

BAKERIAN LECTURE: *The Distortion of an Aluminium Crystal during a Tensile Test.*

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The work described in the following pages was inspired by a paper in which Prof. Carpenter* and Miss Elam described the result of applying tensile tests to specimens of aluminium which had been treated in such a way that they appeared to turn into single crystals. The resulting distortions of the test pieces were very remarkable and clearly suggested that the crystal axes were not orientated in the same direction in different specimens. The uniformity of the distortion in different parts of the same specimen made it seem likely that it would be a straightforward, though possibly laborious, matter to determine the relationship between the orientation of the axes and the distortion produced in a tensile test. And it seemed possible that by examining a number of specimens some general results might be obtained about the forces necessary to produce distortions of this type.

On discussing the matter with Prof. Carpenter and Miss Elam it was found that it would not be possible to determine the distortion from the measurements they had already made. Moreover, no measurements of the orientation of the crystal axes had been made, though Sir W. Bragg had made a few observations indicating that the material retained its crystalline character after it had been distorted. Under these circumstances, it was decided to carry out a test, making all the necessary measurements at various stages during the extension of a specimen.

Before describing the test, however, it is necessary to refer to the work of previous experimenters on the subject.

Previous Work.†

When a metal is strained beyond the elastic limit a microscopic examination of the surfaces of crystals in it frequently shows the existence of lines known as slip bands. These bands have been shown to consist of small steps, and the conclusion is naturally drawn that there are planes inside the crystal, presumably crystal planes, on which slipping takes place. The bands would then mark the intersection of the face of the specimen with these crystal planes.

* These 'Proceedings,' vol. 100 (No. A 704, December, 1921).

† See Note added at end of Paper.

Up to the present, however, the evidence on slipping is purely qualitative. It has not been shown that the deformation of a metallic crystal when the material is strained is such as could be produced by slipping, nor has the relationship between the crystal axes and the slip planes been determined.

A number of experiments have been made on the direction of the crystal axes in drawn wires, and certain conclusions have been reached by Polanyi* in regard to the orientation of the crystal axes with respect to the axis of the wire. References are given in Polanyi's paper.

General Description of Test.

Before the test contemplated could be carried out, it was necessary to enlist the help of an expert in crystal analysis by X-rays. Fortunately, Dr. Alex Müller took up the work and succeeded in devising a satisfactory method of determining the orientation of the crystal axes. This method and some of his results will be described elsewhere.

The results of his and Miss Elam's X-ray analysis of the specimen with which the present work is concerned are given in Table V.

Most of the specimens with which Prof. Carpenter and Miss Elam's experiments had been carried out were flat strips, about $\frac{1}{8}$ inch thick \times 1 inch broad. These strips were unsuitable for the present purpose, partly because of the difficulty of making accurate measurements on the narrow faces and partly because they were so broad in comparison with their length that it was not possible to make sure of getting an evenly stretched parallel piece in the middle.

Specimens could be produced with circular, but not with square sections. On the other hand, there seemed to be no very simple way of making on a round specimen† the measurements which are necessary for calculating its distortion. For this reason, therefore, a round specimen was machined down till its section was square. Its dimensions were then approximately 1.0 \times 1.0 \times 20.0 cm. Each face was marked by a scratch parallel to the length of the specimen or axis, as it will be called, and by cross scratches. The appearance of the specimens so marked is shown in fig. 1. The faces were numbered 1, 2, 3, 4, so that when the specimen was placed upright in the testing machine, the faces appeared in this order when the observer moved round the machine in an anti-clockwise direction. Fig. 1 represents the specimen lying with its top end to the right.

At each successive stage of the test the extension between each pair of cross marks was measured on each face. The ratio of the length at any stage

* Polanyi, 'Die Naturwissenschaften,' April, 1922.

† This difficulty has now been overcome.

to the initial length will be denoted by the symbol ϵ . At the same time the angles between these cross scratches and the longitudinal scratch were measured in each case. These will be denoted by β on face 1 and γ on face 4.

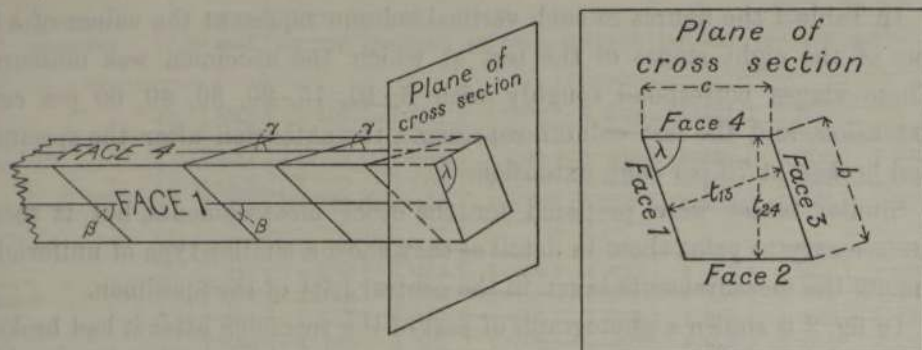


FIG. 1.—Scheme for marking and measuring specimen.

The thickness of the specimen between pairs of opposite faces, t_{13} and t_{24} , and the angles, λ , between neighbouring faces were also measured. These measurements are sufficient to determine the nature of the distortion. The scheme is illustrated in fig. 1.

Methods of Measurement.

The measurements of extension were made with a reading microscope, and the measurements of thickness between opposite faces with a micrometer. The angles between the cross lines and the axis of the specimen were measured with a crystallographer's microscope, with rotating eyepiece, containing cross wires, which was kindly lent us by Mr. A. Hutchinson, F.R.S. The angles between the faces were measured by sticking small pieces of cover-slip glass to them with gum, setting the specimen upright in a goniometer, and observing the reflection in the glass of a distant source of light.

Degree of Uniformity of Measurements.

If the distortion of the specimen had been uniform throughout its entire volume, and if the section had been accurately rectangular to begin with, the angles of the cross scratches measured on pairs of opposite faces would have been the same, the extensions measured on all four faces would have been the same, and the angles between any pair of faces would have been the same as that between the opposite pair.

It turned out that near the ends of the specimen, where it was held in the grips of the testing machine, the measurements were not quite the same as those near the middle, but that the central portion was nearly uniformly strained. The degree of uniformity can be judged from the figures given in Table I, which represent the ratio of the extended to the initial lengths

in each of the five compartments into which the faces were divided by the six cross lines. The ratio will be represented by the symbol ϵ , $\epsilon-1$ being the extension.

In Table I the figures in each vertical column represent the values of ϵ for one of the eight stages of the test at which the specimen was measured. These stages correspond roughly with 5, 10, 15, 20, 30, 40, 60 per cent. extension, and the last column represents the extension after the specimen had broken at 78 per cent. extension.

Similar tables were prepared for the other measurements, but it seems unnecessary to print them in detail as they show a similar type of uniformity among the measurements taken in the central part of the specimen.

In fig. 2 is shown a photograph of part of the specimen after it had broken. It will be seen that its appearance suggests that the distortion is due to a uniform strain at all points, except those which lie in a small region close to the point where the breakage occurred.

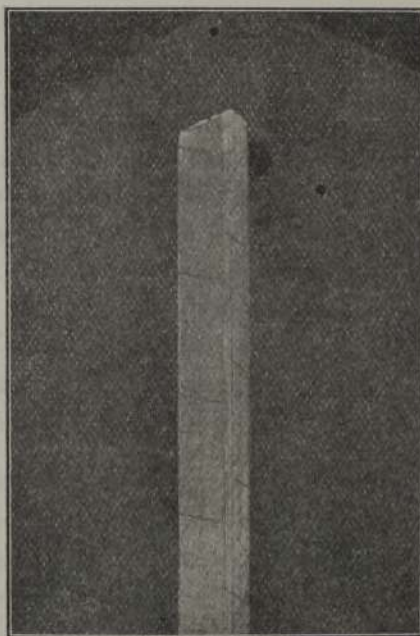


FIG. 2.

Dimensions of Mean Parallelepiped.

On looking at Table I it will be seen that there is a high degree of uniformity among the measurements of extension in the three middle compartments on each face, especially in the earlier stages of the test.

Similar conclusions were reached by inspection of the other measurements ;

Table I.—Ratio of Extended to Initial Length in each of the Five Compartments into which each Face was Divided.

Approximate extension	5 per cent.	10 per cent.	15 per cent.	20 per cent.	30 per cent.	40 per cent.	60 per cent.	78 per cent.
Face 1	1·049	1·103	1·157	1·196	1·300	1·401	1·623	—
	1·047	1·108	1·160	1·199	1·309	1·414	1·645	1·834
	1·058	1·116	1·164	1·200	1·303	1·398	1·604	1·756
	1·052	1·109	1·160	1·199	1·300	1·394	1·595	1·739
	1·052	1·109	1·153	1·278*	1·297	1·390	1·582	1·692
Face 2	1·043	1·100	1·141	1·185	1·283	1·384	1·599	—
	1·048	1·106	1·160	1·199	1·300	1·403	1·628	1·805
	1·053	1·112	1·162	1·201	1·311	1·412	1·645	1·845
	1·051	1·107	1·161	1·198	1·295	1·393	1·649	1·720
	1·052	1·109	1·161	1·202	1·308	1·406	1·561	1·735
Face 3	1·046	1·099	1·153	1·191	1·296	1·392	1·618	1·828
	1·051	1·111	1·162	1·202	1·311	1·424	1·650	1·845
	1·059	1·112	1·163	1·201	1·304	1·401	1·613	1·760
	1·054	1·110	1·163	1·201	1·298	1·397	1·596	1·800
	1·056	1·111	1·150	1·198	1·300	1·390	1·581	1·611
Face 4	1·051	1·103	1·157	1·200	1·304	1·410	1·641	—
	1·050	1·111	1·169	1·203	1·316	1·420	1·648	1·830
	1·056	1·112	1·152	1·200	1·300	1·400	1·615	1·772
	1·054	1·109	1·159	1·198	1·299	1·394	1·594	1·720
	1·058	1·111	1·080	1·201	1·300	1·393	1·591	1·705
Means of middle three	1·053	1·110	1·161	1·200	1·304	1·404	1·623	1·785

* Probably an error in measurement.

accordingly, the mean has been taken in each case of the measurements made in each of the three central compartments. The mean extension, for instance, at any stage is the mean of the twelve figures obtained on all four faces. These are given at the bottom of Table I.

The mean angle of the cross line with the axis of the specimen is found by taking the mean of the eight measurements made on the four middle cross marks on two opposite faces.

In this way the dimensions and angles of a series of mean parallelepipeds have been drawn up, and these have been used in calculating the strain, or distortion, of the specimen at any stage. They are given in Table II.

It is known that when aluminium is worked the change in density is small, at any rate it is less than 1 per cent. The volumes of the strained figures should differ by less than 1 per cent. from the volumes of the unstrained figure. The ratio of the strained to unstrained volumes has been calculated from the figures in Table II. They are given at the bottom of Table II, and on inspecting them it will be seen that none of them differ from 1 by more

than 1 per cent., except the figure in the last column but one corresponding to an extension of 62 per cent. This differs from 1 by nearly 8 per cent. It seems probable, therefore, that some mistake has been made in this case, and all the measurements for this stage have accordingly been rejected.

Table II.—Dimensions of Mean Parallelepiped.

Extension	0	1·053	1·110	1·161	1·200	1·304	1·404	1·623	1·785	
Thickness in mm. $\left\{ \begin{array}{l} t_{13} \dots \\ t_{24} \dots \end{array} \right.$	$t_{13} \dots$	10·16	9·68	9·17	8·82	8·55	7·90	7·38	6·57	6·03
	$t_{24} \dots$	10·35	10·33	10·31	10·28	10·26	10·12	10·00	10·17	9·29
Angle between Faces 1 and 4, λ°	90·6	88·8	87·2	86·7	84·9	81·8	79·2	73·1	70·8	
Angle between cross marks and axis $\left\{ \begin{array}{l} \text{Face 1, } \beta^\circ \\ \text{Face 4, } \gamma^\circ \end{array} \right.$	Face 1, β°	90·0	88·5	87·3	86·7	85·7	84·2	83·0	80·2	78·0
	Face 4, γ°	90·0	90·3	89·7	89·3	88·5	85·2	82·1	73·2	65·5
Volumes	1·000	1·002	0·990	1·003	1·005	1·002	1·003	1·078	1·007	

Analysis of Strain in Mean Parallelepiped.

It is obvious that the measurements given in Table II are sufficient to determine the strain completely. A uniform strain can be specified by giving the directions and magnitudes of the axes of the ellipsoid into which a sphere of unit radius in the unstrained material is transformed by the strain. This ellipsoid, known as the "strain ellipsoid," can be found from the measurements of Table II, but it is not at once obvious how the axes of the strain ellipsoid would be related to the crystal axes, though there would probably be some indirect connection between the two. On the other hand, there are other possible ways in which a uniform strain could be specified, and some of these may be more likely to throw light on the present problem than others.

For this reason it seems desirable to consider what types of strain can be conceived which would satisfy the conditions indicated by experiment, namely, that the material remains a crystal, so that all molecules are orientated in the same direction at any stage in the strain, and that the density is practically unchanged. In the first place, it seems clear that the relative displacements of neighbouring molecules cannot be the same as the relative displacements of particles in a similar position in the material in bulk. If they were the same, that is, if the strain were uniform even when portions of matter of molecular dimensions were examined, the material would remain crystalline when subjected to any uniform strain (it is true) but the crystal symmetry would be altered by the strain. A plane of molecules which lay in a crystal plane in the unstrained material would continue to do so in the

strained material, and the angle between two such planes of molecules would alter continuously during any continuously varying strain. This state of affairs is mathematically conceivable, but it is contrary to all physical experience; moreover, measurements of the inclination of two different crystal planes during the present experiments showed that the angle between them remained practically constant during the whole course of the distortion. (See last column Table V.)

The fact that the material preserves its cubic crystal symmetry during the distortion, leads to the conception that the apparently uniform strain of the material in bulk must be made up of a large number of non-uniform strains or relative displacements between neighbouring molecules.

In order that crystal symmetry may be preserved, the displacement of any molecule relative to a neighbouring molecule must be at least as great as the distance between neighbouring molecules in the direction of its displacement. A small strain in the material in bulk must therefore be due in some way to the occurrence of a small number of these relatively large displacements, the greater part of the molecules preserving their relative positions unchanged.

When a molecule is displaced along any line into the position of a neighbouring molecule, the molecule which previously occupied that space must displace in its turn a third molecule and so on. Owing to the fact that the stress in a material under a uniform tension is constant along any line, it seems likely that all the molecules in a line would be displaced together. By making appropriate combinations of displacements of lines of molecules parallel to a given line in the crystal, the material in bulk might be conceived to assume a great variety of shapes, but they would all be characterised by the fact that lines of particles parallel to this direction would be unchanged in length during the distortion.

These considerations, though admittedly hypothetical, suggest that it would be more promising to analyse the strain, with a view to finding the directions in the material which are unchanged in length than to find the strain ellipsoid. In a uniform strain, the directions which remain unchanged in length lie on a quadric cone in the strained material. This cone evidently passes through the curved line of intersection of the strain ellipsoid and the unit sphere when these two are placed concentrically. The principal axes of this cone therefore coincide in direction with those of the strain ellipsoid.

The particles of this "unstretched cone" have evidently two positions corresponding with the unstrained and the strained material respectively. It is necessary to determine both of them.

Determination of the Cones of Unextended Directions.

The method adopted was to find first a series of corresponding positions in the strained and unstrained material respectively of planes of particles which pass through one edge of the specimen; actually the intersection of faces 1 and 4 was chosen. This was accomplished as follows:—

The cross-sections of the strained and unstrained specimen were set out on a piece of paper in such a way that the lines representing face 1 coincided in direction, while the points representing the edge of intersection of faces 1 and 4 coincided. The figure produced in this way is shown in fig. 3; OPQR is the cross-section of the unstretched specimen, while OSTU is that of the stretched specimen. Evidently, OR, OU, which are the traces of face 1, and OP, OS, which are the traces of face 4, are the traces of two pairs of corresponding planes; the diagonals OQ, OT are another pair. To find

others, take any point V in QR, and let n represent the ratio $RV:RQ$. Divide UT so that $UW = nUT$. Then OV and OW are the traces of corresponding planes.

Let λ_1 and λ_2 be the angles between face 1 and face 4 before and after straining, namely, POR and SOU in fig. 3, let b_1, b_2 represent the breadth of the face 1 before and after straining, so that OR is b_1 and OU is b_2 , and let c_1, c_2 represent the corresponding breadths of face 4. If ϕ_1 and ϕ_2 are the angles between corresponding planes in the unstrained and strained material and face 1 in each case, and if d_1, d_2 represent OV, OW, the breadths of the corresponding longitudinal sections, then

$$\left. \begin{aligned} d_1^2 &= b_1^2 + n^2 c_1^2 + 2nb_1 c_1 \cos \lambda_1 \\ d_2^2 &= b_2^2 + n^2 c_2^2 + 2nb_2 c_2 \cos \lambda_2 \\ \sin \phi_1 &= nc_1 \sin \lambda_1 / d_1 \\ \sin \phi_2 &= nc_2 \sin \lambda_2 / d_2 \end{aligned} \right\} \quad (1)$$

Varying n from $-\infty$ to $+\infty$, all possible pairs of corresponding planes through the edge where faces 1 and 4 intersect are found.

The next step is to find the directions in corresponding planes which are unchanged in length by the strain. These are evidently the intersections of the cones of unstretched directions with this pair of corresponding planes.

At this stage it is necessary to make use of the observed angles between the cross lines on faces 1 and 4 and the axis of the specimen. If corresponding values of these are β_1, β_2 on face 1 and γ_1, γ_2 on face 4, it is possible to find, geometrically or analytically, the angles α_1, α_2 between the axis of the specimen and the lines of intersection of the planes containing the cross lines on faces 1 and 4 and any pair of corresponding planes through the edge where faces 1 and 4 intersect. The relation between $\alpha, \beta, \gamma, \phi$, and λ is shown in fig. 4.

Next by rotation, about the line of intersection of faces 1 and 4, bring the two corresponding planes into coincidence. On these two coincident planes construct two corresponding parallelograms, ABCD, A'EFG (see fig. 5), as follows:—

Draw two lines, BC, EF, parallel to AD (which is on the line of intersection of faces 1 and 4), and at distances d_1 and d_2 from it. At the point A draw two lines, AB, AE, at angles α_1 and α_2 , to AD, so that $\angle BAD = \alpha_1, \angle EAD = \alpha_2$. On AD take any length AD in the unstrained plane. The length AG of the corresponding side of the strained rectangle is found by taking $AG = \epsilon(AD)$, where ϵ is the ratio of the lengths of the specimen after and before stretching. ABCD and A'EFG are the unstrained and strained shapes of a parallelogram of particles in the material.

To find the pairs of lines which are unstretched by the strain:—

Join BE and CF, fig. 5, and let them cut in H. Any straight line through H cuts the lines BC and EF in two points, K_1 and K_2 say, which are such that AK_1, AK_2 are corresponding lines of particles in the unstrained and strained figures respectively.

To find the positions of K_1 and K_2 which are such that $AK_1 = AK_2$, draw a line midway between BC and EF and parallel to them; let it cut a circle described on HA as diameter in I and J. Join HI, HJ, and let these lines

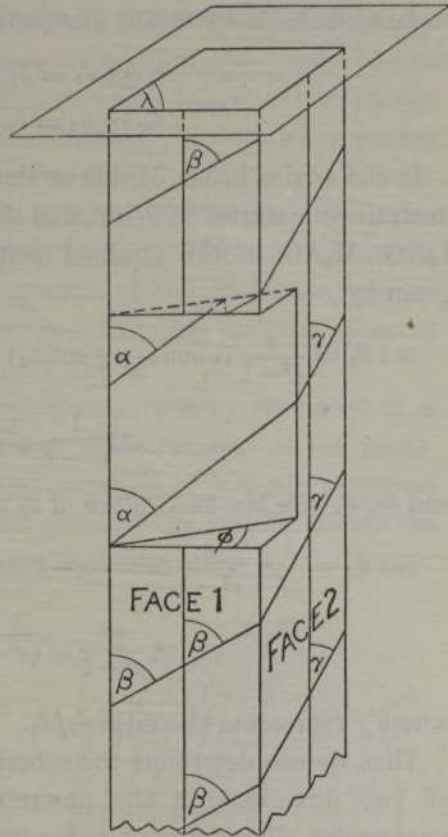


FIG. 4.—Diagram showing how α and ϕ are related to the faces of the specimen.

cut BC, EF in L_1, L_2 , and M_1, M_2 . Evidently $AL_1 = AL_2$ and $AM_1 = AM_2$, so that these lines are in the required directions.

These operations can be performed analytically, and it is sometimes more convenient to do so. Determining d_1, d_2, ϕ_1, ϕ_2 from the measured quantities $b_1, b_2, c_1, c_2, \lambda_1, \lambda_2$ by means of equation (1), the angles α_1, α_2 are given by

$$\left. \begin{aligned} d_1 \cot \alpha_1 &= b_1 \cot \beta_1 + nc_1 \cot \gamma_1 \\ d_2 \cot \alpha_2 &= b_2 \cot \beta_2 + nc_2 \cot \gamma_2 \end{aligned} \right\}. \quad (2)$$

If the angles L_1AD, M_1AD of the two unstretched lines of particles in the unstrained material be θ_1, θ_1' , and if θ_2, θ_2' represent the corresponding angles L_2AG, M_2AG in the strained material, θ_1, θ_1' are the two values of θ_1 given by

$$\cot \theta_1 = \frac{\epsilon}{\epsilon^2 - 1} (\epsilon \cot \alpha_1 - f \cot \alpha_2) \pm \frac{1}{\epsilon^2 - 1} \sqrt{(\epsilon^2 - 1)(1 - f^2) + (\epsilon \cot \alpha_1 - f \cot \alpha_2)^2}, \quad (3a)$$

and θ_2, θ_2' are the two values of θ_2 given by

$$\cot \theta_2 = \frac{1}{\epsilon f} \left\{ \frac{\epsilon}{\epsilon^2 - 1} (\epsilon \cot \alpha_1 - f \cot \alpha_2) \pm \frac{\epsilon^2}{\epsilon^2 - 1} \sqrt{(\epsilon^2 - 1)(1 - f^2) + (\epsilon \cot \alpha_1 - f \cot \alpha_2)^2} \right\}, \quad (3b)$$

where f represents the ratio d_2/d_1 .

Thus we can determine the spherical polar co-ordinates (θ_1, ϕ_1) and (θ_1', ϕ_1) of two directions in the unstrained material which after straining are unextended. Their new co-ordinates are then (θ_2, ϕ_2) and (θ_2', ϕ_2) . By taking all possible values of n in formulæ (1) and (2) we can map out the two corresponding cones of unstretched directions.

Representation of Unstretched Cones.

The natural way to represent these cones is on a sphere, but for convenience it is necessary to use some flat representation of the sphere. The most convenient of these is the stereographic projection, which is the projection from a point on the surface of the sphere on to the plane through the centre parallel to the tangent plane at the point of projection.

The plane of projection is divided by the sphere into two parts. The inner part represents the hemisphere which lies on the opposite side of the plane of projection to the point of projection. As a rule, a convention is adopted which enables the whole sphere to be represented by the area inside the

circle of intersection of the sphere and plane of projection. Assuming the plane to be horizontal, all points of the hemisphere above this are projected from the lowest point of the sphere, while all points of the lower hemisphere are projected from the highest point of the sphere. The representations of the two hemispheres then cover the same area, and they are distinguished by representing points of the upper hemisphere with a dot, and points of the lower hemisphere with a cross. In the diagrams which follow, curves on the upper hemisphere are represented by full lines through the dots, while curves on the lower hemisphere are represented by dotted lines through the crosses. With these conventions a great circle on the sphere is represented by a lens-shaped figure, consisting of two equal circular arcs, which pass through opposite ends of a diameter of the bounding circle of the projection, and lie on opposite sides of it. One of these arcs is a full line and the other is dotted. Several of them will be seen in figs. 7 and 8.

The usefulness of the stereographic projection is greatly increased if a figure known as a "stereographic net" is used. This consists of the representation of a series of meridian circles and parallels of latitude projected from a point on the equator, so that the two poles of the system lie on the bounding circle of the projection. Such a net is shown in fig. 6.

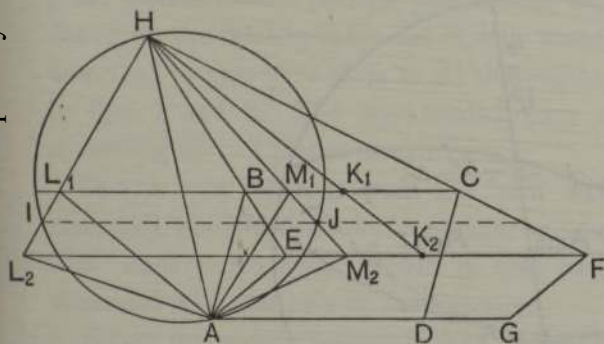


FIG. 5.—Construction for finding directions which are unchanged in length by strain.

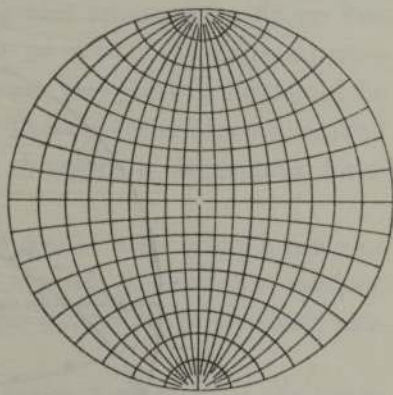


FIG. 6.—Stereographic Net.

To set out a point on a stereographic projection when its spherical polar co-ordinates (θ_1, ϕ_1) are given, let the pole of the spherical co-ordinates, *i.e.*, the point $\theta = 0$, be represented by the centre of the stereographic figure. Draw a circle on a piece of tracing paper equal to the bounding circle of the stereographic net, and mark a radial line on it to represent the meridian circle $\phi = 0$. Next draw another radial line at angle ϕ_1 with the line to represent the meridian $\phi = \phi_1$. Then lay the tracing paper over the stereographic net

so that the line $\phi = \phi_1$ coincides with one of its two principal diameters, and mark off a point to represent the angle $\theta = \theta_1$, using a dot if θ_1 is less than $\frac{1}{2}\pi$ and a cross if it is greater. The points in figs. 7 and 8 are plotted in this way.

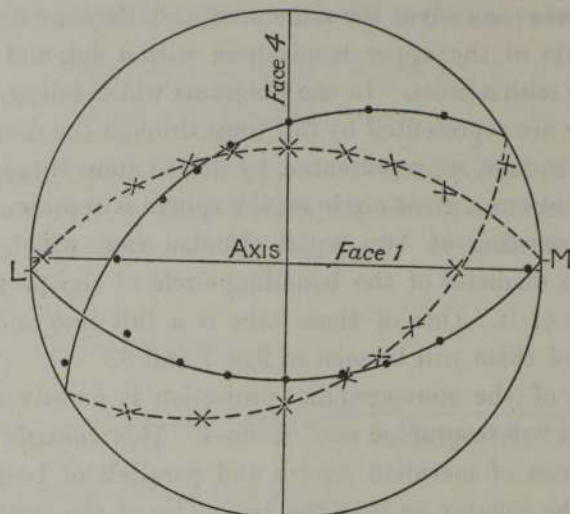


FIG. 7.—Stereographic projection of positions in *unstrained* material of cone of directions which are unstretched after the material has stretched by 30 per cent.

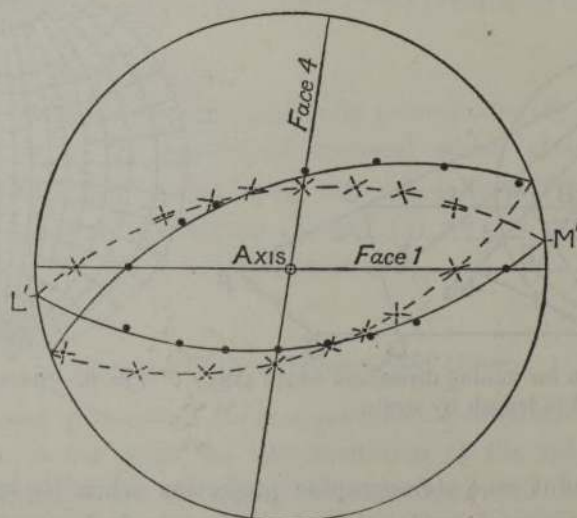


FIG. 8.—*Strained* position of cone shown in fig. 7.

Application of the Method in the present Case.

To analyse a strain by this method, two stereographic figures must be made, showing the undistorted and the distorted positions of the unstretched cone.

In the first instance the distortion produced by a 30-per-cent. extension of the specimen was analysed. Using the following values from Table II,

$b_1 = t_{24}/\sin \lambda_1 = 1.035$, $b_2 = t_{13}/\sin \lambda_2 = 1.022$, $c_1 = 1.016$, $c_2 = 0.798$,
 $\lambda_1 = 90^\circ 35'$, $\lambda_2 = 81^\circ 50'$, $\beta_1 = 90^\circ$, $\beta_2 = 84^\circ 10'$, $\gamma_1 = 90^\circ$, $\gamma_2 = 85^\circ 10'$,
 and taking n successively as 0, $\frac{1}{2}$, 1, 2, ∞ , -2 , -1 , $-\frac{1}{2}$, the values given in Table III were calculated for $\phi_1, \phi_2, \theta_1, \theta_1', \theta_2, \theta_2'$. These values were then set out on the two stereographic diagrams shown in figs. 7 and 8.

Table III.—Spherical Polar Co-ordinates of Directions which are unextended during an increase of 30 per cent. in length of the Specimen.

n .	Unstrained material.			Strained material.		
	ϕ_1° .	θ_1° .	$\theta_1'^\circ$.	ϕ_2° .	θ_2° .	$\theta_2'^\circ$.
-2	-62.6	52.0	126.2	-63.3	37.4	141.4
-1	-44.2	57.8	125.9	-42.1	44.3	138.0
$-\frac{1}{2}$	-25.7	67.7	122.5	-22.7	56.5	130.5
0	0	87.3	112.9	0	80.7	114.5
$\frac{1}{2}$	25.8	111.6	93.0	20.5	111.6	87.0
1	44.7	121.0	81.1	35.2	124.7	71.4
2	63.4	127.5	69.4	51.6	135.3	56.1
∞	90.6	131.0	58.1	81.8	143.7	41.8

It was noticed that the points of these diagrams appeared to lie on two great circles, and on placing the tracing paper on which the diagrams were drawn on the stereographic net it was found that two great circles could be described to pass very close to all the points. These great circles are marked in figs. 7 and 8. The unextended cone is therefore a degenerate form, consisting of two planes in this case.

This is very different from the form which would arise when a bar was stretched which consisted of a number of small crystals orientated at random. The "unextended cone" would in that case be a circular cone whose axis coincided with the axis of the specimen.

Before proceeding further it may be well to notice the nature of the strain which causes the unextended cone to take the form of two planes. All figures described on either of these planes are unchanged both in dimensions and in shape by the strain; that is to say, the strain does not affect the relative positions of particles in either of them. Since the density of the material is practically unchanged by the strain, the distance apart of any two planes of particles which are parallel to either of these planes is also unchanged by the strain. The strain brought about by the extension of 30 per cent. can therefore be regarded as being due to a simple shear parallel to either of the two unstretched planes.

At a later stage we shall see that the X-ray analysis shows that the orientation of the molecules remains constant with respect to one of them, but not with respect to the other. From the external measurements taken at only two stages of the extension, however, we cannot distinguish between them and say that the shear is due to slipping on one plane rather than on the other.

Application of the Method to other Stages of the Extension.

The same method was applied to the other stages in the extension. It was found that in the cases 0-10, 0-15, 0-20, and 0-40 per cent. extension, the same result was obtained as in the case of 0-30 per cent., which has already been studied. In each case the cone of unstretched directions was found to consist of two planes. In the case of the final extension 78 per cent., however, it was found that this was not the case. We shall therefore study first all the stages of the extension up to 40 per cent.

On comparing the stereographic diagrams showing the positions in the unstrained material of the planes which are unstrained at the various stages of the extension up to 40 per cent., it was found that one of the planes in each case consisted always of the same particles.

Table IV.—Spherical Polar Co-ordinates of Directions in unstrained Material which remain unstretched at four stages of the test, namely, 0-10, 0-20, 0-30, 0-40 per cent. extension.

n.	ϕ_1°	θ_1°				$\theta_1'^\circ$			
		0-10.	0-20.	0-30.	0-40.	0-10.	0-20.	0-30.	0-40.
0	0	86.5	87.5	87.3	87.0	114	113	113	111
1	45	122.3	120.5	121.0	120.2	74.3	78.2	81.3	82.2
∞	90.6	132.5	131.3	131.3	130.0	48.8	53.0	58.0	60.2
-1	-45	57.2	58.5	58.2	58.5	133.3	129.2	125.8	123.2

In Table IV are given the spherical polar co-ordinates of eight directions which remain unextended at each of the four stages, 10, 20, 30, and 40 per cent. extension. It will be seen that the figures under heading θ_1 , which give the position before straining of one of the unextended planes, are practically identical for all four stages of the test. The figures under the heading θ_1' , which gives the positions of the other planes, vary according to the particular stage concerned. It appears, therefore, that if the shear which produces the extension be considered as due to slipping on the plane given by the columns under heading θ_1 (Table IV), this plane of particles remains

undeformed and unextended throughout the whole extension from 0 to 40 per cent. If, however, the other planes determined by the figures under the heading θ_1' be considered as planes of slipping, these planes will consist of different particles at different stages of the extension.

It is evident that the former is a simpler physical conception, but one cannot be sure that it is the correct one till one studies the directions of the crystal axes. Before proceeding to do this, however, there is still some more information to be got out of the measurements of the external shape of the specimen, the direction of the shear on the slipping plane can be determined.

Direction of Slipping on Unextended Planes.

Let us now consider how the unextended planes would be situated in the case of a simple shear on a given plane of particles. Let ABCD (fig. 9) be a rectangle in the plane perpendicular to the plane of slip, and let AD be in the plane on which the slipping takes place. Let the sheared position of BC be B'C'. All particles in the line BB'CC' shift through distance equal to BB'. One of the unextended lines of particles is evidently AD. The other is found by taking two points E, E' at distances equal to $\frac{1}{2} BB'$ on opposite sides of B. The line EA in the unstrained material moves to E'A in the strained material and $AE = AE'$, so that AE, AE' are the traces of the second unextended planes.

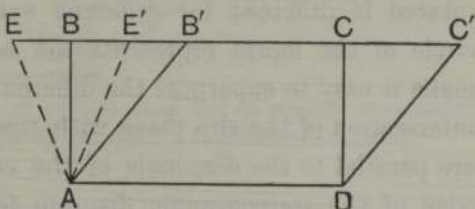


FIG. 9.—Section of material shearing on a slipping plane.

From these considerations it will be seen that:—

(a) The direction of shear is in the line at right angles to the line of intersection of the two unstretched planes.

(b) In a finite shear, due to slipping on one plane, the strained and unstrained positions of the other plane of particles, which is undeformed, lie at equal angles on opposite sides of the plane, which is perpendicular to the plane of slip and the direction of slip, *i.e.*, the plane whose trace is AB. If the slip be regarded as being measured by $\tan BAB'$, fig. 9, this is equal to $2 \tan BAE$, *i.e.*, where $\frac{1}{2} \pi - BAE$ is the angle between the two undeformed planes.

(c) The only plane which remains undeformed during the whole course of a finite shear is the plane of slip.

To determine the direction of shear, therefore, it is necessary to find the line of intersection of the two planes which constitute the "cone of unextended directions." In the present case this was accomplished by constructing a

stereographic diagram in which the plane of slipping, *i.e.*, the plane which was undistorted throughout the extension from 0 to 40 per cent., was represented by the outer circle of the diagram, the normal to the plane of slipping being the centre.

In the case of the figure corresponding with the unstrained material this amounts, in the present case, to rotating the figure through 40° about the line of intersection of the slip plane with the plane perpendicular to the axis of the specimen. This operation can easily be performed with the stereographic net. In the case of the 30 per cent. extension diagram, for instance, the net is placed with its poles on the points L, M (fig. 7). The new positions of the points of the diagram are then found by moving them 40° round on a latitude circle of the net.

In the case of the figure for the strained material the same procedure is adopted, but in this case the angle through which the diagram has to be rotated is different for different extensions. In each case the boundary circle of the figure represents the same plane of particles, and, in order to make it easy to superpose the different figures, the directions of the lines of intersection of the slip plane with the faces and with the axial planes which are parallel to the diagonals of the cross-sections are marked as dots on the edge of the stereographic diagram for each extension. The fact that these dots can be superposed with diagrams obtained from measurements taken at different stages of the extension is confirmatory evidence that the slip plane is undistorted.

In fig. 10 is shown the position in the unstrained material of the second set of planes, which are undistorted at four stages of the extension, namely, 10, 20, 30 and 40 per cent. In fig. 11 is shown the positions of three of these same planes in the strained material.* It will be seen that in both cases all the great circles which represent the second set of planes cut the boundary of the diagram near the same two points. The direction of slip, which, as can be seen from consideration (a) above, is represented by the point on the boundary at 90° from these points, is therefore *practically the same for all the stages of the test up to 40 per cent. elongation of the specimen.*

It will be noticed that the relation between figs. 10 and 11 is what we should expect from consideration (b) above. The fact that the dotted lines in fig. 10 correspond with the full lines in fig. 11 and *vice versa*, shows that the second set of planes make equal angles on opposite sides of the plane perpendicular to the direction of shear.

* The position of the second undistorted plane for 10 per cent. extension has not been shown in this figure because it confused the central part of the diagram.

To summarise, we have shown that, up to 40 per cent. elongation, the distortion of the specimen is due to simple shear in one direction on a

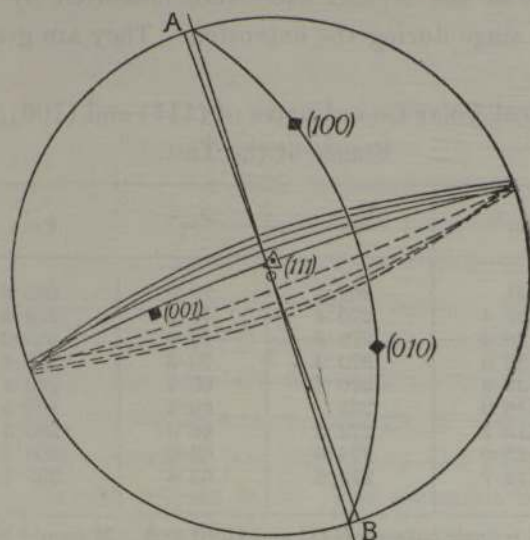


FIG. 10.—Projection on to the plane of slip of crystal axes and of second unstretched planes for extensions 0–10, 0–20, 0–30, 0–40 per cent. *Unstretched material.*

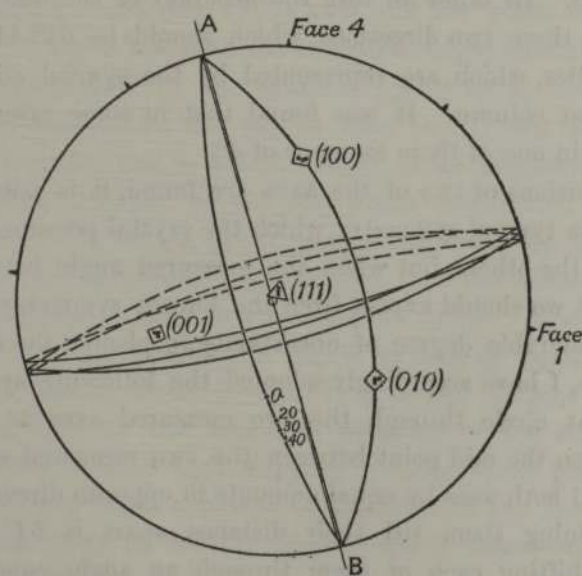


FIG. 11.—Positions in *strained material* of the axes and planes shown in fig. 10. The positions of the axis at 0, 20, 30 and 40 per cent. extension are also shown.

certain plane. We have determined the plane and the direction. It now remains to determine the relation between this plane and the crystal axes.

Relation between Plane of Slip and Crystal Axes.

The co-ordinates of the crystal axes were measured by Dr. Müller and Miss Elam at each stage during the extension. They are given in Table V.

Table V.—Spherical Polar Co-ordinates of (111) and (100) Axes at various Stages of the Test.

ϵ .	θ_{111}° .	ϕ_{111}° .	θ_{100}° .	ϕ_{100}° .	i° .
1·000	131	269·3	79·0	262·2	52·3
1·053	127·4	270·2	75·0	263·8	52·5
1·110	126·0	270·3	73·3	263·2	53·2
1·161	123·5	270·9	70·2	263·8	53·6
1·200	123·2	270·5	69·7	263·2	54·0
1·304	118·9	221	69·5	262·2	50·2
1·404	118·2	272·2	66·0	260·5	53·4
1·623	113·9	274·0	62·0	260·6	52·8
1·785	114·7	272·8	62·4	257·7	54·5

NOTE.— i is angle between (111) and (100) axes. It should be $54^\circ 7'$

It will be seen that one of the (111) planes and one of the (100) planes were determined. In order to test the accuracy of the determinations, the angles between these two directions, which should be $54^\circ 44'$, were worked out. These angles, which are represented by the symbol i in Table V, are given in the last column. It was found that in some cases there was an error of 2° , and in one of them an error of 4° .

When the positions of two of the axes are found, it is possible to use our knowledge of the type of symmetry which the crystal possesses to determine the positions of the others, but when the measured angle between these two axes is not what we should expect from the known symmetry of the crystal, there is a considerable degree of uncertainty in placing the other axes. In placing the axes, I have accordingly adopted the following system:—I have taken the great circle through the two measured axes as being correct. I have then taken the mid point between the two measured axes as correct, and have shifted both axes by equal amounts in opposite directions along the great circle joining them till their distance apart is $54^\circ 44'$. This is equivalent to shifting each of them through an angle equal to half the error in the determination of the angle between them. It is then possible to find the positions of the remaining axes. Such a proceeding is very arbitrary, but something of the kind is necessary, and any system adopted must be arbitrary. The maximum angle through which either of the axes is shifted by this scheme of averaging is $2\cdot2^\circ$ in the case of the 30 per cent. extension. The shifts in all other cases are less than this, the average being $1\cdot0^\circ$.

Fortunately, the distortion of the material is so great that very large changes in the orientation of the crystal axes relative to *some* planes of particles must take place.

To find the relative positions of the axes and the slip plane, the particular triad (111) axis which was measured and the three tetrad (100) axes have been marked on the stereographic diagrams (figs. 10 and 11). Fig. 10, which applies to the unstrained material, of course, has only one set of axes marked on it, but fig. 11 has a set of axes for each stage of the extension.

On looking at these diagrams the following facts will be noticed :—

(1) The (111) axis is close to the pole of the diagram. That is, the plane of slip is nearly coincident with a (111) plane of the crystal.

(2) The positions of all the axes are fixed relatively to the plane of slip during the whole course of the distortion, though, of course, the lines of particles which originally occupied positions along the axes move through very considerable angles with respect to it. The variation in the position of the normal to the (111) plane, for instance, is only about 4° .

(3) The great circle through two of the (100) planes is very nearly parallel to the direction of slip. The angle which the line of intersection of the (100) plane containing these two (100) axes and the plane of slip makes with the direction of slip is less than the margin of experimental error.

If the planes represented by the symbols $(\pm 1 \pm 1 \pm 1)$ be regarded as forming a regular octahedron, the slip is then on a plane which is very nearly parallel to one face, and is almost exactly in the direction of one edge.

The fact that the position of the slip plane determined by external measurements is so close to the (111) plane determined by X-ray analysis makes one suspect that they really coincide, and that the difference between their measured positions is due to experimental error. This idea is confirmed by the fact that the direction of slip is so exactly parallel to one of the three principal lines of atoms in the (111) plane. It may be, however, that the difference between the measured positions of the slip plane and the (111) plane is due to some slight slipping on some other plane. During the test, two sets of slip bands appeared on the faces of the specimen. It is hoped to settle this point in another test, in which improved methods of measurement will be used.

In the remaining part of this paper we shall assume that the slip was actually on the (111) plane, and shall discuss some possible consequences of this assumption.

Changes in Orientation of Specimen relative to Crystal Axes during Extension.

The effect of a shear in any direction on a fixed plane of particles is to move any given line of particles in such a way that it remains parallel to the plane containing its original direction and the direction of slip. In fig. 11, for instance, the positions of the axis of the specimen relative to the slip plane at successive stages of the extension is shown. These are marked 0, 20, 30, 40. It will be seen that a great circle can be drawn to pass through them and through the direction of slip.

As the point representing the direction of the axis moves along this circle the angles between this direction and all the four octahedral planes vary continuously. After a time, it seems likely that the axis of the specimen will move into a position in which it makes an angle with one of the other octahedral planes which is equal to the angle it makes with the particular octahedral plane on which the slipping is taking place. At this stage the force tending to produce slipping on one plane is equal to that tending to produce slipping on the other. It is possible, therefore, that at this stage slipping may take place on both planes. Possibly this is the reason why the cone of unstretched directions in the case of the 78-per-cent. extension no longer consists of two planes.

To examine this point, first turn the axes round so that one of the cubic or tetrad axes is in the centre of the figure. The two remaining cubic axes will then be found on the circumference, and the four octahedral axes will be disposed symmetrically round the central cubic axis.

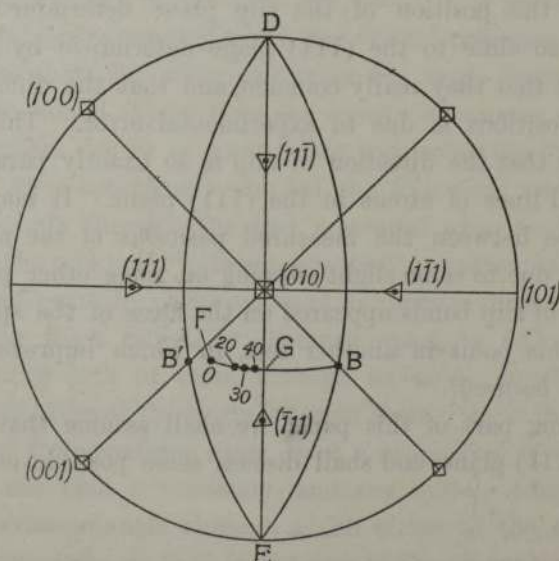


FIG. 12.—Figure showing position of axis of specimen relative to crystal axes at 0, 20, 30, 40 per cent. extension.

In fig. 12 the axes have been rotated so that the cubic axis denoted by (010) in fig. 11 is in the centre. The slip plane is then represented by the great circle DBE (fig. 12), and the point B represents the direction of slip, corresponding with the point B in fig. 11.

We have already shown that the points representing the axis of the specimen lie on a great circle through B. An arc of this circle is shown in fig. 12 as FGB, where G is on the line DE which represents the (101) plane.

As the slipping on the plane DBE proceeds, the point representing the axis of the specimen moves along the arc FGB till, at about 50 per cent. extension, it gets to the point G. At this point, G, it is in such a position that it is equally inclined to the two octahedral planes DBE, DB'E, which are perpendicular to the two axes (111) and ($\bar{1}\bar{1}$ 1).*

It should now be noticed that up to this stage of the extension the slipping has always been along that one of the twenty-four crystallographically similar possible directions of shear which has the maximum component of shearing force tending to set it going. If this condition continues to hold, then directly the point representing the axis has moved beyond G along its path FGB, the force tending to produce a slip on the plane DB'E in the direction of B' is greater than the force tending to continue the slip on DBE. Slipping will then begin on DB'E, and the axis will start moving along a great circle towards B'.

It can easily be seen that the result of successive slips towards B and B' will be that the point representing the axis will move along the line DGE towards the point where the great circle through B and B' cuts it. At this point no further movement of the axis will take place, however much the specimen extends.

These considerations lead to the prediction that there will be a tendency for the axes to orientate themselves, so that two of the octahedral planes make equal angles of $61^{\circ} 52'$ with the axis of the specimen, while their line of intersection makes an angle of $54^{\circ} 44'$ with the axis.

It is hoped to examine this point later, when the distortions and crystal axes of more specimens have been determined.

Verification that Slipping in the Last Stage of the Distortion occurs simultaneously on Two Octahedral Planes.

Though no very satisfactory method has yet been devised for determining the distortion due to simultaneous slipping on two planes, it is possible to apply the methods used in the early part of this paper to test whether the distortion of the specimen at its breaking point (78 per cent. extension)

* No distinction is made here between the planes (111) and ($\bar{1}\bar{1}$ 1).

could have been produced by slipping on the two octahedral planes which are at equal angles to the axis when the extension is about 50 per cent.

The cones of unextended directions in the distortion of the specimen during its extension from 40 to 78 per cent. were first determined by the

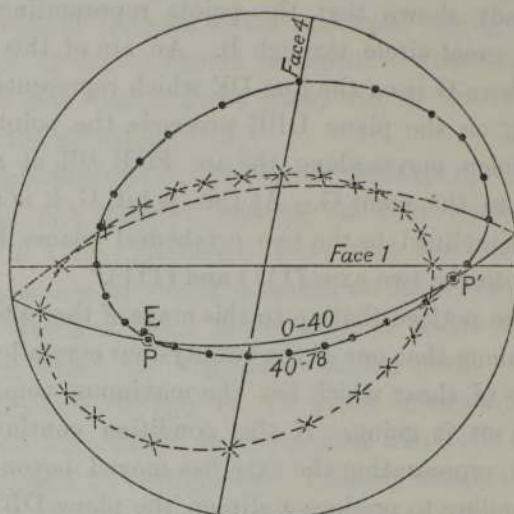


FIG. 13.—Positions in material extended 40 per cent. of unstretched cone for extension from 40 to 78 per cent., and of slip plane for extension 0 to 40 per cent.

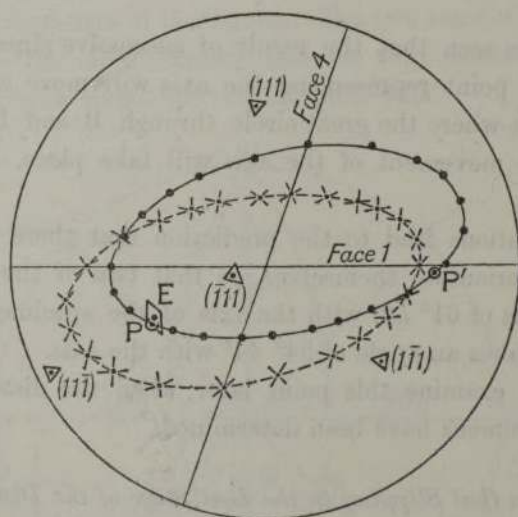


FIG. 14.—Position in material extended 78 per cent. of unstretched cone shown in fig. 13.

equations (1), (2), and (3). The two positions of this cone are plotted on the stereographic figures 13 and 14. Fig. 13 shows that the position of the cone in the material at 40 per cent. extension, while fig. 14 shows its position at 78 per cent. extension. In both these figures the centre spot represents the

axis of the specimen, as it does in figs. 7 and 8. It will be seen that the cone is now very different from the pair of planes which were characteristic of the first 40 per cent. of extension. The distortion is, therefore, no longer due to slipping on one plane.

We have seen that during the last stage of the extension it seems likely that slipping might take place on the second octahedral plane, $DB'E$ (fig. 12). If this is actually the case, then there is one line of particles in the specimen which will not stretch during the whole extension from 0 to 78 per cent., namely, the line of intersection of the two octahedral planes (111) and $(\bar{1}\bar{1}\bar{1})$. The position of this crystal axis, which is represented by the symbol $(10\bar{1})$, can be found from the crystal measurements. It is a line which makes 45° with the plane (100) and 90° with the plane (111) . Using the spherical polar co-ordinates (θ, ϕ) , previously used in Table V for giving the positions of the crystal axes, it was found that the co-ordinates of this crystal axis were $(\theta = 56.5^\circ, \phi = 209.6^\circ)$ in the material at 40 per cent. extension, and $(\theta = 51.5^\circ, \phi = 204.8^\circ)$ in the material at 78 per cent. extension.

To find the directions which were unextended during the extension 0-40 per cent., and remained unextended at 78 per cent. extension, the positions in the material at 40 per cent. extension of both the slip plane for the extension 0 to 40 per cent., and the "unextended cone" for the extension 40 to 78 per cent., were marked on the same diagram (see fig. 13). The points P and P' where these cut then represent the directions required. It is clear that one of them must have remained unextended during the whole of the stretching from 40 to 78 per cent., whereas the other is merely a line of particles which happens to be unextended at the particular stage, 78 per cent. of extension; but it is not possible to prove that this is the case, because there are no reliable measurements available for stages intermediate between 40 and 78 per cent.

One of these two directions should therefore, if our theory is correct, coincide with the diad axis whose position in the 40 per cent. material is $(\theta = 56.5^\circ, \phi = 209.6^\circ)$.

On measuring up fig. 13 it was found that the co-ordinates of P were $(\theta = 57^\circ, \phi = 213^\circ)$, and the co-ordinates of P' were $(\theta = 73^\circ, \phi = 355.5^\circ)$.

The diad axis $(10\bar{1})$ is marked at the point E (fig. 13). It is clear that it is very near to its predicted position.

Verification of Relation between Unstretched Lines of Particles and Crystal Axes in Material at Breaking Point.

The positions in the 78 per cent. material of the unextended lines of particles P and P' are marked on the diagram (fig. 14). The co-ordinates of

the diad axis, which lies on the intersection of the (111) and the ($\bar{1}\bar{1}\bar{1}$) planes, were calculated from the crystal measurements given in the last line of Table V; they are ($\theta = 51.5^\circ$, $\phi = 204.8^\circ$). This diad axis is marked on the diagram (fig. 4) at the point E.

The co-ordinates of the point P (fig. 14) were determined by measuring the diagram; they are ($\theta = 53^\circ$, $\phi = 206.6^\circ$).

The co-ordinates of the diad axis and the unstretched lines P are given in Table VI.

Table VI.—Spherical Polar Co-ordinates of Diad Axis and unextended Direction.

Extension of material	40 per cent.		78 per cent.	
	θ	ϕ	θ	ϕ
Co-ordinates				
Diad axis	56.5°	209.6	51.5°	204.8°
Unextended direction	57°	213.0	53°	206.6°

The agreement between these two is striking, when it is remembered that the diad axis was determined in each case by X-ray analysis of the crystal at one stage only, while the direction of the unextended line of particles was found, using only external measurements of the specimen at a series of successive stages of the distortion.

It will be noticed that this agreement depends only on the fact that the slipping is on the two octahedral planes (111) and ($\bar{1}\bar{1}\bar{1}$). If the slipping on the two planes were of equal amounts (as contemplated on p. 665 above) we should expect the two octahedral planes on which the slipping takes place to remain at equal angles to the axis of the specimen during the last part of the stretching. On calculating these angles from the X-ray measurements it was found that they were 66.1° and 63.5° in the material at 60 per cent. extension, and 65° and 61.5° in the material at 78 per cent. extension. The limiting value should, according to the theory, be $61^\circ 52'$. It will be seen that the agreement is as good as could be expected in view of the uncertainty of the X-ray measurements. The symmetry of the octahedral planes with respect to the axis of the specimens during the last stages of the extension is shown in fig. 14, where the four triad axes, which are normal to the four octahedral planes, have been marked as spots surrounded by triangles.

It will be seen that the two planes (111) and ($\bar{1}\bar{1}\bar{1}$) are at nearly equal angles to the axis, and that the other two triad axes lie on a plane which very nearly passes through the axis of the specimen.

The cone of unextended directions for the extension 40 to 78 per cent. is

not symmetrical with respect to this plane, because in the early part of this range of extension the slipping was only on the original slip plane, but if the external measurements at 62 per cent. extension had been satisfactory I should have expected the cone of unextended directions for the extension 62 to 78 per cent. to be symmetrical with respect to the (101) plane. In this way it is hoped that it will be possible to test whether there is an equal amount of slip on the two slip planes during the final stage of the extension. The fact that the (111) plane and the ($\bar{1}\bar{1}1$) plane are nearly equally inclined to the axis both in the case of 60 per cent. extension and in the case of 78 per cent. extension suggests that this must be the case, but it cannot be proved without further external measurements.

[*Note added, February 8, 1923.*—Since this paper was communicated, an account of some similar work by Mark, Polanyi and Schmid has appeared in the 'Zeitschrift für Physik,' December, 1922. The distortion of a zinc crystal is discussed. The method used is based chiefly on measurements of slip bands (which in the case of aluminium were found to be nearly useless as a basis for distortion measurements). The results obtained are similar to those described in the present work, but, owing to the fact that zinc has hexagonal instead of cubic symmetry, the complication introduced by the existence of a large number of crystallographically similar slip planes does not arise.]
