80-GT-143

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
345 E 47 St., New York, N.Y. 10017

Balancing of a Power-Transmission Shaft with the Application of Axial Torque

E. S. Zorzi
Program Manager,
Mechanical Technology Incorporated,
Latham, N. Y.
Mem. ASME

D. Flemming
Research Engineer,
NASA-Lewis Research Center,
Cleveland, Ohio
Mem. ASME

Evaluation of power transmission shafting for high-speed balancing has shown that when axial torque is applied, the imbalance response is altered. An increase in synchronous excitation always occurs if the axial torque level is altered from the value used during balancing; this was the case even when the shaft was balanced with torque applied. The twisting of the long slender shaft produces a change in the imbalance distribution sufficient to disrupt the balanced state. This paper presents a review of the analytic development of a weighted least squares approach to influence coefficient balancing and a review of experimental results. The analytic approach takes advantage of the fact that the past testing has shown that the influence coefficients are not significantly affected by the application of axial torque. The 3.60-m (12-ft) long aluminum shaft, 7.62 cm (3 in.) in diameter was run through the first flexural critical speed at torque levels ranging from zero-torque to 903.8 N-M (8000 lb-in.) in 112.9 N-M (1000 lb-in.) increments. Good comparison was achieved between predicted and experimental results.

INTRODUCTION

Transmission of power over long distances such as in helicopter tail rotors has generally been achieved by using a series of short shaft segments, each supported on bearing assemblies and requiring properly designed couplings at the end of each shaft segment. Recent advances in balancing and fabrication methods have permitted replacement of the short segmented shaft by a long power transmission shaft. With higher operating speeds, torque levels can be reduced without compromise to higher power levels. Thus shafts with long bearing spans operating at speeds above one or more of the lateral flexural modes are advantageous for low-torque, high-power operation and permit a reduction in number of couplings, bearings, weight and design envelope.

To address the problems associated with power transmission shafting, a test rig was designed and built to simulate high-speed operation under applied axial torque. The primary object of this facility was to evaluate control of synchronous excitation of power transmission shafting using existing and new balancing technology for operation above a number of flexural critical speeds. Previous publications [1] [2] have focused upon these and other aspects of this test program.

TEST FACILITY

The test facility was designed to evaluate supercritical power transmission shafting. Tests were performed to identify any problems, needed technology, or limitations inherent in the use of such shafting. One of the principal areas of interest involved the demonstration of balancing of supercritical shafts. The tests included the balancing of a very flexible shaft with both damped and undamped supports. During the course of these tests, sensitivity of unbalance response to applied torque was observed and documented.

A sketch of the test rig is presented in Figure 1. A 224-kW (300-horsepower) electric motor was the prime mover for this test rig. The output speed of the motor, which drives a variable speed magnetic coupling, was continuously variable from 50 to 3600 rpm. A gearbox with a ratio of approximately 5.7 to 1 was used to produce a drive shaft speed of up to 20,000 rpm. The drive shaft was formed from a section of aluminum tubing 7.62 cm (3 in.) in diameter and 3.66 m (12 ft) long with a 3.175 mm (0.125 in.) wall thickness. Seven balance rings were mounted on the tube at equally spaced intervals to allow the addition of balance weights to the shaft. Adapters were mounted on the ends of the tube for coupling to bearing supports. The coupling adapters were connected to ball bearing-supported stub shafts by means of a disk-type, high-speed flexible coupling. Eddy-current type displacement probes were used to measure the synchronous and nonsynchronous vibration of the test (drive) shaft.

The test rig was initially run in this configuration with no external damping other than that provided by the damping in the ball bearings and the air drag on the test shaft. At a later time, an oil squeeze-film damper was added to the test rig to provide significant external damping. The damper was installed at the far end of the test shaft (away from the driving gearbox). A photograph of the test rig with the damper installed is presented in Figure 2. Subsequent modification to the rig included an elastomer damper to replace the hydraulic mount. The elastomer damper, consisting of six Viton-70 "buttons," was the subject of another study but was operational throughout all phases of this testing. Figure 3 illustrates the squeeze-film damper and

Contributed by the Gas Turbine Division of The American Society of Mechanical Engineers for presentation at the Gas Turbine Conference & Products Show, New Orleans, La., March 10-13, 1980. Manuscript received at ASME Headquarters January 2, 1980.

Copies will be available until December 1, 1980.

elastomer replacement used during this testing. The Viton-70 buttons were 0.635 cm (0.25 in.) in diameter and 0.635 cm (0.25 in.) high. Six buttons, three equally spaced on each end, provided a stiffness of approximately 8.7 x 10^5 N-M (5000 lb-in.) with a loss factor of 0.75 used for design purposes.

ANALYTIC BASIS

One fact surfaced during early testing of the supercritical power transmission shaft, i.e., the influence coefficients did not change when up to 903.8 N-M (8000 lb-in.) torque was applied. Obviously this is a significant result in a number of ways. If a point speed approach [3] was to be used, then the effect of torque, if assessed similar to the method of balancing at multiple speeds, would produce a singular influence coefficient matrix. Also, the methodology developed should take advantage of this fact to minimize the number of trial weights sets applied to the rotor. Certainly the fact that the influence coefficients, critical speeds and mode shapes were not found to be dependent upon torque is a confirmation of the conclusions reached by Zorzi and Nelson [4] and others [5] [6] who demonstrated that low (values much lower than bucking) torque levels do not alter the modes.

Denoting $\tilde{\eta}_1{}^*$ as the imbalance response vector at the first torque level and $\tilde{\eta}_2{}^*$ as the imbalance response vector at a second condition of axial torque, as the influence coefficients are not torque dependent:

$$\begin{aligned} \tilde{\eta}_1{}^* &= \tilde{\eta}_{01}{}^* + [A^*]\ \tilde{T}^* && \text{(a)} \\ \tilde{\eta}_2{}^* &= \tilde{\eta}_{02}{}^* + [A^*]\ \tilde{T}^* && \text{(b)} \end{aligned} \qquad (1)$$

where $\tilde{\eta}_{01}{}^*$, $\tilde{\eta}_{02}{}^*$ vectors are the orbits prior to balancing, $[A^*]$ is the influence coefficient matrix with components α^*_{ij} and $\tilde{T}^*$ is the vector of correction weights.

Equation 2 defines a sum of squares S*, where $[W_1]$ and $[W_2]$ are the diagonal weighting matrices, as:

$$S = \bar{\tilde{\eta}}_1{}^{*T}\ [W_1]\ \tilde{\eta}_1{}^* + \bar{\tilde{\eta}}_2{}^{*T}\ [W_2]\ \tilde{\eta}_2{}^* \qquad (2)$$

Then by an evaluation of Equation (2) and solving $\frac{\delta S}{\delta T^*} = 0$, the results provide a value of the correction weight set $\tilde{T}^*$, Equation (3)

$$\tilde{T}^* = -\{[A^*]^T([W_1 + W_2][A^*])\}^{-1}$$
$$\{[A^*]^T([W_1]\tilde{\eta}_{01}{}^* + [W_2]\tilde{\eta}_{02}{}^*)\} \qquad (3)$$

For the work performed herein, the weighting values for both torque conditions were assumed identical, $[W_1] = [W_2] = [W]$. Thus the same importance was placed upon reducing the synchronous response of a prescribed orbit at all torque levels, Equation (4).

$$\tilde{T} = -\frac{1}{2}\ \{[A^*]^T[W][A^*]\}^{-1}$$
$$\{[A^*]^T[W](\tilde{\eta}_{01}{}^* + \tilde{\eta}_{02}{}^*)\} \qquad (4)$$

Equation (3) was programmed on a Digital Equipment Corporation PDP 11/34 minicomputer for laboratory balancing.

TEST RESULTS

The power transmission shaft was evaluated for rotor dynamic stability prior to testing. The analysis indicated that the rotor supported on the elastomer mount had a critical speed at 803 rpm with a logarithmic decrement calculated at 0.103. The mode shape illustrated in Figure 4 indicated a simple flexural mode as the first critical speed. This mode shape was confirmed by testing. During testing the mode was located at 904 rpm with a calculated logarithmic decrement of 0.129. Displacement probe locations active during testing are shown in Figure 5 with the data from probes #3, 5, 6 and 8 being used for balancing data throughout this testing.

Initially the power transmission shaft was conventionally balanced so that the first bending mode could be traversed without applied torque. The center balance plane was used for this effort and the remaining balancing. As axial torque was applied in 112.9 N-M (1000 lb-in.) increments (Figure 6), the imbalance response through the critical increased until the critical speed could no longer be traversed without risk to the hardware. At a torque of 451.6 N-M (4000 lb-in.), the critical could not be traversed. This was the case as torque was increased beyond this value to a maximum value of 903.8 N-M (8000 lb-in.). To demonstrate the balancing methodology detailed in Equation (3), the no-torque and 903.8 N-M (8000 lb-in.) levels were considered as the two torque conditions for balancing. Base-line data obtained from the runs of Figure 6 are tabulated in Table 1 for the no-torque and 903.8 N-M (8000 lb-in.) torque conditions.

All of the data reported in this work were obtained using a Digital Equipment Corporation PDP 11/03 automated data acquisition system designed by MTI. The data were obtained with a speed envelope ranging from 903 to 907 rpm with the average of ten samples computed for the data reported.

Using the balancing method developed, a predicted correction weight of 1.63 gms at an angle of 310° was applied to the rotor at the center balance plane. The rotor was then run under the same condition of torque loading previously noted. Figure 7 illustrates the data obtained for probe #8. The predicted correction weight when applied to the rotor permitted traversing of the first critical speed for all torque levels. The data obtained for this balance run for the two torque levels reported in Table 1 are presented in Table 2 along with the predicted analytical responses.

From the data reported in Tables 1 and 2, it is evident that the weighted least squares balancing

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* Indicates complex value

\- Indicates complex conjuguate

Table 1
Base-Line Data Figure 6 at 905 rpm
Probe #6

	w/o Torque		903.8 N-M Torque	
Location	Orbit Radius mm (mils)	Orbit Phase degrees	Orbit Radius mm (mils)	Orbit Phase degrees
3	0.1583 (6.233)	290	1.235 (48.626)	22.9
5	0.5096 (20.064)	288	1.404 (55.302)	25.2
7	0.5324 (20.964)	287	1.711 (67.376)	35.5
8	0.3959 (15.588)	290	1.261 (49.649)	21.5

Table 2
Balanced Rotor Figure 7 at 905 rpm
Probe #6

	Measured				Predicted			
	W/O Torque		With 903.8 N-M Torque		W/O Torque		With 903.8 N-M Torque	
Location	Orbit Radius mm (mils)	Orbit Phase degrees	Orbit Radius mm (mils)	Orbit Phase degrees	Orbit Radius mm (mils)	Orbit Phase degrees	Orbit Radius mm (mils)	Orbit Phase degrees
3	0.5819 (22.91)	261	0.6151 (24.217)	343	0.5596 (22.030)	235	0.5973 (23.591)	349
5	0.9050 (35.633)	268	0.8744 (34.427)	329	0.8008 (32.316)	249	0.7833 (30.840)	335
6	0.7166 (28.215)	252	0.7228 (30.405)	347	0.8390 (33.034)	226	0.6755 (26.598)	353
8	0.6174 (24.309)	266	0.7496 (29.512)	333	0.6724 (26.473)	346	0.6920 (27.482)	336

approach required a tradeoff between the two torque conditions. The no-torque orbits before final balancing were smaller and the 903.8 N-M (8000 lb-in.) torque orbits were larger prior to the application of the corrections weights, which was correctly predicted by the analysis.

RECOMMENDATIONS

The balancing work reported here is not intended to be a statement of fully developed and tested technology, only a presentation of a basic effort which has begun investigation of axial torque effects on balancing. Other testing has shown that certain modes are more susceptible to torque effects than are others. Figure 8 illustrates the torque effect on the power transmission shaft for the first and third critical speed (the second critical speed is overdamped). As shown, the third critical is more sensitive to the application of axial torque than the first mode. Therefore, the authors hope to encourage others to investigate this effect, particularly those directly involved with power transmission shafting.

CONCLUSIONS

As a result of this work, the following can be concluded:

- Axial torque can substantially degrade imbalance response of long slender power transmission shafting.
- The effect of axial torque on the influence coefficients appears to be minimum. This fact can be used to reduce trial weight runs.
- A need exists for further efforts in this area to deal with multiple modes and perhaps modes more sensitive to imbalance than reported in this work.

It is hoped that this work leads to safer, higher-speed power transmission shafting and a stronger technology base to support this endeavor.

ACKNOWLEDGMENT

The authors wish to acknowledge NASA-Lewis for funding this study. In addition, the authors gratefully acknowledge the support of Mr. G. Burgess, Mr. S. Azzaro and Mr. D. Smith for their invaluable assistance.

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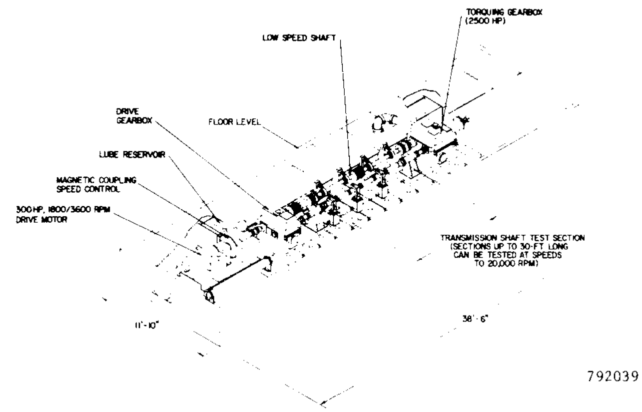

Figure 1 Drive Train Dynamics Technology Test Rig Configuration for High-Speed Shaft Balancing

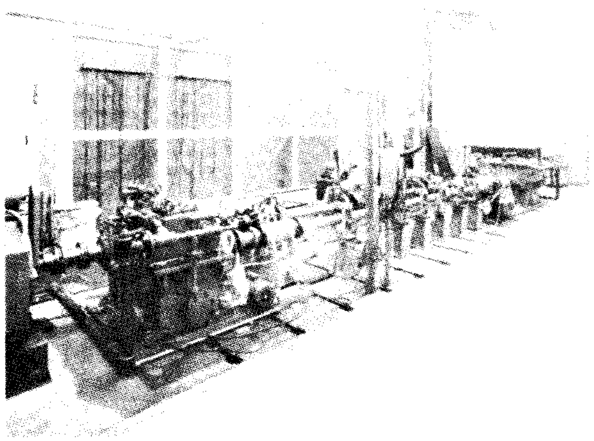

Figure 2 View of Completely Assembled Test Rig Showing High-Speed Side from Drive Gearbox End

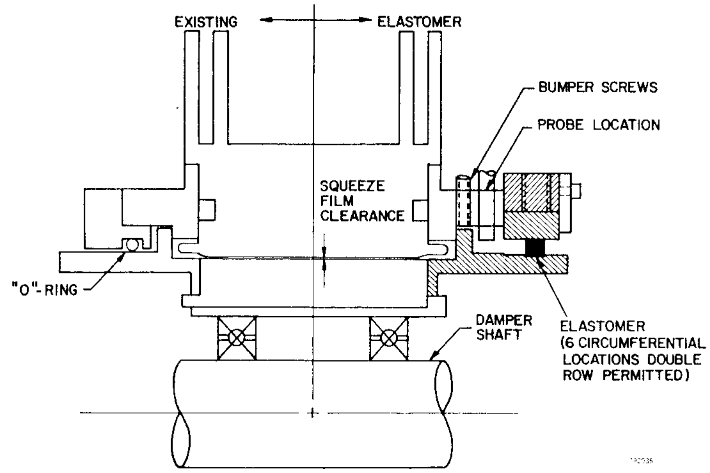

Figure 3 Elastomer Damper/Squeeze-Film Damper Schematic

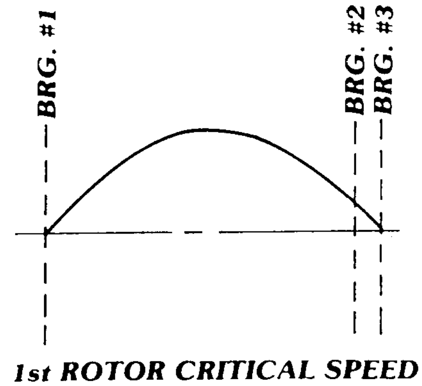

Figure 4 First Rotor Critical Speed

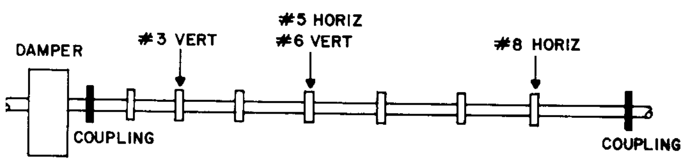

Figure 5 Rig Configuration with Instrumentation

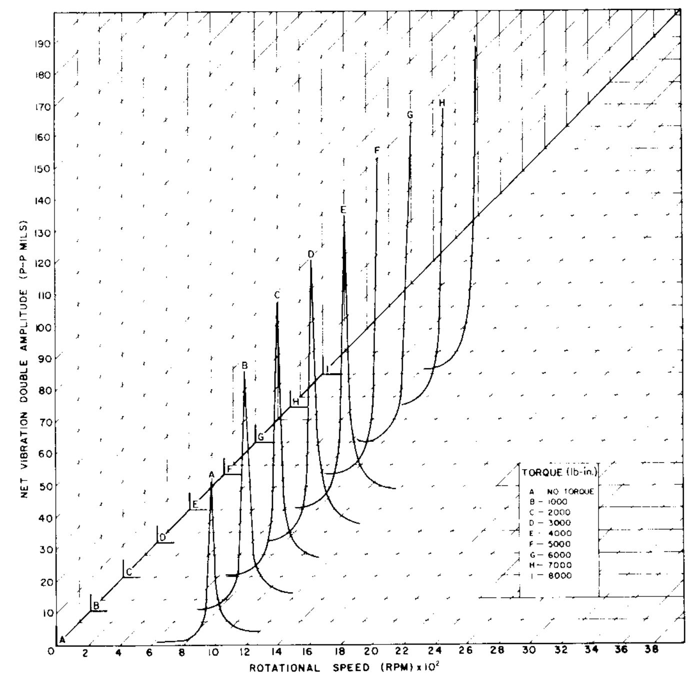

Figure 6 Baseline Test Condition

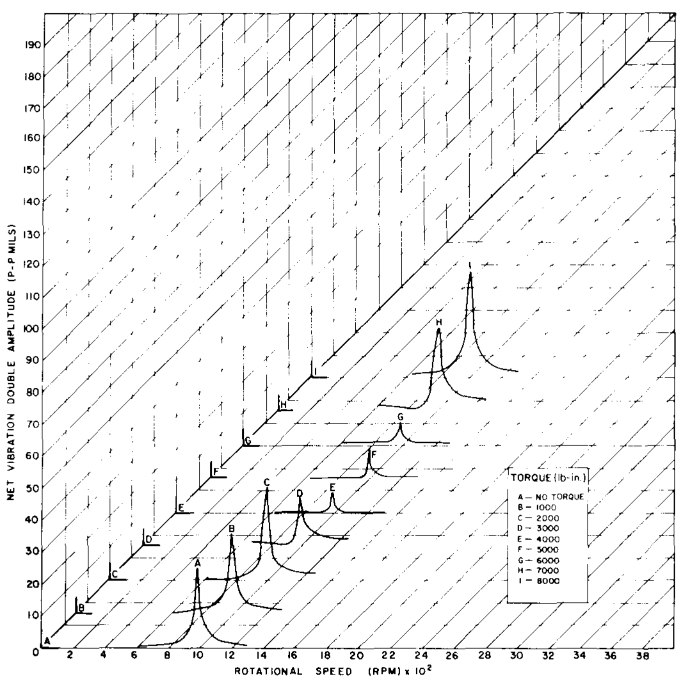

Figure 7 Rotor Response After Balance Correction Applied for Torque Condition

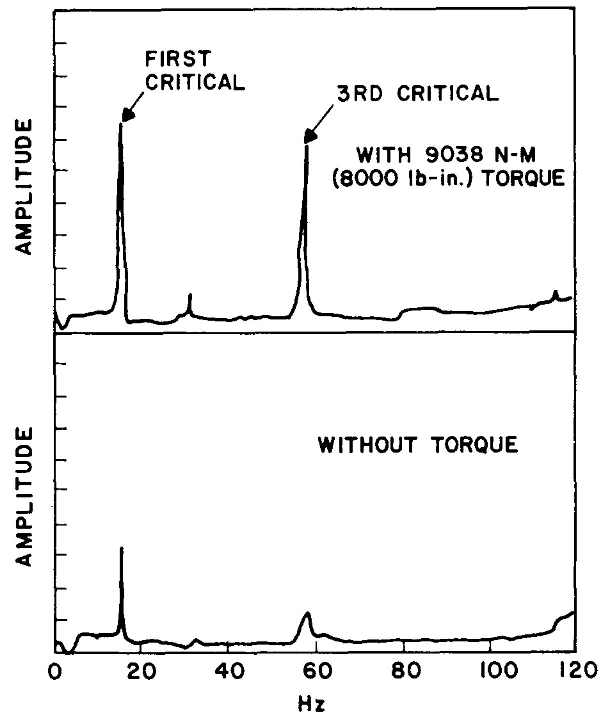

Figure 8 Synchronous Response of Power Transmission Shaft