### Title
BALLISTIC DEFICITS IN PULSE SHAPING AMPLIFIERS




### Authors
Loo, B.W.
Goulding, F.S.
Gao, D.



### Publication Date
1987-09-01

# Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

## Engineering Division



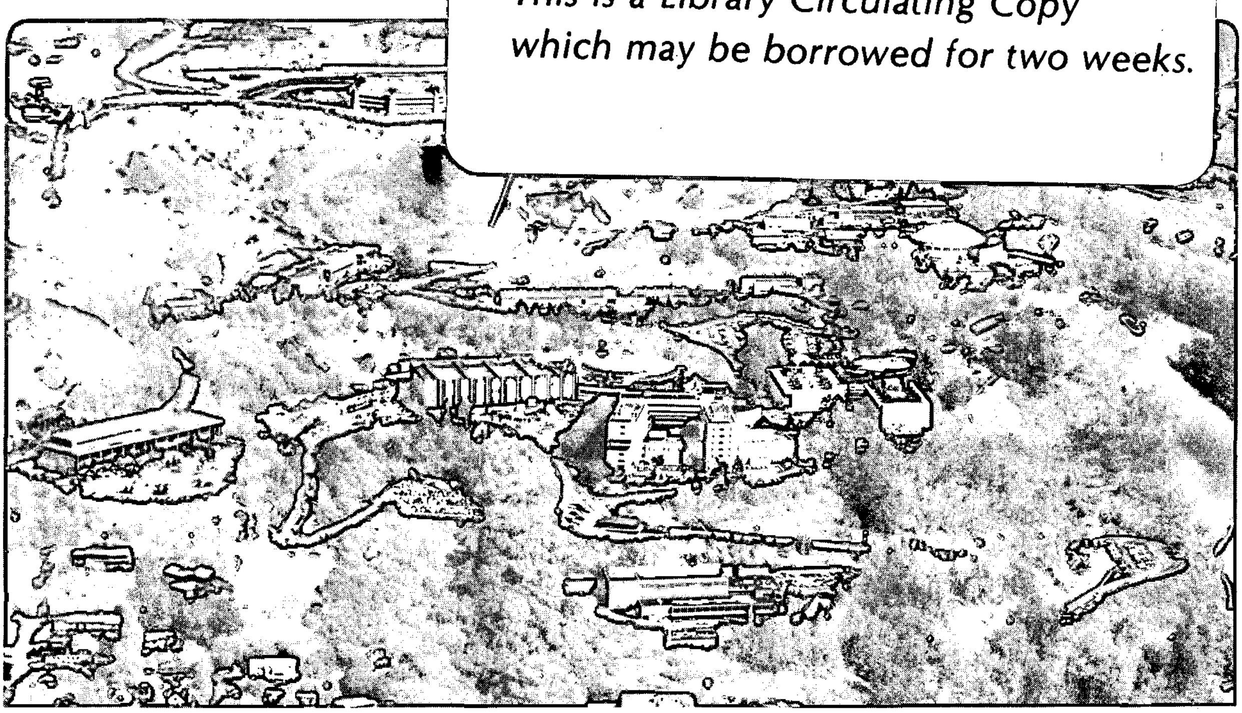
Presented at the 1987 IEEE Nuclear Science Symposium,
San Francisco, CA, October 21–23, 1987, and to
be published in the Proceedings

# Ballistic Deficits in Pulse Shaping Amplifiers

B.W. Loo, F.S. Goulding, and D. Gao

September 1987







Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

# BALLISTIC DEFICITS IN PULSE SHAPING AMPLIFIERS

LBL-23356

Billy W. Loo and Fred S. Goulding
Lawrence Berkeley Laboratory,
University of California, Berkeley CA 94720, U.S.A.
and
Dexi Gao
Institute of Atomic Energy
P.O. Box 275-69, Beijing, P.R.C.

## ABSTRACT

We consider the common problem of ballistic deficit which is the loss of output signal amplitude due to the interplay between the finite charge collection times in a detector and the characteristic time constants of the amplifier. Quantitative estimates of ballistic deficits have been developed by using numerical integration techniques on a microcomputer and by exact calculations using Laplace transforms. A new practical procedure is developed in which an actual amplifier with even an unspecified complex-pole network can be accurately related to an "equivalent" semi-Gaussian shaping network whose exact solution is known. Simple formulas for estimating ballistic deficits and resulting degradation in spectral energy resolutions are given.

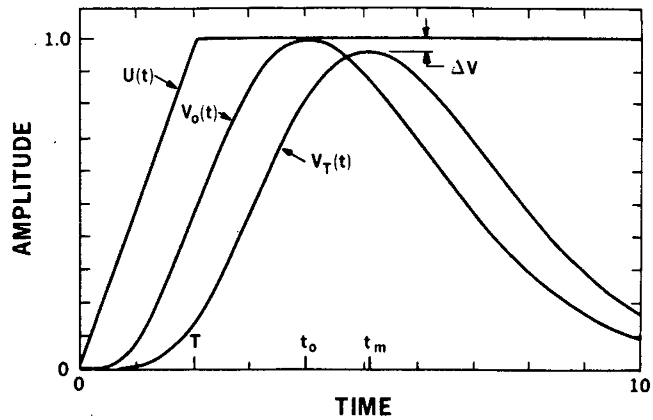



Fig. 1 An Illustration of the Input, Impulse Response and Output Functions and their Peaking Times. ΔV is the Ballistic Deficit.

## INTRODUCTION

By ballistic signal, one is referring to an output signal whose amplitude is proportional to the total charge that appeared on the collection electrode of the detector irrespective of the time profile of charge arrival. The deflection indicated by a ballistic galvanometer is a classic example. However, for most detectors, the signal is usually processed by a pulse-shaping network in a charge amplifier. The optimum choice of pulse shaper for a given application is based on the performance trade-offs among factors such as signal to noise ratio, counting-rate behavior, sensitivity of rise-time fluctuations of the input signal, and the suitability of the output pulse-shape for feeding a pulse height analyzer [1].

For a given pulse shaping network, there is an impulse response output function $V_O(t)$ when all the charges are collected instantly. Fig. 1 shows that $V_O(t)$ has a peaking time of $t_O$. If the charge collection time is finite, the input function U(t) will reach its maximum at time T, and the output $V_T(t)$ will reach its peak amplitude at $t_m$. However, this maximum amplitude is generally less than that of the impulse response function. The ballistic deficit (BD) is therefore defined as

$$BD = \Delta V = V_O(t_O) - V_T(t_m) \qquad (1)$$

and the relative ballistic deficit ( RBD ) is defined as

$$RBD = \Delta V / V_O(t_O) \qquad (2)$$

The effect of ballistic deficit is therefore the imprecision in the determination of the signal in a detector due to variations in the time of arrival at the collection electrodes. In semiconductor detectors, fluctuations in the peaking time of the input signals are often the results of distribution of charge origins within a detector volume, field inhomogeneities, or charge trapping. We know qualitatively that RBD is always large when the ratio ($T/t_O$) is high. But with T being an inherent property of the detector, choosing very large values of $t_O$ to reduce RBD would jeopardize rate performance. A quantitative estimate of RBD is therefore needed for making a judicious choice in order to strike an optimal balance between the effect of sensitivity to rise-time fluctuations in relation to signal-to-noise and count-rate performances.

## EXACT SOLUTIONS

We shall consider the problem of calculating the RBD for a special case in which the charge arrival times are distributed uniformly over a period T. The input function U(t) therefore has a linear rise and reaches a peak value at time T. Once the solution of this special case is known, it can be applied to conditions in real detectors which typically exhibit some variations in T ranging from $T_{min}$ to $T_{max}$.

In the following discussion, we shall assume for simplicity that the characteristic time constants $\tau = RC$ are the same for all the circuits except where noted, and that the times given in mumerical examples are expressed in units of $\tau$.

## (A) NUMERICAL INTEGRATION

If I(t) is the source function (e.g. the current in a detector), then the input function is

$$U(t) = \int_0^t I(z)\, dz \qquad (3)$$

and the output function is a convolution integral representing the superposition of the impulse responses over time.

$$V_T(t) = \int_0^t I(z)\, V_O(t-z)\, dz \qquad (4)$$

$V_T(t_m)$ hence RBD may be determined by numerical integration when I(z) and $V_O(t)$ are specified. Fig. 1 shows the plots of such calculations to illustrate the case where I(z) is constant over a period of T=2. U(t) therefore has a linear rise peaking at amplitude 1.0 at time T. In this example, $V_O(t)$ is a fourth order semi-Gaussian shaping function with $t_O$=4. The resulting output $V_T(t)$ peaks at t=5.08 with a maximum amplitude of 0.960. The RBD is therefore 4.0 %.

## (B) LAPLACE TRANSFORMATION

The output V(t) can be found from the inverse Laplace transformation

$$V(t) = \int_0^\infty f(s)\, e^{st}\, ds \qquad (5)$$

where f(s) is the Laplace transform of the differential equation characterizing the pulse shaping network. Fig. 2 shows the typical circuits used in shaping current pulses with their corresponding Laplace Transforms.

In a semi-Gaussian shaper of order N, a differentiator is followed by N integrator stages, thus

$$f(s) = \frac{s\tau}{(1+s\tau)^{N+1}} \qquad (6)$$

$$V_O(t) = \frac{1}{N!}\left(\frac{t}{\tau}\right)^N \exp\left(\frac{-t}{\tau}\right) \qquad (7)$$

$$\text{and} \quad t_O = N\tau \qquad (8)$$

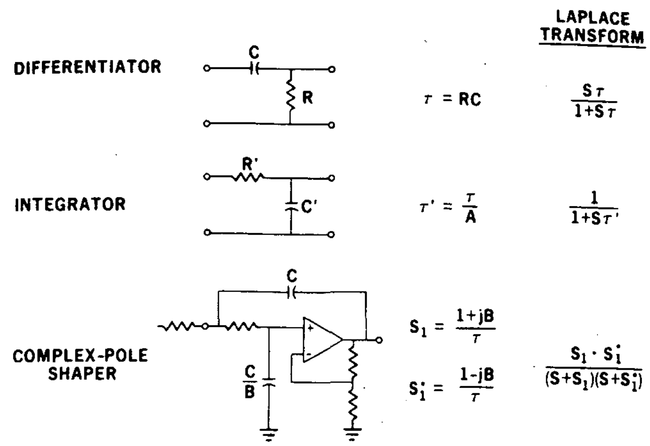



Fig. 2 Typical Pulse Shaping Circuits and Their Laplace Transforms.

Again, for the constant source function of duration T,

$$I(t) = (1/T)\,[\,t - (t-T)\,H(t-T)\,] \qquad (9)$$

$$\text{where} \quad H(t-T) = 0 \ \text{for}\ t<T \ \text{and} \qquad (10)$$
$$= 1 \ \text{for}\ t \geq T$$

The resulting output $V_T(t)$ is

$$V_T(t) = \frac{\tau}{T}\left\{1 - \exp\left(\frac{-t}{\tau}\right)\left(1 + \sum_{m=1}^{N} \frac{t^m}{m!\tau^m}\right) - \left(1 - \exp\left(\frac{T-t}{\tau}\right) - \exp\left(\frac{T-t}{\tau}\right)\sum_{m=1}^{N}\frac{(t-T)^m}{m!\tau^m}\right) H(t-T)\right\} \qquad (11)$$

$$\text{and} \quad t_m = \frac{T}{1 - \exp\left(-\frac{T}{N\tau}\right)} \qquad (12)$$

Similar calculations can be carried out for circuits with complex-poles. For example, a circuit similar to the ORTEC Model 472 amplifier is characterized by

$$f(s) = \frac{s\tau}{1+s\tau}\;\frac{[(1+jB)/\tau][(1-jB)/\tau]}{[s+(1+jB)/\tau][s+(1-jB)/\tau]}\;\frac{A/\tau}{s+A/\tau} \qquad (13)$$

where $j = \sqrt{-1}$, and the corresponding impulse response function is

$$V_O(t) = D\exp\left(\frac{-t}{\tau}\right)\left[E\cos\frac{Bt}{\tau} + F\sin\frac{Bt}{\tau} - \frac{1}{B} + G\exp\left(\frac{-(A-1)t}{\tau}\right)\right] \qquad (14)$$

$$\text{where} \quad D = \frac{A(1+B^2)}{B(1-A)}$$

$$E = \frac{(A-1)^2}{B(B^2+A^2-2A+1)}$$

$$F = \frac{(A-1)}{B^2+A^2-2A+1}$$

and $$G = \frac{B}{B^2+A^2-2A+1}$$

## APPROXIMATE SOLUTIONS

Baldinger and Franzen have shown that, using a second order series expansion, RBD may be expressed as [2]

$$RBD = V_o''(t_o) / 24\ V_o(t_o) \qquad (15)$$

where

$$V_o''(t_o) = \left( \frac{d^2}{dt^2} V_o(t) \right)_{t=t_o} \qquad (16)$$

For all practical purposes, the differences between the second order and the fourth order approximations are insignificant. Thus when $V_o(t)$ is found by eq.(5), eq.(15) will give a direct estimate of the RBD.

In the case of semi-Gaussian shaping,

$$RBD \approx \frac{1}{24N}\left(\frac{T}{\tau}\right)^2 = \frac{N}{24}\left(\frac{T}{t_o}\right)^2 \qquad (17)$$

## A PRACTICAL METHOD

We have shown that if we know the exact form of the impulse response function of the pulse shaping amplifier, we may use the technique of numerical integration; and if we know the details and actual values of the circuit parameters, we may apply the exact albeit often tedious calculations with Laplace Transformations. While these analytical procedures may be of interest to those who design amplifiers, the typical user needs a simpler and more direct way of estimating RBD.

Eq.(15) indicates the important fact that RBD is proportional to $V_o''(t_o)$ which in turn is proportional to the *curvature* of $V_o$ at its peaking time $t_o$. This suggests that if there are simple ways of generating $V_o$ and determining its curvature near its maximum amplitude, we may conveniently arrive at a quantitative estimate of RBD without knowing the specific circuit design of the pulse shaping amplifier.

Since we have already worked out the exact solutions of the semi-Gaussian shaping network for all integer values of N ( equations 7,8,11, and 12 ), we may use them as a set of *base solutions*. The curvature of any impulse response function that has a near parabolic shape at its peak can be compared with those of the semi-Gaussian shapers and an "equivalent" N or n can be assigned to that unknown function and its RBD can be calculated by a modified eq. 17

$$RBD \approx \frac{1}{24n}\left(\frac{T}{\tau}\right)^2 = \frac{n}{24}\left(\frac{T}{t_o}\right)^2 \qquad (18)$$

where $t_o = n\,\tau$, but n is no longer restricted to integer values.

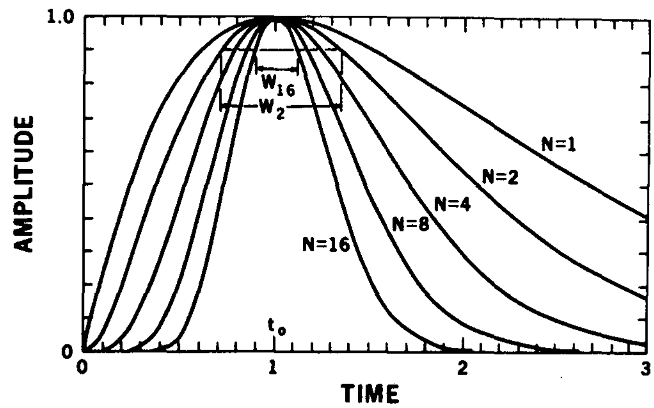



Fig. 3 Semi-Gaussian Shaping Functions of Order N Normalized to the Peaking Time $t_o$. The W's are the Full Width at 90% Maximum Values.

Fig. 3 shows some representative plots of semi-Gaussian $V_o(t)$ for N in the range of 1-16 which are normalized to the same peaking time $t_o$. The curvature near the peak can be related to some readily measurable quantities such as W which is defined as the full width at 0.9 maximum in units of $t_o$. Table 1 lists the calculated values of W for N's in the range of 1-10. A functional relationship between N and W is derived from a least squares fit to these tabulated values with a correlation coefficient of 0.999999:

$$n = 0.8517\ W^{-1.992} \qquad (19)$$

Table 1. Computed W for Semi-Gaussian Shapers of Order N

| N | W | N | W |
|---|---|---|---|
| 1 | .9235 | 6 | .3752 |
| 2 | .6511 | 7 | .3473 |
| 3 | .5311 | 8 | .3248 |
| 4 | .4597 | 9 | .3062 |
| 5 | .4111 | 10 | .2905 |

To estimate the performance of a given amplifier, one needs only to inject charges through the detector with a fast pulse generator. By displaying the output $V_0(t)$ on an oscilloscope (triggered by the pulser), the values of W and $t_0$ can be readily measured, and the resulting RBD easily calculated using equations 19 and 18.

## RESULTS AND DISCUSSION

To test the accuracy of the above procedure(we shall refer to it as the LBL method), we first calculated the exact solutions to the ORTEC Model 472 amplifier by the Laplace Transform (LT) method and the RBD's as determined by eq.15. These results are then compared to those obtained by the LBL method using eqs. 19 and 18 in which the W's were calculated from the exact values of $V_0(0.9t_0)$. Table 2 summarizes the results for $T=\tau$ over an assumed range of circuit parameters. The agreement between these two methods are generally within 2 %. Such agreements are quite remarkable considering the very different paths by which the results were arrived. We have also made a comparison using an amplifier similar to the Canberra Molel 2010 design consisting of a differentiator, an integrator and two complex-pole stages. The agreement between the LBL and the exact LT methods were equally impressive.

Table 2. Comparison Between the LBL and the Laplace Transform (LT) Methods.

| ORTEC 472 | | $t_o$ | n | RBD (LBL) | RBD (LT) | LBL/LT |
|---|---|---|---|---|---|---|
| A = 3 | B = 1.5 | $1.646\ \tau$ | 3.83 | 5.89% | 5.94% | .992 |
| | B = 1.0 | $1.924\ \tau$ | 3.34 | 3.76% | 3.79% | .992 |
| | B = 0.5 | $2.225\ \tau$ | 2.83 | 2.39% | 2.43% | .984 |

We have so far demonstrated the means for determining the RBD for a given value of T. A distribution in T thus will lead to a range of degradation of output pulses and result in an asymmetric broadening in the lower energy side of a spectral line. To translate the effect of RBD into a quantitative measure of degradation in energy resolution, let us consider an example involving a planar or coaxial germanium detector. The peaking time T is the time for all the charge carriers, electrons and holes, to reach their respective electrodes. Under sufficient detector bias voltage and at a temperature near that of liquid nitrogen, the saturation velocities of electrons and holes are about the same. $T_{min}$ therefore corresponds to the case where the charges are generated near the middle of the sensitive volume while $T_{max}$ will be about twice as long for those charges originating near one of the electrodes. The signals for those charges starting between these two extremes will have composite slopes with the later portion having half the initial value when both carriers are in transit. The resulting RBD although not given precisely by the above analysis will nevertheless fall in between the two extremes as determined by $T_{min}$ and $T_{max}$. It is generally reasonable to assume that the frequency distribution is approximately uniform between these two values [3]. Since the RBD is proportional to $T^2$, its frequency distribution will be inversely proportional to $T^2$. Fig. 4 illustrates that, under these approximations, the corresponding increases in spectral resolution are then given by

$$FW = \frac{nE_0}{24}\left[\left(\frac{T_{max}}{t_0}\right)^2 - \left(\frac{T_{min}}{t_0}\right)^2\right] \tag{20}$$

and

$$FWHM = \frac{nE_0}{24}\left(\frac{T_{min}}{t_0}\right)^2 \tag{21}$$

where $E_0$ is the true energy of the gamma-ray, and n is the equivalent N of the semi-Gaussian shaping network.

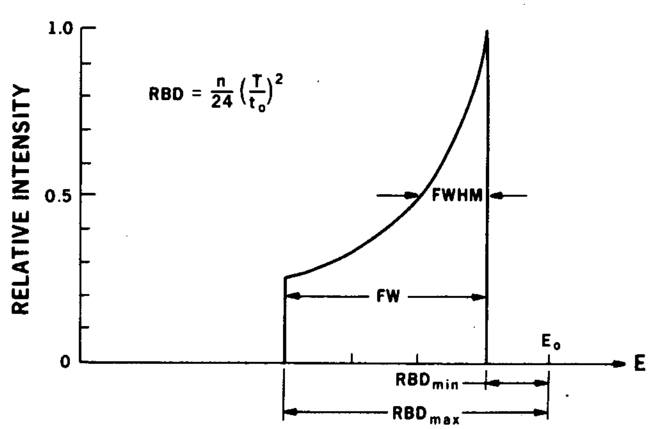



Fig. 4. Asymmetical Spectral Broadening Due to Ballistic Deficits.

We recognize that input signals from real detectors may not have constant slopes during their rise-times so the actual amplifier performance may have slight departures from our analysis. But the extreme values or even the frequency distribution of T of a given detector system may be observed experimentally. Quantitative estimates of RBD are especially valuable where rather large rise-time fluctuations may be expected in detectors such as GaAs, $HgI_2$, CdTe or ionization chambers. On the other hand we are encouraged by the excellent agreement between the LBL and the LT methods even for the very large values of $T/t_0$ used in the examples. In typical germanium detector systems, the $T/t_0$ values are generally much smaller. The procedures described above can therefore give quite resonable estimates of ballistic deficits, and should prove to be a useful guide for optimizing measurement conditions.

## REFERENCES


[1] F. S. Goulding, " Pulse-Shaping in Low-Noise Nuclear Aplifiers: A Physical Approach to Noise Analysis ", Nucl. Instr. and Methods 100, 493 (1972).

[2] E. Baldinger and W. Franzen, " Amplitude and Time Measurements in Nuclear Physics ", *Advances in Electronics and Electron Physics* (Ed. L. Morton), Academic Press, N.Y., 255 (1956).

[3] T.W. Raudorf, M.O. Bedwell and T.J. Paulus, " Pulse Shape and Risetime Distribution Calculations for HPGe Coaxial Detectors ", IEEE Trans. on Nucl. Sci. NS-29, No.1, 764 (1982).


## ACKNOWLEDGMENT


This work was supported by the Office of Health and Environmental Research, U.S. Department of Energy under contract no. DE-AC03-76SF00098. Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.

*LAWRENCE BERKELEY LABORATORY*
*TECHNICAL INFORMATION DEPARTMENT*
*UNIVERSITY OF CALIFORNIA*
*BERKELEY, CALIFORNIA 94720*