Bandwidth Occupancy of Non-Coherent Wideband Fading Channels

Felipe Gómez-Cuba[†], Jinfeng Du[‡♮], Muriel Médard[‡], and Elza Erkip[§]
[†]AtlantTIC, University of Vigo, C.P. 36310 Vigo, España fgomez@gti.uvigo.es
[‡]Massachusetts Institute of Technology, Cambridge, MA 02139, USA {jinfeng,medard}@mit.edu
[§]NYU Polytechnic School of Engineering, Brooklyn, NY 11201, USA elza@nyu.edu
[♮]Royal Institute of Technology (KTH), Stockholm, 10044, Sweden

***Abstract*—Peaky and non-peaky signaling schemes have long been considered species apart in non-coherent wideband fading channels, as the first approaches asymptotically the linear-in-power capacity of a wideband AWGN channel with the same SNR, whereas the second reaches a nearly power-limited peak rate at some finite *critical bandwidth* and then falls to zero as bandwidth grows to infinity. In this paper it is shown that this distinction is in fact an artifact of the limited attention paid in the past to the product between the bandwidth and the fraction of time it is in use. This fundamental quantity, that is termed *bandwidth occupancy*, measures average bandwidth usage over time. The two types of signaling in the literature are harmonized to show that, for any type of signals, there is a fundamental limit—a critical bandwidth occupancy. All signaling schemes with the same bandwidth occupancy approach the capacity of wideband AWGN channels with the same asymptotic behavior as the bandwidth occupancy grows to its critical value. For a bandwidth occupancy above the critical, rate decreases to zero as the bandwidth occupancy goes to infinity.**

***Index Terms*—Wideband regime, non-coherent fading channel, peaky signals, bandwidth occupancy**

I. Introduction and Related Work

Recently there has been great interest in wireless channels with a large bandwidth, owing in part to the prospective investments onto the millimeter wave bands, where vast quantities of new spectrum is readily available [2]. In a frequency selective fading channel where there is no channel state information at the receiver (CSIR) or the transmitter, the wideband capacity regime is affected by the growing uncertainty in the channel impulse response. As bandwidth grows while power is constrained, it becomes infeasible to estimate the channel coefficients to a precision sufficient for coherent detection. Moreover, if one would spread the transmitted signal power across all the available bandwidth and time slots, the desired signal would be buried by this channel uncertainty. Médard and Gallager proved this [3] through an upper bound to rate proportional to the ratio between the fourth moment of the signal ($\mathrm{E}\left[|x|^4\right]$) and its bandwidth ($B$), i.e., $R <\propto \mathrm{E}\left[|x|^4\right]/B$, so that only by making the first infinite —that is, concentrating the power of the signal distribution in a vanishing subset of its coefficients— one could achieve rates above zero when bandwidth goes to infinity. Telatar and Tse [4] related channel uncertainty to the number of independent paths, and showed that in a rich scattering environment the rate grows with B while power per path is sufficient, but it starts decreasing when the number of independent paths is above its critical value. This led to the thought that *peaky* signaling schemes [4]–[8] are imperative to approach the linear-in-power capacity limit of a wideband additive white Gaussian noise (AWGN) channel, which in multi-input multi-output (MIMO) systems is

$$C^{\infty} \triangleq \lim_{B\to\infty} C(B) = \lim_{B\to\infty} BN_{\mathrm{r}}\mathrm{SNR} = N_{\mathrm{r}}P/N_0, \ \text{[nats/s]},$$

where P is the power, N_0 is the noise power spectral density (PSD), N_{r} is the number of receive antennas, and $\mathrm{SNR} = P/(BN_0)$ is the signal-to-noise ratio (SNR) per degree of freedom at each receive antenna.

However, peaky signals may have drawbacks, such as high requirement on hardware and poor *spectral efficiency* (nats/s/Hz). The former comes from the fact a signal with high fourth moment is challenging to synthesize owing to hardware non-linearities. The latter arises from the fact that rate of peaky signalling approaches the capacity limit slowly as $B\to\infty$, thus requiring considerable bandwidth to attain the same rate as a coherent channel. This has been demonstrated in [5] via a second order Taylor series expansion, showing that the second derivative of capacity at $\mathrm{SNR} = 0$ is finite for AWGN and coherent fading channels (which have perfect CSIR) but $-\infty$ for non-coherent scenarios. This abrupt distinction, where either the channel is perfectly known or unknown, contrasts with the intuition that, as the *coherence length* (L_c, determined by coherence time and coherence bandwidth) of a fading channel grows, estimating the channel becomes increasingly rewarding and the capacity of the non-coherent channel converges to the capacity of the coherent channel as $L_c \to \infty$. This seeming conflict has been resolved in [7], [8] by showing that in non-coherent Rayleigh fading channels the capacity $C(B)$ is

$$\frac{C(B)}{B} \simeq N_{\mathrm{r}}\mathrm{SNR} - \frac{N_{\mathrm{r}}(N_{\mathrm{r}}+N_{\mathrm{t}})}{2N_{\mathrm{t}}}\mathrm{SNR}^{1+\alpha} + o(B^{-(1+\alpha)}), \quad (1)$$

where N_{t} is the number of transmit antennas and the exponent $\alpha\in(0,1)$ grows with increasing L_c. The first term is the power limit as in C^{∞} and the second term $\mathrm{SNR}^{1+\alpha}$ vanishes with $B\to\infty$ (dominating the second derivative). The third term captures the fast-vanishing approximation error at large B.

A longer version [1] of this paper, giving detailed proofs and discussions, has been submitted to the IEEE Transactions on Information Theory.

Note that $\mathrm{SNR}^{1+\alpha}$ is sub-quadratic, so the exact same spectral efficiency as coherent schemes can not be achieved because choosing $\alpha{=}1$ would imply an infinite second derivative.

Peaky signaling as in these analyses is compulsory if our requirement is to achieve C^{∞} when $B \to \infty$. However, non-peaky signals can suffice to approach the wideband capacity limit within a bounded gap at some large—but finite—bandwidth, even though the rate vanishes as bandwidth grows. Lozano and Porrat [9] consider non-peaky signaling in the single-input single-output (SISO) channel under a general fading distribution. When bandwidth is not too large there is a transitory first stage where rate grows with B, approaching a maximum value of

$$\mathrm{SNR}\left(1-\tilde{\Delta}\right), \quad \lim_{L_c\to\infty} \tilde{\Delta} = 0, \tag{2}$$

where $\tilde{\Delta}$ vanishes with increasing channel coherence L_c and does not depend on SNR. This maximum is achieved at some critical bandwidth B_{crit}, beyond which rate decreases as B grows, and ultimately rate goes to zero as $B \to \infty$.

As argued above, although both peaky and non-peaky signaling can approach the wideband capacity limit when available bandwidth is abundant, it is not immediately clear how the power-limited rate in [9] (shown in (2), developed for SISO) is related to the polynomial near-power-limited rate in [8] (shown in (1), developed for MIMO).

In this paper, we unify the study of peaky and non-peaky signaling, showing that they are nothing but extreme cases of a more fundamental trade-off that affects all types of signals. We argue that the analyses in [8], [9] are merely two different methods of representing system behavior. Our analysis generalizes [9] to MIMO systems and introduces a transmission duty-cycle to allow arbitrary levels of signal peakiness. The peakiness parameter $\delta \in (0, 1]$ defines the fraction of time the transmitter is active. We show that capacity is a function of only the product of δ and B, namely δB that we call *bandwidth occupancy*, and we prove that capacity $C(\delta B)$ increases as bandwidth occupancy approaches a critical value $(\delta B)_{\text{crit}}$. The capacity[1] at $(\delta B)_{\text{crit}}$ is lower bounded by

$$R^{\text{LB}} = N_{\text{r}} P/N_0 \left(1-\Delta\right), \tag{3}$$

with the same offset Δ for all levels of peakiness $\delta \in (0, 1]$. Using the relation between the main sublinear exponent α used in (1) and the peakiness parameter $\delta{=}\mathrm{SNR}^{1-\alpha}$ in [8], we show that $\Delta{\sim}\mathrm{SNR}^{\alpha}$ at $(\delta B)_{\text{crit}}$. This is, the multiplicative capacity gap Δ in [9] and the sub-linear polynomial approximation SNR^{α} in [8] represent the same behavior. Therefore, it is possible to approach C^{∞} within the same capacity gap at the same convergence speed by any signaling scheme within the family using a bandwidth $B \geq B_{\text{crit}}$ together with the peakiness parameter $\delta \simeq \frac{B_{\text{crit}}}{B}$ as represented in Fig. 1.

The rest of this paper is organized as follows. We present in Sec. II the system model that are essential to prove our main results. Our unified results on wideband limit are presented in Sec. III and conclusions are in IV. Omitted proofs can be found in [1].

[1]The connection between capacity and mutual information bounds (used in our analysis and in [9]) for ergodic channels was established in [10, Prop. 2.1].

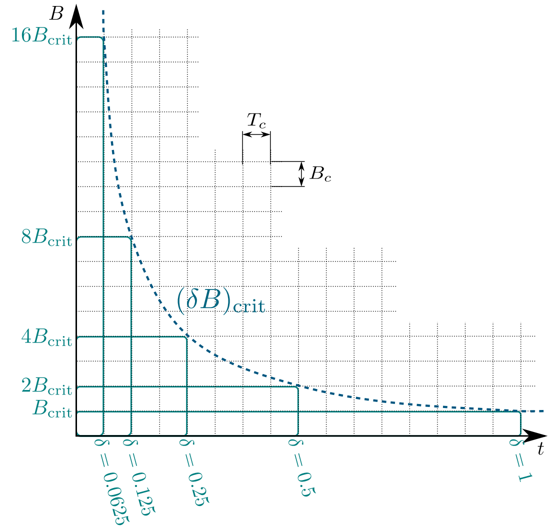


Figure 1. All transmission strategies with the same bandwidth occupancy $\delta B{=}(\delta B)_{\text{crit}}$ achieve the same polynomial approximation of C^{∞}. The transmission time and bandwidth are measured in terms of channel coherence time T_c and coherence bandwidth B_c, respectively.

II. System Model

We consider a rich scattering, frequency selective, block fading, $N_{\text{t}} \times N_{\text{r}}$ MIMO wideband channel with an impulse response $h(t)^{(u,v)}$ between antennas (u, v). For compactness we assume that all channels experience a coherence time T_c and a delay spread D and the channel frequency response becomes uncorrelated for frequencies apart more than one coherence bandwidth $B_c = 1/D$. We focus only on the frequency signaling scheme since it is known [9] that differences between frequency and time signaling only affect the scaling with bandwidth in its vanishing higher order terms.

Our model represents a signaling scheme where every T_c seconds, the transmitted signal $x^{(u)}[n]$ with bandwidth $B/2$ carries $K{=}BT_c$ complex samples on antenna $u \in [0, N_{\text{t}}{-}1]$. Taking a K-point DFT, the transmitted codeword is uniquely defined by the $N_{\text{r}}K \times 1$ vector $\mathbf{x}$ that satisfies

$$\frac{1}{KN_{\text{t}}}\mathrm{E}\left[|\mathbf{x}|^2\right] \leq PT_c.$$

For $i{=}kN_{\text{t}}{+}u$, the i-th coefficient of $\mathbf{x}$, denoted as $x^{(i)}$, corresponds to the transmitted signal on antenna u with DFT index $k \in \{0, 1, \ldots, K{-}1\}$. For each pair of antennas (u, v), the discrete samples of the channel $h^{(u,v)}[n]$ have $M{=}BD$ i.i.d. coefficients, with $M/K{=}D/T_c{=}\frac{1}{B_cT_c}$. After applying K-point DFT to each discrete channel sequence $h^{(u,v)}[n]$, we define a block-diagonal matrix $\mathbf{H}$ with K blocks of size $N_{\text{r}} \times N_{\text{t}}$ matrices,

$$\mathbf{H} = \left(\begin{array}{c|c|c} \mathbf{H}[0] & \ldots & \mathbf{0} \\ \hline \vdots & \ddots & \vdots \\ \hline \mathbf{0} & \ldots & \mathbf{H}[K-1] \end{array}\right), \tag{4}$$

where $\mathbf{H}[k]$ contains in its (v,u)-th element the k-th DFT coefficient of $h^{(u,v)}[n]$. Each channel only has M i.i.d. coefficients and any two blocks $\mathbf{H}[k]$ and $\mathbf{H}[k']$ are correlated if $|k-k'| < B_c T_c$ and independent otherwise. We also define the average gain of the n-th channel coefficient $g_n^{(u,v)} = \mathrm{E}\left[|h^{(u,v)}[n]|^2\right]$ satisfying $\sum_{n=0}^{M-1} g_n^{(u,v)} = 1$.

Assuming $D \ll T_c$, there is no inter-symbol interference and the signal received on each fading realization, T_c, depends only on the state of the channel and signal transmitted during the same realization. Taking K-point DFT of the received signal we can represent the system model as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}, \tag{5}$$

where $\mathbf{y}$ is a $N_\mathrm{r}K \times 1$ vector whose i-th element $y^{(i)}$, $i{=}kN_\mathrm{r}{+}v$, corresponds to the signal received on antenna $v{\in}[0, N_\mathrm{r}{-}1]$ with DFT coefficient index k. Where the $N_\mathrm{r}K \times 1$ noise vector $\mathbf{z}$ follows a Gaussian distribution $\mathcal{CN}(0, \mathbf{I}_{N_\mathrm{r}K} N_0 T_c)$ (with PSD N_0).

Some references, such as [8], use a different type of system model with fewer frequency bins, each experiencing independent fading coefficients that repeat for many consecutive symbols. We can prove (see [1]) that the two models are equivalent at the continuous time level using concepts of Single-Carrier-OFDM modulations, and our results are independent of the model chosen.

The quintessential peaky signal distribution is the on/off distribution. To make our signaling scheme peaky we choose to make active only a fraction δ of the encoding symbols,

$$P_r(|\mathbf{x}|^2 = 0) = 1 - \delta. \tag{6}$$

This converts the system into the time-alternation of an arbitrarily distributed scheme for a fraction δ of the time, achieving a rate $R(\delta)$ with the power gain $P' = \frac{P}{\delta}$, and an idle stage for a fraction $1-\delta$ of the time. When $\frac{1-\delta}{\delta} > D/T_c$ the idle stage serves also as "zero-padding prefix" that justifies our approximation of no ISI. For a random signal $a[n]$ drawn from a stochastic sequence $A[n]$, we will refer to its *kurtosis*

$$\kappa(A) = \frac{\mathrm{E}_A\left[|a(t)|^4\right]}{\mathrm{E}_A\left[|a(t)|^2\right]^2}, \tag{7}$$

to measure the *peakiness* of the random distribution. Notice that when a signal $\mathbf{x}$ is zero a fraction $1{-}\delta$ of the time, its kurtosis can be written as a function of the kurtosis of the distribution of non-zero elements, $\kappa(\mathbf{x}) = \frac{\kappa(\mathbf{x}\neq 0)}{\delta}$, and therefore determining peakiness using the on/off ratio δ and the kurtosis statistic κ are in accordance with each other.

III. The Bandwidth Occupancy Limit

Our analysis is a generalization of the SISO analysis with non-peaky signaling in [9]. The analysis follows four steps, represented in Fig. 2.

1) Find a bell-shaped lower bound $R^{LB}(B) \leq \mathrm{I}(X;Y)$;
2) Determine the unique maximum of $R^{LB}(B)$, $R^{LB}(B^*)$;
3) Find a bell-shaped upper bound $R^{UB}(B) \geq \mathrm{I}(X;Y)$;
4) Determine the two bandwidth values B^+ and B^- such that $B^- \leq B^* \leq B^+$ and $R^{UB}(B^\pm) = R^{LB}(B^*)$.

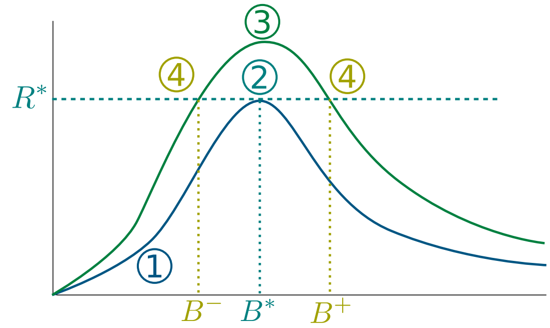


Figure 2. The four-step approach [9] to set the range of critical bandwidth.

The result of [9] is that capacity in a non-coherent fading channel only grows with bandwidth below a *critical bandwidth* B_crit which falls into the range $[B^-, B^+]$. A system operating with insufficient bandwidth $B{<}B_\mathrm{crit}$ is less efficient in converting available energy into data rate due to the convexity of the logarithm function w.r.t. the SNR, and the achievable rate grows with increasing bandwidth.

Our contribution is a generalization of this argument to arbitrary levels of signal peakiness δ and identifying the fundamental quantity *bandwidth occupancy* (δB). We obtain bell-shaped lower and upper bounds on the achievable rate, find the maximum $(\delta B)^*$ for the lower bound, and then determine the range $(\delta B)^\pm$ that contains the unknown *critical bandwidth occupancy* $(\delta B)_\mathrm{crit}$. For any $B{>}B_\mathrm{crit}$ it is possible to operate with peaky signalling with $\delta{=}B_\mathrm{crit}/B$ to bring the system back into the same optimal operation point $(\delta B)_\mathrm{crit}$.

A. Lower bound

Lemma 1. *Achievable rate in a wideband non-coherent fading channel with duty cycle $\delta \in (0,1]$ is lower bounded by*

$$R^{LB}(\delta B) = \frac{PN_\mathrm{r}}{N_0}\left[1 - \frac{P(\kappa - 2 + N_\mathrm{t} + N_\mathrm{r})}{2\delta B N_\mathrm{t} N_0}\right] - \delta\frac{BN_\mathrm{t}N_\mathrm{r}}{B_cT_c}\log\left(1 + \frac{P}{\delta B N_\mathrm{t} N_0}B_cT_c\right), \tag{8}$$

where κ is the kurtosis *of the channel.*

Proof. The proof, detailed in [1, App. B-A], contains three key steps:

- Use $\frac{1}{T_c}\mathrm{I}(X;Y) = \frac{1}{T_c}\mathrm{I}(X,H;Y) - \frac{1}{T_c}\mathrm{I}(H;Y|X)$;
- Lower bound $\frac{1}{T_c}\mathrm{I}(X,H;Y) \geq \frac{1}{T_c}\mathrm{I}(X;Y|H)$;
- Use $\log\det(\mathbf{I} + \mathbf{A}^\dagger\mathbf{A}) \geq \mathrm{tr}(\mathbf{A}^\dagger\mathbf{A}) - \mathrm{tr}((\mathbf{A}^\dagger\mathbf{A})^2)/2$. □

B. Maximum of R^{LB}

Lemma 2. *$R^{LB}(\delta B)$ is maximized at $R^{LB}((\delta B)^*)$ with*

$$(\delta B)^* \simeq \frac{P}{N_0 N_\mathrm{t}}\sqrt{\frac{B_cT_c}{\log(B_cT_c)}(\kappa - 2 + N_\mathrm{t} + N_\mathrm{r})}, \tag{9}$$

and

$$R^{LB}((\delta B)^*) \geq \frac{PN_\mathrm{r}}{N_0}\left[1 - \sqrt{\frac{\log(B_cT_c)}{B_cT_c}(\kappa - 2 + N_\mathrm{t} + N_\mathrm{r})\log\pi}\right]. \tag{10}$$

Proof. Maximize (8) with respect to the joint variable (δB), and follow the inequality of [9, Eq. 60]. See [1, App. B-B]. ☐

Below the optimal bandwidth occupancy $(\delta B)^*$, the third term of (8) is smaller in absolute value than the second. Replacing the third by the second and substituting $\delta{=}\mathrm{SNR}^{1-\alpha}$, $\alpha \in (0,1)$ as in (1), gives the following corollary on sufficient conditions.

Corollary 1. *If $\delta B \leq (\delta B)^*$, the achievable rate is lower bounded by*

$$R^{LB}(\delta B) \geq \frac{PN_\mathrm{r}}{N_0}\left[1-\left(\frac{P}{BN_0}\right)^\alpha \frac{(\kappa{-}2{+}N_\mathrm{t}{+}N_\mathrm{r})}{N_\mathrm{t}}\right]. \quad (11)$$

C. Upper Bound

Lemma 3. *Achievable rate of signalling schemes with duty cycle $\delta{\in}(0,1]$ in a wideband non-coherent Rayleigh fading channel is upper bounded by*

$$R^{UB}(\delta B) = \frac{PN_\mathrm{r}}{N_0}\Bigg[1-\frac{P}{2\delta BN_0} \quad (12)$$

$$-\frac{\delta BN_\mathrm{t}N_0}{PB_cT_c}\mathrm{E}_H\left[\log(1{+}\frac{P}{\delta N_\mathrm{t}BN_0}B_cT_cg_{\min}\psi)\right]\Bigg] + o(\frac{1}{B}),$$

where $g_{\min}{=}\min_{m,u,v}|h^{(u,v)}[m]|^2$ is the minimum non-zero square channel gain among all delays and antenna pairs, and $\psi = \frac{\lambda_*}{K}$ is the eigenvalue, normalized by K, of matrix $\Xi\Xi^\dagger$ that minimizes $\mathrm{E}\left[\log(1+\frac{P}{\delta WN_0}B_cT_cg_{\min}\lambda_m(\Xi\Xi^\dagger)/K)\right]$ for all eigenvalues indexed by m. Here Ξ is a $K\times MN_\mathrm{t}$ circulant matrix that contains in its first column the first K elements of $\mathbf{x}$ after power normalization.

Proof. Letting Ξ be a representation of a-priori known $\mathbf{x}$ as a "pilot" signal, the normalized eigenvalues $\psi_{K,\ell} = \frac{\lambda\{\Xi^\dagger\Xi\}_{K,\ell}}{K}$ are replaced by the one that gives the smallest $\mathrm{I}(H;Y|\Xi)$ for all K and ℓ, following the same trick in [9, Eq. 72]. ☐

D. Critical Bandwidth Occupancy

Lemma 4. *In a wideband non-coherent Rayleigh fading channel, the maximum rate in* (10) *is achievable at a critical bandwidth occupancy $(\delta B)_\mathrm{crit}$ that resides in the range*

$$(\delta B)^- \leq (\delta B)_\mathrm{crit} \leq (\delta B)^+, \quad (13)$$

where

$$(\delta B)^- = \frac{P}{N_0}\frac{1}{2\sqrt{(N_\mathrm{t}+N_\mathrm{r})\log\pi}}\sqrt{\frac{B_cT_c}{\log(B_cT_c)}},$$
$$(\delta B)^+ = \frac{P}{N_0}2\sqrt{\frac{(N_\mathrm{t}+N_\mathrm{r})}{N_\mathrm{t}^2}\log\pi}\sqrt{\frac{B_cT_c}{\log(B_cT_c)}}. \quad (14)$$

Proof. Define a pair of solutions $(\delta B)^-$ and $(\delta B)^+$ such that

$$\frac{P}{(\delta B)^\pm N_0} = \sqrt{\Omega\frac{\log(B_cT_c)}{B_cT_c}} + o\left(\sqrt{\frac{\log(B_cT_c)}{B_cT_c}}\right), \quad (15)$$

and solve for Ω the equality $R^{UB}(\delta B)^\pm = R^{LB}(\delta B)^* + o(\frac{1}{B_cT_c})$. Detailed proof can be found in [1, App. B-D]. ☐

Above the critical bandwidth occupancy $(\delta B)_\mathrm{crit}$, the third term of (8) is greater in absolute value than the second. This means that capacity is smaller than (11), which leads to the following corollary on necessary conditions.

Corollary 2. *In Rayleigh fading (κ=2), if $\delta = \mathrm{SNR}^{1-\alpha}$ and*

$$R(\delta B) \geq \frac{PN_\mathrm{r}}{N_0}\left[1-\left(\frac{P}{BN_0}\right)^\alpha\frac{(N_\mathrm{t}+N_\mathrm{r})}{N_\mathrm{t}}\right], \quad (16)$$

then the bandwidth occupancy satisfies $\delta B < (\delta B)^+$.

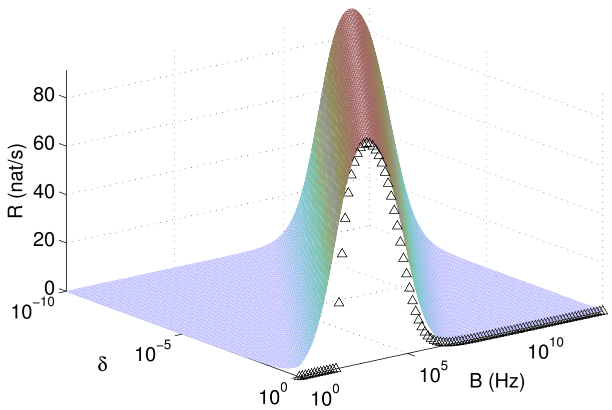
E. Interpretation of the Result

Note that our capacity lower/upper bounds (8) and (12) are both derived from $\mathrm{I}(X;Y){=}\mathrm{I}(X,H;Y){-}\mathrm{I}(H;Y|X)$, which leads to the following capacity expression

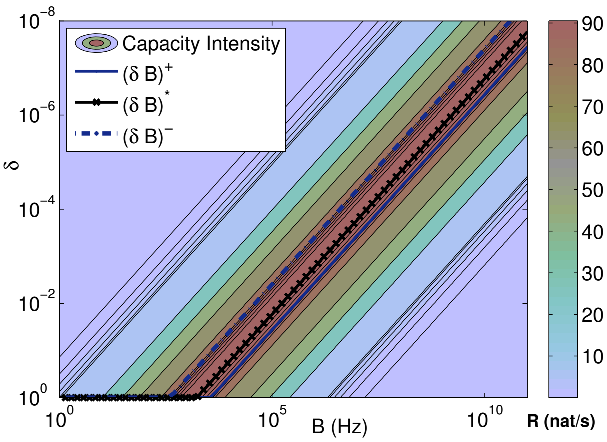
$$\frac{\delta}{T_c}\left[\Theta(K)\log(1{+}\Theta(\frac{P/\delta}{N_0B})) - \Theta(M)\log(1{+}\Theta(\frac{P/\delta}{N_0B}\frac{K}{M}))\right]$$
$$=\Theta(\delta B)\log(1{+}\Theta(\frac{P/\delta}{N_0B})) - \Theta(\frac{\delta B}{B_cT_c})\log(1{+}\Theta(\frac{PB_cT_c}{N_0\delta B})),$$

where the equality is due to substitution of $K{=}BT_c$ and $M{=}BD$. The first term corresponds to the capacity in the wideband regime, and the second term is due to penalty from channel uncertainty. According to our derived channel model, during a period of coherence time T_c, for each spatial dimension we have K i.i.d. input symbols and M i.i.d. channel coefficients. The penalty term resembles a "channel estimation" setup where M unknown channel coefficients are inferred based on K training symbols, resulting in a "power gain" of $\frac{K}{M} = B_cT_c$. As bandwidth B grows, both the number of parallel channels and the number of independent channel coefficients grows linearly with B, but the growth ratio is T_c for the former and D for the later. That is, the penalty term grows B_cT_c times slower than the first term. Since there is also a "power gain" of B_cT_c in "channel estimation", the penalty term "catches up" with the first by an additional factor $\log(B_cT_c)$. This explains the origin of $\sqrt{B_cT_c}$ and $\sqrt{\frac{1}{\log(B_cT_c)}}$ in the critical bandwidth occupancy proved in Lemma 4.

In Fig. 3(a) we represent the upper bound to capacity as a field over the 2D plane (B,δ), and in the vertical cut for $\delta = 1$ we have also represented the lower bound using triangular bullets to illustrate the relation of this representation with Fig. 2. On the B axis, we can see that for fixed values of δ the capacity as a function of bandwidth is bell-shaped, grows at small bandwidth, reaches a maximum and then decreases to zero. Fig. 3(b) provides a better perspective on the value of capacity upper bounds as a function of the bandwidth occupancy, where the optimal $(\delta B)^*$ that maximizes the capacity lower bound R^{LB} and the range $[(\delta B)^-,(\delta B)^+]$ for the critical bandwidth occupancy $(\delta B)_\mathrm{crit}$ are also plotted. For bandwidth occupancy close to $(\delta B)_\mathrm{crit}$, capacity is nearly power-limited. For different level of peakiness δ, the peak values of capacity are the same but appear at different bandwidth value B, and in fact all points with identical value δB have the same lower/upper bounds. Our analysis recovers the previous result for non-peaky signals by selecting $\delta = 1$, producing a finite



(a) Capacity upper bound for (δ, B) and the low bound for δ=1.



(b) Contour plot w.r.t. (δ, B) and levels of bandwidth occupancy.

Figure 3. Capacity upper bound over the plane (δ, B) with $B_cT_c = 10^3$ and $P/N_0 = 20$dB. Range of critical bandwidth occupancy is also shown.

critical bandwidth. It also captures the classical results for infinite-fourth-moment signals by taking $\delta \to 0$, which takes the critical bandwidth occupancy point further into higher bandwidths following $\lim_{\delta\to 0} \frac{(B\delta)_{\rm crit}}{\delta} = \infty$.

IV. Conclusions

We have generalized the analysis in [9] with the introduction of flash-signaling and MIMO. By defining the metric of bandwidth occupancy, δB, it is possible to show that previous results of limited bandwidth with non-peaky signaling [9] and unlimited bandwidth with flash signaling [8], which have been treated as very different phenomena, are merely two extreme points in a family of transmission strategies that can obtain the same nearly-power-limited capacity approximation as long as they have the same amount of bandwidth occupancy.

Our result shows the existence of a fundamental limit on the bandwidth occupancy in non-coherent channels for any level of frequency and time peakiness of the signal. At the critical bandwidth occupancy $(\delta B)_{\rm crit}$, capacity has the same almost-linear-in-power value for all types of signals

$$C \geq \frac{PN_{\rm r}}{N_0}\left[1 - \sqrt{\frac{\log(B_cT_c)}{B_cT_c}(\kappa - 2 + N_{\rm t} + N_{\rm r})\log\pi}\right].$$

Moreover, we provide upper and lower bounds to $(\delta B)_{\rm crit}$. The bounds have the same growth with B_cT_c and $\frac{P}{N_0}$, and they differ by a multiplicative gap that only depends on $N_{\rm t}/N_{\rm r}$ (the ratio between the number of transmit and receive antennas).

We note that any signaling scheme obtains the same asymptotic behavior as long as the product δB remains constant. The near-power-limited capacity can be written as a polynomial of order $1+\alpha$ by representing peakiness as $\delta = {\rm SNR}^{1-\alpha}$. As the bandwidth occupancy approaches the critical value, capacity approaches the power-limited wideband limit with a speed of convergence determined by ${\rm SNR}^{1+\alpha}$. And its speed of convergence catches up with that of coherent channels as the coherence length $L_c \to \infty$. Furthermore, we have shown in [1] that the relationship between polynomial capacity on channel coherence L_c described in [8, Theorems 1-3] can also be established following our analysis.

The criterion for selecting a level of peakiness to transmit $\delta = {\rm SNR}^{1-\alpha}$ is not valid if ${\rm SNR} > 1$ (as $\delta < 1$ by design), whereas the concept of $\delta B < (\delta B)_{\rm crit}$ is well defined for all values of SNR. Below the critical point, the frequency-selective channel is not in the wideband regime and regular non-peaky transmissions with full bandwidth occupancy must be employed. Above the critical point, the amount of peakiness and the bandwidth may be chosen arbitrarily as long as the maximum occupancy level is respected.

Acknowledgment

This work is supported in part by the Swedish Research Council (VR), MIT Wireless Center, and the National Science Foundation under Grant 1302336. This work is also funded by FPU12/01319 MECD, University of Vigo and Xunta de Galicia, Spain.

References

[1] F. Gómez-Cuba, J. Du, M. Médard, and E. Erkip, "Unified Capacity Limit of Non-Coherent Wideband Fading Channels," *submitted to IEEE Transactions on Information Theory, arXiv:1501.04905*, Jan. 2015.

[2] S. Rangan, T. T. S. Rappaport, and E. Erkip, "Millimeter-Wave Cellular Wireless Networks: Potentials and Challenges," *Proceedings of the IEEE*, vol. 102, no. 3, pp. 366–385, Mar. 2014.

[3] M. Médard and R. G. Gallager, "Bandwidth Scaling for Fading Multipath Channels," *IEEE Transactions on Information Theory*, vol. 48, pp. 840–852, Apr. 2002.

[4] I. E. Telatar and D. N. Tse, "Capacity and Mutual Information of Wideband Multipath Fading Channels," *IEEE Transactions on Information Theory*, vol. 46, pp. 1384–1400, Jul. 2000.

[5] S. Verdú, "Spectral Efficiency in the Wideband Regime," *IEEE Transactions on Information Theory*, vol. 48, pp. 1319–1343, Jun. 2002.

[6] C. Luo, M. Medard, and L. Zheng, "On Approaching Wideband Capacity Using Multitone FSK," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 9, pp. 1830–1838, Sep. 2005.

[7] L. Zheng, D. N. C. Tse, and M. Medard, "Channel Coherence in the Low-SNR Regime," *IEEE Transactions on Information Theory*, vol. 53, pp. 976–997, Mar. 2007.

[8] S. Ray, M. Medard, and L. Zheng, "On Noncoherent MIMO Channels in the Wideband Regime: Capacity and Reliability," *IEEE Transactions on Information Theory*, vol. 53, pp. 1983–2009, Jun. 2007.

[9] A. Lozano and D. Porrat, "Non-Peaky Signals in Wideband Fading Channels: Achievable Bit Rates and Optimal Bandwidth," *IEEE Transactions on Wireless Communications*, vol. 11, pp. 246–257, Jan. 2012.

[10] V. Sethuraman and B. Hajek, "Capacity Per Unit Energy of Fading Channels With a Peak Constraint," *IEEE Transactions on Information Theory*, vol. 51, pp. 3102–3120, Sep. 2005.