Bargaining over a finite set of alternatives

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Abstract We analyze bilateral bargaining over a finite set of alternatives. We look for "good" *ordinal* solutions to such problems and show that Unanimity Compromise and Rational Compromise are the only bargaining rules that satisfy a basic set of properties. We then extend our analysis to admit problems with countably infinite alternatives. We show that, on this class, no bargaining rule choosing finite subsets of alternatives can be *neutral*. When rephrased in the utility framework of Nash (1950), this implies that there is no *ordinal* bargaining rule that is *finite-valued*.

1 Introduction

Consider two agents negotiating over a set of alternatives. The outcome is any alternative on which they unanimously agree and, in case of no unanimous agreement, a predetermined "disagreement" alternative is realized. Nash (1950) analyzes this "bargaining problem" under the assumptions that (1) negotiations take place, not only over physical alternatives, but also over their lotteries as well, and (2) the agents' preferences over lotteries satisfy the von Neumann–Morgenstern axioms. Most real-life negotiations violate

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these simplifying assumptions, however. In particular, they take place over a countable (and often finite) number of alternatives.[1]

We study bargaining between two agents who have *complete*, *transitive*, and *antisymmetric* preferences over a finite set of physical alternatives. We focus on the bargaining rules that are *ordinal*, that is, independent from the functional forms chosen to represent the agents' preferences. With uncountably many alternatives, the only such rules on the Nash (1950) domain are the dictatorial rules and the "always-disagreement" rule (Shapley 1969).[2] With a finite number of alternatives, however, many *ordinal* rules exist. Among them, we look for ones that satisfy other desirable criteria. We also extend our analysis to cases where the alternatives are infinite but countable in cardinality.

There are two alternative approaches to modeling cooperative bargaining problems. The first and the most standard in the literature, following Nash (1950), is formulating the problems in utility space and using consistency axioms [such as *scale invariance* (Nash 1950) or *ordinal invariance* (Shapley 1969)] to render the solution independent of the particular utility functions chosen to represent the underlying preferences. The second approach formulates the problems in the space of alternatives along with preferences over these but without any reference to utility functions. With a finite number of alternatives, the two approaches are equivalent.[3] While all our results can be rephrased in the utility framework, here we nevertheless adopt the latter approach as more appropriate to model our ordinal problems.

There is a related literature that considers problems with a finite number of alternatives but focuses on cardinal rules. For example, see Mariotti (1998) or Nagahisa and Tanaka (2002) and the literature cited therein. Anbarcı (2005) alternatively uses an ordinal framework to present a strategic and axiomatic analysis of two real-life arbitration schemes on a finite number of alternatives.

In our analysis, a rule previously proposed by Hurwicz and Sertel (1997) as the "Kant-Rawls Social Compromise" and further analyzed by Brams and Kilgour (2001) under the name of "fallback bargaining" plays a central role. (It is also related to the Majoritarian Compromise social choice rule of Sertel (1985), also studied by Sertel and Yılmaz (1999).) This rule, hereafter the "Unanimity Compromise", is based on the idea that to reach an agreement, both bargainers will simultaneously have to make compromises. If there is no alternative that is a first best for both, the agents also accept their second bests. If there is still no agreement, they proceed to accept their third bests. The procedure continues in this way until an agreement is reached. The Unanimity

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[1] Even in bargaining over monetary payoffs, the number of alternatives is bounded by the indivisibility of the smallest monetary unit.

[2] It is possible to construct other *ordinal* rules if there are more than two bargainers (e.g. see Shubik 1982; Kıbrıs 2004).

[3] With an infinite number of alternatives, this is no more true. Solution rules defined for the latter type of problems do not translate into rules for the former type (Sertel and Yıldız, 2003), that is, unless the set of alternatives is fixed (e.g. as in Rubinstein et al. 1992, who follow the latter approach to redefine the Nash bargaining rule with cardinal preferences).

Compromise rule can equivalently be interpreted as maximizing the welfare of the worst-off agent when each agent's payoff from an alternative x is the cardinality of that agent's lower contour set at x.[4] It is therefore very closely related to the Egalitarian (Kalai 1977) and the Kalai–Smorodinsky (1975) rules, as well as the Shapley–Shubik rule (see Kıbrıs 2002, 2004).

The intuitive procedure that defines the Unanimity Compromise makes it a natural candidate as a prescriptive tool. An evaluation of this rule is thus particularly useful for an arbitrator. The descriptive relevance of the Unanimity Compromise (for real-life bargaining) on the other hand depends on the existence of noncooperative games that implement it and their relevance to real-life bargaining situations. Constructing this relationship, as part of the Nash program, is left for future research.

We axiomatically evaluate the Unanimity Compromise and compare it with other well-known bargaining rules. Additionally to the standard axioms considered in the bargaining literature, we propose new axioms. In particular, we introduce an invariance property related to the monotonicity property of Maskin (1986). It requires that for certain problems, B, the set of chosen alternatives, $F(B)$, is not affected if an agent's preferences are changed so that (1) the lower contour set of his first best in $F(B)$ weakly enlarges and (2) the lower contour sets of the other alternatives in $F(B)$ remain unchanged (see Subsect. 2.2 for a discussion).

In Sect. 2, we introduce our model. In Sect. 3, we discuss solution rules for finite bargaining problems. In particular, we observe that among *neutral* and *anonymous* rules the Unanimity Compromise rule uniquely satisfies *Pareto optimality*, "monotonicity", and "invariance". In Sect. 4, we allow the feasible set to be countably infinite. We show that, when such problems are admitted, no rule choosing finite subsets of alternatives can be *neutral*. When rephrased in the utility framework of Nash, this result states that there is no *ordinal* rule that is *finite-valued*. It is, therefore, closely related to Shapley (1969).

2 Model

There are two bargainers, $N = \{1, 2\}$. Let $\mathbb{S}$ be a finite set of alternatives. Each $i \in N$ is equipped with a linear order L_i on $\mathbb{S}$.[5] Let $\mathbb{L}$ be the class of all such linear orders. Given a linear order L_i, let P_i denote its strict part: sP_it if and only if sL_it and $s \neq t$.

Given $S \subseteq \mathbb{S}$, $i \in N$, and $L_i \in \mathbb{L}$, the "*ranking utility function" that represents L_i with respect to S* assigns each alternative s to the number of alternatives in its strict lower-contour set in S: formally, for each $s \in S$, $v_i(s/S) = |\{t \in S \mid sP_it\}|$. Given $T \subseteq S$, let $v_i(T/S) = \min\{v_i(s/S) \mid s \in T\}$.

[4] For more on the relationship between the two interpretations, see Brams and Kilgour (2001).

[5] A *linear order* L_i on $\mathbb{S}$ is a binary relation that is *complete* (for each $s, t \in \mathbb{S}$, sL_it or tL_is), *transitive* (for each $s, t, r \in \mathbb{S}$, sL_it and tL_ir imply sL_ir), and *antisymmetric* (for each $s, t \in \mathbb{S}$, sL_it and tL_is imply $s = t$).

Two agents with preferences L_1 and L_2 are bargaining over a set of alternatives $S \subseteq \mathbb{S}$. In case of disagreement, an alternative $d \in S$ is realized. To rule out degenerate problems, assume there is $s \in S \setminus \{d\}$ such that for each $i \in N$, sL_id. A bargaining problem**,** simply a *problem,* is a quadruple $B = (S, d, L_1, L_2)$ satisfying these properties. Let $\mathbb{B}$ be the class of all problems.

For each $B \in \mathbb{B}$, let $\mathbf{P}(B) = \{s \in S \mid \text{there is no } t \in S \text{ such that } tP_1s \text{ and } tP_2s\}$ denote the set of *Pareto optimal alternatives in* B, and $\mathbf{I}(B) = \{s \in S \mid sL_1d$ and $sL_2d\}$ denote the set of *individually rational alternatives in* B. Let $\mathbf{IP}(B) = \mathbf{I}(B) \cap \mathbf{P}(B)$. Let $\mathbb{B}_\mathbf{I}$ be the class of problems $B \in \mathbb{B}$ such that every alternative is individually rational: $S = \mathbf{I}(S, d, L_1, L_2)$.

2.1 Bargaining rules

A bargaining rule, simply a *rule*, is a function F assigning to each $B = (S, d, L_1, L_2) \in \mathbb{B}$, a nonempty $F(B) \subseteq S$. The rule which we call the Unanimity Compromise plays an important role in our analysis. Its outcome can be defined by the following simple algorithm. Initially, both agents request their first bests. This is possible when there is a unique Pareto optimal alternative, in which case the Unanimity Compromise rule chooses it. Otherwise, each agent considers his second best. If there are alternatives which are at least second best for both agents, they are chosen by the Unanimity Compromise. Otherwise, the rule chooses the set of alternatives which are at least third best for both, if this set is nonempty. Let k be the smallest integer for which the problem possesses an alternative which is at least kth best for both agents. The Unanimity Compromise picks the set of such alternatives as the solution of the problem at hand.

As Brams and Kilgour (2001; Theorem 3) show, the Unanimity Compromise solution to any problem comprises all alternatives that maximize the minimum ranking of any bargainer.[6] Therefore, the *Unanimity Compromise* rule (UC) is equivalently defined at each $B \in \mathbf{B}$ as follows:

$$\mathrm{UC}(B) = \arg\max_{s \in S} \min_{i \in N} v_i(s/S).$$

The two alternative definitions of the Unanimity Compromise rule are demonstrated in the following example.

Example 1 The feasible set is $S = \{x_1, \ldots, x_5, d\}$ and preferences are as follows (the alternatives are ranked from the best, left-most, to the worst, right-most)

$$L_1 \mid x_1\ x_2\ x_3\ x_4\ x_5\ d$$

$$L_2 \mid x_5\ x_4\ x_3\ x_2\ x_1\ d$$

[6] Sertel and Yılmaz (1999) also utilize a similar equivalence in presenting the Majoritarian Compromise social choice rule of Sertel (1985).

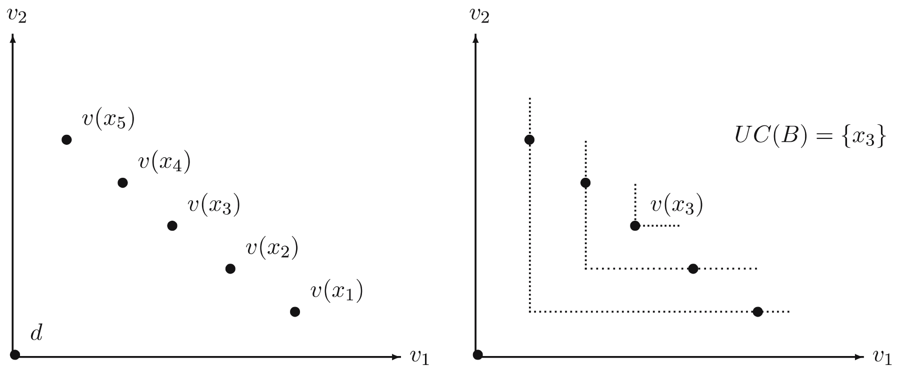

Fig. 1 In Example 1, the utility representation of the problem (*on the left*) and its Unanimity Compromise solution as the maximizer of a Leontief type social welfare function (*on the right*)

Fig. 1 (left) represents the problem $B = (S, d, L_1, L_2)$ in payoff space. To solve B, the Unanimity Compromise procedure follows the following steps:

Step 1 Agent 1 requests his first best, $\{x_1\}$, and Agent 2 requests his first best, $\{x_5\}$. The requests are not compatible.

Step 2 Agent 1 requests alternatives down to his second best, $\{x_1, x_2\}$, and Agent 2 requests alternatives down to his second best, $\{x_5, x_4\}$. The requests are not compatible.

Step 3 Agent 1 requests alternatives down to his third best, $\{x_1, x_2, x_3\}$ and Agent 2 requests alternatives down to his third best, $\{x_5, x_4, x_3\}$. The requests are compatible since the two sets have a nonempty intersection. The procedure stops and the intersection set $\{x_3\}$ is chosen as the Unanimity Compromise solution to this problem: $\mathrm{UC}(B) = \{x_3\}$.

Figure 1 (right) illustrates that the same outcome is also obtained by maximizing the minimum ranking of any bargainer.

The Unanimity Compromise solution to some problems is a doubleton. If the preferences of Agent 2 in Example 1 are instead

$$L_2' \mid x_5 \; d \; x_4 \; x_3 \; x_2 \; x_1$$

the Unanimity Compromise solution is $\mathrm{UC}(S, d, L_1, L_2') = \{x_3, x_4\}$.

Note that $dL_2'x_3$ and $dL_2'x_4$. That is, the Unanimity Compromise violates one of the more important axioms of bargaining: *individual rationality*. However, the same compromise idea, when applied to the set of individually rational alternatives $\mathbf{I}(B)$ guarantees individually rational outcomes. The (*Individually*) Rational (Unanimity) *Compromise* rule (RC) is defined at each $B \in \mathbf{B}$ as follows:

$$\mathrm{RC}(B) = \arg\max_{s \in \mathbf{I}(B)} \min_{i \in N} v_i(s \;/\; \mathbf{I}(B)).$$

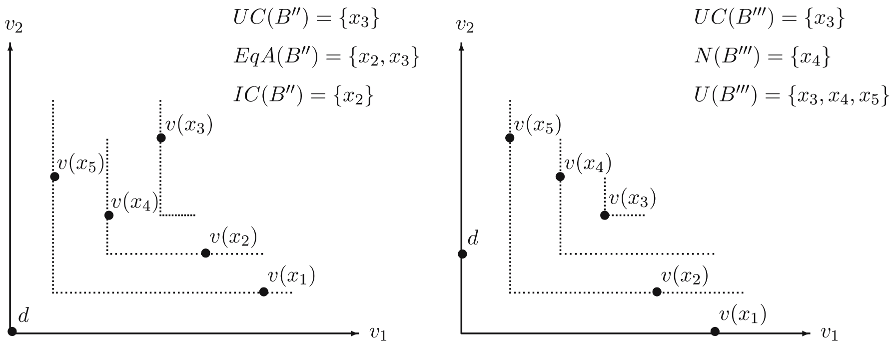

Fig. 2 Example 2 (*on the left*) and Example 3 (*on the right*)

In Example 1, all alternatives are individually rational. Thus, the two rules coincide. However, $\mathrm{RC}(S, d, L_1, L_2') = \{x_5\}$ since x_5 is the only individually rational alternative for (S, d, L_1, L_2').

Restricting the comparison to only individually rational and Pareto optimal (that is, imputational) outcomes leads to an alternative rule which coincides with the finite version of the *Equal Length* (EqL) rule (Thomson 1996). The *Imputational Compromise* (IC) is defined at each $B \in \mathbf{B}$ as follows:

$$\mathrm{IC}(B) = \mathrm{EqL}(B) = \underset{s \in \mathbf{IP}(B)}{\arg\max}\, \min_{i \in N} v_i(s/\mathbf{IP}(B)).$$

The finite version of the Equal Area rule (see Thomson 1994) does not coincide with any of the previous rules; the *Equal Area* solution EqA to a problem chooses those Pareto optimal points at which the difference between the number of better individually rational alternatives for each agent is minimized: for each $s \in \mathbf{P}(B)$, let $\mathrm{Dif}(s/B) = |v_1(s/\mathbf{I}(B)) - v_2(s/\mathbf{I}(B))|$. Then

$$\mathrm{EqA}(B) = \{s \in \mathbf{P}(B) \mid t \in \mathbf{P}(B) \Rightarrow \mathrm{Dif}(s/B) \leqq \mathrm{Dif}(t/B)\}.$$

The following example demonstrates the differences between these two rules and the Unanimity Compromise.

Example 2 (Fig. 2, left) In Example 1, change preferences of Agent 2 to

$$L_2'' \mid x_3\ x_5\ x_4\ x_2\ x_1\ d$$

Let $B'' = (S, d, L_1, L_2'')$. The Equal Area solution to this problem is $\mathrm{EqA}(B'') = \{x_2, x_3\}$, whereas the Unanimity (as well as the Rational) Compromise solution is $\mathrm{UC}(B'') = \mathrm{RC}(B'') = \{x_3\}$. The Imputational Compromise (or the Equal Length) solution is different than either: $\mathrm{IC}(B'') = \mathrm{EqL}(B'') = \{x_2\}$.

Note that for this framework, cardinal rules such as that of Nash (1950) or Kalai and Smorodinsky (1975) as well as the Egalitarian and the Utilitarian rules fail to be well-defined, as they depend on the particular utility representation of the preferences. Once a representation is fixed, however, these rules can be redefined. Here, we take the "utility" of an alternative for an agent as the cardinality of the agent's lower contour set at that alternative. Then the "*Nash-like*" product maximizing rule, N, is defined as

$$N(B) = \underset{s \in \mathbf{I}(B)}{\arg\max} \quad v_1(s/\mathbf{I}(B)) \times v_2(s/\mathbf{I}(B))$$

and the "*Utilitarian-like*" sum maximizing rule, U, is defined as

$$U(B) = \underset{s \in S}{\arg\max} \quad v_1(s/S) + v_2(s/S).$$

Finally note that the "*Egalitarian-like*" rule, E, which maximizes the utility of the worst-off agent coincides with the Rational Compromise:

$$E = \mathrm{RC}.$$

The following example demonstrates the differences between these two rules and the Unanimity Compromise.

Example 3 (Fig. 2, right) In Example 1, change preferences of Agent 2 to

$$L_2''' \mid x_5\ x_4\ x_3\ d\ x_2\ x_1$$

Let $B''' = (S, d, L_1, L_2''')$. Then, the Nash-like solution is $N(B''') = \{x_4\}$ and the Utilitarian-like solution is $U(B''') = \{x_3, x_4, x_5\}$. The Unanimity Compromise solution to the same problem is $\mathrm{UC}(B''') = \{x_3\}$.

2.2 Properties

We focus on rules whose outcomes are independent of the alternatives' names. Let Π be the class of all bijections $\pi : \mathbb{S} \to \mathbb{S}$. For $L_i \in \mathbb{L}$ and $\pi \in \Pi$, let L_i^π be defined as follows: for each $s, t \in \mathbb{S}$, $sL_i^\pi t$ if and only if $\pi^{-1}(s)L_i\pi^{-1}(t)$. For $B = (S, d, L_1, L_2)$, let $\overline{\pi}(B) = (\pi(S), \pi(d), L_1^\pi, L_2^\pi)$. A rule F is *neutral* if for each $B \in \mathbb{B}$ and each $\pi \in \Pi$, we have $F(\overline{\pi}(B)) = \pi(F(B))$. Using standard terminology, we also say that a rule F is *anonymous* if for each $(S, d, L_1, L_2) \in \mathbb{B}$, we have $F(S, d, L_1, L_2) = F(S, d, L_2, L_1)$. A rule F is *regular* if it is both *neutral* and *anonymous*. When possible, we focus on the *regular* rules.

The first set of properties are standard in both the bargaining and social choice literatures. A rule F is *individually rational* if for each $B \in \mathbb{B}$, we have $F(B) \subseteq \mathbf{I}(B)$. It is *Pareto optimal* if for each $B \in \mathbb{B}$, we have $F(B) \subseteq \mathbf{P}(B)$.

The next class of properties relate solutions to a given pair of problems. The first one is a weaker form of a monotonicity property introduced by Nagahisa and Tanaka (2002). These authors note that for standard (infinite) problems, their property is weaker than the monotonicity properties of Kalai (1977) and

Kalai and Smorodinsky (1975). This property requires that, given a problem $B = (S, d, L_1, L_2)$, if the feasible set S expands to a set T in such a way that all added alternatives $t \in T \setminus S$ are considered by every agent better than his worst alternative in $F(B)$, then each agent's worst alternative in $F(T, d, L_1, L_2)$ is better than his worst alternative in $F(B)$. Formally, let $\underline{s}_i(F, B) = \arg\min_{x \in F(B)} v_i(x/S)$ be the worst alternative for i in $F(B)$. Then, a rule F is *monotonic* if for each $B = (S, d, L_1, L_2) \in \mathbb{B}$ and $B' = (T, d, L_1, L_2) \in \mathbb{B}$ satisfying $S \subset T$ and for each $t \in T \setminus S$ and $i \in N$, $tP_i\underline{s}_i(F, B)$, we have $\underline{s}_i(F, B')P_i\underline{s}_i(F, B)$ for each $i \in N$.

Monotonicity can either be interpreted as a solidarity requirement on an impartial arbitrator (a change in the environment that is favorable to both agents should affect the arbitrator's proposal in a similar way), or as a rationality requirement on the bargainers (each agent should refuse to be worse-off by the discovery of an alternative that is better than a current agreement).[7]

The second type of property requires that certain changes in the agents' preferences should not affect the solution. Given a problem at which the worst chosen alternative for each agent is ranked the same, if an agent i's ranking changes so that his top choice $\bar{s}_i(F, B)$ (weakly) improves while the other choices $s \in F(B) \setminus \{\bar{s}_i(F, B)\}$ remain the same in rank, the solution should be the same. Formally, let $\bar{s}_i(F, B) = \arg\max_{x \in F(B)} v_i(x/S)$ be the best alternative for i in $F(B)$. Then, a rule F is *preference replacement invariant* if for each $B = (S, d, L_i, L_j) \in \mathbb{B}$ with $v_i(F(B)/S) = v_j(F(B)/S)$, we have $F(S, d, L_i', L_j) = F(S, d, L_i, L_j)$ so long as L_i' satisfies for each $t \in S$,

1. $\bar{s}_i(F, B)L_i't$ if $\bar{s}_i(F, B)L_it$ and
2. for each $s \in F(B) \setminus \{\bar{s}_i(F, B)\}$, $sL_i't$ if and only if sL_it.

This property is a weaker version of "Maskin monotonicity" (see Maskin 1986). Indeed, the original property is violated by all the rules introduced in the previous section.[8] The reason is quite intuitive. Resolving a bargaining situation (or any conflict for that matter) requires the choice of an agreement that appropriately balances the preferences of the parties. Some of the preference changes allowed by Maskin monotonicity can severely damage this balance.[9] *Preference replacement invariance* is limited to problems where the solution is

[7] An equivalent definition that demonstrates this welfare comparison is as follows: a rule F is monotonic if for each $B = (S, d, L_1, L_2) \in \mathbb{B}$ and $B' = (T, d, L_1, L_2) \in \mathbb{B}$ satisfying $S \subset T$ and for each $t \in T \setminus S$ and $i \in N$, $v_i(t/T) > v_i(F(B)/T)$), we have $v_i(F(B')/T) > v_i(F(B)/T)$ for each $i \in N$.

[8] To see this, consider the problem in Example 2. Moving x_1 up to second rank in L_2'' changes the UC, RC, Nash, and Utilitarian solution from x_3 to x_1. Similar violations can be shown for the Equal Area rule (e.g. moving x_2 up to second rank in L_2'' makes it the unique solution) or the Equal Length rule (e.g. moving $\{x_4, x_5\}$ up to third rank in L_1 makes x_4 the unique solution).

[9] For example, moving x_1 up to second rank in L_2'' (of Example 2) makes it a "better compromise" than x_3 (since in the new problem, x_1 is ranked first by an agent and second by another while x_3 is only ranked first and third).

symmetric (in the sense that the chosen set is ranked the same by both agents) and it rules out preference changes that distort this symmetry.[10]

Preference replacement invariance ignores changes in the individually rational set. As a result, it is violated by the Rational Compromise rule (see Example 4, Part (i)) which, however, satisfies the property on a restricted domain: a rule is *restricted preference replacement invariant* if it is *preference replacement invariant* on $\mathbb{B}_{\mathbf{I}}$. Even on this subdomain however, preference changes can affect the imputation set, and thus the Imputational Compromise rule violates the property (see Example 4, Part (ii)).

Example 4 Consider the problem in Example 1. Note that $\text{RC}(S, d, L_1, L_2) = \text{IC}(S, d, L_1, L_2) = \{x_3\}$.

(i) If preferences of Agent 2 are replaced with

$$L_2''' \mid x_5\ x_4\ x_3\ d\, x_2\ x_1,$$

the lower contour set of x_3 remains unchanged (only d moves from sixth to fourth place). However, $\text{RC}(S, d, L_1, L_2''') = \{x_4\}$. Thus RC violates *preference replacement invariance*.

(ii) If preferences of Agent 2 are replaced with

$$L_2'' \mid x_3\ x_5\ x_4\ x_2\ x_1\ d,$$

x_3 improves in rank. Furthermore, (S, d, L_1, L_2), $(S, d, L_1, L_2'') \in \mathbb{B}_{\mathbf{I}}$. However, $\text{IC}(S, d, L_1, L_2'') = \{x_2\}$. Thus IC violates *restricted preference replacement invariance*.

3 Finite bargaining problems

Our first result is as follows.

Theorem 5 *The* Unanimity Compromise *is the unique* regular *rule that is* Pareto optimal, monotonic, *and* preference replacement invariant.

We prove this result in two steps. However, let us first note that all our three compromise rules, UC, RC, and IC, choose at most two alternatives for each problem.

Lemma 6 For every problem $B \in \mathbb{B}$, $\max\{|\text{UC}(B)|, |\text{RC}(B)|, |\text{IC}(B)|\} \leqq 2$.

[10] In a way, this is reminiscent of "strong monotonicity" in bargaining theory. This property (which says any expansion of the feasible set should make everyone better-off) allows expansions that change a symmetric bargaining problem into a very asymmetric one and is criticized for this reason. As a result, weaker versions that preserve some of the symmetry are proposed (e.g. Roth 1979, achieves this in "restricted monotonicity" by keeping the agents' ideal payoffs fixed).

Also note that *neutrality* and *anonymity* can be weakened to a "welfare-symmetry" property, which is quite standard in the utility-based bargaining literature following Nash (1950). A *set* S *is welfare-symmetric with respect to the profile* L if for each $s \in S$, there is $t \in S$ such that $v_1(s/S) = v_2(t/S)$ and $v_1(t/S) = v_2(s/S)$. A *problem* B *is welfare-symmetric* if (i) $v_1(d/S) = v_2(d/S)$ and (ii) S is welfare-symmetric with respect to L. The welfare-symmetric problems have utility images that are symmetric with respect to the $x_1 = x_2$ line in $\mathbb{R}^2$. A *rule* F *is welfare-symmetric* if for each welfare-symmetric problem B, $F(B)$ is also welfare-symmetric. This property is the counterpart of the symmetry property in Nash (1950) and Kalai and Smorodinsky (1975).

Lemma 7 *If* F *is* regular *then it is* welfare-symmetric.

Proof Let $B = (S, d, L_1, L_2)$ be a welfare-symmetric problem. Let $\pi : S \to S$ be the bijection defined as follows: for each $s \in S$, $\pi(s)$ is such that $v_1(s/S) = v_2(\pi(s)/S)$ and $v_1(\pi(s)/S) = v_2(s/S)$. Note that $\overline{\pi}(B) = (S, d, L_2, L_1)$. Assume that F is *regular.* By *neutrality*, $F(\overline{\pi}(B)) = \pi(F(B))$ and by *anonymity*, $F(\overline{\pi}(B)) = F(B)$. Therefore, $\pi(F(B)) = F(B)$ and so, F is *welfare-symmetric.* □

The following lemma describes the implications of the given properties for welfare-symmetric problems.

Lemma 8 *Let* $B \in \mathbb{B}$ *be a welfare-symmetric problem. If* F *is a* Pareto optimal, monotonic*, and* welfare-symmetric *rule, then* $F(B) = \mathrm{UC}(B)$.

Proof Let $B = (S, d, L_1, L_2)$ be a welfare-symmetric problem and let F be a rule satisfying the given properties. Since the feasible set S is constant throughout the proof, we write $v_i(s)$ instead of $v_i(s/S)$.

Let $\pi : S \to S$ be the bijection defined as follows: for each $s \in S$, $\pi(s)$ is such that $v_1(s) = v_2(\pi(s))$ and $v_1(\pi(s)) = v_2(s)$. Note that $x \in \mathbf{P}(B)$ implies $\pi(x) \in \mathbf{P}(B)$. If $\pi(x) \notin \mathbf{P}(B)$, there is $y \in S$ such that for each $i \in N$ $v_i(y) > v_i(\pi(x))$, which implies that for each $i \in N$, $v_i(\pi(y)) > v_i(x)$, a contradiction.

Let $\mathbf{P}(B) = \{x_1, x_2, \ldots, x_k\}$. Without loss of generality, assume that for each $l \in \{2, \ldots, k\}$ $x_{l-1} L_1 x_l$. Then, Agent 2 has the opposite ranking. That is, for each $l \in \{1, \ldots, k-1\}$ $x_{l+1} L_2 x_l$. To see this, suppose there is $l \in \{1, \ldots, k-1\}$ such that $x_l L_2 x_{l+1}$. Since $x_l L_1 x_{l+1}$, then x_{l+1} Pareto dominates x_l, a contradiction.

Next note that for each $l \in \{1, \ldots, k\}$, $\pi(x_l) = x_{k-l+1}$. To see this, first note that $x_1, \pi(x_1) \in \mathbf{P}(B)$. Therefore, $\pi(x_1) = x_l$ for some $l \in \{2, \ldots, k\}$. Suppose $l < k$. Then $v_1(x_k) < v_1(x_l) = v_2(x_1)$ and $v_2(x_k) > v_2(x_l) = v_1(x_1)$. Since $x_k \in \mathbf{P}(B)$, $\pi(x_k) \in \mathbf{P}(B)$ and by definition of π, $v_1(\pi(x_k)) = v_2(x_k) > v_2(x_l) = v_1(x_1)$. This contradicts x_1 being agent $1's$ top ranked Pareto optimal alternative. Therefore, $\pi(x_1) = x_k$. A similar reasoning shows that $\pi(x_2) = x_{k-1}$. Iterating, one obtains the desired conclusion.

Note that every $x \notin \mathbf{P}(B)$ is Pareto dominated by an $x \in \mathbf{P}(B)$. So, for each $x \in \mathbf{P}(B)$, let $D(x)$ be the union of the set of alternatives that x Pareto dominates, the alternative d, and the alternative x itself. Now, for each $l \in \{1, \ldots, \left\lceil \frac{k}{2} \right\rceil\}$, let

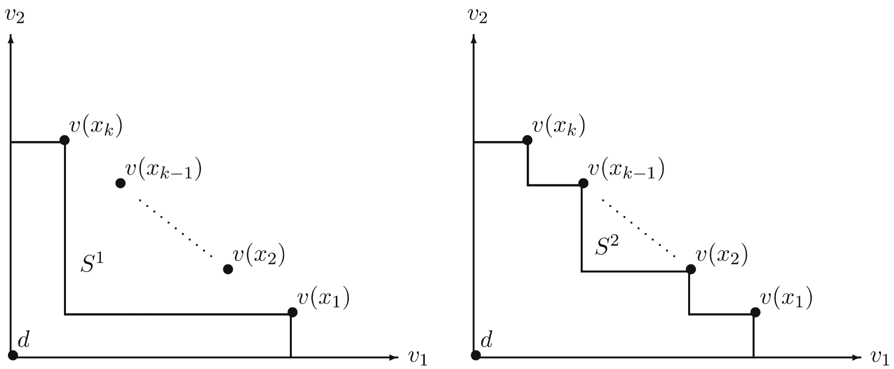

Fig. 3 Construction of the sets S^1 and S^2

$$S^l = \left(\bigcup_{i=1}^{l} D(x_i) \right) \cup \left(\bigcup_{i=1}^{l} D(\pi(x_i)) \right).$$

Note that $S^{\lceil k/2 \rceil} = S$ and each (S^l, d, L_1, L_2) is welfare-symmetric (see Fig. 3).

It follows from *Pareto optimality* and *welfare-symmetry* that $F(S^1, d, L_1, L_2) = \{x_1, x_k\}$. Now note that $S^2 \supset S^1$. Furthermore, for each $x \in S^2 \setminus S^1$, xL_1x_k and xL_2x_1. Therefore, by *monotonicity* of F, for each $i \in N$, $\underline{s}_i(F, (S^2, d, L_1, L_2))P_i\underline{s}_i(F, (S^1, d, L_1, L_2))$. This implies $x_1, x_k \notin F(S^2, d, L_1, L_2)$ and thus, $F(S^2, d, L_1, L_2) = \{x_2, x_{k-1}\}$. Iterating, we obtain $F(S, d, L_1, L_2) = \{x_{\lceil k/2 \rceil}, \pi(x_{\lceil k/2 \rceil})\} = \text{UC}(S, d, L_1, L_2)$. □

All of the rules that we introduced coincide with the Unanimity Compromise rule on welfare-symmetric problems. However, none other satisfies *preference replacement invariance*.

Lemma 9 *Let F be a* preference replacement invariant *rule. If $F = \text{UC}$ on welfare-symmetric problems, then $F = \text{UC}$ (on the whole domain).*

Proof Let F be a rule satisfying the given properties. Let $B \in \mathbb{B}$. By Lemma 6, $|\text{UC}(B)| \leqq 2$. Let $\text{UC}(B) = \{a, b\}$ and assume that aL_1b and bL_2a. Note that $v_1(\{a, b\}) = v_2(\{a, b\})$.

Let L'_1 be obtained from L_1 by moving a down in agent 1's ranking to the spot right above b. Let L'_2 be obtained from L_2 by moving b down in agent 2's ranking to the spot right above a. Let $B' = (S, d, L'_1, L'_2)$. Note that $v'_1(\{a, b\}) = v'_2(\{a, b\})$. For $i \in N$, let

$$U_i = \{t \in S \setminus \{a, b\} \mid tL'_i a \textit{ and } tL'_i b\} \text{ and}$$
$$D_i = \{t \in S \setminus \{a, b\} \mid a\, L'_j t \textit{ and } bL'_j t\}.$$

Note that by *Pareto optimality* of UC, $U_i \subseteq D_j$ for $i \neq j$.

For each $i \in N$ and $j \neq i$, enumerate

$$D_i \cap U_j = U_j = \{t^1(i), t^2(i), \dots, t^n(i)\} \text{ and}$$
$$D_i \cap D_j = \{r^1(i), r^2(i), \dots, r^m(i)\}$$

so that for $l \in \{1, \dots, n-1\}$, $t^{l+1}(i)\, L'_j\, t^l(i)$ and for $l \in \{1, \dots, m-1\}$, $r^l(i)\, L'_j\, r^{l+1}(i)$. By the previous paragraph, n and m are independent of i.

For each $i \in N$, let L''_i be obtained from L'_i by moving alternatives in $D_i \cap U_j$ above those in $D_i \cap D_j$ and reordering alternatives in $D_i \cap U_j$ so that for $l \in \{1, \dots, n-1\}$, $t^l(i)\, L''_i\, t^{l+1}(i)$.

Finally, let $L'''_1 = L''_1$ and let L'''_2 be obtained from L''_2 by reordering alternatives in $D_1 \cap D_2$ so that for $l \in \{1, \dots, m-1\}$, $r^l(2)\, L'''_2\, r^{l+1}(2)$.

The problem $B''' = (S, d, L'''_1, L'''_2)$ is of the following form:

$$L'''_1 \mid t^n(2) \ \dots\ t^1(2)\ a\ b\ t^1(1)\ \dots\ t^n(1)\ r^1(2)\ \dots\ r^m(2)$$

$$L'''_2 \mid t^n(1) \ \dots\ t^1(1)\ b\ a\ t^1(2)\ \dots\ t^n(2)\ r^1(2)\ \dots\ r^m(2)$$

Note that this is a welfare-symmetric problem since

(1) for each $s \in D_1 \cap D_2$, $v'''_1(s) = v'''_2(s)$,
(2) for each $s \in D_i \cap U_j$, there is $t \in U_i \cap D_j$ such that $v'''_i(s) = v'''_j(t)$ and $v'''_i(t) = v'''_j(s)$,
(3) $v'''_1(a) = v'''_2(b)$ and $v'''_1(b) = v'''_2(a)$.

For each $i \in N$ and $j \neq i$, we have $U_i \subseteq D_j$; thus $\mathrm{UC}(B''') = \{a, b\}$. Therefore, by assumption $F(B''') = \mathrm{UC}(B''') = \{a, b\}$.

Now, note that $v'''_1(b) = v'''_2(a)$ and $L'''_1 = L''_1$. Furthermore for each $t \in S$, aL'''_2t if and only if aL''_2t, and bL'''_2t if and only if bL''_2t. Therefore, by *preference replacement invariance*, $F(B'') = F(B''') = \mathrm{UC}(B''') = \mathrm{UC}(B'')$. Similarly, $F(B') = F(B'') = \mathrm{UC}(B'') = \mathrm{UC}(B')$.

Now note that $v'_1(b) = v'_2(a)$. For Agent 1, for each $t \in S$, aL_1t if aL'_1t and bL_1t if and only if bL'_1t. For Agent 2, for each $t \in S$, bL_2t if bL'_2t and aL_2t if and only if aL'_2t. Therefore, by *preference replacement invariance*, $F(B) = F(B') = \mathrm{UC}(B') = \mathrm{UC}(B)$. □

The proof of Theorem 5 then proceeds as follows. It is straightforward to verify that UC satisfies the claimed properties. Conversely, if F is a rule satisfying these properties, by Lemmata 7 and 8, F is equal to UC on welfare-symmetric problems. Then, by Lemma 9, F is equal to UC on every problem.

The next result replaces the *monotonicity* requirement with a minimality condition: a rule F is *minimally connected* if for each $B = (S, d, L_1, L_2) \in \mathbb{B}$, $s, s' \in F(B)$ implies that there is no $t \in S$ such that sP_itP_is' and $s'P_jtP_js$. That is, if s and s' are both chosen and if there is t which both agents rank in between s and s', then s and s' should not have been chosen in the first place since t is a better compromise.

Theorem 10 *The* Unanimity Compromise *is the unique* regular *rule that is* Pareto optimal, minimally connected*, and* preference replacement invariant.

The proof is similar to the previous one except that instead of Lemma 8 it resorts to the following result.

Lemma 11 *Let* $B \in \mathbb{B}$ be a welfare-symmetric problem. If F is a Pareto optimal, minimally connected*, and* welfare-symmetric rule*, then* $F(B) = \mathrm{UC}(B)$.

Proof Let F be a rule satisfying the given properties. Since $B = (S, d, L_1, L_2)$ is welfare-symmetric, let $\pi : S \to S$ be the bijection defined as follows: for each $s \in S$, $\pi(s)$ is such that $v_1(s/S) = v_2(\pi(s)/S)$ and $v_1(\pi(s)/S) = v_2(s/\, S)$.

Let $\mathbf{P}(B) = \{x_1, x_2, \dots, x_k\}$. Without loss of generality, assume that for each $l \in \{2, \dots, k\}$, $x_{l-1} L_1 x_l$. Note that then Agent 2 has the opposite ranking. In the proof of Lemma 8, we established that for each $l \in \{1, \dots, k\}$, $\pi(x_l) = x_{k-l+1}$.

Let $\mathrm{UC}(B) = \{a, \pi(a)\}$ and note that $a = x_{\lceil k/2 \rceil}$. If k is odd, then $a = \pi(a)$; otherwise, $a = x_{k/2}$ and $\pi(a) = x_{(k/2)+1}$. Suppose there is $s \in F(B)$ such that $s \notin \{a, \pi(a)\}$. Then by *welfare-symmetry* of F, $\pi(s) \in F(B)$ as well. Also note that $\{s, \pi(s)\} \subseteq \mathbf{P}(B)$. However then $s = x_l$ for some $l < \lceil k/2 \rceil$. Therefore, $s L_1 a L_1 \pi(s)$ and $\pi(s) L_2 a L_2 s$, contradicting *minimal-connectedness* of F. Therefore, $s \notin F(B)$ for any $s \in \mathbf{P}(B) \setminus \{a, \pi(a)\}$. Since $F(B) \neq \emptyset$, $a \in F(B)$, and by *welfare-symmetry* $\pi(a) \in F(B)$. Thus, $F(B) = \mathrm{UC}(B)$ □

The properties listed in Theorems 5 and 10 are logically independent. To see this, enumerate $\mathbb{S} = \{s_1, \dots, s_K\}$. First, the rule F^1 defined as

$$F^1(B) = \{s_k \in \mathrm{UC}(B) \mid \quad \text{for each } s_l \in \mathrm{UC}(B),\ k \leqq l\}$$

satisfies all properties except *neutrality*. Second, the rule F^2 defined as

$$F^2(B) = \{s_k \in \mathrm{UC}(B) \mid \quad \text{for each } s_l \in \mathrm{UC}(B),\ s_k L_1 s_l\}$$

satisfies all properties except *anonymity*. Let $\mathbb{B}_2 = \{(S, d, L_1, L_2) \in \mathbb{B} \mid |S| = 2$ and $L_1 = L_2\}$. Then the rule F^3 defined as

$$F^3(B) = \begin{cases} S & \text{if} (S, d, L_1, L_2) \in \mathbb{B}_2, \\ \mathrm{UC}(B) & \text{otherwise.} \end{cases}$$

satisfies all properties except *Pareto optimality*. Fourth, the Pareto rule, $\mathbf{P}$, satisfies all properties except *monotonicity* and *minimal connectedness*.[11] Finally, the Rational Compromise rule, RC, satisfies all properties except *preference replacement invariance*.

[11] Note that *monotonicity* and *minimal connectedness* are equivalent for rules that satisfy all the other properties. This equivalence need not hold in general. However, all our other examples satisfy both of these properties.

Results similar to Theorems 5 and 10 are obtained for the Rational Compromise rule if only individually rational alternatives are deemed to be important for the determination of an agreement. A rule F is *independent of nonindividually rational alternatives* if for each $B = (S, d, L_1, L_2) \in \mathbb{B}$, we have $F(S, d, L_1, L_2) = F(\mathbf{I}(B), d, L_1, L_2)$. In the standard framework of Nash (1950), this property is satisfied by all of the well-known rules with the only exception of the Kalai–Rosenthal (1978) rule.

Theorem 12 *The* Rational Compromise *is the unique* regular *rule that is* Pareto optimal, monotonic, restricted preference replacement invariant*, and* independent of nonindividually rational alternatives.

Theorem 13 *The* Rational Compromise *is the unique* regular *rule that is* Pareto optimal, minimally connected, restricted preference replacement invariant*, and* independent of non-individually rational alternatives.

The proofs of these two results proceed similarly. It follows from the proof of Theorem 5 that if F satisfies these properties, then on the subclass $\mathbb{B}_\mathbf{I}$ we have $F = \text{UC} = \text{RC}$. For every $B = (S, d, L_1, L_2) \notin \mathbb{B}_\mathbf{I}$, however, the problem $B' = (\mathbf{I}(B), d, L_1, L_2) \in \mathbb{B}_\mathbf{I}$. Therefore $F(B') = \text{RC}(B')$ and via *independence of nonindividually rational alternatives,* we have $F(B) = F(B') = \text{RC}(B') = \text{RC}(B)$.[12]

The following table compares the discussed rules in terms of the properties they satisfy.[13] We also discuss the Agent-i-Dictatorial rule, D^i, which chooses agent i's first best among individually rational alternatives.

Properties rules	UC	RC = E	IC = EqL	EqA	N	U	D^i
Pareto optimality	+ *Thm* 5, 10	+ *Thm* 12, 13	+	+	+	+	+
Individual rationality	−	+	+	+	+	−	+
Neutrality	+ *Thm* 5, 10	+ *Thm* 12, 13	+	+	+	+	+
Anonymity	+ *Thm* 5, 10	+ *Thm* 12, 13	+	+	+	+	−
Welfare symmetry	+	+	+	+	+	+	−
Monotonicity	+ *Thm* 5	+ *Thm* 12	+	−	+	+	+
Pref. repl. inv.	+ *Thm* 5, 10	−	−	−	−	−	+
Restricted pref. repl. inv.	+	+ *Thm* 12, 13	−	−	−	−	+
Minimal connectedness	+ *Thm* 10	+ *Thm* 13	+	+	−	−	+
Ind. of non-ind. rat. alt.	−	+ *Thm* 12, 13	+	+	+	−	+

[12] The properties stated in these results are also logically independent. Replacing the UC with RC in the definitions of F^1, F^2, and F^3 produces examples of rules that only violate *neutrality, anonymity,* and *Pareto optimality*, respectively. The rule IP (that picks all individually rational and Pareto optimal alternatives) violates only *monotonicity* and *minimal connectedness*. The rule IC violates only *restricted preference replacement invariance*. Finally, UC violates only *independence of nonindividually rational alternatives.*

[13] The superscripts in the table refer to the characterization theorems in which the property corresponding to that row appears.

All of the above rules, except dictatorship, violate Nash's "independence of irrelevant alternatives axiom".[14] These rules also violate *strategy proofness.* Example 4 already demonstrates this point for RC and IC. Similar examples can be constructed for the other rules.

4 Infinite bargaining problems

In this section we allow the universal set $\mathbb{S}$ and the feasible sets S to be countably infinite. This has an important implication. For a finite number of alternatives, the Unanimity Compromise rule can be equivalently defined on either the alternative space or the utility space. For an infinite number of alternatives, however, simply because the agents can now have infinite sized upper or lower contour sets, this equivalence no longer holds.

When the space of alternatives is infinite, even if the analysis is restricted to physical problems, there is no unique way of defining the Unanimity Compromise. If there is an agent who has an infinite sized upper contour set at every alternative, it is not possible to apply the compromise algorithm. Similarly, if there is an agent who has an infinite sized lower contour set at every alternative, one cannot apply the previously equivalent definition of maximizing the ranking of the worst-off agent. If, for example, $S = \{1, 2, 3, \ldots\}$ and L_1, L_2 are such that for each $k \in \{1, 2, 3, \ldots\}$, $kL_1(k+1)$ and $(k+1)L_2k$, neither definition yields an outcome.

On the class of countable problems, *neutrality* turns out to have interesting implications. We discuss them next. Let $B = (S, d, L_1, L_2) \in \mathbb{B}$ and $\pi \in \Pi$. Let $T[S, \pi] = \{s \in S \mid \pi(s) \neq s\}$. The set $T[S, \pi]$ *contains a finite cycle* if there is a finite subset $D = \{s_1, \ldots, s_k\}$ of $T[S, \pi]$ such that for each $l \in \{1, \ldots, k-1\}$ $\pi(s_l) = s_{l+1}$ and $\pi(s_k) = s_1$.

Lemma 14 *Let $B \in \mathbb{B}$ and $\pi \in \Pi$. If $\overline{\pi}(B) = B$ then $T[S, \pi]$* contains no finite cycle.

Proof Suppose $T[S, \pi]$ contains a finite cycle $D = \{s_1, \ldots, s_k\}$. Let $s_1 \in D$ be such that $s_1 L_1 s_l$ for each $l \in \{2, \ldots, k\}$ and let $s_2 = \pi(s_1)$. Now for each $l \in \{2, \ldots, k\}$, $\pi(s_1) L_1^{\pi} \pi(s_l)$ implies $s_2 L_1^{\pi} \pi(s_l)$. But $s_1 = \pi(s_l)$ for some $l \in \{2, .., k\}$. Thus $s_2 L_1^{\pi} s_1$. Since $s_1 L_1 s_2$, $L_1 \neq L_1^{\pi}$, contradicting $\overline{\pi}(B) = B$. □

Note that if $T[S, \pi]$ is finite, it automatically contains a finite cycle. Therefore, $\overline{\pi}(B) = B$ implies that $T[S, \pi]$ is infinite. We use Lemma 14 in the proof of the following result.

[14] To be more precise, a bargaining rule F is *independent of irrelevant alternatives* if for each (S, d, L_1, L_2), $(T, d, L_1, L_2) \in \mathbb{B}$, $S \subset T$ and $s \in F(T, d, L_1, L_2) \cap S$ implies $s \in F(S, d, L_1, L_2)$. A weaker form of this property requires the dropped out alternatives to be ranked below the chosen ones: a bargaining rule F is *weakly independent* if $s \in F(S, d, L_1, L_2)$, $t \in S$, and $t \in L_1 L_2(F(S, d, L_1, L_2))$, then $s \in F(S \backslash \{t\}, d, L_1, L_2)$. This version is satisfied by all the above rules except N and U.

Theorem 15 *Let F be a* neutral *rule. Let $B \in \mathbb{B}$ and $\pi \in \Pi$ be such that $\overline{\pi}(B) = B$. If $F(B) \cap T[S, \pi] \neq \varnothing$ then $|F(B)| = \infty$.*

Proof Assume $F(B) \cap T[S, \pi] \neq \varnothing$. Let $x_1 \in F(B) \cap T[S, \pi]$ and let $x_2 = \pi(x_1)$. Then $x_2 \in \pi(F(B)) = F(\overline{\pi}(B))$. Since $\overline{\pi}(B) = B$, however, $x_2 \in F(B)$. Since $x_2 \in T[S, \pi]$ as well, we have $x_2 \in F(B) \cap T[S, \pi]$.

Now for each $k \in \mathbb{N}$, let $x_k = \pi(x_{k-1})$. By Lemma 14, $T[S, \pi]$ does not contain a finite cycle. Therefore, $x_k \neq x_l$ for $l \in \{1, \ldots, k-1\}$. By iterating the argument of the previous paragraph, we have $x_k \in F(B) \cap T[S, \pi]$ for each $k \in \mathbb{N}$. Thus, $|F(B) \cap T[S, \pi]| = \infty$ establishes the desired conclusion. □

For problems where $T[S, \pi]$ contains a single "infinite chain", the theorem goes further to state that $F(B) \supseteq T[S, \pi]$. This however is not true in general. For instance, let $S = \{s_l\}_{l \in \mathbb{Z}} \cup \{t_l\}_{l \in \mathbb{Z}}$ and for each $l \in \mathbb{Z}$, $\pi(s_l) = s_{l+1}$ and $\pi(t_l) = t_{l+1}$. If for example, $s_1 \in F(B)$, then $\{s_l\}_{l \in \mathbb{Z}} \subseteq F(B)$. However, it might be that $F(B) \cap \{t_l\}_{l \in \mathbb{Z}} = \emptyset$.

Say a rule F is *finite* if for each $B \in \mathbb{B}$, $F(B) \neq \{d\}$ is a finite set. We then have the following corollary to Theorem 15.

Corollary 16 *On the class of two-agent countable problems, no* finite *rule is* neutral.

Proof Let $S = \{d\} \cup \{s^n\}_{n=1}^{\infty} \cup \{t^n\}_{n=1}^{\infty}$ be a countable subset of $\mathbb{S}$. Let L_1 be such that for each $n \in \{1, 2, \ldots\}$

$$s^{n+1} L_1 s^n, \quad t^n L_1 t^{n+1}, \quad s^1 L_1 t^1, \quad \text{and for each } s \in S, sL_1 d.$$

Let L_2 represent the inverse ranking of L_1 on $S \backslash \{d\}$; that is for each $n \in \{1, 2, \ldots\}$

$$t^{n+1} L_2 t^n, \quad s^n L_2 s^{n+1}, \quad t^1 L_2 s_1, \quad \text{and for each } s \in S, sL_2 d.$$

Let $B = (S, d, L_1, L_2)$. Let F be a *finite rule*. Then $F(B)$ is a nonempty and finite subset of S.

Now let $\pi : S \to S$ be a bijection such that for each $n \in \{1, 2, \ldots\}$

$$\pi(s^n) = s^{n+1}, \quad \pi(t^{n+1}) = t^n, \quad \pi(t^1) = s_1, \quad \text{and } \pi(d) = d.$$

By *neutrality*, $F(\overline{\pi}(B)) = \pi(F(B))$. Now note that $\pi(S) = S$, $\pi(d) = d$ and for each $i \in N$ $L_i^{\pi} = L_i$. Therefore, $\overline{\pi}(B) = B$. If $F(B) \neq \{d\}$ then $F(B) \cap T[S, \pi] \neq \emptyset$. By Theorem 15 then, $|F(B)| = \infty$, contradicting *finiteness* of F. □

Note that the statement of Corollary 16 can be strenghtened further. In the construction of the above proof, the only infinite subset of S that is invariant under π is S. Therefore, the only solution a *neutral* rule can suggest for S is the set itself.

This result should be related to Shapley's (1969) finding. *Neutrality* in our framework stands for what Roemer (1996) refers to as *welfarism*: that is, the bargaining outcome should only depend on the problem's utility image. Therefore, a *neutral* rule in our framework corresponds to a (*welfarist*) *ordinal* rule in the Nash framework. Based on this relation, Corollary 16 can be rephrased as follows: on the class of two-agent countable problems, no *finite* rule is *ordinal*.

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