial_\sigma \partial^\sigma, \\
 \psi_4 &\mapsto \psi_4 - g_{\psi_4} \partial_\sigma \partial^\sigma, \\
 \chi_4 &\mapsto \chi_4 - g_{\chi_4} \partial_\sigma \partial^\sigma.
 \end{aligned}
 \tag{A.6}$$

The twist of the new $g_{\psi_{3p}}$ functions corresponds to the twist of their partner ψ_{3p} plus two units of twist. Up to the twist-six level we therefore find twelve independent three-particle LCDAs: one at twist three, three at twist four, four at twist-five, and further four at twist-six; in variance with the ansatz of ref. [63]. Our argument here is in full analogy to the approach to the off-the-light-cone contributions for two-particle LCDAs in form of g_+ and g_- introduced in ref. [23].

In order to evaluate numerically the form factors, we use the exponential models proposed in ref. [104] and adapted in ref. [23] to the LCDAs ϕ_+ , ϕ_- , g_+ , ϕ_3 , ϕ_4 , ψ_4 and χ_4 . Since g_- receives contributions from the three-particle DA ψ_5 , for which no model is given in ref. [23], we approximate g_- in the Wandzura-Wilczek (WW) limit. We use

$$g_-^{WW}(\omega) = \frac{1}{4} \int_0^\omega d\eta_2 \int_0^{\eta_2} d\eta_1 [\phi_+(\eta_1) - \phi_-^{WW}(\eta_1)] - \frac{1}{2} \int_0^\omega d\eta_1 (\eta_1 - \bar{\Lambda}) \phi_-^{WW}(\eta_1) \tag{A.7}$$

$$= \frac{3\omega}{4} e^{-\omega/\lambda_{B,+}}, \tag{A.8}$$

where in the second line we use the Grozin-Neubert relation $2\lambda_{B,+} = 4\bar{\Lambda}/3$.

B Coefficients of the LCSR formula

Here we list all the coefficients of eq. (2.18). The normalization factors are:

$$\begin{aligned}
 K(f_+^{B \rightarrow P}) &= K(f_{+/-}^{B \rightarrow P}) = f_P, & K(f_T^{B \rightarrow P}) &= \frac{f_P(m_B^2 - m_P^2 - q^2)}{m_B(m_B + m_P)}, \\
 K(V^{B \rightarrow V}) &= \frac{2f_V m_V}{m_B(m_B + m_V)}, & K(A_1^{B \rightarrow V}) &= \frac{2f_V m_V(m_B + m_V)}{m_B^2}, \\
 K(A_2^{B \rightarrow V}) &= \frac{2f_V m_V}{m_B + m_V}, & K(A_{30}^{B \rightarrow V}) &= \frac{4f_V m_V^2}{q^2}, \\
 K(T_1^{B \rightarrow V}) &= K(T_{23A}^{B \rightarrow V}) = K(T_{23B}^{B \rightarrow V}) = \frac{2f_V m_V}{m_B}.
 \end{aligned}
 \tag{B.1}$$

In the next subsections we give the $C_n^{(F,\psi)}$ coefficients of eqs. (2.20) and (2.21). For all the form factors, the following relations hold among the three-particle contributions:

$$C_n^{(F,\bar{\psi}_4)} = -C_n^{(F,\bar{\phi}_3)} - C_n^{(F,\bar{\phi}_4)}, \quad C_n^{(F,\bar{\chi}_4)} = C_n^{(F,\bar{\phi}_3)} - C_n^{(F,\bar{\phi}_4)}. \tag{B.2}$$

B.1 $B \rightarrow P$

B.1.1 Two-particle contributions

The coefficients of eq. (2.20), for the two-particle DAs, are listed in the following. For $f_+^{B \rightarrow P}$ we find the non-vanishing coefficients:

$$\begin{aligned}
 C_1^{(f_+^{B \rightarrow P}, \phi_+)} &= -\bar{\sigma}, \\
 C_2^{(f_+^{B \rightarrow P}, \bar{\phi})} &= -m_B \bar{\sigma}^2, \\
 C_2^{(f_+^{B \rightarrow P}, g_+)} &= -4\bar{\sigma}, & C_3^{(f_+^{B \rightarrow P}, g_+)} &= 8m_1^2 \bar{\sigma}, \\
 C_3^{(f_+^{B \rightarrow P}, \bar{g})} &= -8m_B \bar{\sigma}^2, & C_4^{(f_+^{B \rightarrow P}, \bar{g})} &= 24m_1^2 m_B \bar{\sigma}^2.
 \end{aligned} \tag{B.3}$$

For $f_{+/-}^{B \rightarrow P}$ we find:

$$\begin{aligned}
 C_1^{(f_{+/-}^{B \rightarrow P}, \phi_+)} &= 2\sigma - 1, \\
 C_2^{(f_{+/-}^{B \rightarrow P}, \bar{\phi})} &= 2m_B \sigma \bar{\sigma} - m_1, \\
 C_2^{(f_{+/-}^{B \rightarrow P}, g_+)} &= 4(2\sigma - 1), & C_3^{(f_{+/-}^{B \rightarrow P}, g_+)} &= -8m_1^2 (2\sigma - 1), \\
 C_3^{(f_{+/-}^{B \rightarrow P}, \bar{g})} &= 16m_B \sigma \bar{\sigma}, & C_4^{(f_{+/-}^{B \rightarrow P}, \bar{g})} &= 24m_1^2 (m_1 - 2m_B \sigma \bar{\sigma}).
 \end{aligned} \tag{B.4}$$

For $f_T^{B \rightarrow P}$ we find:

$$\begin{aligned}
 C_1^{(f_T^{B \rightarrow P}, \bar{\phi})} &= \frac{1}{m_B}, & C_2^{(f_T^{B \rightarrow P}, \bar{\phi})} &= \frac{-(m_B^2 \bar{\sigma}^2 - m_1^2 + 2q^2 \sigma - q^2)}{m_B}, \\
 C_2^{(f_T^{B \rightarrow P}, \bar{g})} &= \frac{8}{m_B}, & C_3^{(f_T^{B \rightarrow P}, \bar{g})} &= \frac{-8(m_B^2 \bar{\sigma}^2 + 2m_1^2 + 2q^2 \sigma - q^2)}{m_B}, \\
 C_4^{(f_T^{B \rightarrow P}, \bar{g})} &= \frac{24m_1^2 (m_B^2 \bar{\sigma}^2 - m_1^2 + 2q^2 \sigma - q^2)}{m_B}.
 \end{aligned} \tag{B.5}$$

B.1.2 Three-particle contributions

The coefficients of eq. (2.21), for the three-particle DAs, for $f_+^{B \rightarrow P}$ follow. For ϕ_3 :

$$\begin{aligned}
 C_2^{(f_+^{B \rightarrow P}, \phi_3)} &= -\frac{2m_1}{m_B} - u\bar{\sigma}, \\
 C_2^{(f_+^{B \rightarrow P}, \bar{\phi}_3)} &= \frac{u}{m_B}, & C_3^{(f_+^{B \rightarrow P}, \bar{\phi}_3)} &= -\frac{2}{m_B} (u(m_B^2 \bar{\sigma}^2 + q^2) + 4m_B m_1 \bar{\sigma} + u m_1^2), \\
 & & C_4^{(f_+^{B \rightarrow P}, \bar{\phi}_3)} &= -6m_1 \bar{\sigma} (2m_B \bar{\sigma} + m_1 (2u - 1)).
 \end{aligned} \tag{B.6}$$

For ϕ_4 :

$$\begin{aligned}
 C_2^{(f_+^{B \rightarrow P}, \phi_4)} &= \bar{\sigma}(1-u), \\
 C_2^{(f_+^{B \rightarrow P}, \bar{\phi}_4)} &= \frac{u-1}{m_B}, \\
 C_3^{(f_+^{B \rightarrow P}, \bar{\phi}_4)} &= 2um_B\bar{\sigma}^2 + 4m_1\bar{\sigma} + 2\frac{(1-u)(m_1^2 + q^2)}{m_B}, \\
 C_3^{(f_+^{B \rightarrow P}, \bar{\bar{\phi}}_4)} &= \frac{2}{m_B}(m_B\bar{\sigma}(2u-1) + 2m_1), \\
 C_4^{(f_+^{B \rightarrow P}, \bar{\bar{\phi}}_4)} &= \frac{6}{m_B}(m_B^2\bar{\sigma}^2 - q^2)(m_B\bar{\sigma}(2u-1) + 2m_1).
 \end{aligned} \tag{B.7}$$

For ψ_4 :

$$C_2^{(f_+^{B \rightarrow P}, \bar{\psi}_4)} = \frac{1-2u}{m_B}, \quad C_3^{(f_+^{B \rightarrow P}, \bar{\psi}_4)} = \frac{2}{m_B}(2u-1)(m_1^2 - m_B^2\bar{\sigma}^2 + q^2). \tag{B.8}$$

For χ_4 :

$$\begin{aligned}
 C_2^{(f_+^{B \rightarrow P}, \bar{\chi}_4)} &= \frac{1}{m_B}, \\
 C_3^{(f_+^{B \rightarrow P}, \bar{\chi}_4)} &= -\frac{2}{m_B}(m_B^2\bar{\sigma}^2(2u-1) + 4m_Bm_1\bar{\sigma} + m_1^2 + q^2).
 \end{aligned} \tag{B.9}$$

The coefficients of eq. (2.21), for the three-particle DAs, for $f_{+/-}^{B \rightarrow P}$ follow. For ϕ_3 :

$$\begin{aligned}
 C_2^{(f_{+/-}^{B \rightarrow P}, \phi_3)} &= (3-2\bar{\sigma})u - \frac{4m_1}{m_B}, \\
 C_2^{(f_{+/-}^{B \rightarrow P}, \bar{\phi}_3)} &= 2u\frac{(\bar{\sigma}-1)}{m_B\bar{\sigma}}, \\
 C_3^{(f_{+/-}^{B \rightarrow P}, \bar{\phi}_3)} &= -\frac{2}{m_B\bar{\sigma}}(m_B^2\bar{\sigma}^2(2\bar{\sigma}-3)u \\
 &\quad + (2\bar{\sigma}-1)(4m_Bm_1\bar{\sigma} + uq^2) + um_1^2(2\bar{\sigma}+1)), \\
 C_4^{(f_{+/-}^{B \rightarrow P}, \bar{\bar{\phi}}_3)} &= -6m_1(4m_B(\bar{\sigma}-1)\bar{\sigma} + m_1(2\bar{\sigma}+1)(2u-1)).
 \end{aligned} \tag{B.10}$$

For ϕ_4 :

$$\begin{aligned}
 C_2^{(f_{+/-}^{B \rightarrow P}, \phi_4)} &= (1-u)(2\bar{\sigma}+1), \\
 C_2^{(f_{+/-}^{B \rightarrow P}, \bar{\phi}_4)} &= \frac{2}{m_B\bar{\sigma}}(\bar{\sigma}-1)(u-1), \\
 C_3^{(f_{+/-}^{B \rightarrow P}, \bar{\phi}_4)} &= \frac{2}{m_B\bar{\sigma}}(m_B^2\bar{\sigma}^2(2\bar{\sigma}u - u - 1) + m_Bm_1\bar{\sigma}(4\bar{\sigma}-1) \\
 &\quad + m_1^2(2\bar{\sigma}+1)(1-u) + q^2(2\bar{\sigma} - 2\bar{\sigma}u + u - 1)), \\
 C_3^{(f_{+/-}^{B \rightarrow P}, \bar{\bar{\phi}}_4)} &= \frac{2}{m_B\bar{\sigma}}(2m_B(\bar{\sigma}-2)\bar{\sigma}(2u-1) + m_1(4\bar{\sigma}-3)), \\
 C_4^{(f_{+/-}^{B \rightarrow P}, \bar{\bar{\phi}}_4)} &= \frac{6}{m_B\bar{\sigma}}(m_1(4\bar{\sigma}-1)(m_B^2\bar{\sigma}^2 - q^2) + 2m_B(\bar{\sigma}-1)\bar{\sigma}(2u-1)(m_B^2\bar{\sigma}^2 - q^2)) \\
 &\quad + \frac{6}{m_B\bar{\sigma}}(m_Bm_1^2\bar{\sigma}(2u-1) - m_1^3).
 \end{aligned} \tag{B.11}$$

For ψ_4 :

$$\begin{aligned}
 C_2^{(f_{+/-}^{B \rightarrow P}, \bar{\psi}_4)} &= \frac{2}{m_B \bar{\sigma}} (\bar{\sigma} - 1)(1 - 2u), \\
 C_3^{(f_{+/-}^{B \rightarrow P}, \bar{\psi}_4)} &= \frac{2}{m_B \bar{\sigma}} ((2\bar{\sigma} - 1)(2u - 1)(q^2 - m_B^2 \bar{\sigma}^2) - 2m_B m_1 \bar{\sigma} + m_1^2 (2\bar{\sigma} + 1)(2u - 1)).
 \end{aligned} \tag{B.12}$$

For χ_4 :

$$\begin{aligned}
 C_2^{(f_{+/-}^{B \rightarrow P}, \bar{\chi}_4)} &= \frac{2}{m_B \bar{\sigma}} (\bar{\sigma} - 1), \\
 C_3^{(f_{+/-}^{B \rightarrow P}, \bar{\chi}_4)} &= -\frac{2}{m_B \bar{\sigma}} (m_B^2 \bar{\sigma}^2 (4u(\bar{\sigma} - 1) - 2\bar{\sigma} + 1) \\
 &\quad + 4m_B m_1 \bar{\sigma} (2\bar{\sigma} - 1) + m_1^2 (2\bar{\sigma} + 1) + q^2 (2\bar{\sigma} - 1)).
 \end{aligned} \tag{B.13}$$

The coefficients of eq. (2.21), for the three-particle DAs, for $f_T^{B \rightarrow P}$ follow. For ϕ_3 :

$$\begin{aligned}
 C_1^{(f_T^{B \rightarrow P}, \phi_3)} &= \frac{2u}{m_B^2 \bar{\sigma}}, \\
 C_2^{(f_T^{B \rightarrow P}, \phi_3)} &= -\frac{2u}{m_B^2 \bar{\sigma}} (m_B^2 \bar{\sigma}^2 - m_1^2 - 2q^2 \bar{\sigma} + q^2), \\
 C_2^{(f_T^{B \rightarrow P}, \bar{\phi}_3)} &= \frac{4}{m_B^2 \bar{\sigma}} (m_B \bar{\sigma} u + m_1), \\
 C_3^{(f_T^{B \rightarrow P}, \bar{\phi}_3)} &= -\frac{4}{m_B^2 \bar{\sigma}} (m_B^2 \bar{\sigma}^2 - m_1^2 - 2q^2 \bar{\sigma} + q^2) (m_B \bar{\sigma} u + m_1), \\
 C_3^{(f_T^{B \rightarrow P}, \bar{\bar{\phi}}_3)} &= 12 \frac{m_1}{m_B}, \\
 C_4^{(f_T^{B \rightarrow P}, \bar{\bar{\phi}}_3)} &= -12 \frac{m_1}{m_B} (m_B^2 \bar{\sigma}^2 - m_1^2 - 2q^2 \bar{\sigma} + q^2).
 \end{aligned} \tag{B.14}$$

For ϕ_4 :

$$\begin{aligned}
 C_2^{(f_T^{B \rightarrow P}, \bar{\phi}_4)} &= -\frac{2}{m_B}, \\
 C_3^{(f_T^{B \rightarrow P}, \bar{\phi}_4)} &= \frac{2}{m_B} (m_B^2 \bar{\sigma}^2 - m_1^2 - 2q^2 \bar{\sigma} + q^2), \\
 C_2^{(f_T^{B \rightarrow P}, \bar{\bar{\phi}}_4)} &= -\frac{4}{m_B^2 \bar{\sigma}} (2u - 1), \\
 C_3^{(f_T^{B \rightarrow P}, \bar{\bar{\phi}}_4)} &= \frac{2}{m_B^2 \bar{\sigma}} (2u - 1) (m_1^2 - m_B^2 \bar{\sigma}^2 + q^2 (5 - 4\bar{\sigma})), \\
 C_4^{(f_T^{B \rightarrow P}, \bar{\bar{\phi}}_4)} &= \frac{6}{m_B^2 \bar{\sigma}} (2u - 1) (m_1^2 - m_B^2 \bar{\sigma}^2 + q^2) (m_1^2 - m_B^2 \bar{\sigma}^2 + q^2 (2\bar{\sigma} - 1)).
 \end{aligned} \tag{B.15}$$

For ψ_4 :

$$\begin{aligned}
 C_2^{(f_T^{B \rightarrow P}, \bar{\psi}_4)} &= -\frac{4m_1}{m_B^2 \bar{\sigma}}, \\
 C_3^{(f_T^{B \rightarrow P}, \bar{\psi}_4)} &= \frac{4m_1}{m_B^2 \bar{\sigma}} (m_B^2 \bar{\sigma}^2 - m_1^2 - 2q^2 \bar{\sigma} + q^2).
 \end{aligned} \tag{B.16}$$

For χ_4 :

$$\begin{aligned} C_2^{(f_T^{B \rightarrow P}, \bar{\chi}_4)} &= \frac{4}{m_B^2 \bar{\sigma}} (m_B \bar{\sigma} u + m_1), \\ C_3^{(f_T^{B \rightarrow P}, \bar{\chi}_4)} &= -\frac{4}{m_B^2 \bar{\sigma}} (m_B \bar{\sigma} u + m_1) (m_B^2 \bar{\sigma}^2 - m_1^2 - 2q^2 \bar{\sigma} + q^2). \end{aligned} \quad (\text{B.17})$$

B.2 $B \rightarrow V$

B.2.1 Two-particle contributions

The coefficients of eq. (2.20), for the two-particle DAs, are listed in the following. For $V^{B \rightarrow V}$ we find:

$$\begin{aligned} C_1^{(V^{B \rightarrow V}, \phi_+)} &= -\frac{1}{m_B}, \\ C_2^{(V^{B \rightarrow V}, \bar{\phi})} &= -\frac{m_1}{m_B}, \\ C_2^{(V^{B \rightarrow V}, g_+)} &= -\frac{4}{m_B}, \quad C_3^{(V^{B \rightarrow V}, g_+)} = \frac{8m_1^2}{m_B}, \\ C_4^{(V^{B \rightarrow V}, \bar{g})} &= \frac{24m_1^3}{m_B}. \end{aligned} \quad (\text{B.18})$$

For $A_1^{B \rightarrow V}$ we find:

$$\begin{aligned} C_1^{(A_1^{B \rightarrow V}, \phi_+)} &= \frac{q^2 - (m_B \bar{\sigma} + m_1)^2}{m_B^2 \bar{\sigma}}, & C_2^{(A_1^{B \rightarrow V}, \bar{\phi})} &= \frac{m_1 (q^2 - (m_B \bar{\sigma} + m_1)^2)}{m_B^2 \bar{\sigma}}, \\ C_1^{(A_1^{B \rightarrow V}, \bar{\phi})} &= -\frac{m_1}{m_B^2 \bar{\sigma}}, & C_2^{(A_1^{B \rightarrow V}, g_+)} &= \frac{4(q^2 - m_B^2 \bar{\sigma}^2 + m_1^2)}{m_B^2 \bar{\sigma}}, \\ C_1^{(A_1^{B \rightarrow V}, g_+)} &= -\frac{4}{m_B^2 \bar{\sigma}}, & C_3^{(A_1^{B \rightarrow V}, g_+)} &= \frac{8m_1^2 ((m_B \bar{\sigma} + m_1)^2 - q^2)}{m_B^2 \bar{\sigma}}, \\ C_3^{(A_1^{B \rightarrow V}, g_+)} &= \frac{8m_1^2 ((m_B \bar{\sigma} + m_1)^2 - q^2)}{m_B^2 \bar{\sigma}}, & C_2^{(A_1^{B \rightarrow V}, \bar{g})} &= -\frac{8}{m_B}, \\ C_2^{(A_1^{B \rightarrow V}, \bar{g})} &= -\frac{8}{m_B}, & C_3^{(A_1^{B \rightarrow V}, \bar{g})} &= \frac{8m_1^2 (2m_B \bar{\sigma} + 3m_1)}{m_B^2 \bar{\sigma}}, \\ C_4^{(A_1^{B \rightarrow V}, \bar{g})} &= \frac{24m_1^3 ((m_B \bar{\sigma} + m_1)^2 - q^2)}{m_B^2 \bar{\sigma}}. \end{aligned} \quad (\text{B.19})$$

For $A_2^{B \rightarrow V}$ we find:

$$\begin{aligned} C_1^{(A_2^{B \rightarrow V}, \phi_+)} &= 2\sigma - 1, \\ C_2^{(A_2^{B \rightarrow V}, \bar{\phi})} &= 2m_B \sigma \bar{\sigma} - m_1, \\ C_2^{(A_2^{B \rightarrow V}, g_+)} &= 4(2\sigma - 1), \quad C_3^{(A_2^{B \rightarrow V}, g_+)} = -8m_1^2 (2\sigma - 1), \\ C_3^{(A_2^{B \rightarrow V}, \bar{g})} &= 16m_B \sigma \bar{\sigma}, \quad C_4^{(A_2^{B \rightarrow V}, \bar{g})} = 24m_1^2 (m_1 - 2m_B \sigma \bar{\sigma}). \end{aligned} \quad (\text{B.20})$$

For $A_{30}^{B \rightarrow V}$ we find:

$$\begin{aligned}
 C_1^{(A_{30}^{B \rightarrow V}, \phi_+)} &= 2\sigma + 1, \\
 C_2^{(A_{30}^{B \rightarrow V}, \bar{\phi})} &= m_1 - 2m_B\sigma(\sigma + 1), \\
 C_2^{(A_{30}^{B \rightarrow V}, g_+)} &= 4(2\sigma + 1), & C_3^{(A_{30}^{B \rightarrow V}, g_+)} &= -8m_1^2(2\sigma + 1), \\
 C_3^{(A_{30}^{B \rightarrow V}, \bar{g})} &= -16m_B\sigma(\sigma + 1), & C_4^{(A_{30}^{B \rightarrow V}, \bar{g})} &= 24m_1^2(2m_B\sigma(\sigma + 1) - m_1).
 \end{aligned} \tag{B.21}$$

For $T_1^{B \rightarrow V}$ we find:

$$\begin{aligned}
 C_1^{(T_1^{B \rightarrow V}, \phi_+)} &= -\frac{(m_B\bar{\sigma} + m_1)}{m_B}, \\
 C_2^{(T_1^{B \rightarrow V}, \bar{\phi})} &= -m_1 \frac{(m_B\bar{\sigma} + m_1)}{m_B}, \\
 C_2^{(T_1^{B \rightarrow V}, g_+)} &= -4\bar{\sigma}, & C_3^{(T_1^{B \rightarrow V}, g_+)} &= 8m_1^2 \frac{(m_B\bar{\sigma} + m_1)}{m_B}, \\
 C_2^{(T_1^{B \rightarrow V}, \bar{g})} &= -\frac{4}{m_B}, & C_3^{(T_1^{B \rightarrow V}, \bar{g})} &= \frac{8m_1^2}{m_B}, \\
 C_4^{(T_1^{B \rightarrow V}, \bar{g})} &= 24m_1^3 \frac{(m_B\bar{\sigma} + m_1)}{m_B}.
 \end{aligned} \tag{B.22}$$

For $T_{23A}^{B \rightarrow V}$ we find:

$$\begin{aligned}
 C_1^{(T_{23A}^{B \rightarrow V}, \phi_+)} &= -\frac{(m_B\bar{\sigma} + m_1)}{m_B}, \\
 C_2^{(T_{23A}^{B \rightarrow V}, \bar{\phi})} &= -\frac{(m_1(m_B\bar{\sigma} + m_1) - 2q^2\sigma)}{m_B}, \\
 C_2^{(T_{23A}^{B \rightarrow V}, g_+)} &= -4\bar{\sigma}, & C_3^{(T_{23A}^{B \rightarrow V}, g_+)} &= 8m_1^2 \frac{(m_B\bar{\sigma} + m_1)}{m_B}, \\
 C_2^{(T_{23A}^{B \rightarrow V}, \bar{g})} &= -\frac{4}{m_B}, & C_3^{(T_{23A}^{B \rightarrow V}, \bar{g})} &= \frac{8(m_1^2 + 2q^2\sigma)}{m_B}, \\
 C_4^{(T_{23A}^{B \rightarrow V}, \bar{g})} &= 24m_1^2 \frac{(m_1(m_B\bar{\sigma} + m_1) - 2q^2\sigma)}{m_B}.
 \end{aligned} \tag{B.23}$$

For $T_{23B}^{B \rightarrow V}$ we find:

$$\begin{aligned}
 C_1^{(T_{23B}^{B \rightarrow V}, \phi_+)} &= \frac{(m_B\sigma - m_1)}{m_B}, \\
 C_1^{(T_{23B}^{B \rightarrow V}, \bar{\phi})} &= \frac{\sigma}{m_B\bar{\sigma}}, & C_2^{(T_{23B}^{B \rightarrow V}, \bar{\phi})} &= \frac{m_B m_1 \sigma \bar{\sigma} - m_B^2 \sigma \bar{\sigma}^2 + (2\sigma - 1)(m_1^2 - q^2\sigma)}{m_B\bar{\sigma}}, \\
 C_2^{(T_{23B}^{B \rightarrow V}, g_+)} &= 4\sigma, & C_3^{(T_{23B}^{B \rightarrow V}, g_+)} &= 8m_1^2 \frac{(m_1 - m_B\sigma)}{m_B}, \\
 C_2^{(T_{23B}^{B \rightarrow V}, \bar{g})} &= 4 \frac{(3\sigma - 1)}{m_B\bar{\sigma}}, & C_3^{(T_{23B}^{B \rightarrow V}, \bar{g})} &= -8 \frac{\sigma(m_B^2\bar{\sigma}^2 + 3m_1^2 + q^2(2\sigma - 1)) - m_1^2}{m_B\bar{\sigma}}, \\
 C_4^{(T_{23B}^{B \rightarrow V}, \bar{g})} &= \frac{24m_1^2}{m_B\bar{\sigma}} (m_B^2\sigma\bar{\sigma}^2 - m_B m_1 \sigma \bar{\sigma} + (2\sigma - 1)(q^2\sigma - m_1^2)).
 \end{aligned} \tag{B.24}$$

Where $T_{23A}^{B \rightarrow V}$ and $T_{23B}^{B \rightarrow V}$ are defined in eqs. (2.24) and (2.25).

B.2.2 Three-particle contributions

The coefficients of eq. (2.21), for the three-particle DAs, for $V^{B \rightarrow V}$ follow. For ϕ_3 :

$$\begin{aligned}
 C_2^{(V^{B \rightarrow V}, \phi_3)} &= \frac{u}{m_B}, \\
 C_2^{(V^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{2u}{m_B^2 \bar{\sigma}}, \\
 C_3^{(V^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{2u}{m_B^2 \bar{\sigma}} (m_B^2 \bar{\sigma}^2 + m_1^2 - q^2), \\
 C_4^{(V^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{6m_1^2}{m_B} (2u - 1).
 \end{aligned}
 \tag{B.25}$$

For ϕ_4 :

$$\begin{aligned}
 C_2^{(V^{B \rightarrow V}, \phi_4)} &= \frac{u - 1}{m_B}, \\
 C_2^{(V^{B \rightarrow V}, \bar{\phi}_4)} &= 2 \frac{(u - 1)}{m_B^2 \bar{\sigma}}, \\
 C_3^{(V^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{2}{m_B^2 \bar{\sigma}} (u - 1) (m_1^2 - q^2) - \frac{2m_1}{m_B} + 2\bar{\sigma}(u - 1), \\
 C_3^{(V^{B \rightarrow V}, \bar{\bar{\phi}}_4)} &= -\frac{6m_1}{m_B^2 \bar{\sigma}}, \\
 C_4^{(V^{B \rightarrow V}, \bar{\bar{\phi}}_4)} &= -\frac{6m_1}{m_B^2 \bar{\sigma}} (m_B^2 \bar{\sigma}^2 + m_B m_1 \bar{\sigma} (1 - 2u) + m_1^2 - q^2).
 \end{aligned}
 \tag{B.26}$$

For ψ_4 :

$$\begin{aligned}
 C_2^{(V^{B \rightarrow V}, \bar{\psi}_4)} &= \frac{2 - 4u}{m_B^2 \bar{\sigma}}, \\
 C_3^{(V^{B \rightarrow V}, \bar{\psi}_4)} &= \frac{2}{m_B^2 \bar{\sigma}} ((2u - 1)(q^2 - m_B^2 \bar{\sigma}^2) + 2m_B m_1 \bar{\sigma} + m_1^2 (1 - 2u)).
 \end{aligned}
 \tag{B.27}$$

For χ_4 :

$$\begin{aligned}
 C_2^{(V^{B \rightarrow V}, \bar{\chi}_4)} &= \frac{2}{m_B^2 \bar{\sigma}}, \\
 C_3^{(V^{B \rightarrow V}, \bar{\chi}_4)} &= \frac{2}{m_B^2 \bar{\sigma}} (m_1^2 - q^2) + 2\bar{\sigma}.
 \end{aligned}
 \tag{B.28}$$

The coefficients of eq. (2.21), for the three-particle DAs, for $A_1^{B \rightarrow V}$ follow. For ϕ_3 :

$$\begin{aligned}
 C_1^{(A_1^{B \rightarrow V}, \phi_3)} &= \frac{u}{m_B^2 \bar{\sigma}}, \\
 C_2^{(A_1^{B \rightarrow V}, \phi_3)} &= u \frac{(m_1^2 - q^2)}{m_B^2 \bar{\sigma}} + \frac{2m_1}{m_B} + u\bar{\sigma}, \\
 C_1^{(A_1^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{2u}{m_B^3 \bar{\sigma}^2}, \\
 C_2^{(A_1^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{1}{m_B^3 \bar{\sigma}^2} (4m_B m_1 \bar{\sigma} - 2um_B^2 \bar{\sigma}^2 + 4m_1^2 u - 4q^2 u),
 \end{aligned}
 \tag{B.29}$$

$$\begin{aligned}
 C_3^{(A_1^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{2}{m_B^3 \bar{\sigma}^2} (m_B^2 \bar{\sigma}^2 + m_1^2 - q^2) (m_B^2 \bar{\sigma}^2 u + 2m_B m_1 \bar{\sigma} + m_1^2 u - q^2 u), \\
 C_3^{(A_1^{B \rightarrow V}, \bar{\bar{\phi}}_3)} &= \frac{6m_1}{m_B^2 \bar{\sigma}} (2m_B \bar{\sigma} + m_1(2u - 1)). \\
 C_4^{(A_1^{B \rightarrow V}, \bar{\bar{\phi}}_3)} &= \frac{6m_1^2}{m_B^2 \bar{\sigma}} (m_B^2 \bar{\sigma}^2 (2u - 1) + 2m_B m_1 \bar{\sigma} + (2u - 1)(m_1^2 - q^2)).
 \end{aligned}$$

For ϕ_4 :

$$\begin{aligned}
 C_1^{(A_1^{B \rightarrow V}, \phi_4)} &= \frac{u - 1}{m_B^2 \bar{\sigma}}, \\
 C_2^{(A_1^{B \rightarrow V}, \phi_4)} &= \frac{u - 1}{m_B^2 \bar{\sigma}} (m_B^2 \bar{\sigma}^2 + m_1^2 - q^2), \\
 C_1^{(A_1^{B \rightarrow V}, \bar{\phi}_4)} &= 2 \frac{(u - 1)}{m_B^3 \bar{\sigma}^2}, \\
 C_2^{(A_1^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{1}{m_B^3 \bar{\sigma}^2} (2m_B^2 \bar{\sigma}^2 u - 2m_B m_1 \bar{\sigma} + 4(u - 1)(m_1^2 - q^2)), \\
 C_3^{(A_1^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{2}{m_B^3 \bar{\sigma}^2} ((m_B \bar{\sigma} + m_1)^2 - q^2) (m_B^2 \bar{\sigma}^2 (u - 1) \\
 &\quad + m_B m_1 \bar{\sigma} (1 - 2u) + (u - 1)(m_1^2 - q^2)), \\
 C_2^{(A_1^{B \rightarrow V}, \bar{\bar{\phi}}_4)} &= \frac{2}{m_B^3 \bar{\sigma}^2} (2m_B \bar{\sigma} (2u - 1) - 3m_1), \\
 C_3^{(A_1^{B \rightarrow V}, \bar{\bar{\phi}}_4)} &= \frac{12m_1}{m_B^3 \bar{\sigma}^2} (q^2 - m_1^2) - \frac{2}{m_B^2 \bar{\sigma}} (2u - 1)(m_1^2 + 2q^2) - \frac{4m_1}{m_B} + 4\bar{\sigma}(2u - 1), \\
 C_4^{(A_1^{B \rightarrow V}, \bar{\bar{\phi}}_4)} &= -\frac{6m_1}{m_B^3 \bar{\sigma}^2} (m_B m_1 \bar{\sigma} (2u - 1)(m_B^2 \bar{\sigma}^2 - q^2) + (q^2 - m_B^2 \bar{\sigma}^2)^2 \\
 &\quad + m_1^3 m_B \bar{\sigma} (2u - 1) + m_1^4 - 2m_1^2 q^2).
 \end{aligned} \tag{B.30}$$

For ψ_4 :

$$\begin{aligned}
 C_1^{(A_1^{B \rightarrow V}, \bar{\psi}_4)} &= -\frac{2}{m_B^3 \bar{\sigma}^2} (2u - 1), \\
 C_2^{(A_1^{B \rightarrow V}, \bar{\psi}_4)} &= -\frac{2}{m_B^3 \bar{\sigma}^2} (2u - 1) (m_B^2 \bar{\sigma}^2 + 2m_1^2 - 2q^2), \\
 C_3^{(A_1^{B \rightarrow V}, \bar{\psi}_4)} &= -\frac{2}{m_B^3 \bar{\sigma}^2} (2u - 1) ((q^2 - m_B^2 \bar{\sigma}^2)^2 - 2m_1^2 (m_B^2 \bar{\sigma}^2 + q^2) + m_1^4).
 \end{aligned} \tag{B.31}$$

For χ_4 :

$$\begin{aligned}
 C_1^{(A_1^{B \rightarrow V}, \bar{\chi}_4)} &= \frac{2}{m_B^3 \bar{\sigma}^2}, \\
 C_2^{(A_1^{B \rightarrow V}, \bar{\chi}_4)} &= \frac{2}{m_B^3 \bar{\sigma}^2} (m_B^2 \bar{\sigma}^2 (1 - 2u) + 2m_B m_1 \bar{\sigma} + 2m_1^2 - 2q^2), \\
 C_3^{(A_1^{B \rightarrow V}, \bar{\chi}_4)} &= \frac{2}{m_B^3 \bar{\sigma}^2} (m_B^4 \bar{\sigma}^4 + 2m_B^3 m_1 \bar{\sigma}^3 - 2m_B^2 \bar{\sigma}^2 (m_1^2 (1 - 2u) + q^2) \\
 &\quad + 2m_B m_1 \bar{\sigma} (m_1^2 - q^2) + (m_1^2 - q^2)^2).
 \end{aligned} \tag{B.32}$$

The coefficients of eq. (2.21), for the three-particle DAs, for $A_2^{B \rightarrow V}$ follow. For ϕ_3 :

$$\begin{aligned}
 C_2^{(A_2^{B \rightarrow V}, \phi_3)} &= \frac{4m_1}{m_B} - (2\bar{\sigma} + 1)u, \\
 C_2^{(A_2^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{2u}{m_B \bar{\sigma}} (\bar{\sigma} - 1), \\
 C_3^{(A_2^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{1}{m_B \bar{\sigma}} (2u(m_B^2(3 - 2\bar{\sigma})\bar{\sigma}^2 - 2q^2\bar{\sigma} + q^2) \\
 &\quad + 8m_B m_1 \bar{\sigma} (2\bar{\sigma} - 1) - 2m_1^2 (2\bar{\sigma}u + u)), \\
 C_4^{(A_2^{B \rightarrow V}, \bar{\bar{\phi}}_3)} &= 6m_1(4m_B(\bar{\sigma} - 1)\bar{\sigma} + m_1(2\bar{\sigma} + (6 - 4\bar{\sigma})u - 3)).
 \end{aligned} \tag{B.33}$$

For ϕ_4 :

$$\begin{aligned}
 C_2^{(A_2^{B \rightarrow V}, \phi_4)} &= (1 - u)(2\bar{\sigma} - 3), \\
 C_2^{(A_2^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{2}{m_B \bar{\sigma}} (u - 1)(\bar{\sigma} - 1), \\
 C_3^{(A_2^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{2}{m_B \bar{\sigma}} (m_B^2 \bar{\sigma}^2 (2\bar{\sigma}u - u - 1) + m_B m_1 (3 - 4\bar{\sigma})\bar{\sigma} + m_1^2 (2\bar{\sigma} + 1)(1 - u) \\
 &\quad + q^2 (2\bar{\sigma} - 2\bar{\sigma}u + u - 1)), \\
 C_3^{(A_2^{B \rightarrow V}, \bar{\bar{\phi}}_4)} &= \frac{2}{m_B \bar{\sigma}} (2m_B(\bar{\sigma} - 2)\bar{\sigma}(2u - 1) + m_1(9 - 4\bar{\sigma})), \\
 C_4^{(A_2^{B \rightarrow V}, \bar{\bar{\phi}}_4)} &= \frac{6}{m_B \bar{\sigma}} (2m_B(\bar{\sigma} - 1)\bar{\sigma}(2u - 1)(m_B^2 \bar{\sigma}^2 - q^2) \\
 &\quad + 3m_B m_1^2 \bar{\sigma} (1 - 2u) + 3m_1^3 + m_1(4\bar{\sigma} - 3)(q^2 - m_B^2 \bar{\sigma}^2)).
 \end{aligned} \tag{B.34}$$

For ψ_4 :

$$\begin{aligned}
 C_2^{(A_2^{B \rightarrow V}, \bar{\psi}_4)} &= \frac{2}{m_B \bar{\sigma}} (1 - 2u)(\bar{\sigma} - 1), \\
 C_3^{(A_2^{B \rightarrow V}, \bar{\psi}_4)} &= \frac{1}{m_B \bar{\sigma}} (2(2\bar{\sigma} - 1)(2u - 1)(q^2 - m_B^2 \bar{\sigma}^2) \\
 &\quad - 4m_B m_1 \bar{\sigma} + 2m_1^2 (2\bar{\sigma} + 1)(2u - 1)).
 \end{aligned} \tag{B.35}$$

For χ_4 :

$$\begin{aligned}
 C_2^{(A_2^{B \rightarrow V}, \bar{\chi}_4)} &= \frac{2}{m_B \bar{\sigma}} (\bar{\sigma} - 1), \\
 C_3^{(A_2^{B \rightarrow V}, \bar{\chi}_4)} &= -\frac{2}{m_B \bar{\sigma}} (m_B^2 \bar{\sigma}^2 (-2\bar{\sigma} + 4(\bar{\sigma} - 1)u + 1) \\
 &\quad + 4m_B m_1 (1 - 2\bar{\sigma})\bar{\sigma} + m_1^2 (2\bar{\sigma} + 1) + q^2 (2\bar{\sigma} - 1)).
 \end{aligned} \tag{B.36}$$

The coefficients of eq. (2.21), for the three-particle DAs, for $A_{30}^{B \rightarrow V}$ follow. For ϕ_3 :

$$\begin{aligned}
 C_2^{(A_{30}^{B \rightarrow V}, \phi_3)} &= \frac{4m_1}{m_B} + (5 - 2\bar{\sigma})u, \\
 C_2^{(A_{30}^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{2u}{m_B \bar{\sigma}} (\bar{\sigma} - 3), \\
 C_3^{(A_{30}^{B \rightarrow V}, \bar{\phi}_3)} &= -\frac{2}{m_B \bar{\sigma}} (u(m_B^2 \bar{\sigma} (\bar{\sigma} (2\bar{\sigma} - 9) + 8) + q^2 (2\bar{\sigma} - 3)) \\
 &\quad + 4m_B m_1 (3 - 2\bar{\sigma})\bar{\sigma} + m_1^2 (2\bar{\sigma} + 3)u), \\
 C_4^{(A_{30}^{B \rightarrow V}, \bar{\bar{\phi}}_3)} &= 6m_1(4m_B(\bar{\sigma} - 2)(\bar{\sigma} - 1) + m_1(2\bar{\sigma} + (2 - 4\bar{\sigma})u - 1)).
 \end{aligned} \tag{B.37}$$

For ϕ_4 :

$$\begin{aligned}
 C_2^{(A_{30}^{B \rightarrow V}, \phi_4)} &= 2\bar{\sigma} - 2\bar{\sigma}u + u - 1, \\
 C_2^{(A_{30}^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{2}{m_B \bar{\sigma}} (u - 1)(\bar{\sigma} - 3), \\
 C_3^{(A_{30}^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{1}{m_B \bar{\sigma}} (2m_B^2 \bar{\sigma} (2\bar{\sigma}^2 u - 3\bar{\sigma}(u + 1) + 4) + 2m_B m_1 (5 - 4\bar{\sigma}) \bar{\sigma} \\
 &\quad - 2m_1^2 (2\bar{\sigma} + 3)(u - 1) - 2q^2 (2\bar{\sigma} - 3)(u - 1)), \\
 C_3^{(A_{30}^{B \rightarrow V}, \bar{\bar{\phi}}_4)} &= \frac{2}{m_B \bar{\sigma}} (2m_B ((\bar{\sigma} - 6)\bar{\sigma} + 6)(2u - 1) + m_1 (15 - 4\bar{\sigma})), \\
 C_4^{(A_{30}^{B \rightarrow V}, \bar{\bar{\phi}}_4)} &= \frac{6}{m_B \bar{\sigma}} (2m_B^3 (\bar{\sigma} - 2)(\bar{\sigma} - 1)\bar{\sigma}^2 (2u - 1) + m_B^2 m_1 (5 - 4\bar{\sigma}) \bar{\sigma}^2 \\
 &\quad - m_B (2u - 1)(m_1^2 (\bar{\sigma} - 4) + 2q^2 (\bar{\sigma} - 2)(\bar{\sigma} - 1)) \\
 &\quad + m_1 (5m_1^2 + q^2 (4\bar{\sigma} - 5))).
 \end{aligned} \tag{B.38}$$

For ψ_4 :

$$\begin{aligned}
 C_2^{(A_{30}^{B \rightarrow V}, \bar{\psi}_4)} &= \frac{2}{m_B \bar{\sigma}} (1 - 2u)(\bar{\sigma} - 3), \\
 C_3^{(A_{30}^{B \rightarrow V}, \bar{\psi}_4)} &= \frac{2}{m_B \bar{\sigma}} ((2\bar{\sigma} - 3)(2u - 1)(q^2 - m_B^2 \bar{\sigma}^2) + 2m_B m_1 \bar{\sigma} + m_1^2 (2\bar{\sigma} + 3)(2u - 1)).
 \end{aligned} \tag{B.39}$$

For χ_4 :

$$\begin{aligned}
 C_2^{(A_{30}^{B \rightarrow V}, \bar{\chi}_4)} &= \frac{2}{m_B \bar{\sigma}} (\bar{\sigma} - 3), \\
 C_3^{(A_{30}^{B \rightarrow V}, \bar{\chi}_4)} &= -\frac{2}{m_B \bar{\sigma}} (m_B^2 \bar{\sigma} ((3 - 2\bar{\sigma})\bar{\sigma} + 4(\bar{\sigma} - 2)(\bar{\sigma} - 1)u) \\
 &\quad + 4m_B m_1 (3 - 2\bar{\sigma})\bar{\sigma} + m_1^2 (2\bar{\sigma} + 3) + q^2 (2\bar{\sigma} - 3)).
 \end{aligned} \tag{B.40}$$

The coefficients of eq. (2.21), for the three-particle DAs, for $T_1^{B \rightarrow V}$ follow. For ϕ_3 :

$$\begin{aligned}
 C_2^{(T_1^{B \rightarrow V}, \phi_3)} &= \frac{m_1}{m_B} + u\bar{\sigma}, \\
 C_2^{(T_1^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{1}{m_B^2 \bar{\sigma}} (2m_1 - m_B \bar{\sigma} u), \\
 C_3^{(T_1^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{2}{m_B^2 \bar{\sigma}} (m_B^2 \bar{\sigma}^2 + m_1^2 - q^2)(m_B \bar{\sigma} u + m_1), \\
 C_3^{(T_1^{B \rightarrow V}, \bar{\bar{\phi}}_3)} &= 6 \frac{m_1}{m_B}, \\
 C_4^{(T_1^{B \rightarrow V}, \bar{\bar{\phi}}_3)} &= 6 \frac{m_1^2}{m_B} (m_B \bar{\sigma} (2u - 1) + m_1).
 \end{aligned} \tag{B.41}$$

For ϕ_4 :

$$\begin{aligned}
 C_2^{(T_1^{B \rightarrow V}, \phi_4)} &= (u - 1)\bar{\sigma}, \\
 C_2^{(T_1^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{u}{m_B},
 \end{aligned}$$

$$\begin{aligned}
 C_3^{(T_1^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{2}{m_B} (-(m_B \bar{\sigma} + m_1)(m_1 u - m_B \bar{\sigma}(u-1)) - q^2 u + q^2), \\
 C_2^{(T_1^{B \rightarrow V}, \bar{\bar{\phi}}_4)} &= \frac{2}{m_B^2 \bar{\sigma}} (2u - 1), \\
 C_3^{(T_1^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{1}{m_B^2 \bar{\sigma}} (-2(2u-1)(q^2 - m_B^2 \bar{\sigma}^2) - 2m_B m_1 \bar{\sigma} + m_1^2(4-8u)), \\
 C_4^{(T_1^{B \rightarrow V}, \bar{\bar{\phi}}_4)} &= \frac{6m_1}{m_B^2 \bar{\sigma}} (-m_B^3 \bar{\sigma}^3 + m_B q^2 \bar{\sigma} - m_1(2u-1)(m_1^2 - q^2)).
 \end{aligned} \tag{B.42}$$

For ψ_4 :

$$\begin{aligned}
 C_2^{(T_1^{B \rightarrow V}, \bar{\psi}_4)} &= \frac{1}{m_B^2 \bar{\sigma}} (m_B \bar{\sigma}(1-2u) - 2m_1), \\
 C_3^{(T_1^{B \rightarrow V}, \bar{\psi}_4)} &= \frac{2}{m_B^2 \bar{\sigma}} (m_B^3 \bar{\sigma}^3(1-2u) + m_B^2 m_1 \bar{\sigma}^2 + m_B \bar{\sigma}(2u-1)(m_1^2 + q^2) - m_1^3 + m_1 q^2).
 \end{aligned} \tag{B.43}$$

For χ_4 :

$$\begin{aligned}
 C_2^{(T_1^{B \rightarrow V}, \bar{\chi}_4)} &= \frac{1}{m_B^2 \bar{\sigma}} (m_B \bar{\sigma}(1-2u) + 2m_1), \\
 C_3^{(T_1^{B \rightarrow V}, \bar{\chi}_4)} &= 2 \left(\frac{m_1^3 - m_1 q^2}{m_B^2 \bar{\sigma}} - \frac{-2m_1^2 u + m_1^2 + q^2}{m_B} + m_B \bar{\sigma}^2 + m_1 \bar{\sigma} \right).
 \end{aligned} \tag{B.44}$$

The coefficients of eq. (2.21), for the three-particle DAs, for $T_{23A}^{B \rightarrow V}$ follow. For ϕ_3 :

$$\begin{aligned}
 C_2^{(T_{23A}^{B \rightarrow V}, \phi_3)} &= \frac{m_B m_1 - 4q^2 u}{m_B^2} + \bar{\sigma} u, \\
 C_2^{(T_{23A}^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{1}{m_B^2 \bar{\sigma}} (2m_1 - m_B \bar{\sigma} u), \\
 C_3^{(T_{23A}^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{2}{m_B^2 \bar{\sigma}} (m_B^3 \bar{\sigma}^3 u + \bar{\sigma}(m_B u(m_1^2 + 3q^2) + 4m_1 q^2) \\
 &\quad + m_B \bar{\sigma}^2 (m_B m_1 - 4q^2 u) + m_1^3 - m_1 q^2), \\
 C_3^{(T_{23A}^{B \rightarrow V}, \bar{\bar{\phi}}_3)} &= 6 \frac{m_1}{m_B}, \\
 C_4^{(T_{23A}^{B \rightarrow V}, \bar{\bar{\phi}}_3)} &= 6 \frac{m_1}{m_B} (m_B m_1 \bar{\sigma}(2u-1) + m_1^2 + 4q^2(\bar{\sigma}-1)).
 \end{aligned} \tag{B.45}$$

For ϕ_4 :

$$\begin{aligned}
 C_2^{(T_{23A}^{B \rightarrow V}, \phi_4)} &= \bar{\sigma}(u-1), \\
 C_2^{(T_{23A}^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{u}{m_B}, \\
 C_3^{(T_{23A}^{B \rightarrow V}, \bar{\phi}_4)} &= -\frac{2}{m_B} ((m_B \bar{\sigma} + m_1)(m_1 u - m_B \bar{\sigma}(u-1)) + q^2(-2\bar{\sigma} + u + 1)), \\
 C_2^{(T_{23A}^{B \rightarrow V}, \bar{\bar{\phi}}_4)} &= \frac{2}{m_B^2 \bar{\sigma}} (2u-1),
 \end{aligned} \tag{B.46}$$

$$\begin{aligned}
 C_3^{(T_{23A}^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{2}{m_B^2 \bar{\sigma}} ((2u-1)(m_B^2 \bar{\sigma}^2 + q^2(4\bar{\sigma} - 7)) - m_B m_1 \bar{\sigma} + m_1^2(2-4u)), \\
 C_4^{(T_{23A}^{B \rightarrow V}, \bar{\phi}_4)} &= -\frac{6}{m_B^2 \bar{\sigma}} (m_1(m_B^3 \bar{\sigma}^3 - m_B q^2 \bar{\sigma}) + 2q^2(\bar{\sigma} - 1)(2u-1)(q^2 - m_B^2 \bar{\sigma}^2) \\
 &\quad + m_1^4(2u-1) + m_1^2 q^2(2\bar{\sigma} + 1)(2u-1)).
 \end{aligned}$$

For ψ_4 :

$$\begin{aligned}
 C_2^{(T_{23A}^{B \rightarrow V}, \bar{\psi}_4)} &= \frac{1}{m_B^2 \bar{\sigma}} (m_B \bar{\sigma}(1-2u) - 2m_1), \\
 C_3^{(T_{23A}^{B \rightarrow V}, \bar{\psi}_4)} &= \frac{2}{m_B^2 \bar{\sigma}} (m_B^3 \bar{\sigma}^3(1-2u) + m_B^2 m_1 \bar{\sigma}^2 \\
 &\quad + \bar{\sigma}(m_B(2u-1)(m_1^2 + q^2) - 4m_1 q^2) - m_1^3 + m_1 q^2).
 \end{aligned} \tag{B.47}$$

For χ_4 :

$$\begin{aligned}
 C_2^{(T_{23A}^{B \rightarrow V}, \bar{\chi}_4)} &= \frac{1}{m_B^2 \bar{\sigma}} (m_B \bar{\sigma} - 2m_B \bar{\sigma} u + 2m_1), \\
 C_3^{(T_{23A}^{B \rightarrow V}, \bar{\chi}_4)} &= \frac{2}{m_B^2 \bar{\sigma}} (m_B^3 \bar{\sigma}^3 + \bar{\sigma}(2m_B u(m_1^2 + 2q^2) - m_B(m_1^2 + q^2) + 4m_1 q^2) \\
 &\quad + m_B \bar{\sigma}^2(m_B m_1 - 4q^2 u) + m_1^3 - m_1 q^2).
 \end{aligned} \tag{B.48}$$

The coefficients of eq. (2.21), for the three-particle DAs, for $T_{23B}^{B \rightarrow V}$ follow. For ϕ_3 :

$$\begin{aligned}
 C_1^{(T_{23B}^{B \rightarrow V}, \phi_3)} &= -2 \frac{u}{m_B^2 \bar{\sigma}}, \\
 C_2^{(T_{23B}^{B \rightarrow V}, \phi_3)} &= \frac{1}{m_B^2 \bar{\sigma}} (u(m_B^2 \bar{\sigma}(3\bar{\sigma} - 1) + q^2(2 - 4\bar{\sigma})) + m_B m_1 \bar{\sigma} - 2m_1^2 u), \\
 C_2^{(T_{23B}^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{1}{m_B^2 \bar{\sigma}} (m_B(2 - 5\bar{\sigma})u + 6m_1), \\
 C_3^{(T_{23B}^{B \rightarrow V}, \bar{\phi}_3)} &= \frac{2}{m_B^2 \bar{\sigma}} (m_1(q^2(4\bar{\sigma} - 3) - m_B^2 \bar{\sigma}^2) + m_B(\bar{\sigma} - 1)u(3m_B^2 \bar{\sigma}^2 - 4q^2 \bar{\sigma} + q^2) \\
 &\quad - m_B m_1^2(\bar{\sigma} - 1)u + 3m_1^3), \\
 C_3^{(T_{23B}^{B \rightarrow V}, \bar{\bar{\phi}}_3)} &= 6 \frac{m_1}{m_B \bar{\sigma}} (3\bar{\sigma} - 2), \\
 C_4^{(T_{23B}^{B \rightarrow V}, \bar{\bar{\phi}}_3)} &= 6 \frac{m_1}{m_B \bar{\sigma}} ((3\bar{\sigma} - 2)m_1^2 + m_B(2u-1)(\bar{\sigma} - 1)\bar{\sigma} m_1 \\
 &\quad - 2(\bar{\sigma} - 1)(m_B^2 \bar{\sigma}^2 - 2q^2 \bar{\sigma} + q^2)).
 \end{aligned} \tag{B.49}$$

For ϕ_4 :

$$\begin{aligned}
 C_2^{(T_{23B}^{B \rightarrow V}, \phi_4)} &= (\bar{\sigma} - 1)(u - 1), \\
 C_2^{(T_{23B}^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{1}{m_B \bar{\sigma}} (2\bar{\sigma} + (\bar{\sigma} - 2)u), \\
 C_3^{(T_{23B}^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{2}{m_B \bar{\sigma}} ((\bar{\sigma} - 1)(m_B^2 \bar{\sigma}^2(u - 2) + 2q^2 \bar{\sigma} - q^2 u) - m_B m_1(\bar{\sigma} - 1)\bar{\sigma} \\
 &\quad - m_1^2(\bar{\sigma}(u - 1) + u)),
 \end{aligned}$$

$$\begin{aligned}
 C_2^{(T_{23B}^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{6}{m_B^2 \bar{\sigma}^2} (\bar{\sigma} - 1)(2u - 1), \\
 C_3^{(T_{23B}^{B \rightarrow V}, \bar{\phi}_4)} &= \frac{1}{m_B^2 \bar{\sigma}^2} (4(2u - 1)(m_B^2 \bar{\sigma}^3 + q^2(2(\bar{\sigma} - 3)\bar{\sigma} + 3)) \\
 &\quad - 2m_B m_1 (\bar{\sigma} - 3)\bar{\sigma} - 6m_1^2 (\bar{\sigma} + 2)(2u - 1)), \\
 C_4^{(T_{23B}^{B \rightarrow V}, \bar{\phi}_4)} &= -\frac{6}{m_B^2 \bar{\sigma}^2} ((2u - 1)(2\bar{\sigma} + 1)m_1^4 - m_B \bar{\sigma} m_1^3 + (2u - 1)(m_B^2 (1 - 2\bar{\sigma})\bar{\sigma}^2 \\
 &\quad + q^2(2\bar{\sigma}^2 + \bar{\sigma} - 2))m_1^2 + m_B (\bar{\sigma} - 1)\bar{\sigma} (m_B^2 \bar{\sigma}^2 - q^2)m_1 \\
 &\quad + (2u - 1)(\bar{\sigma} - 1)(m_B^2 \bar{\sigma}^2 - q^2)(m_B^2 \bar{\sigma}^2 - 2q^2 \bar{\sigma} + q^2)).
 \end{aligned} \tag{B.50}$$

For ψ_4 :

$$\begin{aligned}
 C_2^{(T_{23B}^{B \rightarrow V}, \bar{\psi}_4)} &= \frac{1}{m_B^2 \bar{\sigma}} (m_B (\bar{\sigma} - 2(\bar{\sigma} - 2)u - 2) - 6m_1), \\
 C_3^{(T_{23B}^{B \rightarrow V}, \bar{\psi}_4)} &= \frac{2}{m_B^2 \bar{\sigma}} (m_B^3 \bar{\sigma}^2 (\bar{\sigma} - 2(\bar{\sigma} - 1)u - 1) + m_B^2 m_1 \bar{\sigma} (3\bar{\sigma} - 2) \\
 &\quad + m_B (2u - 1)(m_1^2 (\bar{\sigma} + 1) + q^2 (\bar{\sigma} - 1)) - 3m_1^3 + m_1 q^2 (3 - 4\bar{\sigma})).
 \end{aligned} \tag{B.51}$$

For χ_4 :

$$\begin{aligned}
 C_2^{(T_{23B}^{B \rightarrow V}, \bar{\chi}_4)} &= \frac{1}{m_B^2 \bar{\sigma}} (m_B (\bar{\sigma} + (4 - 6\bar{\sigma})u - 2) + 6m_1), \\
 C_3^{(T_{23B}^{B \rightarrow V}, \bar{\chi}_4)} &= \frac{2}{m_B^2 \bar{\sigma}} (m_B^3 (\bar{\sigma} - 1)\bar{\sigma}^2 (2u + 1) - m_B^2 m_1 \bar{\sigma}^2 + m_B (m_1^2 (-\bar{\sigma} + 2u - 1) \\
 &\quad - q^2 (\bar{\sigma} - 1)((4\bar{\sigma} - 2)u + 1)) + 3m_1^3 + m_1 q^2 (4\bar{\sigma} - 3)).
 \end{aligned} \tag{B.52}$$

C Plots of the form factors

This appendix is dedicated to illustrate our numerical results for the form factors in relation to previous results obtained from LCSR with B -meson LCDAs [6–8] and to results obtained from LQCD.

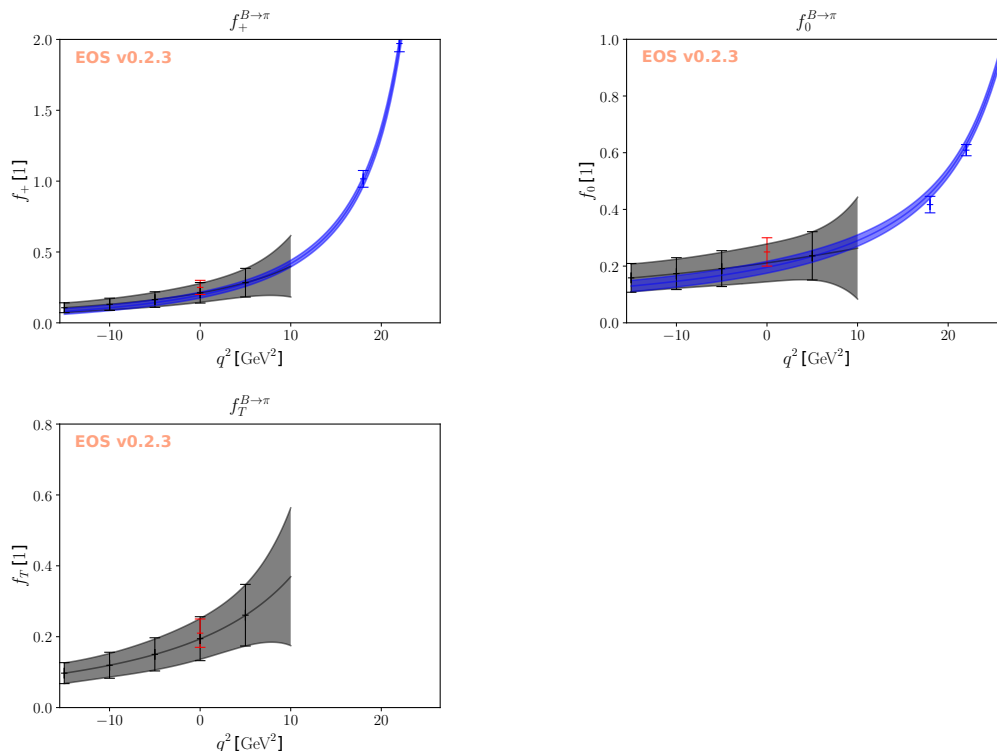


Figure 2. Plots of our results (gray points) and LQCD results from ref. [69] (blue points) for the $B \rightarrow \pi$ form factors. Central values and 68% probability envelopes as functions of q^2 from fits to our results only (gray) and a combination of our results and LQCD results (blue) are shown as well. Previous results from LCSRs using B -LCDAs [7] at $q^2 = 0$ are not used in the fits and shown in red for illustrative purpose only. Solid lines represent the central values, and shaded areas illustrate the 68% probability envelope.

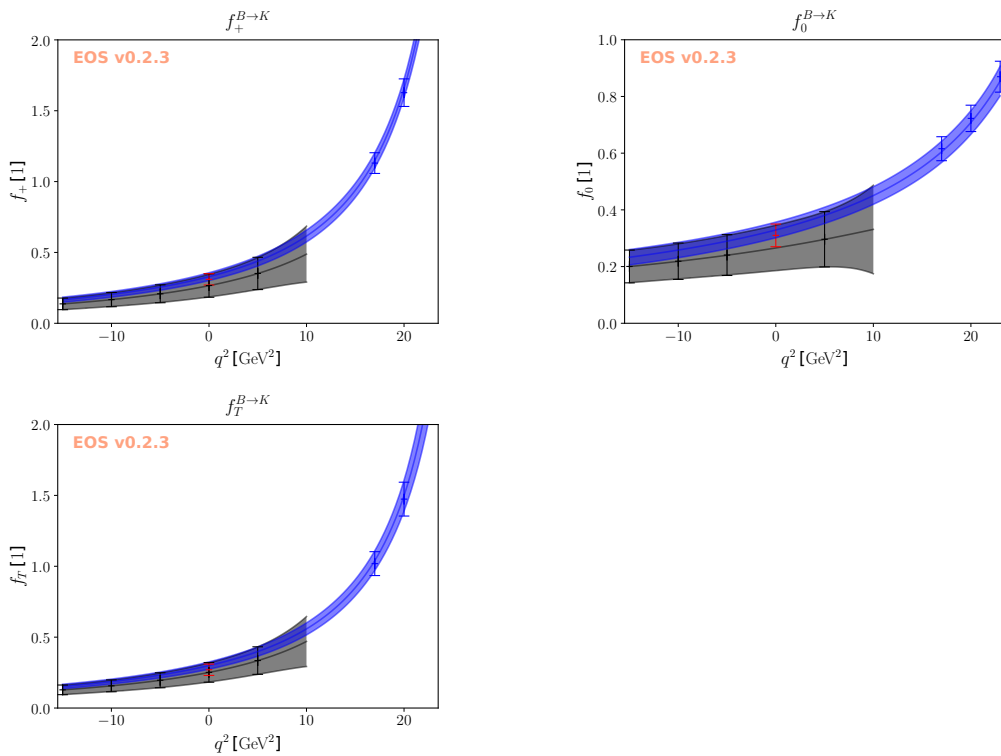


Figure 3. Plot of $B \rightarrow K$ form factors, LQCD results from ref. [64]. For a description see figure 2.

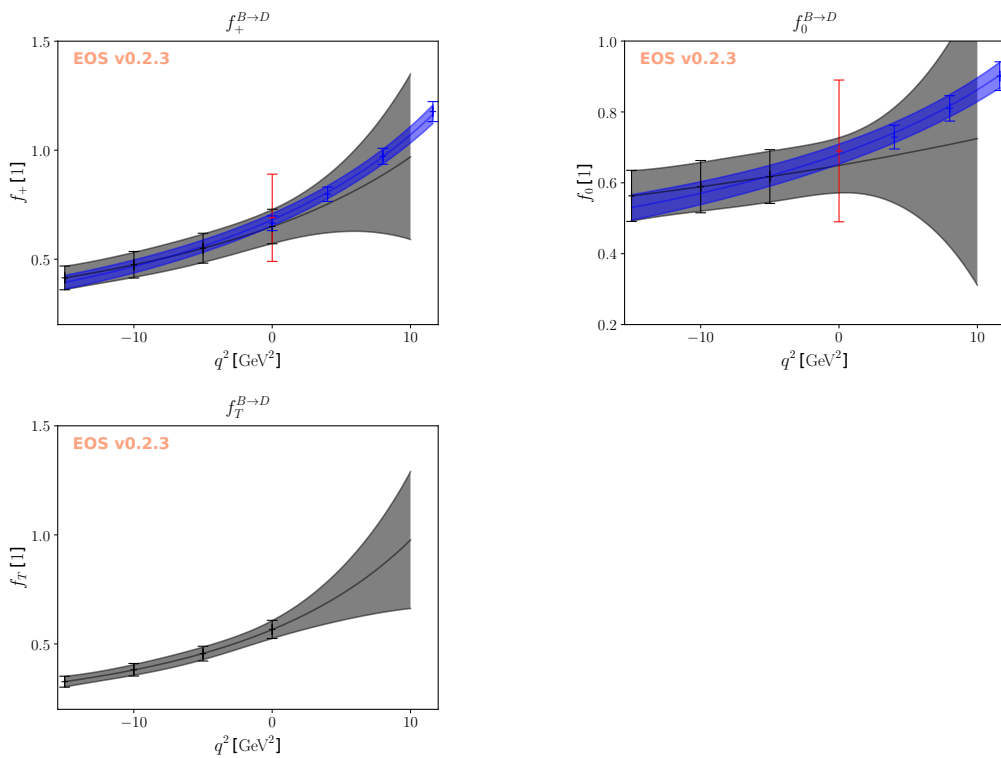


Figure 4. Plot of $B \rightarrow \bar{D}$ form factors, LQCD results from ref. [68]. For a description see figure 2.

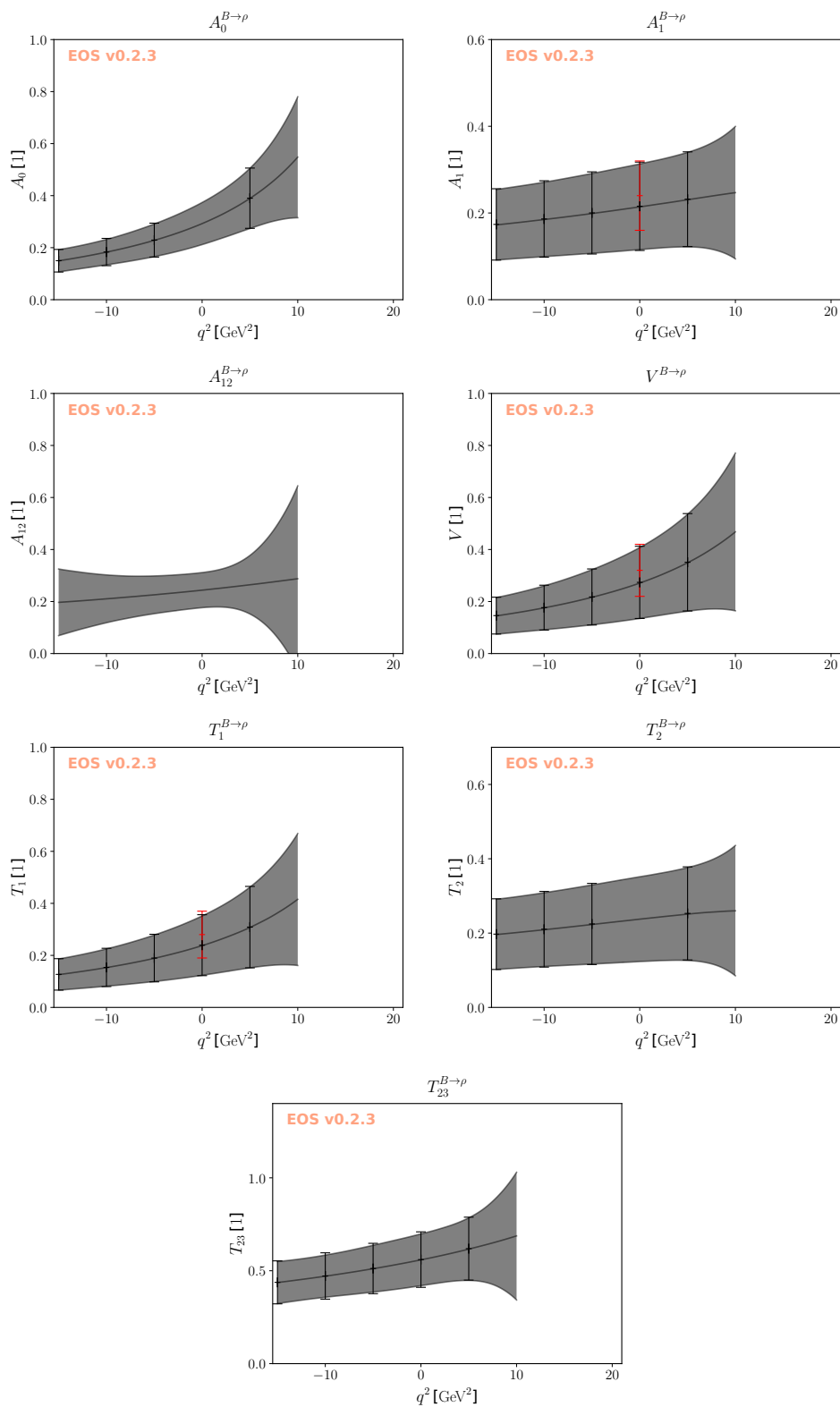


Figure 5. Plot of $B \rightarrow \rho$ form factors, no LQCD results available. For a description see figure 2.

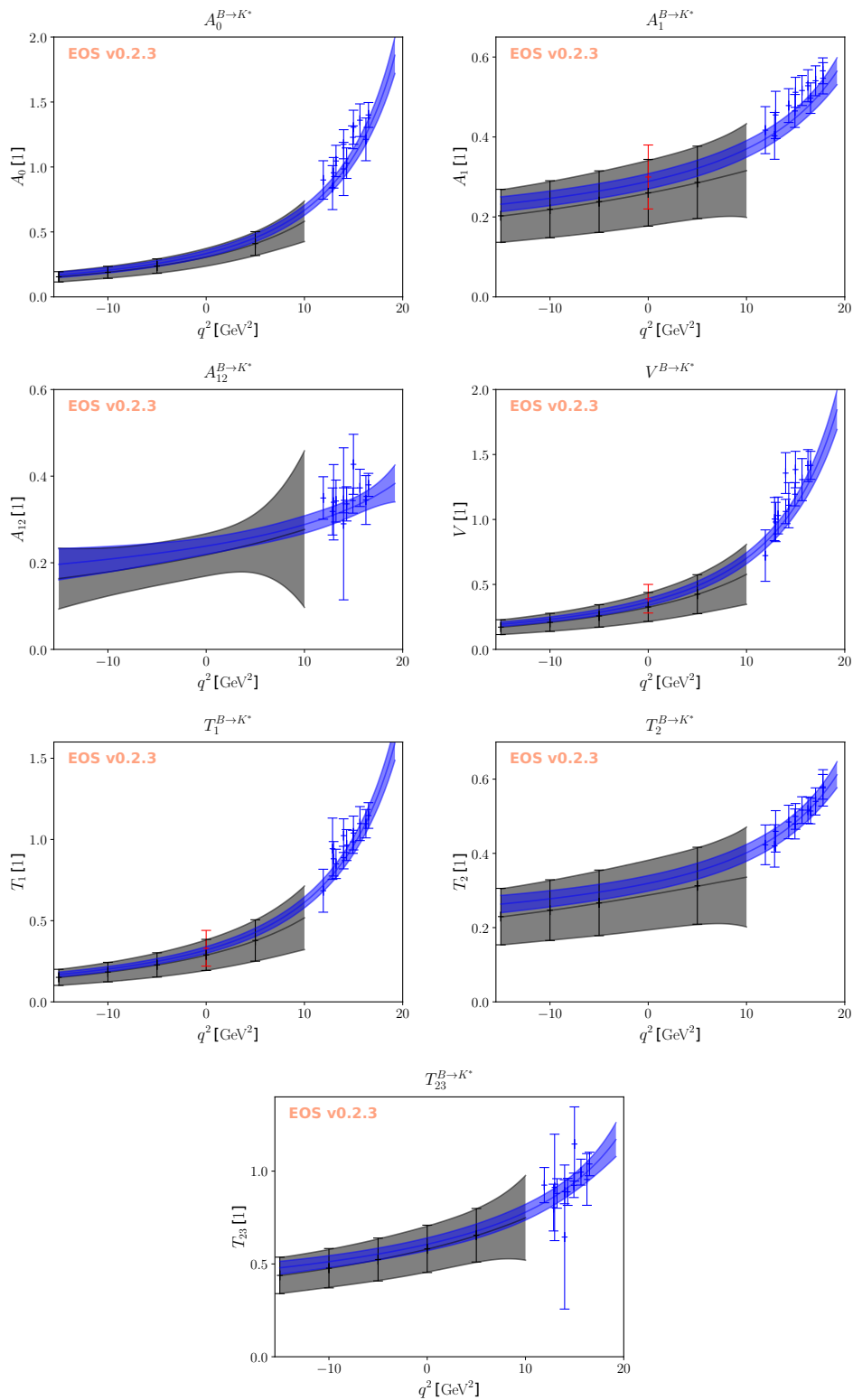


Figure 6. Plot of $B \rightarrow K^*$ form factors, LQCD results from refs. [65, 67]. For a description see figure 2.

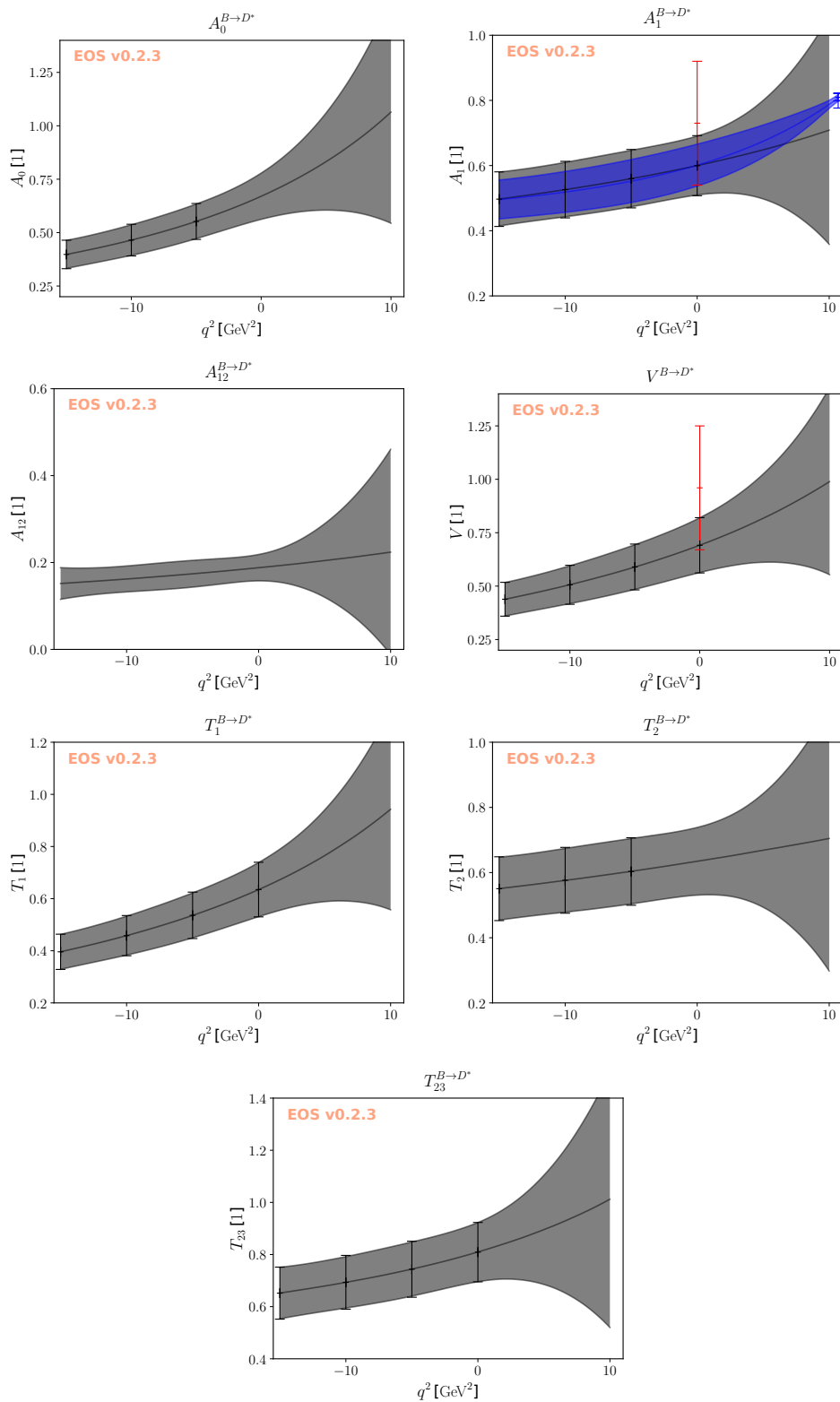


Figure 7. Plot of $B \rightarrow \bar{D}^*$ form factors, LQCD results from refs. [66, 70]. For a description see figure 2.

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