Back Stepping Sliding Mode Control of Blood Glucose for Type I Diabetes

N. Tadrisi Parsa, A. R. Vali, R. Ghasemi

Abstract—Diabetes is a growing health problem in worldwide. Especially, the patients with Type 1 diabetes need strict glycemic control because they have deficiency of insulin production. This paper attempts to control blood glucose based on body mathematical body model. The Bergman minimal mathematical model is used to develop the nonlinear controller. A novel back-stepping based sliding mode control (B-SMC) strategy is proposed as a solution that guarantees practical tracking of a desired glucose concentration. In order to show the performance of the proposed design, it is compared with conventional linear and fuzzy controllers which have been done in previous researches. The numerical simulation result shows the advantages of sliding mode back stepping controller design to linear and fuzzy controllers.

Keywords—Back stepping, Bergman Model, Nonlinear control, Sliding mode control.

I. INTRODUCTION

DIABETES is one of the most important chronic diseases. It is a metabolic illness created by inadequate generation or shortage counteraction to insulin. According to the World Health Organization more than 180 million people suffer from diabetes worldwide, and it is estimated that this number will double in the next 30 years [1], [2]. It is also predicted that approximately 9% of all deaths worldwide are caused by diabetes [3]. Inside the human body, there are plenty of natural feedback loops in charge of maintaining the homeostasis within the body. The failure or malfunction of one of these loops can induce severe illnesses with short- and long-term complications. In this context, insulin is a hormone responsible for regulating the glucose uptake by the body cells. If the blood glucose level not properly controlled, diabetes can results side difficulties like nerve hurt, brain hurt, mutilation and finally death. Diabetes dependent side difficulties are a pandemic around the world which has high medical, economic and social costs [4].There are two types of diabetes: type I and type II. Type I diabetes or insulin dependent and type 2 diabetes or non-insulin dependent [5].

In Type 1 diabetes, the pancreas does not release insulin since pancreatic beta cells are destroyed. So the effect of hormone glucagon raising blood glucose level cannot be suppressed by insulin. Thus, the patients need exogenous insulin injection to regulate their glucose concentration to a normal level. There are two situations depending on glucose concentration, namely, hyperglycemia and hypoglycemia.

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Hyperglycemia occurs if the glucose concentration is much higher than normal level (higher than $8 mmol/L$) [6]. Hypoglycemia, on the other hand, occurs when the glucose concentration is lower than normal level (lower than $3 mmol/L$) [6]. Both, hyperglycemia and hypoglycemia can affect the patient's health and life.

For a diabetic person it is necessary to perform the procedures of blood glucose regulation manually. Therefore, a system that automatically monitors and controls the blood glucose level of a diabetic individual permits the patient to have more participation in the ordinary daily activities with risk reduction of long-term side effects.

In recent years, many studies have been made for intelligent control of blood glucose. Among them we focus the 3rd order minimal model of Bergman [7].

Several methods have been previously employed to design the feedback controller for insulin delivery, such as classical methods like proportional-integral-derivatives (PID) [8], [9] and pole placement [10], which require a linearized model for the design, as well as model predictive control (MPC) [11], [12]. In [13] a PID controller based on BP neural networks proposed in order to reduce the time of lowering blood glucose. In [14], the parameters of Hammerstein controller were optimized in order to minimizing the time that takes for blood glucose to come back to its basal level. Also there are some efforts to use non-model based controller such as fuzzy controllers [15], [16]. In [15], a closed-loop control system according to fuzzy logic control introduced and the performance of this controller is tested on three different patients. Reference [16] tried to tune the PD fuzzy controller with PSO algorithm. These fuzzy controllers just control the glucose concentration, and suffer from lack of insulin and pump control.

Modeling the patients and the size and structure of the models, as well as nonlinear model versus linear models have been addressed in several works. Adaptation of the model is discussed as an improvement to the performance [17] that can result in imposing some limitations on the model parameters. On the other hand, if linear models are employed for the patients, control algorithms like H_∞ control technique can guarantee some levels of performance but full robustness cannot be achieved via this algorithm. Some applications of this method can be found in [18]-[20] introduced a nonlinear robust controller for blood glucose control, in this controller only the glucose level is assumed to be available for control development and other states such as insulin concentration and plasma insulin concentration are assumed to be unavailable for measurement. In [21] a higher-order sliding mode control

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techniques, in specific prescribed convergence law, quasi-continuous and super-twisting control algorithms, are used to robustly stabilize the glucose concentration level of a diabetic patient in presence of the parameter variations and meal disturbance. Normalizing the equations for using sliding mode control is the main disadvantage of this method, since some uncertainties derivative will appear in equation that cannot be determined in reality. In this paper to tackle this problem we introduce a back stepping sliding mode control which prepare the ability to control each parameter separately. In this method the need of normalizing is also eliminated. Although the accuracy of this controller is not comparable with sliding mode control, but being applicable, the less of complexity, and the domain of desired glucose concentration made this method totally preferable than sliding mode control.

The paper is organized as follows: in Section II we show a brief overview of mathematical model of Bergman body model, whereas the control designs procedure is illustrated in Section III. To proof the advantages of introduced method, numerical simulation is done in Section IV. Finally the conclusions are provided in Section V.

II. Bergman Insulin-Glucose Regulation Model

Many efforts have been made to model diabetes [22]-[24]. One of the pioneers in this task was Dr. Richard Bergman, who developed the so-called 'Minimal Model.' Bergman minimal model, which is a commonly referenced model in the literature, approximates the dynamic response of a diabetic patient's blood glucose concentration to the insulin injection. Compared with the other relevant studies in the literature, main advantages of the Bergman minimal model are the number of parameters is minimum and it describes the interaction between main components such as insulin and glucose concentrations without getting into biological complexity. Therefore in this paper the Bergman nonlinear model is considered [7]:

$$\dot{G}(t) = -p_1[G(t) - G_b] - X(t)G(t) + D(t)$$
$$\dot{X}(t) = -p_2 X(t) + p_3[I(t) - I_b]$$
$$\dot{I}(t) = -n[I(t) - I_b] + \gamma[G(t) - H]^+ t + u(t)$$
(1)

where $G(t) \in R$ is the plasma glucose concentration in $mmol/L$ (or mg/dL) (above basal value), $X(t) \in R$ is proportional to the insulin concentration in the remote compartment in mU/L, $I(t) \in R$ is the plasma insulin concentration in mU/L (above basal value), and $u(t) \in R$ is injected insulin rate in $mU/\min$. G_b is the basal pre-injection level of glucose (mg/dl), I_b is the basal pre-injection level of insulin $\mu U/ml$, p_1 is the insulin independent rate constant of glucose uptake in muscles and liver (1/min), p_2 is the rate for decrease in tissue glucose uptake ability ($1/\min$), p_3 is the insulin-dependent increase in glucose uptake ability in tissue per unit of insulin concentration above the basal level [$(\mu U/ml)/\min^2$], n is the first-order decay rate for insulin in

blood (1/min), h is the threshold value of glucose above which the pancreatic β cells release insulin (mg/dl), g is the rate of the pancreatic β cells' release of insulin after the glucose injection with glucose concentration above the threshold [$(\mu U/ml)/\min^2/(mg/dl)$]. $D(t)$ shows the rate at which glucose is absorbed to the blood from the intestine, following food intake. Since in diabetic patients, the normal insulin regulatory system does not exist, this glucose absorption is considered as a disturbance for the system dynamics presented in (1). This disturbance can be modeled by a decaying exponential function of the following form [25]:

$$D(t) = A\exp(-Bt),\ B > 0$$
(2)

where t is in (min) and $D(t)$ is in ($mg/dl/min$). We modeled the pump as a first order delay:

$$\dot{u}(t) = \frac{1}{\tau}[-u(t) + w(t)]$$
(3)

where $w(t)$ insulin rate command in pump input, and τ is pump time constant. As it can be seen in (1, 3) this model is a four variable nonlinear system and isn't normalized. Also the uncertainties have an affine relation with main input. The system output is $G(t)$ and the control input is pump input ($w(t)$).

III. Control Design

As we mentioned previously, in this paper due to nonlinearities and uncertainties of system, we used B-SMC. Thus, four decouple dynamic model are assumed. The first dynamic part considered as follows:

$$\dot{G}(t) = -p_1[G(t) - G_b] - X(t)G(t) + D(t)$$
(4)

where $X(t)$ is considered as a pseudo control input which should control $G(t)$ for tracking desired G_b in presence of $D(t)$ uncertainties. Thus according to Sliding Mode Control (SMC) theory, the sliding variable introduced as follows:

$$S_1(t) = G(t) - G_b$$
(5)

According to SMC theory, when there is no uncertainties in system and all of the paths are in sliding variable, $X_{eq}(t)$ is obtained by establishing $\dot{S}_1 = 0$, which is obtained as follows [26]:

$$\dot{S}_1 = \dot{G}(t) = -p_1[G(t) - G_b] - X_{eq}(t)G(t) = 0 \Rightarrow$$
$$X(t)_{eq} = \frac{1}{G(t)}[-p_1[G(t) - G_b]]$$
(6)

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In order to consider the uncertainties, (7) is added, and it is guarantee the stability of sliding variable when the paths are deviated from the sliding surface.

$$X(t) = X(t)_{eq} - ksign(S_1) \quad (7)$$

In this part for considering the uncertainties, the Lyapunov function assumed as follows:

$$V = \frac{1}{2} S_1^2 \quad (8)$$

According to the Lyapunov theorem, to prove the stability of the control design, V derivative should be negative definite that. For ensuring the limited time stability, these conditions must be maintained:

$$\dot{V} \le -cV^\beta \qquad \forall V(0) \ge 0, c > 0, \beta \in (0,1) \quad (9)$$

The rising time for achieving sliding surface will be adjusted as follows:

$$t_r \le \frac{V(0)^{1-\beta}}{c(1-\beta)} \quad (10)$$

By establishing (10), after passing t_r second, the control variables will get to the $S = 0$ and considering (7) the rising time can be adjusted by changing the amounts of c and β. For some certain cases, to establish the sliding condition, for $\beta = 0.5$ the amount of k is determined by using (4), (5) and (7) as follows:

$$\dot{V} = S_1 \dot{S}_1 =$$
$$S_1[-p_1[G(t) - G_b] - \left[X(t)_{eq} - ksign(S_1)\right]G(t) + D(t)]$$
$$S_1[ksign(S_1)G(t) + D(t)] \le -\frac{c}{\sqrt{2}}|S_1| \Rightarrow \quad (11)$$
$$-k \ge \frac{1}{G(t)}\left[D(t)\frac{S_1}{|S_1|} + \frac{c}{\sqrt{2}}\right]$$

By choosing $-k$ as the largest possible amount in (11), the sliding condition (9) will always be maintained, so we have:

$$-k = \frac{1}{G(t)}\left[\alpha + \frac{c}{\sqrt{2}}\right] \quad (12)$$

where α is the upper bound of uncertainties in the system. Finally the desired value of $X(t)$ is considered as an input of $G(t)$ pseudo controller which has to track the desired value of G_b in the presence of $D(t)$ uncertainty. By using (5)-(7) and (12), the desired $X(t)$ which is denoted by $X(t)_d$ can be obtained as:

$$X(t)_d = \frac{1}{G(t)}\left[-p_1[G(t) - G_b]\right] + \quad (13)$$
$$\frac{1}{G(t)}\left(\alpha + \frac{c}{\sqrt{2}}\right)sign\left(G(t) - G_b\right)$$

The chattering phenomenon is generated because of sign function in (13). To alleviate this problem alternative function can be used such as $sat(S/\varphi)$, and $tanh(\theta S)$. By using these functions, ϕ, and θ parameters will determine the boundary layer width. Now the second part of the system model is defined as follows:

$$\dot{X}(t) = -p_2 X(t) + p_3[I(t) - I_b] \quad (14)$$

In this part $I(t)$ is assumed as a virtual control input and is designed such that the output X(t) follows the desired $X(t)_d$ that is designed in (13). Due to the sliding mode control theory, the second sliding variable is defined by using the system error as follows:

$$S_2(t) = X(t) - X(t)_d \quad (15)$$

By using sliding mode control theory, the desired value of $I(t)$ will result as follows:

$$I(t)_d = I(t)_{eq} - k_2 sign(S_2) \quad (16)$$

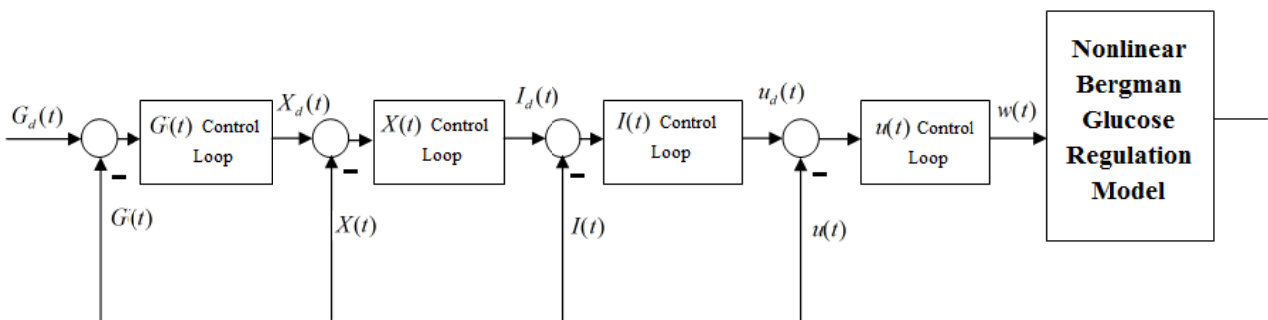where $I(t)_{eq}$ is designed for removing certain sentences and k_2 is established for sliding condition.

Fig. 1 Overall Structure of Blood Glucose Back stepping Sliding Mode Control Design

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$$\dot{I}(t) = -n[I(t) - I_b] + \gamma[G(t) - H]^+ t + u(t) \qquad (17)$$

The sliding variables are defined in (18):

$$S(t)_3 = I(t) - I(t)_d$$
$$S(t)_4 = u(t) - u(t)_d \qquad (18)$$

The $u(t)$ pseudo controller is designed by (19):

$$u(t)_d = u(t)_{eq} - k_3 sign(S_3) \qquad (19)$$

Finally, by using sliding mode control theory $w(t)$ is designed as follows:

$$w(t) = w(t)_{eq} - k_4 sign(S_4) \qquad (20)$$

With replacement (13), (16) and (19) in (20), the reference control input can be obtained. By exerting this signal to the system, the system state variables will converge quickly without the normalization of system uncertainties. The overall diagram of designed controller is shown on Fig. 1.

IV. NUMERICAL SIMULATION

In this paper we used nonlinear Bergman glucose regulation model for designing nonlinear B-SMC. The control system designed in this paper will be next used as an autonomous blood glucose controller for type I diabetes patient. In this section we evaluate the proposed design with numerical simulation, and compared them with linear control and the fuzzy-PSO method which is introduced on [16]. The specification of Bergman parameters is available on Table I.

TABLE I
BODY MODEL SPECIFICATION

Parameter	Healthy	Patient
p_1	0.0317	0
p_2	0.0123	0.0123
p_3	8.2×10^{-8}	8.2×10^{-8}
γ	6.5×10^{-5}	0
n	0.2659	0.2659
h	79.0353	0
G_b	70	70
I_b	7	7
G_0	140	140
I_0	20	20

The simulation results are depicted on Figs. 2-5. As it can be seen in Fig. 2 the proposed controller design (BSMC) successfully controlled the Blood Glucose and decreased the glucose from the critical area, although it has not enough accuracy, but according to the glucose sensitivity it is acceptable. Also as it can clearly be seen, the BSMC controller is more accurate than PID and Fuzzy controller.
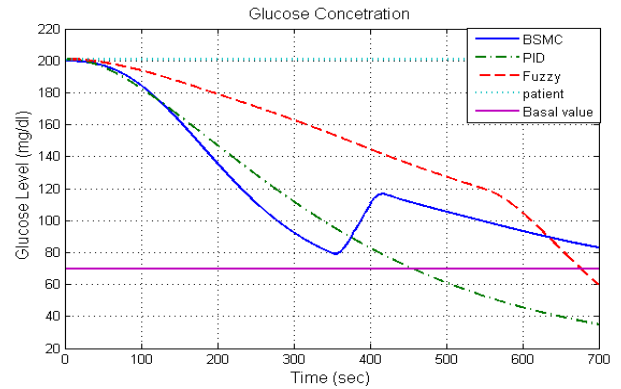

Fig. 2 Blood Glucose Level in various control type

Fig. 3 shows the X(t) variation, as it can be seen, the proposed controller has better performance in this case either. Figs. 4 and 5 show the insulin profile and the control function for designed controllers, it can be seen the BSMC controller has the lowest variable in comparison of other method. It is demonstrated that the insulin concentration must not exceed than 100 mU, that only the BSMC controller fulfilled this condition.
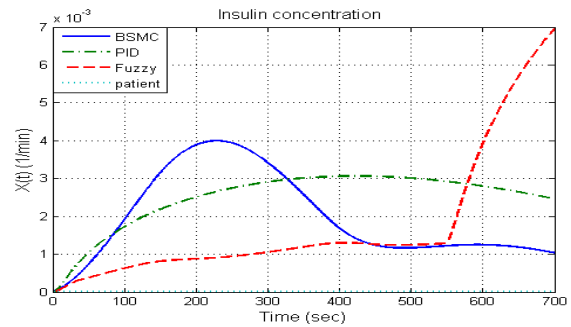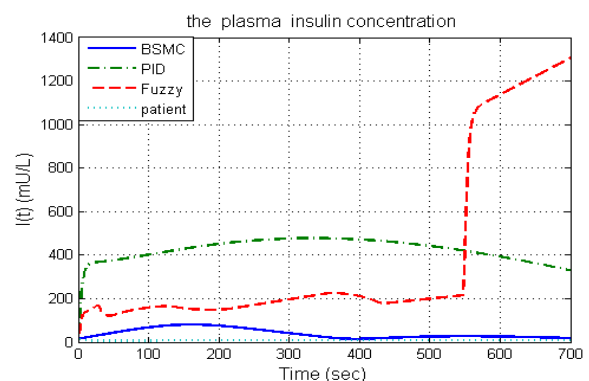

Fig. 3 Insulin Concentration in various control type

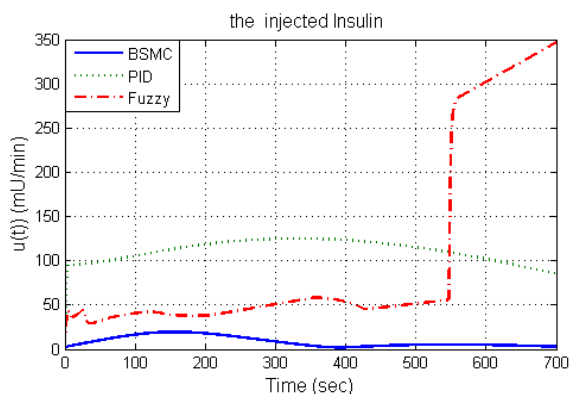Fig. 4 Plasma Insulin Injection (I(t))

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Fig. 5 Control Input (Injected Insulin)

V. CONCLUSION

The diabetes management as one of the challenging control problems in human regulatory systems has been discussed. The treatment of the disease via robust feedback control design has been considered. Stabilization of blood glucose has been discussed in presence of the external disturbances such as food intake. With this aim, a back stepping design in four separated part with sliding mode control is done. The designed controller is checked and confirmed by computer simulations. To show the advantages of introduced design, it is compared with conventional PID and fuzzy controllers. The introduced controller has complete advantage over fuzzy and PID controllers. Also the ability for being applicable, the less of complexity, and the domain of desired glucose concentration made this introduced method totally preferable than sliding mode control.

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