Backward Monte Carlo Calculations of the Polarization Characteristics of the Radiation Emerging from Spherical-Shell Atmospheres

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A Monte Carlo procedure, designated as FLASH, was developed for use in computing the intensity and polarization of the radiation emerging from spherical-shell atmospheres and is especially useful for investigating the sunlit sky at twilight time. The procedure utilizes the backward Monte Carlo method and is capable of computing the Stokes parameters for discrete directions at the receiver position. Both molecular and aerosol scattering are taken into account as well as ozone, aerosol, water vapor, and carbon dioxide absorption within the atmosphere. The curvature of the light path due to the changing index of refraction with altitude is taken into account. Some comparisons are made between FLASH calculations for a pure Rayleigh atmosphere, a combined Rayleigh and aerosol atmosphere, and calculations reported by other authors for plane-parallel atmospheres. The comparisons show that the FLASH calculations for spherical-shell atmospheres are in good agreement with those for plane-parallel atmospheres within the range of zenith angles for which no differences could be attributed to the difference in the geometries of the two atmospheric models.

Introduction

During the past decade several Monte Carlo codes have been developed for use in studying the effects of multiple scattering of light in plane-parallel atmospheres. Skumanich and Bhattachargie[1] used this method to calculate time-dependent light scattering by a cloud layer for a pulsed point source. Wells and Collins[2] developed the LITE codes to study problems in atmospheric optics; these codes were later modified to include the effects of water vapor and carbon dioxide absorption[3] and to allow for molecular and aerosol polarization.[4] Plass and Kattawar[5] have also reported Monte Carlo calculations that were obtained from a computer code that is similar to that reported in Ref. 2. Subsequently Kattawar and Plass[6] extended their code to allow for molecular and aerosol polarization. Kastner[7] has reported Monte Carlo calculations of resonance-line polarization of sodium D and hydrogen Lyman-α in the upper atmosphere.

The Monte Carlo method is especially well adapted to situations involving atmospheres whose properties change with altitude and for phase functions that are highly anisotropic. The restriction to plane-parallel geometry is not a limitation on the method but is done for mathematical simplicity and to enable the computer runs to be of moderate duration. However, it is necessary to resort to spherical geometry in order to solve certain problems, e.g., the illumination of the sky at twilight or during a total solar eclipse or the extension of the cusps of Venus at inferior conjunction.

One of the earliest reported applications of the Monte Carlo method to problems involving the scattering of sunlight in a spherical-shell nonhomogeneous atmosphere is that reported by Marchuk and Mikhailov.[8] These authors used the Monte Carlo method to estimate the intensity of atmospheric scattered sunlight at a satellite positioned above the atmosphere. Later, Collins and Wells[9] developed the FLASH Monte Carlo procedure that treated the scattering of sunlight in a spherical-shell nonhomogeneous atmosphere. Their procedure uses the backward Monte Carlo method and treats the effects of polarization on the scattered intensity and the absorption of light by water vapor, carbon dioxide, ozone, and aerosol particles in the atmosphere. FLASH calculations of light scattering under twilight conditions are reported in Ref. 10. Subsequently, Kattawar et al.[11] have modified their Monte Carlo procedure to treat the reflection of light by a symmetric planetary atmosphere.

The application of the Monte Carlo method to spherical-shell atmospheric models is not as simple as the application to plane-parallel models because of the interdependence of the position of entrance into the atmosphere and the angle of incidence at that position. The

net flux incident at the top of a spherical-shell atmosphere is not uniformly distributed. Therefore, in a forward Monte Carlo approach, it is necessary to sample either a position of entrance for each photon history or an angle of incidence for the photon history. This introduces a problem that is not easily solved, viz., determining the bounds of the surface area within which the incident radiation significantly contributes to the scattered intensity at a given receiver location. A related question is whether some importance functions should be developed to concentrate the sampling of the entrance positions of the photon histories in those areas in which the incident radiation is most likely to scatter into the receiver. Utilization of a backward Monte Carlo approach eliminates the necessity of having to solve the above problems.

This paper describes a backward Monte Carlo procedure, designated as FLASH, which computes the Stokes parameters I, Q, U, and V for a spherical-shell atmosphere. Comparisons are shown between FLASH calculations for a Rayleigh atmosphere and a combined Rayleigh and aerosol atmosphere and data computed by other authors for plane-parallel atmospheres.

Backward Monte Carlo

We shall adopt the notation of Chandrasekhar[12] and characterize a beam of polarized light by the Stokes vector (I_l, I_r, U, V), where l and r refer to two mutually orthogonal directions in the plane transverse to the oncoming light. It is convenient to define the propagation plane as the plane containing the earth's center and the direction of a light ray. The line of intersection of the propagation plane with the transverse plane at a given point determines the direction l, and the intensity observed for an analyzer oriented in this direction is I_l; I_r is then the intensity for the analyzer oriented at right angles to the l direction.

The sunlight incident on the top of the atmosphere is assumed to be unpolarized. The Stokes vector I_0 used in FLASH to describe the characteristics of the source radiation is defined by

$$I_0 = (I_l = 0.5, I_r = 0.5, U = 0.0, V = 0.0). \quad (1)$$

We shall define the integral over solid angle at the receiver position of the Stokes vector $I(\mathbf{R}, \mathbf{\Omega})$, which describes the characteristics of the polarized scattered light at position $\mathbf{R}$ in the direction $\mathbf{\Omega}$, by

$$II(\mathbf{R}) = \int_{4\pi} I(\mathbf{R}, \mathbf{\Omega}) d\Omega.$$

Let us write in symbolic integral form the transport equation giving the integrated Stokes vector, $II_n(\mathbf{R})$, for photons that have undergone exactly n collisions within the atmosphere, for the case where the source is a plane monodirectional, monochromatic source emitting one photon per unit area in the direction $\mathbf{\Omega}_0$.

$$II_n(\mathbf{R}) = \int_A \int_{r_1} \int_{\Omega_1} \cdots \int_{r_{n-1}} \int_{\Omega_{n-1}} \int_{r_n} |\mathbf{R} - \mathbf{R}_n|^{-2}$$
$$\times \sigma_{T^n} P(\mathbf{\Omega}_n \to \mathbf{\Omega}) T_g(\mathbf{R}_n \to \mathbf{R}) T(\mathbf{R}_n \to \mathbf{R}) P(\mathbf{\Omega}_{n-1} \to \mathbf{\Omega}_n)$$
$$T_g(\mathbf{R}_{n-1} \to \mathbf{R}_n) T(\mathbf{R}_{n-1} \to \mathbf{R}_n) \cdots P(\mathbf{\Omega}_1 \to \mathbf{\Omega}_2) T_g(\mathbf{R}_1 \to \mathbf{R}_2)$$
$$T(\mathbf{R}_1 \to \mathbf{R}_2) P(\mathbf{\Omega}_0 \to \mathbf{\Omega}_1) T_g(\mathbf{R}_0 \to \mathbf{R}_1) T(\mathbf{R}_0 \to \mathbf{R}_1)$$
$$I_0 dr_n d\Omega_{n-1} dr_{n-1} \cdots d\Omega_2 dr_2 d\Omega_1 dr_1 dA. \quad (2)$$

A is the area of the source plane, $\mathbf{R}_0$ is a position on the source plane, and $\mathbf{\Omega}_0$ is the direction of emission of the source. The position of the nth collision is designated by $\mathbf{R}_n$, and the distance between the $(n-1)$th collision and the nth collision, $|\mathbf{R}_{n-1} - \mathbf{R}_n|$, is r_n. The receiver is positioned at $\mathbf{R}$, and the direction of incidence of the scattered radiation at the receiver is $\mathbf{\Omega}$. The parameter σ_T is the pseudoextinction coefficient of the atmosphere. It represents all the interactions undergone by photons in the atmosphere except gaseous absorption. The scalar function $T_g(\mathbf{R}_{n-1} \to \mathbf{R}_n)$ describes the absorption of photons by gaseous absorption along the distance between the $(n-1)$th and the nth collision, and the scalar function $T(\mathbf{R}_{n-1} \to \mathbf{R}_n)$ describes the attenuation of the photons along the distance between the $(n-1)$th and nth collision that results from scattering interactions and absorption interactions other than gaseous absorption. $P(\mathbf{\Omega}_{n-1} \to \mathbf{\Omega}_n)$ is the phase matrix for polarized scattering interactions. For atmospheres in which aerosol particle and Rayleigh scattering occur, the scattering matrix is given by

$$S(\mathbf{\Omega}_{n-1} \to \mathbf{\Omega}_n) = \frac{\sigma_S}{\sigma_T}$$
$$\times \left[\frac{\sigma_P}{\sigma_S} S_P(\mathbf{\Omega}_{n-1} \to \mathbf{\Omega}_n) + \frac{\sigma_R}{\sigma_S} S_R(\mathbf{\Omega}_{n-1} \to \mathbf{\Omega}_n) \right], \quad (3)$$

where σ_S is the sum of the aerosol scattering coefficient, σ_P, and the Rayleigh scattering coefficient, σ_R and $S_P(\mathbf{\Omega}_{n-1} \to \mathbf{\Omega}_n)$, and $S_R(\mathbf{\Omega}_{n-1} \to \mathbf{\Omega}_n)$ are the aerosol particle and Rayleigh scattering matrices, respectively. The difference between σ_T and σ_S represents the absorption coefficient for all absorption processes other than gaseous absorption.

The integrated Stokes's vector at the receiver is given by

$$II(\mathbf{R}) = \sum_{n=1}^{\infty} II_n(\mathbf{R}). \quad (4)$$

Equation (2) can be evaluated by the use of Monte Carlo techniques by sampling a source position $\mathbf{R}_0$ from the density function for the source area, A, and n distances between collision, r_n, from the density function:

$$\sigma_T T(\mathbf{R}_{n-1} \to \mathbf{R}_n) = \sigma_T \exp(-\sigma_T |\mathbf{R}_{n-1} - \mathbf{R}_n|).$$

The type of scattering interaction at each of the n collisions can be determined at random from a knowledge of the Rayleigh and aerosol particle scattering coefficients. The directions after each of the first $(n-1)$ collisions can then be selected from either the Rayleigh or aerosol particle phase matrix, depending on the type of scattering event selected for that collision. The direction from the nth collision to the receiver position is fixed

once the nth collision position is selected. An estimate of the integrated Stokes vector, $II_n(\mathbf{R})$, is then given by

$$EST1 = \pi\rho_{max}^2|\mathbf{R} - \mathbf{R}_n|^{-2}\left(\frac{\sigma_S}{\sigma_T}\right)^n \exp(-\sigma_T|\mathbf{R} - \mathbf{R}_n|)$$

$$\times T_g(\mathbf{R}_n \to \mathbf{R})\left[\prod_{i=1}^{n} T_g(\mathbf{R}_{i-1} \to \mathbf{R}_i)\right] P'(\mathbf{\Omega}_n \to \mathbf{\Omega})I_0, \quad (5)$$

where the phase matrix P' is taken as either P_P or P_R depending on the type of scattering event selected for the nth collision, and ρ_{max} is the largest radius on the source plane for which any significant contribution to $II_n(\mathbf{R})$ can be expected.

The Stokes vector, $I(\mathbf{R},\mathbf{\Omega})$, characterizing the polarization for scattered radiation at the receiver as a function of direction can be obtained by accumulating the estimates for each of the polarization parameters, I_l, I_r, U,V, given by Eq. (5) into a set of solid angles defined about the receiver position and then dividing the accumulated values by the size of the solid angles and the number of histories.

Equation (2) can be transformed into another integral equation in which the variables of integration defining the source area, A, and the distance to the first collision, r_1, are replaced with the variables Ω and r defining the direction of incidence at the receiver and the distance from the nth collision position to the receiver position. Applying such a transformation to Eq. (2) results in the following equation:

$$II_n(\mathbf{R}) = \int_{r_2}\int_{\Omega_2} \cdots \int_{r_n}\int_{\Omega_n}\int_r\int_\Omega \sigma_T{}^n P(\mathbf{\Omega}_n \to \mathbf{\Omega})$$

$$\times T(\mathbf{R}_n \to \mathbf{R})T_g(\mathbf{R}_n \to \mathbf{R}) \cdot P(\mathbf{\Omega}_{n-1} \to \mathbf{\Omega}_n)T(\mathbf{R}_{n-1} \to \mathbf{R}_n)$$

$$\times T_g(\mathbf{R}_{n-1} \to \mathbf{R}_n)\cdots P(\mathbf{\Omega}_1 \to \mathbf{\Omega}_2) \cdot T(\mathbf{R}_1 \to \mathbf{R}_2)$$

$$\times T_g(\mathbf{R}_1 \to \mathbf{R}_2)P(\mathbf{\Omega}_0 \to \mathbf{\Omega}_1)T(\mathbf{R}_0 \to \mathbf{R}_1)T_g(\mathbf{R}_0 \to \mathbf{R}_1)I_0$$

$$\cdot d\Omega dr dr_n d\Omega_n dr_{n-1}d\Omega_{n-1}\cdots dr_2 d\Omega_2. \quad (6)$$

Equation (6) can be evaluated by Monte Carlo techniques by sampling (1) a direction of incidence at the receiver, $\mathbf{\Omega}$, from an isotropic distribution, (2) a distance r from the nth collision to the receiver from the density $\sigma_T T(\mathbf{R}_n \to \mathbf{R})$, (3) $n - 1$ distances between collisions, r_n, from the density $\sigma_T T(\mathbf{R}_{n-1} \to \mathbf{R}_n)$, (4) the type of scattering event for each collision, and (5) $n - 1$ directions after collision from either P_R or P_P depending on the type of scattering event selected for each collision and then evaluating the estimating function

$$EST2 = 4\pi\left(\frac{\sigma_S}{\sigma_T}\right)^n T_g(\mathbf{R}_n \to \mathbf{R})\left[\prod_{i=1}^{n} T_g(\mathbf{R}_{i-1} \to \mathbf{R}_i)\right]$$

$$\times P'(\mathbf{\Omega}_0 \to \mathbf{\Omega}_1) \exp(-\sigma|\mathbf{R}_0 - \mathbf{R}_1|)I_0 \quad (7)$$

for the sampled value. The value of $\mathbf{\Omega}_1$ is determined by a knowledge of $\mathbf{\Omega}_0$, $\mathbf{R}_1$, and $\mathbf{\Omega}_2$.

The phase matrix $P(\mathbf{\Omega}_{n-1} \to \mathbf{\Omega}_n)$ is a bivariate distribution in the polar and azimuthal scattering angles. Since it is not possible in the backward Monte Carlo method to sample $\mathbf{\Omega}_n$ directly from this distribution, a method similar to that later used by Kattawar and Plass[6] was developed for use in the FLASH program in which the polar and azimuthal scattering angles are selected from a biased distribution. The Stokes vector is then divided by this distribution function. A discussion of the sampling technique is given in the next section.

The Stokes vector for the scattered radiation that reaches the receiver in the solid angle Ω' after exactly n collisions can be obtained from Eq. (6) by changing the limits of integration over Ω to correspond to the limits of the solid angle Ω'. When the limits for θ and ϕ that define Ω are changed in Eq. (6) to correspond to the limits of a given solid angle interval (θ_1,θ_2 and ϕ_1,ϕ_2), the normalized density functions for θ and ϕ are

$$\sin\theta/(\cos\theta_1 - \cos\theta_2) \quad (8)$$

and

$$1/(\phi_2 - \phi_1), \quad (9)$$

respectively. The estimating function for Eq. (6) becomes

$$EST3 = (EST2/4\pi)(\cos\theta_1 - \cos\theta_2)(\phi_2 - \phi_1). \quad (10)$$

An estimate of the Stokes vector for the scattered radiation in the solid angle defined by $\theta_1,\theta_2,\phi_1,\phi_1$ is obtained by dividing the estimate defined in Eq. (10) by the size of the solid angle. If the receiver solid angle is defined by a discrete direction, θ,ϕ, the estimating function

$$EST4 = (EST2/4\pi) \quad (11)$$

gives an estimate of the Stokes vector for the scattered radiation at the receiver in the discrete direction θ,ϕ for photons that have undergone exactly n collisions. The method described above for evaluating Eq. (6) is denoted as the backward Monte Carlo method since the sampling of the photon trajectory for each history is started at the receiver position, and the histories are traced backward along the random path.

From an examination of Eq. (2) it is seen that prior knowledge of the maximum size of the radius of the source area is required in order to be able to select positions of emission on the source plane. Also, the estimating function of Eq. (2) as given by Eq. (5) has infinite variance since $|\mathbf{R} - \mathbf{R}_n|^{-2}$ increases without limit as $|\mathbf{R} - \mathbf{R}_n|$ approaches zero. It is not feasible to place limits on the integration variables in Eq. (2) so as to compute the Stokes vector for the scattered radiation for a finite solid angle or a discrete direction at the receiver.

The use of the backward Monte Carlo method to evaluate the scattered intensity removes the necessity of determining the maximum radius of the source area that needs to be considered and results in the use of an estimating function that has finite variance. In addition, the backward Monte Carlo method allows limits to be placed on the directions at which the photons enter the receiver. The capability of computing the scattered intensity for a discrete direction at the receiver position eliminates the argument advanced by Dave[13] and Hansen[14] against the use of finite solid angle intervals in Monte Carlo calculations for recording the angular distribution of the polarization parameters at the
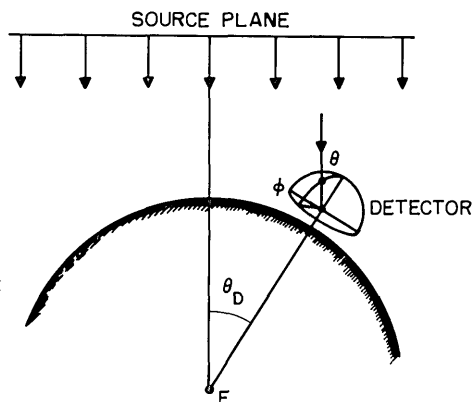
Fig. 1. Geometry for spherical-shell atmosphere.

receiver. Although the random distribution of the collision centers generated by the forward Monte Carlo programs developed by the authors,[4] Plass and Kattawar,[5] and Kastner[7] for plane-parallel atmospheres, do not allow one to control the direction of incidence at the receiver position, it is possible to control the direction of incidence at the receiver position in forward Monte Carlo programs for plane-parallel problems by using statistical estimation methods. Use of the backward Monte Carlo approach in a spherical shell geometry allows all the photon histories to be concentrated in a given field of view of particular interest at the receiver position. Thus, no computer time is wasted in computing the polarization parameters outside that field of view.

Flash Program

A computer program employing the backward Monte Carlo approach has been developed to aid in light transport studies involving spherical-shell geometries. The program evaluates the four Stokes parameters, I, Q, U, V, characterizing the scattered radiation incident at a receiver position within each of a set of finite solid angle intervals (view angles); the size and direction of the view angles are arbitrary and may be limited to a set of discrete directions if desired. The program assumes that a plane-parallel unpolarized source of monochromatic radiation is incident at the top of a spherical-shell atmosphere (Fig. 1). The receiver polar position angle, θ_D, between the radial through the receiver position and the radial that is normal to the source plane, is equivalent to the zenith angle of incidence for a plane-parallel atmosphere.

The FLASH program treats the scattering of monochromatic radiation by molecules and aerosol particles and the absorption of monochromatic radiation by aerosols, ozone, water vapor, and carbon dioxide in the atmosphere. The spherical-shell atmosphere is divided into concentric spherical-shell layers, each having different scattering and absorption coefficients. This method for describing an atmosphere allows for arbitrary and individual variation of each of the scattering and absorption coefficients with altitude, which is essential for twilight studies. In addition to the variation of the aerosol scattering coefficient with altitude, the

aerosol phase matrix may also be varied with altitude to define the different angular scattering distributions expected in haze layers, clouds, and clear regions of the atmosphere.

Another feature of the program that enhances its application to twilight conditions is the consideration of the bending of the photon paths by the change in the atmospheric index of refraction with altitude. The index of refraction is specified for each of the concentric spherical-shell layers, and the direction of the photon's path is modified using Snell's formula at each crossing of a boundary between the spherical-shell layers.

The polarization parameters at the receivers are computed as a function of the ground albedo where the albedo may vary from 0.0 to 1.0. The ground surface may be defined in FLASH as a Lambert type reflecting surface or alternatively by an arbitrary reflection distribution.

Water Vapor and CO_2 Absorption

Processes such as aerosol scattering and absorption, Rayleigh scattering, and ozone absorption are treated separately in FLASH from processes such as water vapor and carbon dioxide absorption which are dependent on the amount of the gas present and the atmospheric temperature and pressure along the photon's path. An altitude dependent pseudoextinction coefficient, σ_T, which is taken to be the sum of the aerosol and Rayleigh scattering coefficients and the ozone and aerosol absorption coefficients, is used to compute optical distances between collisions. The attenuation between collisions in each of the concentric spherical layers through which the photon path passes that results from water vapor and carbon dioxide absorption is used to account for these processes. The transmission through the water vapor contained in each of the atmospheric layers is based on the use of a transmission equation developed by Howard et al.[15] The transmission through the CO_2 in each of the atmospheric layers is computed with a transmission equation developed by Green and Griggs.[16]

Selection of Scattering Event

At each collision in the atmosphere a scattering event is forced to occur. The photon weight before collision is multiplied by the ratio of the scattering coefficient to the pseudoextinction coefficient at the collision altitude to correct for the bias introduced by forcing all collisions to be scattering collisions. The type of scattering collision, Rayleigh or aerosol particle, is determined at random from a knowledge of the ratio of the Rayleigh scattering coefficient to the sum of the Rayleigh and aerosol scattering coefficients at the collision altitude.

Optical Distance Between Collisions

In the FLASH program optical distances are defined as the integral of the pseudoextinction coefficient, σ_T, over the physical distance, taking into account the fact that the pseudoextinction coefficient varies with altitude.

Two methods are used to sample the distances between the receiver and the nth collision point and the successive collisions that occur along the photon's backward path. If the photon's extended path intersects the ground surface, the optical distance is sampled from the distribution,

$$\tau = -\ln(1 - RN), \qquad (12)$$

where τ is the optical distance between collisions and RN is a random number sampled from a uniform distribution between 0.0 and 1.0. If the optical distance sampled is greater than the optical distance along the path to the ground surface, a reflection is forced at the point where the photon path intersects the ground surface. The weight parameter is then multiplied by the albedo for the ground surface. If the sampled pathlength is less than the optical distance to the ground surface, a collision is taken to occur at the point on the path where the optical distance equals the sampled optical distance. The photon is forced to scatter at this point, and the photon weight is multiplied by the ratio of the scattering-to-pseudoextinction coefficient evaluated for the collision altitude.

If the photon's extended path intersects the upper bound of the spherical shell atmosphere before intersecting the ground surface, a collision is forced before the photon escapes the atmosphere by selecting the optical distance to the next collision from the truncated exponential distribution,

$$\tau = -\ln\{1 - RN[1 - \exp(-\tau_{\max})]\}, \qquad (13)$$

where $\tau_{\max}$ is the optical distance along the path to the upper bound of the atmosphere. When the collision is forced to occur before the photon escapes the atmosphere, the photon weight before collision, W_{n-1}, is adjusted after the collision to remove the bias introduced by forcing the collision with the expression

$$W_n = W_{n-1}[1 - \exp(-\tau_{\max})]. \qquad (14)$$

A program option is available for systematically sampling the positions of the first order collisions and for using the combined Rayleigh and aerosol phase function to determine the Stokes parameters at the receiver resulting from first order scattering. Use of this option minimizes the statistical fluctuations in the FLASH calculations since the Stokes' parameters for single order scattering are calculated in a straightforward numerical method.

Scattering of Polarized Light

The bivariate phase matrix $P(\Omega_{n-1} \to \Omega_n)$ for polarized light is defined by the relation[12]

$$P(\Omega_{n-1} \to \Omega_n) = R(\phi_{n,2})S(\psi_n)R(\phi_{n,1}), \qquad (15)$$

where $R(\phi_{n,1})$ is the matrix for rotation of the Stokes vector from the plane of propagation before collision into the scattering plane, $S(\psi_n)$ is the scattering matrix, and $R(\phi_{n,2})$ is the matrix for rotation of the Stokes vector from the scattering plane to the plane of propagation after scattering. The scattering matrix $S(\psi)$ is given by

$$S(\psi) = (\sigma_S/\sigma_T)[(\sigma_R/\sigma_S)S_R(\psi) + (\sigma_P/\sigma_S)S_P(\psi)] \qquad (16)$$

for atmospheric scattering and by

$$S(\psi) = S_G(\psi) \qquad (17)$$

for ground reflection. The matrices $S_R(\psi)$, $S_P(\psi)$, and $S_G(\psi)$ are the scattering matrices for Rayleigh scattering, aerosol particle scattering, and ground reflection, respectively. For Rayleigh scattering, $S_R(\psi)$ is given by

$$S_R(\psi) = \frac{3}{8\pi}\begin{vmatrix} \cos^2\psi & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\psi & 0 \\ 0 & 0 & 0 & \cos\psi \end{vmatrix}. \qquad (18)$$

For aerosol particle scattering, $S_P(\psi)$ is given by

$$S_P(\psi) = \begin{vmatrix} P2(\psi) & 0 & 0 & 0 \\ 0 & P1(\psi) & 0 & 0 \\ 0 & 0 & P3(\psi) & -P4(\psi) \\ 0 & 0 & P4(\psi) & P3(\psi) \end{vmatrix}, \qquad (19)$$

where $P1(\psi)$, $P2(\psi)$, $P3(\psi)$, and $P4(\psi)$ are the four elements of the normalized aerosol phase matrix computed with Mie theory. When light reflects from a Lambert surface, $S_G(\psi)$ is defined by

$$S_G(\psi) = \frac{\cos\psi}{\pi}\begin{vmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}. \qquad (20)$$

The selection of a polar and azimuthal scattering angle from the phase matrix $P(\Omega_{n-1} \to \Omega_n)$ is accomplished with the use of a biased sampling scheme. Random values of ψ_n and α_n, the polar and azimuthal scattering angles at the nth collision, are sampled from the density $F(\psi,\alpha)$, where $F(\psi,\alpha)$ is the phase function for unpolarized light. The Stokes vector after scattering is then given by

$$I_n = C_n R(\phi_{n,2})S(\psi_n)R(\phi_{n,1})I_{n-1}, \qquad (21)$$

where

$$\phi_{n,2} = \alpha_n,$$
$$C_n = 1/F(\psi_n,\alpha_n), \qquad (22)$$

and I_{n-1} is the Stokes vector in the reference plane before the nth collision. The rotation angle $\phi_{n,1}$ is calculated from a knowledge of the direction of propagation in the reference plane after scattering and the scattering angles ψ_n and α_n.

The density function $F(\psi,\alpha)$ is defined in FLASH for Rayleigh scattering by the Rayleigh phase function

$$F(\psi,\alpha) = (3/16\pi)(1 + \cos^2\psi) \qquad (23)$$

for unpolarized light. For aerosol scattering $F(\psi,\alpha)$ is defined by the normalized aerosol particle phase function for unpolarized light as given by

$$F(\psi,\alpha) = [P1(\psi) + P2(\psi)]/2, \qquad (24)$$

and for reflection by a Lambert surface $F(\psi,\alpha)$ is defined by

$$F(\psi,\alpha) = \cos\psi/\pi. \qquad (25)$$

The correction factor C_n for removing the bias introduced in the sampling scheme outlined above is

$$C_n = 1/F(\psi_n, \alpha_n), \quad (26)$$

where ψ_n and α_n are the polar and azimuthal scattering angles selected at random from the density function $F(\psi, \alpha)$. The estimating function used in the FLASH procedure to determine the Stokes vector for the jth history in the discrete direction Ω at the receiver for photons that have undergone exactly n collisions is given by

$$I_{j,n} = \left[\prod_{i=1}^{n} W_i \right] T_\sigma(\mathbf{R}_n \rightarrow \mathbf{R}) \left[\prod_{i=1}^{n} T_\sigma(\mathbf{R}_{n-1} \rightarrow \mathbf{R}_n) \right]$$
$$\times \exp(-\sigma_T |\mathbf{R}_0 - \mathbf{R}_1|) \left[\prod_{i=2}^{n} C_i \right] \cdot P(\Omega_{n-1} \rightarrow \Omega)$$
$$\times P(\Omega_{n-2} \rightarrow \Omega_{n-1}) \cdots P(\Omega_1 \rightarrow \Omega_2) P(\Omega_0 \rightarrow \Omega_1) I_0. \quad (27)$$

The parameter W_n is a weight correction factor to account for the use of biased sampling in selecting the optical distance to collision and to account for forcing the interaction at the collision to be a scattering event. When the photon's direction of motion after the $(n-1)$th collision is upward, the parameter W_n is given by

$$W_n = (\sigma_S/\sigma_T)[1 - \exp(-\tau_{\max})],$$

where $\tau_{\max}$ is the optical distance along the path to the upper bound of the atmosphere. When the photon direction of motion after the $(n-1)$th collision is downward and the nth collision position is within the atmosphere, W_n is given by $W_n = \sigma_S/\sigma_T$. If the photon hits the ground surface, $W_n = A$, where A is the magnitude of the ground albedo. The phase matrix $P(\Omega_{n-1} \rightarrow \Omega_n)$ used in Eq. (27) for each of the n orders of scattering is the phase matrix for either Rayleigh scattering, aerosol scattering, or ground reflection, depending on which type of scattering event was selected for the nth collision. It should be noted that the parameters giving the collision positions as computed for the nth collision for $n \geq 1$ become the parameters for the $(n + 1)$th collision when the above estimator is used to compute $I_{j,n+1}$. This results from the fact that after each estimate is made a new position for the first collision and a new direction of motion before the second collision are selected at random. The position of the first collision for the previous estimate becomes the position of the second collision. The position of the second collision for the previous estimate becomes the collision position for the third collision etc.

All the quantities in Eq. (27) are scalars except the matrix product

$$I = P(\Omega_{n-1} \rightarrow \Omega) P(\Omega_{n-2} \rightarrow \Omega_{n-1}) \cdots P(\Omega_1 \rightarrow \Omega_2)$$
$$\times P(\Omega_0 \rightarrow \Omega_1) I_0, \quad (28)$$

which is a vector. The product vector is obtained in FLASH by multiplying the P matrices in Eq. (28) from the left to the right. The matrix resulting from the product of the P matrices is then multiplied by the vector I_0.

The estimator functions for both the forward and backward Monte Carlo methods when the density function $F(\psi, \alpha)$ is used to select the polar and azimuthal scattering angles at each collision will both contain the matrix product given by Eq. (28). From an examination of Eq. (28) it is seen that the order in which the polar and azimuthal scattering angles were selected is not important as long as Eq. (28) is evaluated in the manner described above. This results from the fact that the matrix product is associative.

When the Stokes vector is computed by FLASH for single scattering and the option for systematically sampling of the first order collision positions is used, the estimator for the jth history is

$$I_{j,1} = L\sigma_S \exp[-\sigma_T(|\mathbf{R} - \mathbf{R}_1| + |\mathbf{R}_0 - \mathbf{R}_1|)]$$
$$\times T_\sigma(\mathbf{R}_0 \rightarrow \mathbf{R}_1) T_\sigma(\mathbf{R}_1 \rightarrow \mathbf{R}) P(\Omega_0 \rightarrow \Omega) I_0, \quad (29)$$

where L is the distance through the atmosphere along the direction Ω from the receiver position and $P(\Omega_0 \rightarrow \Omega)$ is defined by Eq. (15). Distances r between the first collision and the receiver are selected systematically from the density L^{-1}.

The Stokes vector for the scattered radiation in the direction Ω at the receiver position is obtained from the estimates [Eq. (27)] by use of the relation

$$\mathbf{I}(I_l, I_r, U, V) = \frac{1}{j} \sum_j \sum_n \mathbf{I}_{j,n}(I_l, I_r, U, V). \quad (30)$$

The Stokes's parameters I and Q are then given by

$$I = I_l + I_r, \quad (31)$$

$$Q = I_r - I_l. \quad (32)$$

The degree of polarization (%) is computed with the expression

$$\delta = 100(Q^2 + U^2 + V^2)^{\frac{1}{2}}/I, \quad (33)$$

and the orientation of the plane of polarization is computed with the expression

$$\chi = 0.5 \tan^{-1}(U/-Q). \quad (34)$$

It should be noted that the above expression for δ differs from the expression

$$\delta = 100Q/I, \quad (35)$$

as used for certain purposes by Plass and Kattawar.[17] As pointed out by Dave,[13] serious errors can result if one confuses the degree of polarization given by the use of Eq. (33) with that given by the use of Eq. (35) when the direction of propagation is not parallel or perpendicular to the plane containing the direction of the scattered radiation. At receiver azimuthal angles of $\phi = 30°$ and $150°$, for example, the values of Q and U are of the same magnitude (see results reported in a later section of this paper). Use of Eq. (33) would therefore give values of the degree of polarization that are approximately 40% higher than those obtained using Eq. (35).
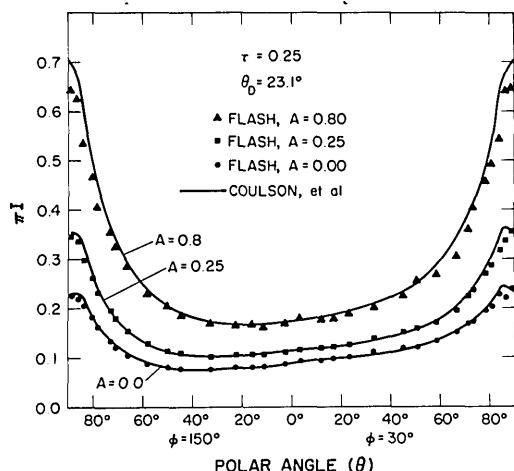
Fig. 2. Scattered intensity transmitted through a Rayleigh atmosphere with an optical thickness of 0.25 as a function of the receiver zenith angle: $\theta_D = 23.1°$, $\phi = 30°$ and $150°$.

Program Efficiency

The FLASH program is written in FORTRAN IV and has been compiled and executed on the CDC-6600 computer. The execution time is highly dependent upon the number of spherical-shell atmospheric layers used to define the atmosphere and on the total number of photon histories run. For the homogeneous Rayleigh atmosphere with an optical thickness of 0.25, calculations of the polarization parameters for sixty-four different receiver directions of view at each of two receiver positions were performed in approximately 20 min of CDC-6600 computer time. Each of the problems run to calculate the transmitted or reflected polarization parameters were for 12,800 histories. This represented 200 histories per receiver view angle. (One history provides an estimate of the polarization parameters incident at a given direction for all receiver positions being considered.) The polarization parameters for up to ten ground albedos may be computed simultaneously with effectively no increase in computer time.

Calculations for a Rayleigh Atmosphere

Several calculations of the polarization parameters for the scattered radiation transmitted and reflected from a spherical-shell Rayleigh atmosphere have been made with the FLASH program. Comparison of the results of these calculations with those obtained with plane-parallel atmospheric models[18,19] reveals the ability of the program to calculate accurately the transmitted and reflected polarization parameters and indicates the differences one would expect to find between calculations for these two atmospheric models.

Receivers were placed at the bottom and top of an homogeneous molecular atmosphere having an optical thickness of $\tau = 0.25$. The ground surface was assumed to be a Lambert reflector. Calculations were performed for receiver positions of $\theta_D = 23.1°$ and $84.3°$ (receiver position in the spherical-shell atmo-

spheric model corresponds to zenith angle of incidence in the plane-parallel models). The scattered polarization parameters were computed as a function of the receiver zenith angle for receiver azimuthal directions of $\phi = 0°$, $30°$, $150°$, and $180°$, and the ground albedo A. The azimuthal angle toward the incident radiation was taken to be $0°$. Two hundred photon histories were run for each receiver view angle.

The polarization parameters I, Q, and U, as plotted in the following figures, are defined having a dimension of intensity or radiance (photons cm^{-2} sr^{-1}) with the incident flux density taken as one (photon cm^{-2}) normal to the direction of incidence. The calculated quantities are given as a function of the receiver zenith angle, θ, and the receiver azimuthal angle, ϕ. The $0°$ azimuth direction is contained in the plane of the incident radiation. The zenith angle for the transmitted radiation varies between $0°$ and $90°$. For the reflected radiation, the receiver zenith angle varies between $90°$ and $180°$.

Intensity Calculations

Calculations of the receiver zenith angle distribution of the transmitted scattered intensities in the $30°$ and $150°$ azimuthal planes for several values of the ground albedo are compared in Figs. 2 and 3 with data for a plane-parallel Rayleigh atmosphere by Coulson et al.[18] for incident angles of $\theta_D = 23.1°$ and $\theta_D = 84.3°$, respectively. The transmitted scattered intensities in the $0°$ and $180°$ azimuthal plane for ground albedos of 0.0 and 0.8 are compared in Fig. 4 with the data computed by Coulson et al. for an incident zenith angle of $84.3°$. For comparison purposes, smooth curves have been drawn through Coulson's et al. data for a plane-parallel atmosphere, while the calculations for the spherical-shell geometry are shown as point data. The agreement with the data for a plane-parallel atmosphere is excellent when $\theta_D = 23.1°$, and indicates that,
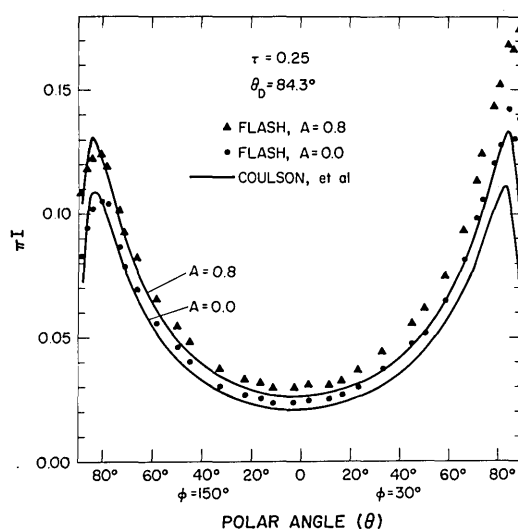

Fig. 3. Scattered intensity transmitted through a Rayleigh atmosphere with an optical thickness of 0.25 as a function of the receiver zenith angle: $\theta_D = 84.3°$, $\phi = 30°$ and $150°$.
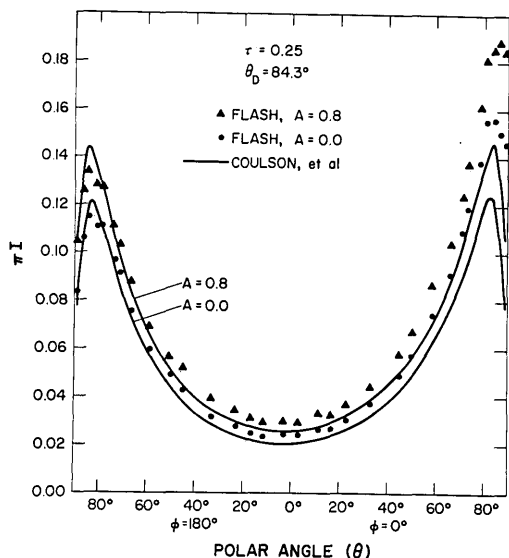
Fig. 4. Scattered intensity transmitted through a Rayleigh atmosphere with an optical thickness of 0.25 as a function of the receiver zenith angle: $\theta_D = 84.3°$, $\phi = 0°$ and $180°$.
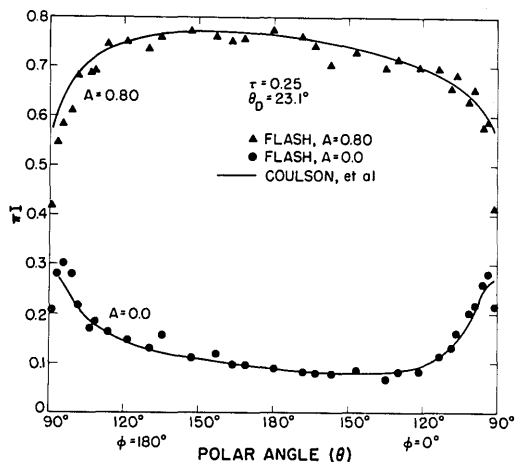

Fig. 5. Intensity reflected from a Rayleigh atmosphere with an optical thickness of 0.25 as a function of the receiver zenith angle: $\theta_D = 23.1°$, $\phi = 0°$ and $180°$.

except for receiver zenith angles close to 90°, there is very little difference between the scattered intensities transmitted through the plane-parallel and spherical-shell atmospheres for that angle of incidence.

Good agreement between the transmitted scattered intensities for the plane-parallel and the spherical-shell atmospheric models have been obtained for other azimuthal planes and other values of θ_D less than 70°.

A difference in the transmitted scattered intensities computed for the plane-parallel and spherical-shell atmospheric models is noted in Figs. 3 and 4, for radiation incident at $\theta_D = 84.3°$. The scattered intensities transmitted through the spherical shell atmosphere are higher at all receiver zenith angles, except for those greater than 70° in the 150° and 180° azimuthal planes. As seen in Fig. 4, the most significant differences between the results for the two atmospheric models occur

at receiver zenith angles between 80° and 90° in the 0° azimuthal plane. The scattered intensities computed for these angles in the spherical-shell atmosphere exceed those computed by Coulson et al. for the plane-parallel atmosphere by more than 40%. Bearing in mind the differences in the plane-parallel and spherical-shell geometries, one would expect the differences in the transmitted scattered intensities to be larger in the 0° azimuthal plane when the receiver zenith angle approaches 90°.

The receiver zenith angle distribution of the intensity reflected in the 0°, 30°, 150°, and 180° azimuthal planes from 0.25 optically thick homogeneous plane-parallel and spherical-shell Rayleigh atmospheres for ground albedos of 0.0 and 0.8 are compared in Figs. 5–7 for
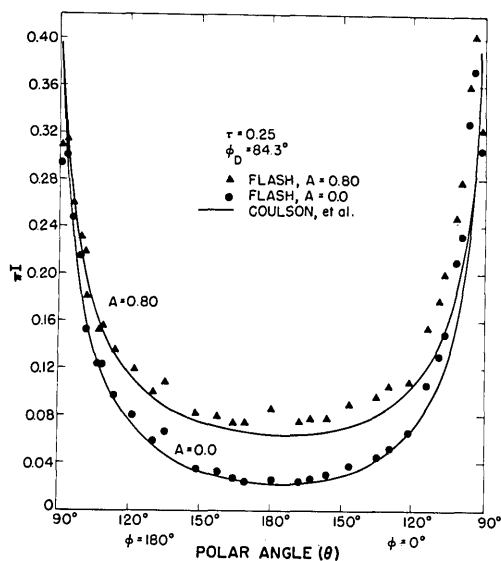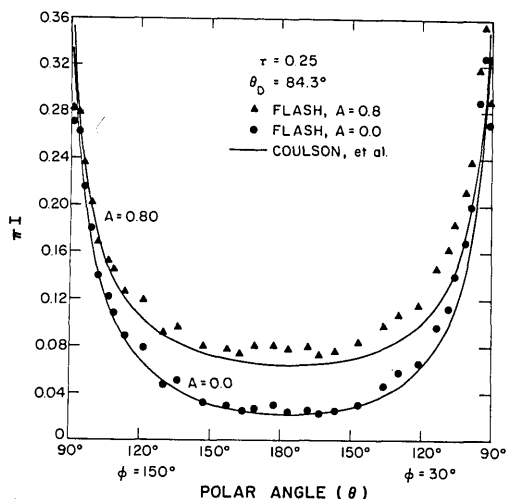

Fig. 6. Intensity reflected from a Rayleigh atmosphere with an optical thickness of 0.25 as a function of the receiver zenith angle: $\theta_D = 84.3°$, $\phi = 0°$ and $180°$.

Fig. 7. Intensity reflected from a Rayleigh atmosphere with an optical thickness of 0.25 as a function of the receiver zenith angle: $\theta_D = 84.3°$, $\phi = 30°$ and $150°$.
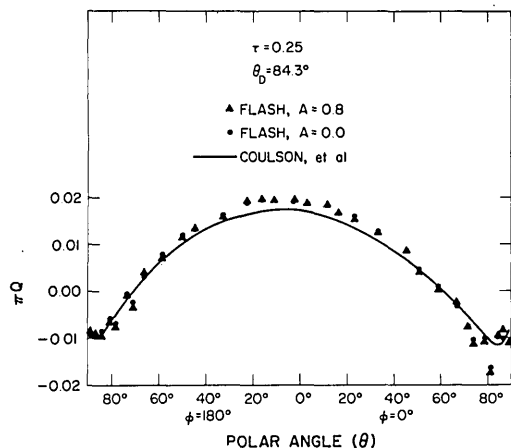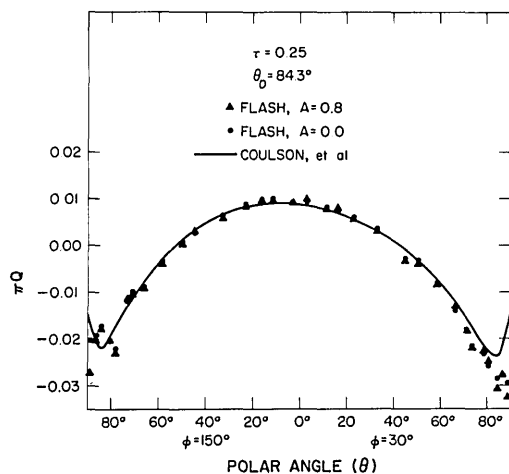
Fig. 8. Parameter Q for the scattered intensity transmitted through a Rayleigh atmosphere with an optical thickness of 0.25 as a function of the receiver zenith angle: $\theta_D = 84.3°, \phi = 0°$ and 180°.

Fig. 9. Parameter Q for the scattered intensity transmitted through a Rayleigh atmosphere with an optical thickness of 0.25 as a function of the receiver zenith angle: $\theta_D = 84.3°, \phi = 30°$ and 150°.

incident zenith angles of 23.1° and 84.3°. As noted previously for the transmitted scattered intensities, the reflected intensities for the two atmospheric models agree reasonably well for an incident zenith angle of $\theta_D = 23.1°$ (see Fig. 5), except for receiver zenith angles near 90° where the intensities reflected from the spherical-shell atmosphere underpredict those reflected from the plane-parallel atmosphere. The reflected intensity for the radiation incident upon the spherical-shell atmosphere at a zenith angle of 84.3° tends to overpredict that reflected from the plane-parallel model (Fig. 6, for the 0°, 180° azimuthal plane and Fig. 7 for the 30° and 150° azimuthal planes), particularly for receiver directions oriented toward the sun.

Parameters Q and U

Values of the Stokes parameters Q and U computed for the 0.25 optically thick spherical-shell homogeneous

Rayleigh atmosphere have also been compared with corresponding parameters computed for the plane-parallel atmosphere by Coulson et al. For a zenith angle of incidence of $\theta_D = 23.1°$, good agreement was obtained between the two different calculations of the parameters Q and U for the transmitted and reflected radiation, except for receiver zenith and receiver nadir angles, respectively, near 90°.

Figures 8 and 9 show comparisons of the parameter Q for the transmitted scattered radiation when the source radiation is incident at 84.3° to the top of the two atmospheric models and the ground albedo was either 0.0 or 0.8. The parameter Q for the transmitted scattered radiation in the 0° to 180° azimuthal plane, as shown in Fig. 8 for the spherical-shell atmosphere, appears
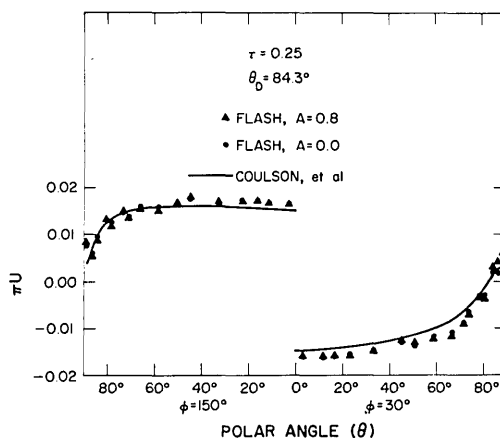

Fig. 10. Parameter U for the scattered intensity transmitted through a Rayleigh atmosphere with an optical thickness of 0.25 as a function of the receiver zenith angle: $\theta_D = 84.3°, \phi = 30°$ and 150°.
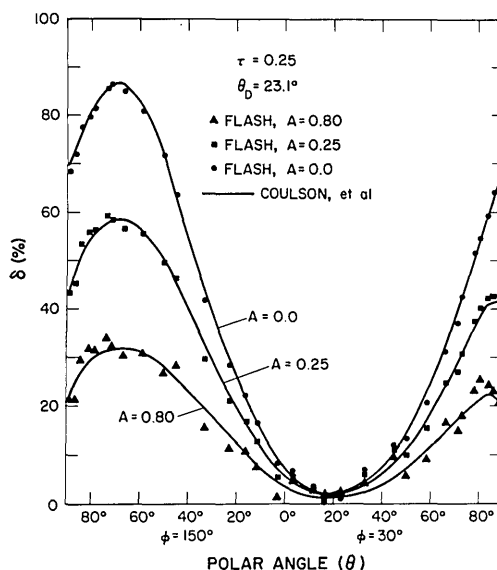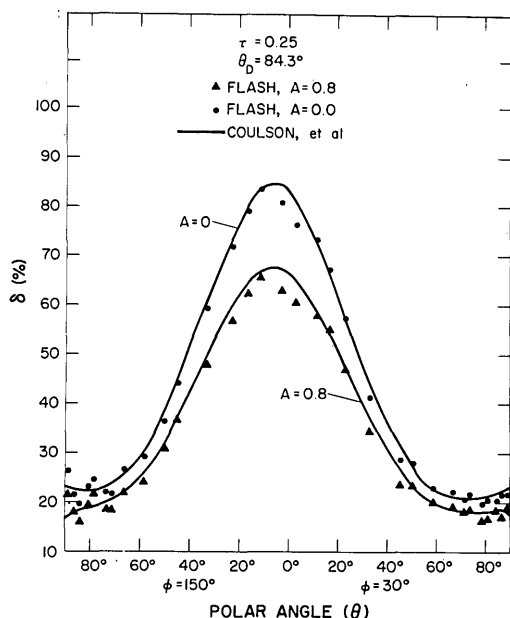

Fig. 11. Percent of polarization of the scattered intensity transmitted through a Rayleigh atmosphere with an optical thickness of 0.25 as a function of the receiver zenith angle: $\theta_D = 23.1°$, $\phi = 30°$ and 150°.

Fig. 12. Percent of polarization of the scattered intensity transmitted through a Rayleigh atmosphere with an optical thickness of 0.25 as a function of the receiver zenith angle: $\theta_D = 84.3°$, $\phi = 30°$ and $150°$.

to be slightly larger in absolute value than the parameter Q for the plane-parallel atmosphere. However, the transmitted scattered parameter Q in the 30° and 150° azimuthal planes is seen in Fig. 9 to be in better agreement with the parameter Q for the plane-parallel atmosphere at all receiver zenith angles, except for those zenith angles close to 90°. It should be noted that the parameter Q, as given in Figs. 8 and 9, is independent of the magnitude of the ground albedo.

The transmitted scattered parameter U in the 0–180° azimuthal plane for radiation incident to the spherical-shell atmosphere at zenith angles of $\theta_D = 23.1°$ and 84.3° was computed to be approximately zero for all receiver zenith angles. This agrees favorably with the value of zero for the parameter U in the 0–180° azimuthal plane as given by the plane-parallel atmosphere data reported by Coulson et al. The transmitted scattered parameter U in the 30° and 150° azimuthal planes that is shown in Fig. 10 when the source radiation is incident at a zenith angle of 84.3° to the spherical-shell atmosphere appears to be slightly larger in absolute value than the corresponding parameter U for the plane-parallel atmosphere. It is seen in Fig. 10 that the magnitude of the parameter U is independent of the value of the ground albedo.

Degree and Plane of Polarization

Comparisons of calculations of the percent of polarization, δ, of the scattered transmitted radiation in the 30° and 150° azimuthal planes when $\theta_D = 23.1°$ and 84.3° for the spherical-shell and plane-parallel atmospheres are shown in Figs. 11 and 12 for several values of the ground albedo. Except for the statistical fluctuation in the percent of polarization data shown in

Fig. 11 for the spherical-shell atmosphere when the ground albedo was 0.8, the agreement with the plane-parallel atmospheric data for an incident zenith angle of $\theta_D = 23.1°$ is excellent. It appears that the percent of polarization of the transmitted radiation is not as highly affected by the differences in the spherical-shell and plane-parallel atmosphere as were the intensities and the parameters Q and U when the angle of incidence is 84.3° (see Fig. 12). Differences in the percent of polarization near its maximum value are attributed to the statistical fluctuation of the Monte Carlo results rather than to the difference in the atmospheric geometries.

Comparisons of data giving the angle χ between the plane of polarization and the vertical plane in the appropriate azimuth for the transmitted scattered radiation in the 30° and 150° azimuthal planes for the two atmospheric geometries when the ground albedo is 0.0 and 0.8 are shown in Fig. 13 for radiation incident at $\theta_D = 84.3°$. The agreement between the data for the two atmospheric models is excellent, and thus, it appears that the atmospheric geometry has little influence on the values of the angle χ for all incident zenith angles.

The FLASH calculations of the plane of polarization in the 0° and 180° azimuthal plane were erratic, alternating between approximately 0° and 90°. This is due to the fact that the values computed for the parameter U, while insignificant in absolute value, fluctuated about 0, and therefore, the ATAN2(U, $-Q$) function used in FLASH for computing the plane of polarization gave values of approximately 0 where $U/-Q$ was slightly greater than 0 and values of approximately π when $U/-Q$ was slightly less than 0.

It should be pointed out that the FLASH calculations for the spherical-shell atmospheric model give the opposite sign of the angle χ than that reported by Coulson et al.[18] Using the equation given in Ref. 18 for calcu-
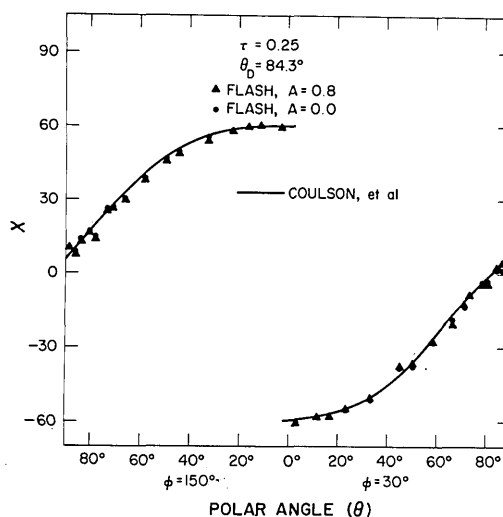

Fig. 13. Plane of polarization for the scattered intensity transmitted through a Rayleigh atmosphere with an optical thickness of 0.25 as a function of the receiver zenith angle: $\theta_D = 84.3°$, $\phi = 30°$ and $150°$.
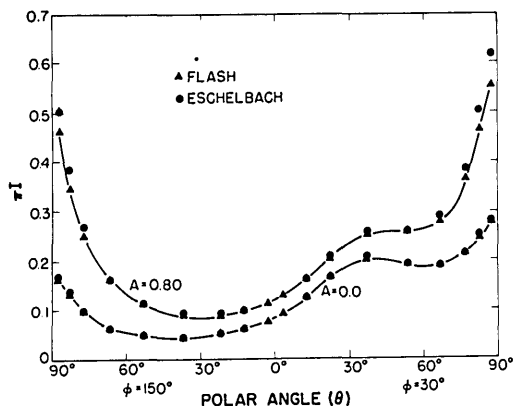
Fig. 14. Scattered intensity transmitted through a model atmosphere with a turbidity of 2 as a function of the receiver zenith angle: $\theta_D = 36.9°$, $\phi = 30°$ and 150°.
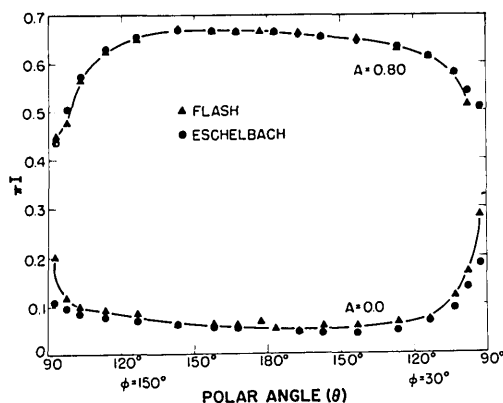

Fig. 15. Intensity reflected from a model atmosphere with a turbidity of 2 as a function of the receiver zenith angle: $\theta_D = 36.9°$, $\phi = 30°$ and 150°.

lating the angle χ and the calculated values for the parameters U and Q, the FORTRAN statement

CHI = 0.5*ATAN2($U,-Q$)*180.0/PI,

where PI = π, will also give a sign of the angle χ (CHI) opposite to that reported in Ref. 18.

Calculations for an Atmosphere Containing Both Molecules and Aerosol Particles

The FLASH program was also used to calculate polarization data for a spherical-shell atmosphere containing both molecules and aerosol particles. The atmosphere was assumed to be homogeneous with a turbidity factor of $T = 2$, i.e., the total optical thickness of the atmosphere was taken to be twice that of a Rayleigh atmosphere for 0.55-μ wavelength light.

The size distribution of the aerosol particles was assumed to be described by $dN(r)/dr = r^{-4}$ with limiting radii of 0.04 μ and 10.0 μ. The index of refraction for the aerosol particles was taken to be 1.50–0.00i, the wavelength of the source was chosen to be 0.55 μ. Since the optical thickness for a Rayleigh atmosphere at that wavelength is 0.09874, the total optical thick-

ness of the atmosphere was taken to be 0.19748. Calculations were performed for a zenith angle of the incident light of 36.9°. Receivers were placed at the bottom and the top of the atmosphere for recording the transmitted scattered and reflected radiation in the 30° and 150° azimuthal planes for several receiver zenith directions; 350 histories were run for each receiver view angle.

The results calculated for the spherical-shell atmosphere with a turbidity of $T = 2$ have been compared with results reported by Eschelbach[19] for a plane-parallel atmosphere using the same aerosol scattering data. Eschelbach employed a modified version of the method reported by Herman,[20] where the mathematical approach is derived from the integral form of the equation of radiative transfer as given by Chandrasekhar,[12] which, without any circuitous series development, is directly transformed into a system of linear equations for the upward and downward radiation.

Figures 14 and 15 show the transmitted and reflected scattered intensity in the 30° and 150° azimuthal planes as a function of the receiver zenith angle for ground albedo values of 0.0 and 0.8. The circled points are
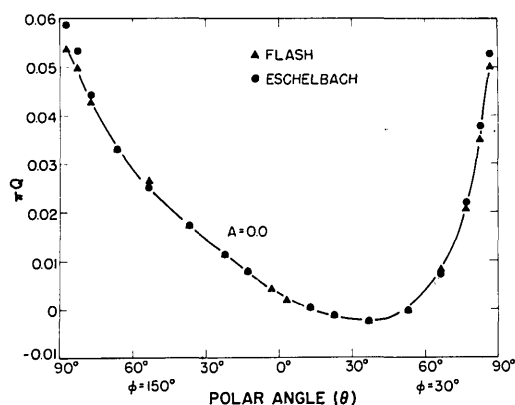

Fig. 16. Parameter Q for the scattered intensity transmitted through a model atmosphere with a turbidity of 2 as a function of the receiver zenith angle: $\theta_D = 36.9°$, $\phi = 30°$ and 150°.
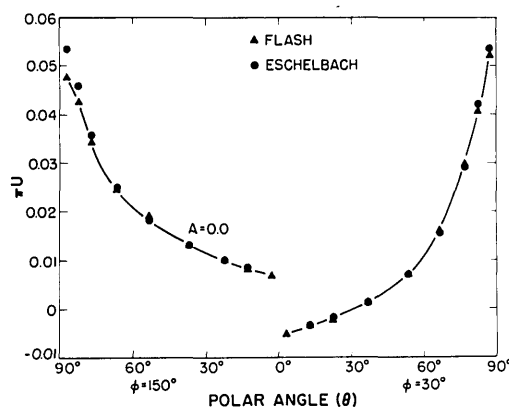

Fig. 17. Parameter U for the scattered intensity transmitted through a model atmosphere with a turbidity of 2 as a function of the receiver zenith angle: $\theta_D = 36.9°$, $\phi = 30°$ and 150°.
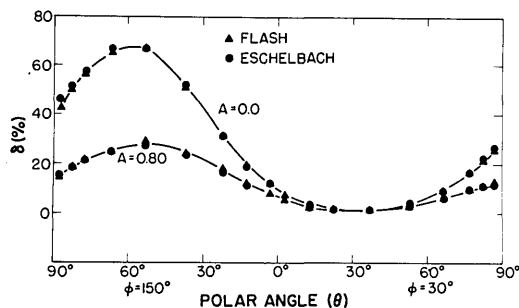
Fig. 18. Percentage of polarization of the scattered intensity transmitted through a model atmosphere with a turbidity of 2 as a function of the receiver zenith angle: $\theta_D = 36.9°$, $\phi = 30°$ and $150°$.
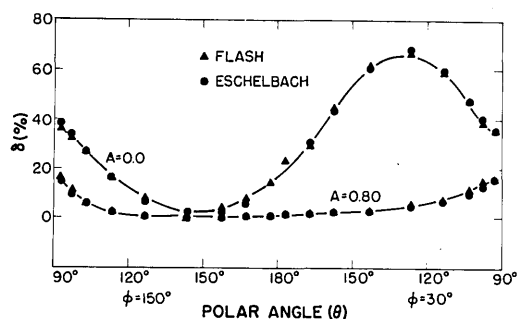

Fig. 19. Percentage of polarization of the intensity reflected from a model atmosphere with a turbidity of 2 as a function of the receiver zenith angle: $\theta_D = 36.9°$, $\phi = 30°$ and $150°$.

the results computed by Eschelbach for the plane-parallel atmospheric model, and the triangles give the data computed for the spherical-shell atmospheric model. The agreement between the two calculations is excellent, except for receiver zenith angles close to 90° where the spherical-shell model seems to underpredict slightly the data for the plane-parallel model. (Better statistics were obtained for the combined Rayleigh and aerosol atmosphere than for the pure Rayleigh atmosphere due to the larger sample size of 350 histories per receiver view angle.)

Figures 16 and 17 show a comparison of the parameters Q and U for the scattered radiation transmitted through the plane-parallel and spherical-shell atmospheres for a ground albedo of 0.0. Again an excellent agreement was observed. Similar to the calculated intensity data, the parameters Q and U as computed for the spherical-shell model seem to underpredict slightly the data obtained for the plane-parallel model for receiver zenith angles near 90°. Similar comparison was found for an albedo of $A = 0.8$. The parameters Q and U as calculated for both atmosphere geometries were found to be nearly independent of the ground albedo. For clarity, the points for $A = 0.8$ were therefore not plotted in Figs. 16 and 17.

No favorable agreement was found for the parameter V as computed for the two atmospheric models. Where Eschelbach[19] claims that his values for the parameter

V are at least of the correct magnitude, the FLASH data show large statistical variations about Eschelbach's data. In the 30° and 150° azimuthal planes the parameter V, however, is very small when compared with the parameters Q and U, and therefore no statistically better data for V can be expected using Monte Carlo methods.

Excellent agreement was obtained between the FLASH data and Eschelbach's calculations of the percentage of polarization (Fig. 18 for the transmitted and Fig. 19 for the reflected radiation). There seems to be no apparent difference in the percentage of polarization when calculated for the spherical-shell and plane-parallel models for a zenith angle of incidence of 36.9°.

Remarks

Additional comparisons between the results reported by Coulson et al.[18] and Eschelbach[19] and FLASH calculations have been made for incident angles other than those reported in this paper and for a molecular atmosphere with an optical thickness of 1.0 and atmospheres with turbidities of 4 and 6. These comparisons also showed good agreement for angles of incidence less than 80°.

The FLASH calculations show that orders of scattering greater than 1 contribute significantly to the sky radiation. The relative importance of multiple scattering is, however, dependent upon several parameters, i.e., the receiver polar position angle, θ_D, the optical thickness of the atmosphere, the ground albedo, and the coordinates of the receiver view angle. A detailed description of the influence of multiple scattering on the sky radiation in a molecular atmosphere is given by de Bary and Bullrich[21] and by Dave.[22] For the atmospheric models considered in this paper, the ratio of the total to the single scattered intensities varies between 1.1 and 1.9 for a ground albedo of 0.0 and between 1.3 and 5.3 for a ground albedo of 0.8.

Conclusions

The good agreement found between the polarization calculations obtained with the FLASH program and the data for plane-parallel atmospheres reported in Refs. 18 and 19 for zenith angles of incidence $\theta_D = 23.1°$ and $\theta_D = 36.9°$, indicates that the FLASH program correctly accounts for the polarization of atmospheric scattered light. Where one may be doubtful about the accuracy of polarization calculations averaged over rather large angle increments, i.e., 30° over azimuth (as used in Refs. 4 and 17), the utilization of the backward Monte Carlo method allows one to compute polarization data for discrete directions at a receiver position.

It appears, from the comparisons shown in this paper, that the parameters I, Q, and U at low sun elevations are more dependent upon the geometric model being used than are the degree and plane of polarization. The most significant differences between the polarization parameters computed for the two atmospheric models occur for receiver view angles oriented near the sun's direction.

It should be pointed out that the differences in the diffuse transmitted and reflected polarization parameters for plane-parallel and spherical-shell atmospheres shown in this paper would have been smaller if realistic atmospheric models had been considered. That is, if the scattering properties of the atmospheres had been defined to decrease with altitude, the differences between the results for the two atmospheric models would be expected to be smaller for the low sun elevations. Such being the case, however, the FLASH program still provides a means for identifying the limitations of plane-parallel atmospheric models.

The FLASH program, therefore, provides a computation tool for performing studies of the transmittance and reflectance of twilight radiation from realistic spherical-shell atmospheric models and will extend computations of the sky radiation to the twilight sky. The program is currently being used in studies of the effect of dust layers upon the color ratios computed for various sun depression angles.[10]

The flexibility of the program also allows the placement of receivers above the spherical shell atmosphere and therefore will prove a valuable method for an optical analysis of planetary atmospheres and for the interpretation of data taken by satellites and other spacecraft.

The backward Monte Carlo approach is not limited to spherical-shell geometries or to plane-parallel sources. Programs[23] similar to the FLASH program have been developed to treat both point and volume sources.

References

1. A. Skumanich and K. Bhattachargie, J. Opt. Soc. Am. **51**, 484 (1961).

2. D. G. Collins and M. B. Wells, *Monte Carlo Codes for Study of Light Transport in the Atmosphere*, Report RRA-T54 (Radiation Research Associates, Inc., Fort Worth, Texas, 1965), Vols. 1 and 2.

3. D. G. Collins, K. Cunningham, and M. B. Wells, *Monte Carlo Studies of Light Transport*, Report RRA-T74 (Radiation Research Associates, Inc., Fort Worth, Texas, 1967).

4. D. G. Collins, J. Opt. Soc. Am. **57**, 1423 (1967); also see D. G. Collins, Report RRA-T86 (Radiation Research Associates, Inc., Fort Worth, Texas, 1968).

5. G. N. Plass and G. W. Kattawar, Appl. Opt. **7**, 361 (1968).

6. G. W. Kattawar and G. N. Plass, Appl. Opt. **7**, 1519 (1968).

7. S. O. Kastner, J. Quant. Spectrosc. Rad. Transfer **6**, 317 (1966).

8. G. I. Marchuk and G. A. Mikhailov, Bull. Acad. Sci. USSR, Atmos. Oceanic Phys. Ser. **3**, No. 2 (1967).

9. D. C. Collins and M. B. Wells, FLASH, *a Monte Carlo Procedure for Use in Calculating Light Scattering in a Spherical-Shell Atmosphere*, Report RRA-T704 (Radiation Research Associates, Inc., Fort Worth, Texas, 1970).

10. W. G. Blättner, D. G. Collins, and M. B. Wells, *Monte Carlo Calculations in Spherical-Shell Atmospheres*, Report RRA-T7104 (Radiation Research Associates, Inc., Fort Worth, Texas, 1971).

11. G. W. Kattawar, G. N. Plass, and C. N. Adams, Astrophys. J. **170**, 371 (1971).

12. S. Chandrasekhar, *Radiative Transfer* (Clarendon Press, Oxford, 1950).

13. J. V. Dave, Appl. Opt. **9**, 2673 (1970).

14. J. E. Hansen, J. Atmos. Sci. **28**, 123 (1971).

15. J. N. Howard, D. E. Burch, and D. Williams, *Sci. Rept. 1*, GRD, AFCRL (Ohio State University Research Foundation, 1954).

16. A. E. S. Green and M. Griggs, Appl. Opt. **6**, 561 (1963).

17. G. N. Plass and G. W. Kattawar, Appl. Opt. **9**, 1122 (1970).

18. K. L. Coulson, J. V. Dave, and Z. Sekera, *Tables Related to Radiation Emerging from a Planetary Atmosphere with Rayleigh Scattering* (University of California Press, Berkeley, 1960).

19. G. Eschelbach, private communication (University of Mainz, 1971). The method is described in: K. Bullrich, R. Eiden, G. Eschelbach, K. Fischer, G. Hänel, K. Heger, H. Schollmayer, and G. Steinhorst, *Research on Atmospheric Radiation Transmission*, University of Mainz, Final Report, AFCRL-69-0266 (January, 1969).

20. B. M. Herman, J. Geophys. Res. **70**, 1212 (1965).

21. E. de Bary and K. Bullrich, J. Opt. Soc. Am. **54**, 1413 (1964).

22. J. V. Dave, J. Opt. Soc. Am. **54**, 307 (1964).

23. D. G. Collins and M. B. Wells, *Computer Procedures for Calculating Time Dependent Light Scattering in Spherical-Shell Atmospheres*, Report RRA-T7017 (Radiation Research Associates, Inc., Fort Worth, Texas, 1970).