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# Bank Behavior and the Interbank Rate in an Oligopolistic Market

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## Abstract

The well-known Klein-Monti model of bank behavior considers a monopolistic bank. We demonstrate that this model's results on the comparative static effects of a change in the exogenous interbank market interest rate do not necessarily hold in oligopolistic Cournot or Stackelberg generalizations. Introducing asymmetries in the cost functions of the banks, or in their way of conduct, may imply counterintuitive effects on the individual banks' volumes of loans and deposits.

**Keywords:** Bank behavior, Cournot oligopoly, Stackelberg oligopoly

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## 1. Introduction

This paper investigates extensions of the well-known Klein-Monti model of a representative, profit-maximizing bank, originally introduced by Klein (1971) and Monti (1972). The Klein-Monti model is a prototype model of the so-called Industrial Organization approach to banking, in which banks are considered as profit-maximizing firms that offer services to agents; see e.g. the recent book by Freixas and Rochet (1997). These services are described by the securities that banks buy from agents (i.e. loans) and sell to agents (i.e. deposits). The difference between the volume of deposits and the volume of loans is the bank's (net) position on the interbank market.

The Klein-Monti model is described and compared to alternative models of banking in surveys by Baltensperger (1980) and Santomero (1984). It has been generalized and extended by many authors, for example by Dermine (1986) and Prisman et al. (1986). Hannan (1991) shows that the model can be used to derive various empirical predictions. For that reason, it has been the (implicit) starting point for a number of empirical studies, for instance in Molyneux et al. (1994), Neuberger and Zimmerman (1990), and Suominen (1994). The model is also discussed in detail by Freixas and Rochet (1997).

Although the original Klein-Monti model concentrates on the case of a single, monopolistic bank, which might apply in countries with only one (state) bank, the situation of several banks is more interesting. In fact, as Molyneux et al. (1994) observe for the case of Europe, in many countries the banking industry is very concentrated, which suggests that oligopoly models are relevant for banking. In order to extend the Klein-Monti model to the case of more than one bank, the standard oligopoly models from the theory of Industrial Organization (Martin, 1993) can be used as a starting point. In particular, the extension towards a symmetric Cournot oligopoly, in which all banks are assumed to have the same linear management-cost function, is straightforward, as shown by Freixas and Rochet (1997). These authors examine some comparative static properties of both the original model and the symmetric Cournot version with respect to changes in the exogenous interbank market interest rate. Such changes can be made by the central bank in order to influence the volumes of loans and deposits of banks and the corresponding interest rates. We extend their analysis to other forms of market structure.

Intuitively, one would expect an increase in the interbank market rate to lead to a decrease in a bank's volume of loans, an increase in its volume of deposits, and increases in the interest rates on loans and deposits. This is exactly what occurs both in the original, monopolistic Klein-Monti model and in the symmetric Cournot version of Freixas and Rochet (1997). In this paper we demonstrate that this result does not

necessarily hold in asymmetric oligopolistic generalizations of the model. In order to show this we introduce asymmetries either in the management-cost functions of the banks or in their way of conduct. For simplicity, we concentrate on the situation with two banks. In particular, we investigate the Cournot case with asymmetric management-cost functions and, as an example of asymmetric conduct, the Stackelberg case. It turns out that in both cases we can obtain counterintuitive comparative static effects of a change in the interbank market interest rate on individual banks' volumes of loans (deposits).

The observation that comparative static effects in oligopolistic markets might be counterintuitive is also made in some related studies. In particular, Dixit (1986) investigates a general quantity-setting conjectural variations oligopoly. The conjectural variations, as well as the cost functions, may be different for different firms. The Cournot case and the Stackelberg case can be obtained as special cases by choosing the conjectural variations in an appropriate way. However, Dixit focuses mainly on the general methodology of comparative statics and on the effects of parameter changes on the profits of the firms, whereas we focus on the output (loan and deposit volumes) and price (interest rate) effects as these are more relevant in our context. Moreover, Dixit only mentions the Stackelberg case in passing, without further analysing it. Katz and Rosen (1985) consider a similar kind of oligopoly in which the conjectural variations as well as the cost functions are identical for all firms. As a result, our analysis does not fit within their framework. Kimmel (1992) investigates the effects of common cost changes in a Cournot oligopoly. However, Kimmel focuses the attention on the effects of these changes on the profits and market shares of the firms, i.e. not on the absolute size of the output of each firm as we do. Finally, Caputo (1996) discusses comparative static properties of Nash equilibria by using a so-called dual methodology which is based on the Envelope Theorem. Using the same methodology, Caputo (1998) analyses comparative statics of a Stackelberg equilibrium. However, these papers focus mainly on the general methodology, and our results do not readily follow from them.

The next section introduces the original Klein-Monti model of a monopolistic bank, and summarizes its comparative static properties. This model will be considered as our benchmark case. Section 3 presents the generalized version of the model and its comparative static properties in the situation where the two banks are Cournot oligopolists with asymmetric management-cost functions. Section 4 examines the Stackelberg case, in which conduct is asymmetric. Section 5 concludes.

## 2. The Klein-Monti Model

Assume that there is a single, monopolistic bank, that chooses its outputs in order to maximize profits. The bank operates on the market for loans as well as on the market for deposits. The difference between the volume of loans  $L$  and the volume of deposits  $D$  of the bank can be borrowed (or lent, if negative) on an interbank market. Denote the interest rates on the loan market and deposit market by  $r_L$  and  $r_D$ , respectively. The inverse demand function for loans is given by  $r_L(L)$ , with derivative  $r'_L(L) < 0$ , and the inverse supply function of deposits is  $r_D(D)$ , with derivative  $r'_D(D) > 0$ . The cost of managing an amount  $L$  of loans and an amount  $D$  of deposits is given by the convex management-cost function  $C(L, D)$ . The functions  $r_L(\cdot)$ ,  $r_D(\cdot)$  and  $C(L, D)$  are continuously differentiable up to any order.

Let  $r$  denote the exogenous interest rate on the interbank market, and  $\alpha$  be the exogenous fraction of deposits that is required as a non-interest bearing reserve ( $0 \leq \alpha < 1$ ). Both  $r$  and  $\alpha$  are set by the central bank.

The bank's decision problem is to maximize its profits  $\pi(L, D)$ , i.e.

$$\max_{(L,D)} \pi(L, D) = [r_L(L) - r]L + [r(1 - \alpha) - r_D(D)]D - C(L, D)$$

We assume that  $\pi(L, D)$  is strictly concave. The first-order conditions are

$$\frac{\partial \pi}{\partial L} = r'_L(L)L + r_L(L) - r - \frac{\partial}{\partial L}C(L, D) = 0 \quad (1)$$

$$\frac{\partial \pi}{\partial D} = r(1 - \alpha) - r'_D(D)D - r_D(D) - \frac{\partial}{\partial D}C(L, D) = 0 \quad (2)$$

From (1) and (2), the unique (positive) solution  $(\widehat{L}, \widehat{D})$  can be derived. The corresponding interest rates are given by  $\widehat{r}_L$  and  $\widehat{r}_D$ . If the cost function is separable, i.e.  $C(L, D) = C_L(L) + C_D(D)$ , the maximization problem is separable. That is, the optimal volume of loans  $\widehat{L}$  (and the corresponding interest rate  $\widehat{r}_L$ ) is independent of the properties of the deposit market, and the optimal volume of deposits  $\widehat{D}$  (and the corresponding interest rate  $\widehat{r}_D$ ) is independent of the properties of the loan market.

Freixas and Rochet (1997, p. 59) discuss the comparative static effects of a change of the interbank interest rate  $r$  in the Klein-Monti model, assuming separability. They show that  $d\widehat{L}/dr < 0$ ,  $d\widehat{D}/dr > 0$ ,  $d\widehat{r}_L/dr > 0$ , and  $d\widehat{r}_D/dr > 0$ , which we will refer to as the benchmark case.

### 3. Asymmetric Management Costs

Next, we consider the case in which there is Cournot competition with two banks on both markets. Let the index  $i$  denote bank  $i$ ,  $i = 1, 2$ . Define total loan and deposit volumes by  $L \equiv L_1 + L_2$  and  $D \equiv D_1 + D_2$ . Bank  $i$  maximizes its profit function  $\pi_i(L_i, D_i)$ , which is assumed to be strictly concave. The maximization problem for bank  $i$  is

$$\begin{aligned} \max_{(L_i, D_i)} \pi_i(L_i, D_i) = & [r_L(L_i + L_j) - r]L_i \\ & + [r(1 - \alpha) - r_D(D_i + D_j)]D_i - C_i(L_i, D_i) \end{aligned}$$

where  $i, j = 1, 2, i \neq j$ . Assume that the cost function  $C_i(L_i, D_i)$  is linear,

$$C_i(L_i, D_i) = \gamma_{L,i}L_i + \gamma_{D,i}D_i \quad (3)$$

in order to keep the analysis manageable. Note that the cost function (3) is not necessarily equal for the two banks, i.e. we allow for asymmetric costs.

We assume that a unique (positive) Nash-Cournot equilibrium,  $(L_i^*, D_i^*)$ ,  $i = 1, 2$ , exists, with corresponding interest rates  $r_L^*$  and  $r_D^*$ . It is given by the simultaneous solution of the first-order conditions

$$\frac{\partial \pi_i}{\partial L_i} = r'_L(L_i + L_j)L_i + r_L(L_i + L_j) - r - \gamma_{L,i} = 0 \quad (4)$$

$$\frac{\partial \pi_i}{\partial D_i} = r(1 - \alpha) - r'_D(D_i + D_j)D_i - r_D(D_i + D_j) - \gamma_{D,i} = 0 \quad (5)$$

with  $i, j = 1, 2, i \neq j$ . In case the two banks have the same cost function, the solution is symmetric, i.e.  $L_1^* = L_2^*$  and  $D_1^* = D_2^*$ . On the other hand, in the asymmetric costs case we have  $L_i^* > L_j^*$  if and only if  $\gamma_{L,i} < \gamma_{L,j}$ , and  $D_i^* > D_j^*$  if and only if  $\gamma_{D,i} < \gamma_{D,j}$ .

Proceeding, we observe that (4) and (5) implicitly define the reaction functions  $L_1 = f_1(L_2)$ ,  $L_2 = f_2(L_1)$ ,  $D_1 = g_1(D_2)$  and  $D_2 = g_2(D_1)$ . Let us consider the derivatives of the reaction functions, and concentrate on bank 2. For the loan side we obtain:

$$f'_2(L_1) = -\frac{r'_L(\cdot) + r''_L(\cdot)f_2(L_1)}{2r'_L(\cdot) + r''_L(\cdot)f_2(L_1)} \quad (6)$$

where the first-order and second-order derivatives of  $r_L(\cdot)$  are evaluated in the point  $(L_1 + f_2(L_1))$ . The denominator is identical to the second-order derivative of bank 2's (strictly concave) profit function with respect to  $L_2$  and therefore is negative. This

shows that  $f'_2(L_1) > -1$ . We assume that  $r''_L(\cdot) < -r'_L(\cdot)/f_2(L_1)$ , i.e. the inverse demand function for loans is not too convex. Consequently,

$$-1 < f'_2(L_1) < 0 \quad (7)$$

Similarly, for the deposit side we assume  $r''_D(\cdot) > -r'_D(\cdot)/g_2(D_1)$  i.e. the inverse supply function of deposits is not too concave, which implies that

$$-1 < g'_2(D_1) < 0 \quad (8)$$

For bank 1, a similar result holds. Decreasing reaction functions can be considered as the normal case with quantity strategies (Shapiro, 1989). Note that with linear inverse loan demand, we have  $f'_i(L_j) = -\frac{1}{2}$ , and with linear inverse deposit supply, we have  $g'_i(D_j) = -\frac{1}{2}$ ,  $i, j = 1, 2, i \neq j$ .

Now let us turn to the comparative static effects of a change in the interbank interest rate  $r$  in this Cournot version of the Klein-Monti model. This question is also considered by Freixas and Rochet (1997, p. 60), who assume symmetric, linear management-cost functions. Also, for simplicity, they assume constant elasticities of demand of loans and supply of deposits. We do not use the latter assumption. We remark that we will only discuss the details here for the loan side. Details for the deposit side are similar.

By totally differentiating (4) with respect to  $r$  for  $i = 1, 2$ , and next solving the resulting two equations, we obtain

$$\frac{dL_i^*}{dr} = \frac{r'_L(L^*) + r''_L(L^*)[L_j^* - L_i^*]}{r'_L(L^*)[3r'_L(L^*) + r''_L(L^*)L^*]} \quad i, j = 1, 2, i \neq j \quad (9)$$

$$\frac{dL^*}{dr} = \frac{2}{3r'_L(L^*) + r''_L(L^*)L^*} \quad (10)$$

$$\frac{dr_L^*}{dr} = \frac{2r'_L(L^*)}{3r'_L(L^*) + r''_L(L^*)L^*} \quad (11)$$

where  $L^* \equiv L_1^* + L_2^*$ . A similar result can be obtained for deposits. Next, let  $\epsilon_L(L_i)$  be the elasticity of  $r'_L(L_i + L_j)$  with respect to  $L_i$ , i.e.  $\epsilon_L(L_i) = r''_L(L)L_i/r'_L(L)$ . Similarly,  $\epsilon_D(D_i)$  is the elasticity of  $r'_D(D_i + D_j)$  with respect to  $D_i$ . Using this, we can present Proposition 3.1.

**Proposition 3.1** *In the Cournot version of the model the following holds:*

$$(a) \quad \frac{dL^*}{dr} < 0 \text{ and } \frac{dr_L^*}{dr} > 0.$$

- (b) Let  $r_L''(L^*) \geq 0$  and/or the marginal management costs of loans be identical for both firms, i.e.  $\gamma_{L,1} = \gamma_{L,2}$ . Then  $\frac{dL_i^*}{dr} < 0$ ,  $i = 1, 2$ .
- (c) Let  $r_L''(L^*) < 0$  and the marginal management costs of loans be different for both banks, with  $\gamma_{L,1} < \gamma_{L,2}$  say. Then  $\frac{dL_2^*}{dr} < 0$ . Moreover,  $\frac{dL_1^*}{dr} \leq 0$  if and only if  $1 + \epsilon_L(L_2^*) \geq \epsilon_L(L_1^*)$ .
- (d)  $\frac{dD^*}{dr} > 0$  and  $\frac{dr_D^*}{dr} > 0$ .
- (e) Let  $r_D''(D^*) \leq 0$  and/or the marginal management costs of deposits be identical for both firms, i.e.  $\gamma_{D,1} = \gamma_{D,2}$ . Then  $\frac{dD_i^*}{dr} > 0$ ,  $i = 1, 2$ .
- (f) Let  $r_D''(D^*) > 0$  and the marginal management costs of deposits be different for both banks, with  $\gamma_{D,1} < \gamma_{D,2}$  say. Then  $\frac{dD_2^*}{dr} > 0$ . Moreover,  $\frac{dD_1^*}{dr} \geq 0$  if and only if  $1 + \epsilon_D(D_2^*) \geq \epsilon_D(D_1^*)$ .

PROOF. Recall that the profit function  $\pi_i(\cdot)$  is strictly concave, and notice that  $r_L'(L^*) + r_L''(L^*)L_j^* < 0$ , as the reaction function  $f_j(L_i)$  is downward sloping,  $i, j = 1, 2$ ,  $j \neq i$ . Using this, part (a) easily follows from (10) and (11). Next, we see that  $dL_i^*/dr \geq 0$  if and only if the numerator of (9) is nonnegative. In turn, since  $r_L'(L^*) + r_L''(L^*)L_j^* < 0$ , the latter condition implies that  $-r_L''(L^*)L_i^* > 0$ , which cannot hold if  $r_L''(L^*) \geq 0$ . This proves part (b). Part (c) follows from (9), the fact that  $L_1^* > L_2^*$  if and only if  $\gamma_{L,1} < \gamma_{L,2}$ , and the observation that  $r_L'(L^*) + r_L''(L^*)[L_2^* - L_1^*] \geq 0$  if and only if  $1 + \epsilon_L(L_2^*) \leq \epsilon_L(L_1^*)$ . The parts (d), (e) and (f) can be proven similarly.

We remark first that parts (a) and (d) of Proposition 3.1 also follow easily from Kimmel (1992, Proposition 1). Second, from parts (a) and (b) of Proposition 3.1 we conclude that in the symmetric case with identical management-cost functions, the comparative static effects of a change in  $r$  on  $L_1^*$ ,  $L_2^*$  and  $L^*$  all have the ‘normal’ negative sign, directly comparable to the result of the original Klein-Monti model. Moreover, it appears that this also holds for asymmetric cost functions as long as the inverse loan demand function is convex in the Nash-Cournot equilibrium (note that a linear inverse loan demand function satisfies this requirement).

On the contrary, part (c) learns that if the marginal costs of the banks are different, and moreover the inverse loan demand is strictly concave in the Nash-Cournot equilibrium, then the sign of the effect of a change in  $r$  on the loan volume of the bank with the smallest marginal loan costs depends on the relative sizes of  $\epsilon_L(L_1)$  and  $\epsilon_L(L_2)$ . In particular, if bank 1 has the smallest marginal loan costs, then the loan volume of bank 1 changes in the *same* direction as the interbank market rate, i.e.  $dL_1^*/dr > 0$ , if and only if  $1 + \epsilon_L(L_2^*) < \epsilon_L(L_1^*)$ , i.e. if the elasticity  $\epsilon_L(L_1^*)$  is ‘large’ as compared to the elasticity  $\epsilon_L(L_2^*)$ . This stands in contrast to the intuitive, benchmark result of the monopolistic Klein-Monti model and the symmetric Cournot version.



We make two remarks here. First, recalling that  $L_1^* > L_2^*$  as  $\gamma_{L,1} < \gamma_{L,2}$ , we see that the counterintuitive  $dL_1^*/dr > 0$  change applies to the bank with the largest volume of loans. Second,  $dL_1^*/dr > 0$  implies that  $r_L''(L^*)[L_2^* - L_1^*] > 0$ . It can be verified by using (6) that  $r_L''(L^*)[L_2^* - L_1^*] > 0$  if and only if  $f_2'(L_1^*) < f_1'(L_2^*)$ , i.e. the derivative of the reaction function of bank 2 is in the equilibrium smaller than the derivative of the reaction function of bank 1. Finally, the deposit side can be discussed in a similar way.

#### 4. Asymmetric Conduct

Now consider the Stackelberg model of quantity leadership. Suppose that bank 1 is the leader (i.e., it can set its quantities  $L_1$  and  $D_1$  first), and bank 2 is the follower. As in the previous section, assume that the management-cost functions of the banks are linear, but now also assume that they are equal. That is, the only asymmetry is now caused by the way of conduct.

This two-stage model is solved backwards. In the second stage, bank 2 maximizes its profits, taking as given the output  $(L_1, D_1)$  of bank 1. This maximization problem is the same as that of a Cournot bank. The first-order conditions for the follower are given by (4) and (5), assuming  $\gamma_{L,i} = \gamma_L$  and  $\gamma_{D,i} = \gamma_D$ ,  $i = 1, 2$ . In this section, it is convenient to write the reaction function related to the loans of firm 2 as  $L_2 = f(L_1, r)$ , i.e. we include  $r$  explicitly as an argument and omit the subscript '2' of  $f(\cdot)$ . As a matter of notation, the first-order partial derivatives of  $f(\cdot)$  with respect to respectively  $L_1$  and  $r$  will be abbreviated as  $f_L'(\cdot)$  and  $f_r'(\cdot)$ . In a similar way, we write the reaction function for the deposit side of bank 2 as  $g(D_1, r)$ , with first-order partial derivatives  $g_D'(\cdot)$  and  $g_r'(\cdot)$ . With regard to bank 2 we make the same assumptions as made in the previous section with respect to the Cournot banks. In particular, we have  $-1 < f_L'(\cdot) < 0$  and  $-1 < g_D'(\cdot) < 0$ .

Next, look at the first stage of the model. Bank 1 wants to choose the amounts  $L_1$  and  $D_1$  such that its profit is maximized, taking into account how bank 2 will respond to its choice. The problem for bank 1 is therefore

$$\begin{aligned} \max_{(L_1, D_1)} \pi_1(L_1, D_1) = & [r_L(L_1 + f(L_1, r)) - r]L_1 \\ & + [r(1 - \alpha) - r_D(D_1 + g(D_1, r))]D_1 - C(L_1, D_1) \end{aligned}$$

where we assume that the profit function  $\pi_1(\cdot)$  is strictly concave. The first-order conditions for the leader are

$$\frac{\partial \pi_1}{\partial L_1} = r_L'(L_1 + f(L_1, r))[1 + f_L'(\cdot)]L_1$$

$$+ r_L(L_1 + f(L_1, r)) - r - \gamma_L = 0 \quad (12)$$

$$\begin{aligned} \frac{\partial \pi_1}{\partial D_1} &= r(1 - \alpha) - r'_D(D_1 + g(D_1, r))[1 + g'_D(\cdot)]D_1 \\ &\quad - r_D(D_1 + g(D_1, r)) - \gamma_D = 0 \end{aligned} \quad (13)$$

We assume that a unique (positive) Stackelberg equilibrium exists. It is characterized by the four first-order conditions and denoted by  $\tilde{L}_1$ ,  $\tilde{L}_2$ ,  $\tilde{D}_1$  and  $\tilde{D}_2$ . The corresponding total equilibrium volumes are  $\tilde{L} \equiv \tilde{L}_1 + \tilde{L}_2$  and  $\tilde{D} \equiv \tilde{D}_1 + \tilde{D}_2$ , and the corresponding interest rates are  $\tilde{r}_L$  and  $\tilde{r}_D$ .

It follows from (4) and (12) and the assumption that the marginal management costs of loans of the two banks are equal that

$$f(\cdot) = \tilde{L}_2(\cdot) = [1 + f'_L(\cdot)]\tilde{L}_1 \quad (14)$$

As a result,  $\tilde{L}_1 > \tilde{L}_2$ , i.e. the volume of loans of the leader bank is largest. Similarly, it can be shown that  $\tilde{D}_1 > \tilde{D}_2$ .

Now consider the effects of a change in the interbank interest rate  $r$ . We first observe that for the follower, the definition of  $f(\cdot)$  shows that

$$\frac{d\tilde{L}_2}{dr} = f'_L(\cdot) \frac{d\tilde{L}_1}{dr} + f'_r(\cdot)$$

which implies

$$\frac{d\tilde{L}}{dr} = \frac{d\tilde{L}_1}{dr} + \frac{d\tilde{L}_2}{dr} = [1 + f'_L(\cdot)] \frac{d\tilde{L}_1}{dr} + f'_r(\cdot) \quad (15)$$

It is easy to verify that  $f'_r(\cdot) < 0$ . Thus, we conclude directly that if  $d\tilde{L}_1/dr < 0$ , then  $d\tilde{L}_2/dr < 0$  as well. In a similar way, if  $d\tilde{D}_1/dr > 0$ , then  $d\tilde{D}_2/dr > 0$  as well. Next, we present the following helpful lemma.

**Lemma 1** *In the Stackelberg version of the model, where bank 1 is the leader and bank 2 the follower, we have  $\frac{d\tilde{L}_1}{dr} = \frac{A_1}{A_2}$  and  $\frac{d\tilde{L}}{dr} = \frac{A_3}{A_2}$ , with  $A_2 < 0$  the second-order derivative of the (strictly concave) profit function of bank 1 with respect to  $L_1$ , and*

$$\begin{aligned} A_1 &= -6(f'_L(\cdot))^2 - 6f'_L(\cdot) - 1 + H_L(\cdot) \\ A_3 &= -(f'_L(\cdot))^2 + 1 > 0 \end{aligned}$$

where

$$H_L(\cdot) = \frac{[1 + f'_L(\cdot)]r_L'''(\cdot)(f(\cdot))^2}{2r'_L(\cdot) + r_L''(\cdot)f(\cdot)}$$

All expressions are evaluated in the Stackelberg equilibrium.

PROOF. See the appendix.

Observe that the sign of  $H_L(\cdot)$  is minus the sign of  $r_L'''(\cdot)$ , the third-order derivative of  $r_L(\cdot)$ . Similarly, for the deposit side there holds:

**Lemma 2** *In the Stackelberg version of the model, where bank 1 is the leader and bank 2 is the follower, we have  $\frac{d\tilde{D}_1}{dr} = \frac{B_1}{B_2}$  and  $\frac{d\tilde{D}}{dr} = \frac{B_3}{B_2}$ , with  $B_2 < 0$  the second-order derivative of the (strictly concave) profit function of bank 1 with respect to  $D_1$ , and*

$$\begin{aligned} B_1 &= (1 - \alpha)[-6(g'_D(\cdot))^2 - 6g'_D(\cdot) - 1 + H_D(\cdot)] \\ B_3 &= (1 - \alpha)[-(g'_D(\cdot))^2 + 1] > 0 \end{aligned}$$

where

$$H_D(\cdot) = \frac{[1 + g'_D(\cdot)]r_D'''(\cdot)(g(\cdot))^2}{2r_D'(\cdot) + r_D''(\cdot)g(\cdot)}$$

All expressions are evaluated in the Stackelberg equilibrium.

Observe that the sign of  $H_D(\cdot)$  is the same as the sign of  $r_D'''(\cdot)$ . Using (7), (8), and Lemma's 1 and 2, we easily obtain the following proposition on the effects of a change in the interbank market rate  $r$ .

**Proposition 4.1** *In the Stackelberg version of the model, where bank 1 is the leader and bank 2 is the follower, the following holds:*

- (a)  $\frac{d\tilde{L}}{dr} < 0$  and  $\frac{d\tilde{r}_L}{dr} > 0$ .
- (b) Let  $r_L'''(\tilde{L}_1 + \tilde{L}_2) = 0$ . Then  $\frac{d\tilde{L}_1}{dr} < 0$  if and only if  $c_1 < f'_L(\tilde{L}_1, r) < c_2$ .
- (c) Let  $r_L'''(\tilde{L}_1 + \tilde{L}_2) < 0$ . Then  $f'_L(\tilde{L}_1, r) \in (c_1, c_2)$  implies  $\frac{d\tilde{L}_1}{dr} < 0$ .
- (d) Let  $r_L'''(\tilde{L}_1 + \tilde{L}_2) > 0$ . Then  $f'_L(\tilde{L}_1, r) \in (-1, c_1]$  or  $f'_L(\tilde{L}_1, r) \in [c_2, 0)$  implies  $\frac{d\tilde{L}_1}{dr} > 0$ .
- (e)  $\frac{d\tilde{D}}{dr} > 0$  and  $\frac{d\tilde{r}_D}{dr} > 0$ .
- (f) Let  $r_D'''(\tilde{D}_1 + \tilde{D}_2) = 0$ . Then  $\frac{d\tilde{D}_1}{dr} > 0$  if and only if  $c_1 < g'_D(\tilde{D}_1, r) < c_2$ .
- (g) Let  $r_D'''(\tilde{D}_1 + \tilde{D}_2) > 0$ . Then  $g'_D(\tilde{D}_1, r) \in (c_1, c_2)$  implies  $\frac{d\tilde{D}_1}{dr} > 0$ .
- (h) Let  $r_D'''(\tilde{D}_1 + \tilde{D}_2) < 0$ . Then  $g'_D(\tilde{D}_1, r) \in (-1, c_1]$  or  $g'_D(\tilde{D}_1, r) \in [c_2, 0)$  implies  $\frac{d\tilde{D}_1}{dr} < 0$ .

where  $c_1 = -\frac{1}{2} - \frac{1}{6}\sqrt{3} \approx -0.79$  and  $c_2 = \frac{1}{2} + \frac{1}{6}\sqrt{3} \approx -0.21$ .

Part (a) of Proposition 4.1 shows that the comparative static effect on the total volume of loans  $\tilde{L}$  has the ‘normal’ negative sign. However, parts (b) and (d) point out that there are situations where the effect on the volume of loans of the leader bank 1 is positive, i.e.  $d\tilde{L}_1/dr > 0$ . We notice that the critical values  $c_1$  and  $c_2$  are located symmetrically around  $-\frac{1}{2}$ . Recall that if the inverse loan demand is linear, we have  $f'_L(\tilde{L}_1, r) = -\frac{1}{2}$ . Thus, in the situations of parts (b) and (d) with  $d\tilde{L}_1/dr > 0$ , the value of  $f'_L(\tilde{L}_1, r)$  is sufficiently different from its value in the linear case. Intuitively speaking, we can say that counterintuitive effects can occur if we are sufficiently far away from the linear case. We further remark that  $d\tilde{L}_1/dr > 0$  implies that  $d\tilde{L}_2/dr < 0$ , because  $d\tilde{L}/dr < 0$ . Recalling that  $\tilde{L}_1 > \tilde{L}_2$ , we see that, just as in the asymmetric Cournot case, the counterintuitive effect applies to the bank with the largest volume of loans. Finally, we remark again that the results of the deposit side can be discussed in a similar way.

## 5. Conclusions

In the original, monopolistic Klein-Monti bank model and the corresponding Cournot generalization with symmetric management costs, a change in the exogenous interbank market interest rate leads to the intuitive result of a decrease in a bank’s volume of loans, an increase in its volume of deposits, and increases in the interest rates on loans and deposits. This paper demonstrates that for the Cournot version with asymmetric costs as well as for the Stackelberg version of the model, the same results hold for the *total* volumes of loans and deposits, and the corresponding interest rates.

However, in the asymmetric-cost Cournot version the changes in the *individual* volumes of loans and deposits of the bank with the smallest costs may change direction. The same holds for the *individual* volumes of the leader in the Stackelberg version. That is, we have shown that for oligopolistic generalizations of the Klein-Monti model, when there are asymmetries, either in the cost functions of the banks or in the way of conduct, a change in the interbank rate may lead to counterintuitive results for the individual loan and deposit volumes of the banks, even in the case of only two banks. In both cases, the bank for which the counterintuitive effect occurs is the one with the largest volume of loans (or deposits).

## Appendix: Proof of Lemma 1

In this appendix we briefly discuss the proof of Lemma 1. In order to provide the proof, the following equations are useful:

$$f'_L(\cdot) = -\frac{r'_L(\cdot) + r''_L(\cdot)f(\cdot)}{2r'_L(\cdot) + r''_L(\cdot)f(\cdot)} \quad (\text{A.1})$$

$$1 + f'_L(\cdot) = \frac{r'_L(\cdot)}{2r'_L(\cdot) + r''_L(\cdot)f(\cdot)} \quad (\text{A.2})$$

$$f'_r(\cdot) = \frac{1}{2r'_L(\cdot) + r''_L(\cdot)f(\cdot)} \quad (\text{A.3})$$

$$\frac{\partial^2 f}{\partial L_1 \partial r} = \frac{-[r''_L(\cdot)f(\cdot) + 2r'_L(\cdot)][1 + f'_L(\cdot)]f'_r(\cdot) - r''_L(\cdot)f'_L(\cdot)f'_r(\cdot)}{2r'_L(\cdot) + r''_L(\cdot)f(\cdot)} \quad (\text{A.4})$$

$$\frac{\partial^2 f}{\partial L_1^2} = \frac{-[r''_L(\cdot)f(\cdot) + r'_L(\cdot)][1 + f'_L(\cdot)]^2 - 2r''_L(\cdot)f'_L(\cdot)[1 + f'_L(\cdot)]}{2r'_L(\cdot) + r''_L(\cdot)f(\cdot)} \quad (\text{A.5})$$

where the derivatives of  $f(\cdot)$  have been computed by differentiating the first-order condition (4) of the follower.

Let us first concentrate on the leader. Differentiating the first-order condition (12) with respect to  $r$ , and solving for  $dL_1/dr$  gives the result that in the Stackelberg equilibrium we have  $d\tilde{L}_1/dr = A_1/A_2$ , where

$$A_1 = 1 - r'_L(\cdot)f'_r(\cdot) - r''_L(\cdot)[1 + f'_L(\cdot)]\tilde{L}_1 f'_r(\cdot) - r'_L(\cdot)\tilde{L}_1 \frac{\partial^2 f(\cdot)}{\partial L_1 \partial r} \quad (\text{A.6})$$

$$A_2 = 2r'_L(\cdot)[1 + f'_L(\cdot)] + r''_L(\cdot)[1 + f'_L(\cdot)]^2 \tilde{L}_1 + r'_L(\cdot)\tilde{L}_1 \frac{\partial^2 f(\cdot)}{\partial L_1^2} < 0 \quad (\text{A.7})$$

Here, all derivatives of  $r_L(\cdot)$  are evaluated in the point  $(\tilde{L}_1 + f(\tilde{L}_1, r))$ , and  $f(\cdot)$  and its derivatives are evaluated in  $(\tilde{L}_1, r)$ . Using (A.1), (A.2), (A.3), and (A.4), it can be verified that (A.6) can be rewritten as

$$A_1 = -f'_L(\cdot) - \frac{r'_L(\cdot)r''_L(\cdot)[1 + f'_L(\cdot)]\tilde{L}_1}{[2r'_L(\cdot) + r''_L(\cdot)f(\cdot)]^2} - \frac{2(r''_L(\cdot))^2 f(\cdot)[1 + f'_L(\cdot)]\tilde{L}_1}{[2r'_L(\cdot) + r''_L(\cdot)f(\cdot)]^2} + H_L(\cdot) \quad (\text{A.8})$$

where

$$H_L(\cdot) \equiv \frac{[1 + f'_L(\cdot)]r'''_L(\cdot)(f(\cdot))^2}{2r'_L(\cdot) + r''_L(\cdot)f(\cdot)}$$

Recalling (14), we substitute  $f(\cdot) = [1 + f'_L(\cdot)]\tilde{L}_1$  into (A.8). Rewriting the resulting expression using (A.1) and (A.2) shows that  $A_1$  can be written as

$$A_1 = -6(f'_L(\cdot))^2 - 6f'_L(\cdot) - 1 + H_L(\cdot) \quad (\text{A.9})$$

which proves the part concerning  $A_1$  of Lemma 1.

Next, in order to demonstrate the part concerning  $A_3$ , we observe that it follows from (15) that  $d\tilde{L}/dr = A_3/A_2$ , where

$$A_3 = (1 + f'_L(\cdot))A_1 + f'_r(\cdot)A_2 \quad (\text{A.10})$$

By making use of (A.5) and applying the same methods as above,  $f'_r(\cdot)A_2$  can be shown to satisfy

$$f'_r(\cdot)A_2 = 6(f'_L(\cdot))^3 + 11(f'_L(\cdot))^2 + 7f'_L(\cdot) + 2 - [1 + f'_L(\cdot)]H_L(\cdot) \quad (\text{A.11})$$

Substituting (A.9) and (A.11) into (A.10) gives

$$A_3 = -(f'_L(\cdot))^2 + 1 \quad (\text{A.12})$$

which completes the proof.

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