BACTERICIDAL EFFECTS OF THE PARTIAL IRRADIATION OF A ROOM WITH ULTRA-VIOLET LIGHT

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(With 7 Figures in the Text)

Bacteria-carrying particles introduced into the air of a room may be removed in a number of ways. These include ventilation, sedimentation of the particles on to floors and other horizontal surfaces, and any bactericidal action which may be taking place. If the air of the room is continually and effectively mixed, these processes, taken singly or in combination, will lead to the disappearance of the bacteria-carrying particles from the air at a rate proportional to the number present at any given time, i.e. of a number N_0 present at zero time a number $N = N_0 e^{-Kt}$ will remain after time t. If there is a steady introduction of bacteria-carrying particles into the air of the room, an equilibrium state will be reached when the number of such particles present $= B/K$, where B is the rate of introduction of the particles. The constant K in the above expression is under these conditions additive for any number of processes taking place simultaneously, provided these individually lead to a logarithmic decay rate.

When ultra-violet irradiation of a portion of the room space only is taking place, as, for example, when it is necessary to prevent the radiation reaching persons in the room, and only the portion of the room extending above the 6 or 7 ft. level can be irradiated, the above equations may still apply, but the rate of circulation of the air between the irradiated and the unirradiated zones may often be too small to maintain adequate mixing. Under these circumstances, which probably represent the majority of cases when natural circulation only is involved, the effects of ventilation, sedimentation, and irradiation may no longer be additive, and it may not be possible to characterize both equilibrium and die-away rates by a single constant K as above. This point is of some importance since, as will be seen, the divergence from simple conditions may be large, and in many environments the emission of bacteria is not continuous, but varies greatly in intensity in an irregular manner in time. The effective exposure to infection is then governed as much by the rate of disappearance of sudden concentrations of bacteria-carrying particles as by the equilibrium levels.

In evaluating the effects of the non-uniformity introduced by partial irradiation of the room space, it is necessary to make some simplifying assumptions as to the nature of the circulation between the irradiated and non-irradiated zones. The actual air movements in any given case will usually be complex and variable. The two simplest, and in one sense limiting, types of circulation are: circulation of air through both zones without mixing, and interchange of air between the two zones, the air in each zone remaining effectively mixed at all times. Luckiesh & Holladay (1942*b*) have tabulated data for equilibrium conditions based essentially on the first of these. Buttolph (1945) has similarly considered equilibrium conditions for the second of the above types of circulation. In both of these papers the data are presented in the form of charts, and details of the methods of calculation are not given. In the present paper formulae are derived for the rates of removal of concentrations of bacteria-carrying particles as well as for the equilibrium levels reached when the rate of emission of these into the room is constant. The effect of simultaneous ventilation and sedimentation is also taken into account. Before presenting the calculations and experimental data, which form the main body of this paper, it will be convenient to discuss some of the assumptions made and also to define the principal symbols used.

The effect of irradiation on a bacterial population is assumed to be described by the equation

$$\frac{dN}{dt} = -kNI,$$

where N is the population at time t, I is the intensity of irradiation and k is a constant dependent on the bacterial species and on physical factors such as temperature, relative humidity, etc. The unirradiated portion of the room is designated as zone 1, and the mean density of bacteria-carrying particles present at any time as $\bar{N}_1$. The irradiated portion of the room is designated zone 2, and the mean density of bacteria-carrying particles as $\bar{N}_2$.

The fraction of the room irradiated is α, and the circulation of air between the two zones is described in terms of a circulation time T. When the air is assumed circulating through the two zones without mixing, then T is the time of a complete circuit, i.e. a volume of air equal to the total volume of the room passes through each zone in this time. When air is assumed to be exchanged between the two zones while complete mixing is maintained in each, then T is defined as the time necessary for a volume of air equal to the volume of the room to be interchanged between the two zones. The unit of volume throughout, except when otherwise stated, is the volume of the room.

Ventilation is expressed by the symbol R representing that number of air changes per unit time expressed for the whole volume of the room even when the ventilation is considered as introduced via one zone only. Sedimentation is expressed by the symbol S and the rate of introduction of bacteria into the room by B, both being expressed in terms of the whole volume even when considered as introduced via one zone only.

Finally, the irradiation intensity has been assumed uniform throughout the irradiated space. This is far from true in most cases and non-uniform irradiation combined with an uneven distribution of the air circulation may lead to considerable divergences from the calculated behaviour, but the only alternative seems to be the individual analysis of particular cases.

I. *Equilibrium conditions with air interchange between two zones in both of which mixing is continuous and complete.*

Bacteria-carrying particles are introduced at a uniform rate, and lost by ventilation and sedimentation from zone 1 only. In equilibrium

Rate of loss from zone 1

$$= \left\{\frac{\bar{N}_1}{T} - \frac{\bar{N}_2}{T} - B + (R+S)\bar{N}_1\right\}\frac{1}{(1-\alpha)} = 0,$$

Rate of loss from zone 2

$$= \left\{\frac{\bar{N}_2}{T} - \frac{\bar{N}_1}{T}\right\}\frac{1}{\alpha} + kI\bar{N}_2 = 0,$$

whence
$$\bar{N}_1 = \frac{B}{R+S+\dfrac{kI\alpha}{1+kI\alpha T}}. \tag{1}$$

Any ventilation or sedimentation occurring in zone 2 can be allowed for in this or subsequent calculations by addition to $kI\alpha$.

II. *Die-away rates with air interchange between two zones in both of which mixing is continuous and complete.*

Ventilation and sedimentation are occurring in zone 1 only. In time δt

Loss from zone 1

$$= -(1-\alpha)\,\delta\bar{N}_1 = \left\{\frac{\bar{N}_1}{T} - \frac{\bar{N}_2}{T} + (R+S)\bar{N}_1\right\}\delta t,$$

Loss from zone 2

$$= -\alpha\delta\bar{N}_2 = \left\{\frac{\bar{N}_2}{T} - \frac{\bar{N}_1}{T} + kI\alpha\bar{N}_2\right\}\delta t,$$

whence

$$(1-\alpha)\,T\frac{d\bar{N}_1}{dt} = \bar{N}_2 - \bar{N}_1(1+(R+S)\,T), \tag{2}$$

$$\alpha T\frac{d\bar{N}_2}{dt} = \bar{N}_1 - \bar{N}_2(1+kI\alpha T), \tag{3}$$

and
$$\frac{\alpha}{1-\alpha}\frac{d\bar{N}_2}{d\bar{N}_1} = \frac{\bar{N}_2 - \bar{N}_2(1+kI\alpha T)}{\bar{N}_2 - \bar{N}_1(1+(R+S)\,T)}, \tag{4}$$

putting
$$\bar{N}_2 = v\bar{N}_1,$$
$$\frac{d\bar{N}_2}{d\bar{N}_1} = v + \bar{N}_1\frac{dv}{d\bar{N}_1},$$

and by substitution in (4) and rearrangement

$$\frac{d\bar{N}_1}{\bar{N}_1} = -\frac{z-b}{z^2-c^2}dz, \tag{5}$$

where $z = \dfrac{\bar{N}_2}{\bar{N}_1} + a, \qquad c^2 = \dfrac{1-\alpha}{\alpha} + a^2,$

$$2a = \frac{1-\alpha}{\alpha}(1+kI\alpha T) - (1+(R+S)\,T),$$

$$2b = \frac{1-\alpha}{\alpha}(1+kI\alpha T) + (1+(R+S)\,T).$$

Integrating and putting $\bar{N}_1 = \bar{N}_2 = N_0$ at $t = 0$,

$$2\ln\frac{\bar{N}_1}{N_0} = \frac{b}{c}\ln\left(\frac{z/c-1}{z/c+1}\right) - \ln\left(\frac{z^2}{c^2}-1\right)$$
$$-\frac{b}{c}\ln\left(\frac{\dfrac{1+a}{c}-1}{\dfrac{1+a}{c}+1}\right) + \ln\left(\frac{(1+a)^2}{c^2}-1\right) \tag{6}$$

$$= F\left(\frac{b}{c}, \frac{z}{c}\right) - F\left(\frac{b}{c}, \frac{1+a}{c}\right), \tag{6a}$$

where $F(w,x) = w\ln\left(\dfrac{x-1}{x+1}\right) - \ln(x^2-1);$

from (2)

$$\frac{1}{\bar{N}_1}\frac{d\bar{N}_1}{dt} = \frac{1}{(1-\alpha)\,T}\left\{\frac{\bar{N}_2}{\bar{N}_1} - (1+(R+S)\,T)\right\}$$
$$= \frac{1}{(1-\alpha)\,T}(z-b). \tag{7}$$

In any given case a, b and c are known, so that $\dfrac{1}{\bar{N}_1}\dfrac{d\bar{N}_1}{dt}$ can be evaluated by means of equations (6) and (7) for any given value of $\dfrac{\bar{N}_1}{N_0}$. Where $1+a>c$, equation (6) may conveniently be rearranged in the form (6a). (This condition will apply in any case where the ultra-violet irradiation is potentially useful.) From equation (6) it will be seen that if $1+a>c$ then $b>z>c$ and $\dfrac{1}{\bar{N}_1}\dfrac{d\bar{N}_1}{dt}$ tends to a limiting value, as $\bar{N}_1 \to 0$, given by

$$\underset{t\to\infty}{\text{Lt}}\left(-\frac{1}{\bar{N}_1}\frac{d\bar{N}_1}{dt}\right) = \frac{1}{(1-\alpha)\,T}(b-c), \tag{8}$$

when $1+a<c\,\dfrac{1}{\bar{N}_1}\dfrac{d\bar{N}_1}{dt}$ will also tend to this same limit.

Since $\frac{1+a}{c} < \frac{b}{c}$ and $\frac{1}{\bar{N}_1}\frac{d\bar{N}_1}{dt}$ reaches 90 % of its maximum value when $\frac{z}{c} = 1+0\cdot1\left(\frac{b}{c}-1\right)$, it is clear from the nature of the function $F(w,x)$, which is shown graphically in Fig. 1, that the die-away rate approaches its limiting value rapidly in most cases.

III. *Equilibrium conditions with air circulating through the two zones without mixing.*

Bacteria-carrying particles are introduced at a uniform rate into, and lost by ventilation and sedimentation from, zone 1 only.

Let N_1 be the density of bacteria-carrying particles as the air leaves zone 1 and N_2 the value on leaving zone 2.

Then

$$N_2 = N_1 e^{-kI\alpha T} \text{ (passage through zone 2).} \tag{9}$$

In passing through zone 1

$$\frac{dN}{dt} = \frac{B}{1-\alpha} - \frac{R+S}{1-\alpha}N,$$

$$\frac{d}{dt}\left\{N\exp\left[\frac{R+S}{1-\alpha}t\right]\right\} = \frac{B}{1-\alpha}\exp\left[\frac{R+S}{1-\alpha}t\right].$$

Integrating and putting $N = N_2$ when $t = 0$

$$N = \frac{B}{R+S} + \left\{N_2 - \frac{B}{R+S}\right\}\exp\left[-\frac{(R+S)}{1-\alpha}t\right]; \tag{10}$$

equilibrium conditions are defined by N reaching a value N_1 when $t = (1-\alpha)T$, i.e. no change in the density of the bacterial population on completing the circulation cycle.

Then

$$N_1 = \frac{B}{R+S} + \left(N_2 - \frac{B}{R+S}\right)e^{-(R+S)T},$$

and combining this with (9)

$$N_2 = \frac{B(1-e^{-(R+S)T})}{(R+S)(e^{kI\alpha T} - e^{-(R+S)T})}. \tag{11}$$

Now $$\bar{N}_1 = \frac{1}{(1-\alpha)T}\int_0^{(1-\alpha)T} N\,dt,$$

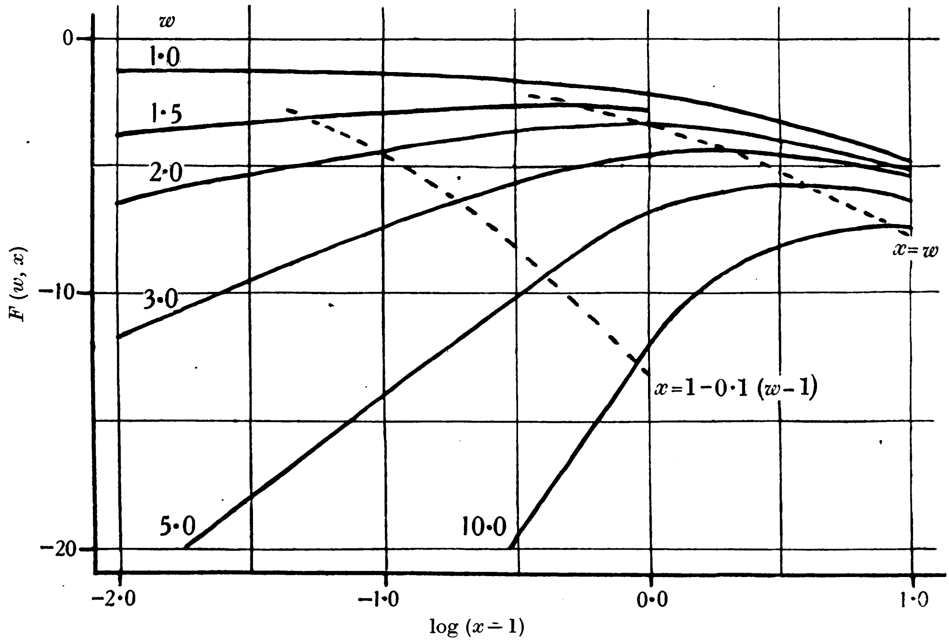

Fig. 1.

so that putting $e^{-kI\alpha T} = p$ and $e^{-(R+S)T} = q$ and using (10) and (11) we have on integrating

$$\bar{N}_1 = \frac{B}{R+S} - \left\{\frac{Bp(1-q)}{(R+S)(1-pq)} - \frac{B}{R+S}\right\}\left\{\frac{q}{(R+S)T} - \frac{1}{(R+S)T}\right\}$$

and $$\bar{N}_1 = \frac{B\{(R+S)T(1-pq) - (1-p)(1-q)\}}{(R+S)^2 T(1-pq)}. \tag{12}$$

This may be written

$$\bar{N}_1 = \frac{B}{\lambda(R+S) + \frac{2(1-p)}{T(1+p)}}. \tag{12a}$$

Fig. 2 shows the form of the function λ.

IV. *Die-away rates with air circulation through the two zones without mixing.*

Ventilation and sedimentation are occurring in zone 1 only.

After n complete cycles, i.e. after a time $T = nt$,

$$N = N_0 e^{-(kI\alpha nT+(R+S)nT)} + A,$$

where A is a constant depending on the position of the representative point in the cycle.

Hence the mean value of the die-away rate at any point is given by

$$-\frac{1}{N}\frac{dN}{dt} = kI\alpha + R + S. \qquad (13)$$

The formulae derived above are summarized in Table 1 together with the limiting values for $T = 0$ and $kI = \infty$, these representing the limiting effects for complete mixing throughout the whole room, i.e. maximum air movement, and for maximum irradiation. Expressions for equilibrium levels and die-away rates under other conditions can be deduced in similar ways. The quantity needed in these formulae which is likely to be most difficult to obtain is the value for the circulation time, T. Two methods of estimating this quantity suggest themselves. The actual circulation of the air may be traced in detail either by anemometer measurements or by timing the circulation of smoke or meta snow (made by holding a piece of 'meta' solid fuel against a hot metal surface, e.g. a soldering iron), and an average value for the circulation time in a vertical plane deduced. Alternatively, a sudden puff of a tracer substance may be liberated at some suitable point in the room, and the variation in concentration of this substance at a point just 'upwind' of the liberating point observed. Usually the concentration will remain near zero for some time and then rise to a maximum value after a total interval equal to the circulation time. Thereafter the concentration at this point will usually follow a course resembling a damped oscillation with a period equal to the circulation time, but the picture may be complicated by secondary circulations. Equating the time deduced in this way to T in the above formulae assumes that the principal circulation involves sufficient vertical movement to cross the dividing plane between the two zones; this will usually be so, but should be verified if doubtful. The values of T used in obtaining the calculated die-away rates for the experiments described later were obtained in this way, using acetyl acetone as a tracer substance and following the concentration by its ultra-violet absorption (Lidwell & Lovelock, 1946). A curve of this kind is shown in Fig. 3.

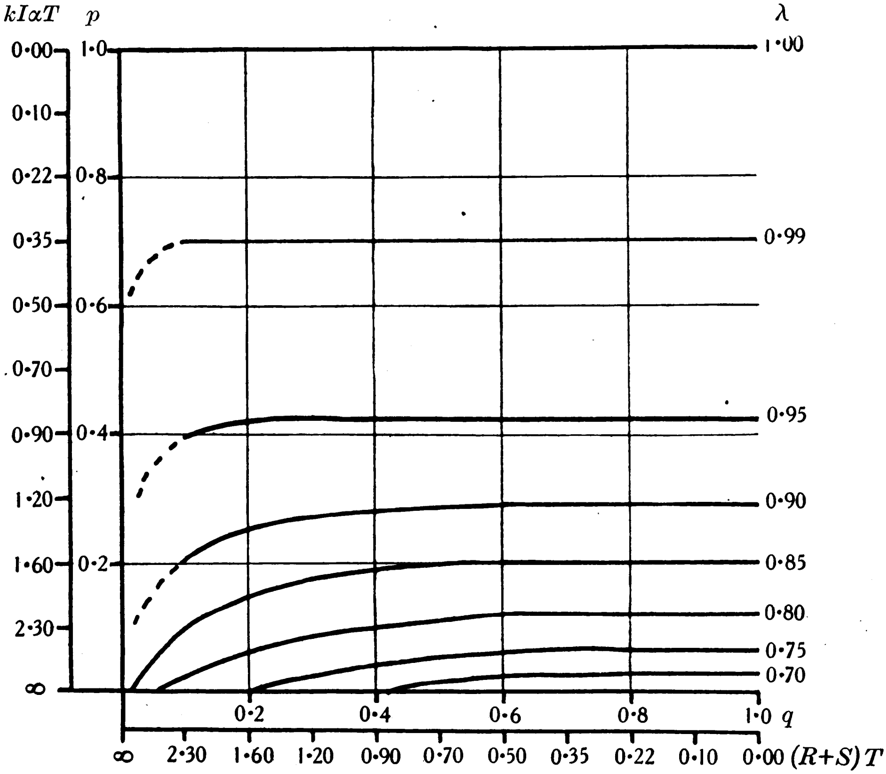

Fig. 2.

Table 1

	Type of circulation	
	Interchange complete mixing	Circulation no mixing
Equilibrium level	$\dfrac{B}{R+S+\dfrac{kI\alpha}{1+kI\alpha T}}$	$\dfrac{B}{\lambda(R+S)+\dfrac{2(1-p)}{T(1+P)}}$
$\operatorname{Lt}_{T\to 0}$	$\dfrac{B}{R+S+kI\alpha}$	$\dfrac{B}{R+S+kI\alpha}$
$\operatorname{Lt}_{kI\to\infty}$	$\dfrac{B}{R+S+\dfrac{1}{T}}$	$\dfrac{B}{R+S+\dfrac{2}{T}}$
Die-away rate	$\dfrac{1}{(1-\alpha)\,T}(b-c)$ (steady value)	$R+S+kI\alpha$ (mean value)
$\operatorname{Lt}_{T\to 0}$	$R+S+kI\alpha$	$R+S+kI\alpha$
$\operatorname{Lt}_{kI\to\infty}$	$\dfrac{1}{(1-\alpha)}\left(R+S+\dfrac{1}{T}\right)$	—

Where $p=e^{-kI\alpha T}$,

$q=e^{-(R+S)T}$,

λ is shown in Fig. 2,

$$b=\frac{1}{2}\left(\frac{1-\alpha}{\alpha}(1+kI\alpha T)+(1+(R+S)\,T)\right)$$

and $$c^2=\frac{1-\alpha}{\alpha}+\frac{1}{4}\left(\frac{1-\alpha}{\alpha}(1+kI\alpha T)-(1+(R+S)\,T)\right)^2.$$

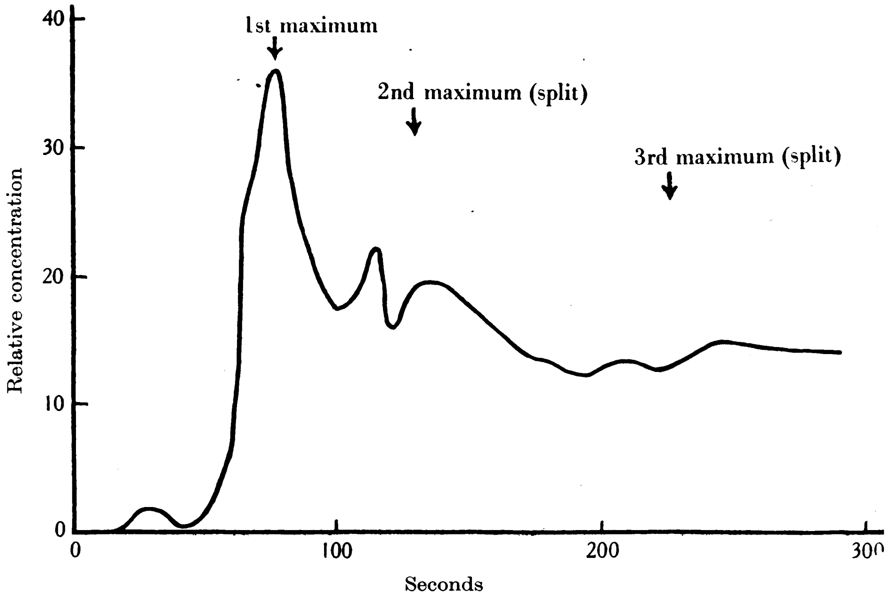

Fig. 3. The first main maximum occurs in just under 80 sec. After this the peaks are split owing, presumably, to the interaction of two main circulations at slightly differing rates. Estimated value of T, 1·3 min.

EXPERIMENTAL RESULTS

In order to obtain some kind of a check on the above calculations, a short series of bactericidal tests was carried out. The room used had a volume of approximately 3750 cu.ft.; the shape was somewhat irregular, but the irradiated portion, above $7\frac{1}{4}$ ft., was roughly 26 × 10 × 5 ft. and the ultra-violet lamps, 18 W. G.E.Co. (America) low-pressure type, were arranged as shown in Fig. 4, which also gives some other details. This corresponds to a value of $\alpha = 0{\cdot}32$. The mean irradiation intensities were calculated by the method described in the Appendix. For convenience, measurements of die-away rates were made and the general procedure in any experiment was as follows.

Bacteria from the mouth of a healthy subject were sprayed into the air of the room by a series of simulated sneezes. After allowing 5 min. for air mixing, sampling was begun at 1 cu.ft./min. on to serum agar plates in a slit sampler (Bourdillon, Lidwell & Thomas 1941) from a point in the centre of the room 6 ft. above the floor. Four minutes later the ultra-violet lamps in use, which had been running for at least 10 min. previously, were uncovered. After incubation for 24 hr. the colonies developed on the plates were counted. The results of such a test are shown in Fig. 5 and the results of the whole series are given in Table 2, together with the die-away rates calculated for the two types of air circulation. It will be seen that, having regard to the uncertainties in the determination of T, the experimental results are in good agreement with those calculated on the basis of air interchange between the two zones accompanied by complete mixing in each zone.

The value taken for k, the sensitivity of the bacteria to ultra-violet irradiation, was derived from a series of experiments carried out in a room 8 × 10 × 8 ft. completely irradiated by a single 18 W. low-pressure lamp placed at the centre of the 10 × 8 ft. wall. The general procedure was similar to that described above, except that when broth cultures of a salivary *Streptococcus* were used, they were sprayed from an 'Atomozon' spray which produces bacteria-carrying particles with a similar size range to the simulated sneeze, i.e. a falling rate of about 30 ft./hr., which leads to a die-away rate due to sedimentation only in a room 8 ft. high of about 4/hr. The air of the room was continually mixed by a small 12 in. table fan in one corner pointing diagonally upwards across the room. The results of these tests are shown in Fig. 6. The numerical values of k are very similar to those obtained for *B. coli* in duct experiments by Luckiesh & Holladay (1942*a*).

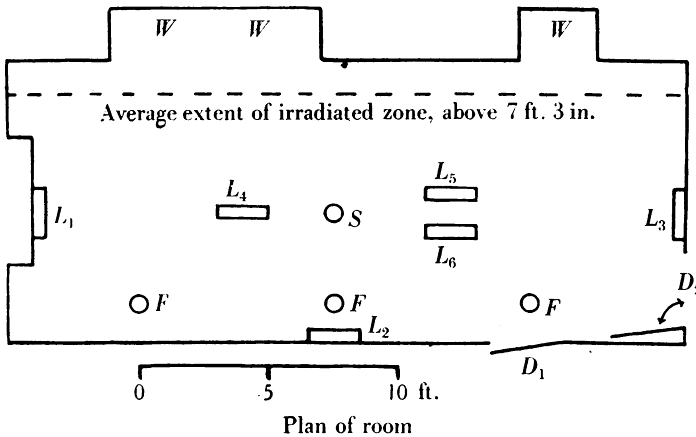

Fig. 4. L_1, L_2, L_3, L_4, L_5, L_6, 18 W. G.E.Co. (America) low-pressure lamps in suitable reflectors to screen all radiation below $7\frac{1}{4}$ ft. Total emission from each lamp at 2537 A. approximately 2 W. W, W, W, windows, shut throughout experiments; D_1, door to passage, shut throughout experiments; D_2, door to adjacent kitchen, open throughout experiments; S, sampling point; F, F, F, fans when used.

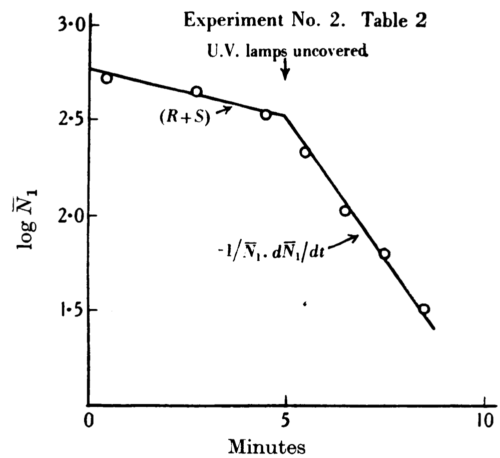

Fig. 5.

Table 2

No.	Lamp in use	I (mW./ft.^{2})	Circulation	T (min.)	$R+S$ (hr.$^{-1}$)	Die-away rate; limiting value of $-\frac{1}{\bar{N}_1}\frac{d\bar{N}_1}{dt}$ (hr.$^{-1}$): Observed	Calculated: Interchange with mixing	Calculated: Circulation without mixing
1	L_1, L_2, L_3	24	Natural	3·0	6	28	32	72
2	L_1, L_2, L_3	24	One fan	1·5	7½	37½	43	72
3	All six	53	Natural	3·0	7	32	36	150
4	All six	53	One fan	1·5	8	40½	55	150
5	None	Zero	Natural	3·0	6	(6)	—	—
6	None	Zero	One fan	1·5	7½	(7½)	—	—
7	All six	53	Natural	1·3	13	62	64	153
8	All six	53	Three fans	0·6	9	92	109	153
9	None	Zero	Natural	1·3	10	(10)	—	—

In nos. 1–6 the ventilation, R, was approximately 3 changes/hr., and the calculated figures have been based on a mean value of $R+S$ of 7/hr. or 0·12/min.

In nos. 7–9 the ventilation was about 6 changes/hr., and the calculated figures have been based on a mean value of $R+S$ of 10/hr. or 0·17/min. It is noticeable that the greater ventilation in this series, due to a windy day outside, is accompanied by a considerable increase in the rate of air circulation in the room.

The relative humidity was about 54 % and the temperature 60–65° F. throughout.

In the calculations k has been taken as 0·14 sq.ft./mW./min. and α as 0·32.

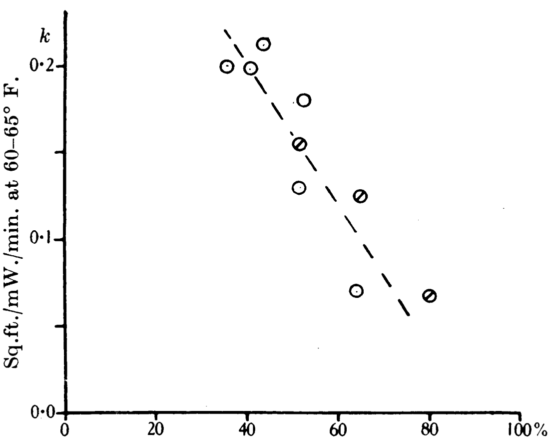

Fig. 6.

APPENDIX

Calculation of mean irradiation intensity in irradiated zone

With a knowledge of the polar diagram for the lamps used in any given case (including the effect of reflectors, etc.), it is easy in principle to evaluate the mean intensity of radiation over any given volume by a process of summation of elementary volumes. Wells (1944) has carried out a number of such evaluations based on the idea of mean ray length, using as elements of volume conical wedges enclosed between suitable angular deviations from the vertical through the centre of the lamp. Luckiesh & Holladay (1942*b*) have tabulated data based on this procedure assuming a continuous line source of ultra-violet irradiation along the complete length or width of the irradiated space. It is convenient to derive a general expression for the mean radiation intensity based on treating the lamps as point sources radiating uniformly in all directions into a rectangular volume. The errors involved in such a procedure are not often serious, and the gain in ease in evaluating a given case involving a number of lamps is considerable.

All cases of the above kind can be derived from the case of a source at one angle of a rectangular parallelopiped. At any point whose co-ordinates relative to the source O are x, y, z,

$$\text{Intensity} = \frac{\mathscr{I}}{x^2+y^2+z^2},$$

so that the mean intensity over the whole volume, having sides of length x, y, and z, is given by

$$I = \frac{\mathscr{I}}{xyz}\int^x\int^y\int^z \frac{dx\,dy\,dz}{x^2+y^2+z^2}$$

$$= \frac{\mathscr{I}}{xyz}\int^y\int^z \frac{1}{(y^2+z^2)^{\frac{1}{2}}}\tan^{-1}\frac{x}{(y^2+z^2)^{\frac{1}{2}}}\,dy\,dz,$$

putting $y' = y/x$ and $z' = z/x$

$$I = \frac{\mathscr{I}}{x^2}\frac{1}{y'z'}\int^{y'}\int^{z'} (y'^2+z'^2)^{\frac{1}{2}}\tan^{-1}(y'^2+z'^2)^{-\frac{1}{2}}\,dy'\,dz'.$$

If E = total watts entering volume then

$$\mathscr{I} = \frac{2E}{\pi}$$

and
$$I = \frac{2E}{\pi x^2} F(y', z').$$

The function $F(y',z')$ cannot be directly computed since it becomes infinite when $y' = z' = 0$. The difference $F(y',z') - F(0{\cdot}1, 0{\cdot}1)$, however, can be so evaluated, and a value for $F(0{\cdot}1, 0{\cdot}1)$ estimated by summation of elementary volumes. The chief uncertainty in the estimation of $F(0{\cdot}1, 0{\cdot}1)$ is the contribution of the element, conveniently cubical, containing the source. This, however, is the same, with suitable allowance for change of scale, as the value of $F(1{\cdot}0, 1{\cdot}0)$, so that by an iterative process the value assumed for $F(0{\cdot}1, 0{\cdot}1)$ can be improved to any desired extent. The function $F(y',z')$ is shown graphically in Fig. 7 for values of y' and z' ranging from 0·1 to 10·0.

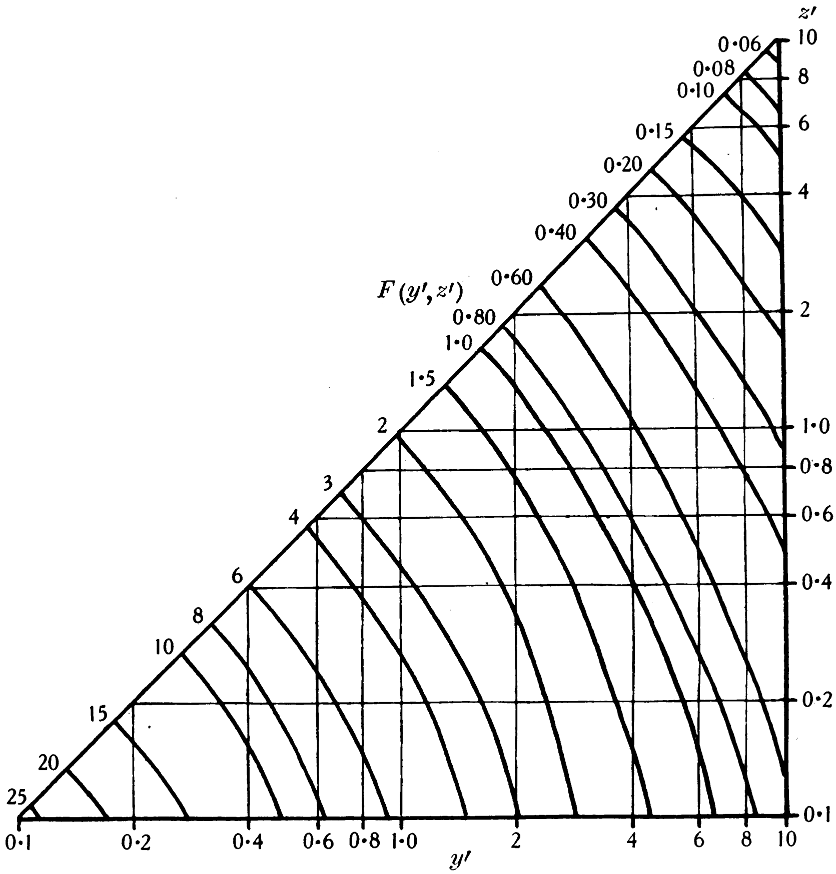

Fig. 7.

SUMMARY

The process of removal of bacteria from the air of a room of which part only is irradiated with ultra-violet light is discussed.

Formulae are derived for the rate of disappearance of bacteria from the air in such circumstances and for the equilibrium levels reached when bacteria-carrying particles are being continuously introduced into the air at a constant rate. These formulae include terms for the effect of simultaneous ventilation and for the effect of sedimentation of the bacteria-carrying particles.

The results of a short series of tests are compared with those calculated from these formulae and the two found to agree reasonably well on the assumption that the air within both the irradiated and non-irradiated zones is effectively mixed.

Figures are given over a range of humidities for the sensitivity of salivary organisms to ultra-violet

irradiation when suspended as small particles in the air.

A chart is presented for the evaluation of the mean radiation intensity within a rectangular volume produced by a point source situated within or on the boundaries of the volume.

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