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Baryons from Instantons in Holographic QCD

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We consider aspects of dynamical baryons in a holographic dual of QCD that is formulated on the basis of a D4/D8-brane configuration. We construct a soliton solution carrying a unit baryon number and show that it is obtained as an instanton solution of four-dimensional Yang-Mills theory with fixed size. The Chern-Simons term on the flavor D8-branes plays a crucial role of protecting the instanton from collapsing to zero size. By quantizing the collective coordinates of the soliton, we derive the baryon spectra. Negative-parity baryons as well as baryons with higher spins and isospins can be obtained in a simple manner.

§1. Introduction

Since the discovery of the AdS/CFT correspondence[1)–3)] (For a review, see Ref. 4).), it has been recognized that a gravity description is a promising framework for understanding non-perturbative aspects of gauge theory. The application of this idea to realistic models, like QCD, has attracted much attention. (See, for example, Refs. 5)–9) for recent progress along this line.)

In Refs. 10) and 11), a holographic dual of QCD with N_f massless quarks is constructed using a D4/D8-brane configuration in type IIA string theory. It has been argued that the low energy phenomena of QCD, such as chiral symmetry breaking, can be derived from this model. The key components of the D4/D8 model are the $G = U(N_f)$ five-dimensional Yang-Mills (YM) and Chern-Simons (CS) theory on a curved background, both of which originate from the low energy effective action on the probe D8-branes embedded into the D4 background presented in Ref. 12). In this model, the massless pion and an infinite tower of massive (axial-)vector mesons are interpreted as Kaluza-Klein states associated with the fifth (or holographic) direction, and the masses and couplings of the mesons are found to be in good agreement with experiments. In addition to the mesonic states, dynamical baryons are also studied in Ref. 10), where it is demonstrated that the baryon number can be identified with the instanton number of the 5d YM, and hence it is concluded that baryons can be described by a soliton with a non-trivial instanton number. (See also Ref. 5).)

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In the context of the AdS/CFT correspondence, it has been argued that baryons are constructed from D-branes wrapped on non-trivial cycles.[13)–19)] In the case of the D4/D8 model, baryons are identified as D4-branes wrapped on a non-trivial four-cycle in the D4 background. Such a D4-brane is realized as a small instanton configuration in the world-volume gauge theory on the probe D8-brane. Also, it has been found that the pion effective action obtained from the 5d YM theory is identically that of the Skyrme model, in which baryons appear as solitons, called Skyrmions.[20)–22)] It can be shown that the baryon number of a Skyrmion, which is defined as the winding number carried by the pion field, is equivalent to the instanton number in the 5d YM theory. In this way, the D4/D8 model connects various descriptions of baryons. (For further studies of baryons in the AdS/CFT or AdS/QCD, see Refs. 23)–28). Also, closely related works are presented in Refs. 5) and 29).)

The purpose of this paper is to investigate aspects of baryons described as instantons in the 5d YM-CS theory formulated in the D4/D8 model. For brevity, we restrict ourselves to the two-flavor case, $N_f = 2$. We first construct a soliton solution of the 5d YM that carries a unit baryon number. We show that for $\lambda = g_{\rm YM}^2 N_c \gg 1$, which ensures the validity of the supergravity approximation, the soliton is represented by a BPST one-instanton solution[30)] with a fixed size of order $\lambda^{-1/2}$ located at the origin in the holographic direction. Here, the CS term and the $U(1)$ part of the gauge field play an important role in stabilizing the instanton size. We next quantize the soliton by formulating a quantum-mechanical system that governs the collective motion. A baryon is identified with a quantum state of this system. Note that this procedure is a natural extension of the old, well-known idea of Adkins, Nappi and Witten[31)] in the context of the Skyrme model.[20)–22)] In the original work, appearing in Ref. 31), only the massless pion is taken into account. Therefore it is natural to extend the analysis to include the contribution from massive (axial-)vector mesons. Such an extension has been studied in Refs. 32)–35) (See Refs. 36) and 37) for reviews and references therein.) using phenomenological effective actions including the (axial-)vector mesons, such as the ρ, ω and a_1 mesons. This paper proposes a new approach for incorporating vector mesons. This approach utilizes the fact that, in the D4/D8 model, the pion and an infinite number of the massive (axial-)vector mesons are unified in a single 5d gauge field with a reasonably simple effective action. Thus, it is expected that a thorough study of this model will allow us to gain some new insight into baryon physics that cannot be captured by the Skyrme model.

The idea of describing baryons in terms of YM instantons was previously investigated in Ref. 38), in which it is argued that the pion field configuration corresponding to the Skyrmion is accurately approximated by integrating the one-instanton solution along an artificial fifth direction. Our approach is a manifestation of this idea, although the motivation is completely different. An interesting point here is that the introduction of the fifth direction is not just a mathematical trick. Rather, this direction has a physical interpretation as one of the spatial directions in the holographic description of QCD.

Unfortunately, because the instanton size is of order $\lambda^{-1/2}$, it is necessary to incorporate an infinite number of higher-derivative terms into the 5d YM-CS theory in order to derive quantitatively precise results concerning baryon physics. In this paper, we do not attempt to resolve this issue. Instead, we mainly consider the 5d YM theory with the CS term (although in Appendix B, we also analyze the non-Abelian DBI action). For this reason, it may be the case that quantitative comparisons of our results with experiments, which are made below for several examples, are of limited physical meaning. However, even if this is the case, we believe that the qualitative picture of baryon physics investigated in this paper is rather interesting and can help us to gain deeper insight into it. In fact, the baryon spectrum obtained in this paper seems to capture some characteristics of the baryon spectra observed in experiments, although the predicted masses are not very close to the experimental values.

The organization of this paper is as follows. In §2, we formulate the 5d YM-CS system that we treat throughout this paper. In §3, we show that baryons are described by an instanton solution whose size is fixed by taking into account the effect of the CS term. Section 4 is devoted to the construction of the Lagrangian of the collective motion of the soliton. Quantization of the Lagrangian is performed in §5, where the correspondence between each quantum state and a baryon is established. There, we also make a quantitative comparison of our results with experimental results for several cases. We end this paper with conclusions in §6. Some technical details are summarized in the Appendices.

§2. The model

Our model consists of the following YM-CS theory with gauge group $U(N_f)$ in a five-dimensional curved background:

$$\begin{aligned} S &= S_{\rm YM} + S_{\rm CS} \ , \\ S_{\rm YM} &= -\kappa \int d^4x dz \ \mathrm{tr} \left[\frac{1}{2} h(z) \mathcal{F}_{\mu\nu}^2 + k(z) \mathcal{F}_{\mu z}^2 \right] \ , \\ S_{\rm CS} &= \frac{N_c}{24\pi^2} \int_{M^4 \times \mathbb{R}} \omega_5^{U(N_f)}(\mathcal{A}) \ . \end{aligned} \tag{2·1}$$

Here, $\mu, \nu = 0, 1, 2, 3$ are four-dimensional Lorentz indices, and z is the coordinate of the fifth-dimension. The quantity $\mathcal{A} = \mathcal{A}_\mu dx^\mu + \mathcal{A}_z dz$ is the 5-dimensional $U(N_f)$ gauge field, and $\mathcal{F} = d\mathcal{A} + i\mathcal{A} \wedge \mathcal{A}$ is its field strength. The constant κ is related to the 't Hooft coupling λ and the number of colors N_c as*)

$$\kappa = \frac{\lambda N_c}{216\pi^3} \equiv a\lambda N_c \ . \tag{2·2}$$

The functions $h(z)$ and $k(z)$ are given by

$$h(z) = (1 + z^2)^{-1/3} \ , \quad k(z) = 1 + z^2 \ , \tag{2·3}$$

*) In Refs. 10) and 11), we used $\kappa = \lambda N_c/(108\pi^3)$, which is due to the misleading factor of 2 appearing in Eq. (5.1) of Ref. 10).

and $\omega_5^{U(N_f)}(\mathcal{A})$ is the CS 5-form for the $U(N_f)$ gauge field defined as

$$\omega_5^{U(N_f)}(\mathcal{A}) = \mathrm{tr}\left(\mathcal{A}\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right) . \tag{2·4}$$

This theory is obtained as the effective action of N_f probe D8-branes placed in the D4-brane background studied in Ref. 12) and is supposed to be an effective theory of mesons, including an infinite number of (axial-)vector mesons as well as the massless pion, in four-dimensional QCD with N_f massless quarks. In Refs. 10) and 11), it is argued that much of the low energy behavior of QCD is reproduced by this simple action. Here we employ units in which $M_{\rm KK} = 1$, where $M_{\rm KK}$ is the single mass parameter of the model, which specifies the Kaluza-Klein mass scale. The $M_{\rm KK}$ dependence can easily be recovered through dimensional analysis.

Note that it is also possible to extend our investigation to cases of more general functions $h(z)$ and $k(z)$, as in the phenomenological approach to holographic QCD given in Ref. 5). However, in this paper we use the functional forms given in Eq. (2·3) for definiteness.

It is useful to decompose the $U(N_f)$ gauge field $\mathcal{A}$ into its $SU(N_f)$ part A and its $U(1)$ part $\widehat{A}$ as

$$\mathcal{A} = A + \frac{1}{\sqrt{2N_f}}\widehat{A} = A^aT^a + \frac{1}{\sqrt{2N_f}}\widehat{A} , \tag{2·5}$$

where T^a $(a = 1, 2, \cdots, N_f^2 - 1)$ are the generators for $SU(N_f)$ normalized as

$$\mathrm{tr}(T^aT^b) = \frac{1}{2}\delta^{ab} . \tag{2·6}$$

The action is then written as

$$\begin{aligned} S_{\rm YM} &= -\kappa\int d^4xdz\ \mathrm{tr}\left[\frac{1}{2}h(z)F_{\mu\nu}^2 + k(z)F_{\mu z}^2\right] \\ &\quad -\frac{\kappa}{2}\int d^4xdz\left[\frac{1}{2}h(z)\widehat{F}_{\mu\nu}^2 + k(z)\widehat{F}_{\mu z}^2\right] , \end{aligned} \tag{2·7}$$

$$\begin{aligned} S_{\rm CS} &= \frac{N_c}{24\pi^2}\int\left[\omega_5^{SU(N_f)}(A) + \frac{3}{\sqrt{2N_f}}\widehat{A}\,\mathrm{tr}\,F^2 + \frac{1}{2\sqrt{2N_f}}\widehat{A}\,\widehat{F}^2\right. \\ &\quad \left. + \frac{1}{\sqrt{2N_f}}d\left(\widehat{A}\,\mathrm{tr}\left(2FA - \frac{i}{2}A^3\right)\right)\right] . \end{aligned} \tag{2·8}$$

As mentioned above, we consider only the $N_f = 2$ case in the present paper. In this case, $\omega_5^{SU(2)}(A)$ vanishes, and the CS term reduces to

$$\begin{aligned} S_{\rm CS} &= \frac{N_c}{24\pi^2}\int\left[\frac{3}{2}\widehat{A}\,\mathrm{tr}\,F^2 + \frac{1}{4}\widehat{A}\,\widehat{F}^2 + (\text{total derivatives})\right] \\ &= \frac{N_c}{24\pi^2}\epsilon_{MNPQ}\int d^4xdz\left[\frac{3}{8}\widehat{A}_0\,\mathrm{tr}(F_{MN}F_{PQ}) - \frac{3}{2}\widehat{A}_M\,\mathrm{tr}(\partial_0A_NF_{PQ})\right. \end{aligned}$$

$$+\frac{3}{4}\widehat{F}_{MN}\,\mathrm{tr}(A_0F_{PQ})+\frac{1}{16}\widehat{A}_0\widehat{F}_{MN}\widehat{F}_{PQ}-\frac{1}{4}\widehat{A}_M\widehat{F}_{0N}\widehat{F}_{PQ}+(\text{total derivatives})\Bigg]\ , \tag{2·9}$$

with $M, N = 1, 2, 3, z$ and $\epsilon_{123z} = +1$.

§3. Classical solution

3.1. *Soliton solutions for S_{YM}*

In our model, λ is assumed to be large, and we employ the $1/\lambda$ expansion. Since $S_{\rm YM} \sim \mathcal{O}(\lambda^1)$ and $S_{\rm CS} \sim \mathcal{O}(\lambda^0)$, it is expected that the leading contribution to the soliton mass comes from $S_{\rm YM}$. Let us first consider the system without the CS term. In this case, the $U(1)$ part, $\widehat{A}$, of the gauge field is decoupled from the $SU(2)$ part, and thus it is consistent to set $\widehat{A} = 0$. We are interested in the minimal energy static configuration carrying a unit baryon number, $N_B = 1$, where the baryon number N_B is equal to the instanton number and is given by

$$N_B = \frac{1}{32\pi^2}\int d^3x dz\, \epsilon_{MNPQ}\,\mathrm{tr}(F_{MN}F_{PQ})\ . \tag{3·1}$$

If the five-dimensional space-time were flat and the functions $h(z)$ and $k(z)$ were trivial (i.e. $h(z) = k(z) = 1$), the solution would be given by the BPST instanton solution[30)] of arbitrary size ρ and position in the four-dimensional space parameterized by x^M $(M = 1, 2, 3, z)$. However, in the present case with Eq. (2·3), it can be shown that the minimal energy configuration is given by a small instanton with infinitesimal size, $\rho \to 0$.

To illustrate this fact, we first examine the ρ dependence of the energy calculated by inserting the BPST instanton configuration as a trial configuration. The BPST instanton configuration is given by

$$A_M(x) = -if(\xi)\, g\partial_M g^{-1}\ , \tag{3·2}$$

where

$$f(\xi) = \frac{\xi^2}{\xi^2+\rho^2}\ , \quad \xi = \sqrt{(\vec{x}-\vec{X})^2+(z-Z)^2}\ , \tag{3·3}$$

$$g(x) = \frac{(z-Z)-i(\vec{x}-\vec{X})\cdot\vec{\tau}}{\xi}\ , \tag{3·4}$$

and its field strengths are

$$F_{ij} = \frac{2\rho^2}{(\xi^2+\rho^2)^2}\epsilon_{ija}\tau^a\ , \quad F_{zj} = \frac{2\rho^2}{(\xi^2+\rho^2)^2}\tau_j\ . \tag{3·5}$$

Here $\vec{\tau} = (\tau^1, \tau^2, \tau^3)$ are the Pauli matrices, and we have $\vec{x} = (x^1, x^2, x^3)$ and $a, i, j = 1, 2, 3$. The constants $(\vec{X}, Z)$ and ρ denote the position and the size of the instanton, respectively. This is the one-instanton solution for the $SU(2)$ Yang-Mills theory in flat four-dimensional space. Assuming $\vec{X}=0$ and $Z=0$ for simplicity, the energy of this configuration is calculated as

$$\begin{aligned} E(\rho) &= \kappa \int d^3x dz \ \mathrm{tr}\left[\frac{1}{2}h(z)F_{ij}^2 + k(z)F_{iz}^2\right] \\ &= 3\pi^2\kappa\rho^4 \int dz\,(z^2+\rho^2)^{-5/2}(h(z)+k(z)) \\ &= 3\pi^2\kappa\left[\frac{\sqrt{\pi}\,\Gamma(7/3)}{\Gamma(17/6)}\,F\left(\frac{1}{3},\frac{1}{2},\frac{17}{6};1-\rho^2\right)+\frac{4}{3}+\frac{2}{3}\rho^2\right] . \end{aligned} \tag{3·6}$$

It can be shown that $E(\rho)$ is a monotonically increasing function of ρ whose minimal value is $E(\rho=0)=8\pi^2\kappa$.

It is also possible to show that the minimal value of the energy $E=8\pi^2\kappa$ found above is actually the absolute minimum in the sector with a unit instanton number. In fact, the $SU(2)$ part of the YM action has the following bound for any static configuration:

$$\begin{aligned} \kappa\int d^3x dz\,\mathrm{tr}\left[\frac{1}{2}h(z)F_{ij}^2 + k(z)F_{iz}^2\right] &\geq \frac{\kappa}{2}\int d^3x dz\sqrt{h(z)k(z)}\left|\epsilon^{ijk}F_{jk}^aF_{iz}^a\right| \\ &\geq 8\pi^2\kappa|N_B| . \end{aligned} \tag{3·7}$$

Here we have used the relation $h(z)k(z)\geq h(0)k(0)=1$. The lower bound of (3·7) is realized only in the case of a (anti-)self-dual instanton with an infinitesimal size located at $z=0$.

It is interesting that the minimal value $8\pi^2\kappa$ is equal to the baryon mass obtained in Ref. 10) from the mass of a D4-brane wrapped around an S^4 that surrounds the color D4-branes. This fact suggests that the soliton mass $8\pi^2\kappa$ is not modified even if we include higher derivative terms in the DBI action, because the wrapped D4-brane can be regarded as a small instanton on the probe D8-branes.[39),40)] More evidence supporting this conjecture is given in Appendix B.

3.2. *Contribution from S_{CS}*

Let us next consider the contribution from the CS term, (2·9). It is important to note that this term includes a term of the form

$$\epsilon_{MNPQ}\int d^4x dz\,\widehat{A}_0\,\mathrm{tr}(F_{MN}F_{PQ}) . \tag{3·8}$$

This shows that the instanton configuration induces an electric charge coupled to the $U(1)$ gauge field $\widehat{A}$. As is well known from the theory of electrodynamics, the energy possessed by the electric field of a point charge diverges. In the 1+4 dimensional case, the energy behaves as $E\sim\rho^{-2}$ for a charged particle of radius ρ. Then, taking this contribution into account, it follows that the minimal energy configuration representing a baryon must have a finite size. This reasoning is analogous to that used to argue the stability of a Skyrmion via the ω meson presented in Ref. 32).

In fact, as we show below, the classical solution at leading order in the $1/\lambda$ expansion is given by a BPST instanton in the flat space whose size ρ is of order

$\lambda^{-1/2}$. For this reason, in order to carry out a systematic $1/\lambda$ expansion, it is convenient to rescale the coordinates x^M as well as the $U(2)$ gauge field $\mathcal{A}_M$ as

$$\begin{aligned}
&\widetilde{x}^M = \lambda^{+1/2}x^M \ , \quad \widetilde{x}^0 = x^0 \ , \\
&\widetilde{\mathcal{A}}_0(t,\widetilde{x}) = \mathcal{A}_0(t,x) \ , \quad \widetilde{\mathcal{A}}_M(t,\widetilde{x}) = \lambda^{-1/2}\mathcal{A}_M(t,x) \ , \\
&\widetilde{\mathcal{F}}_{MN}(t,\widetilde{x}) = \lambda^{-1}\mathcal{F}_{MN}(t,x) \ , \quad \widetilde{\mathcal{F}}_{0M}(t,\widetilde{x}) = \lambda^{-1/2}\mathcal{F}_{0M}(t,x) \ ,
\end{aligned} \tag{3·9}$$

and regard the quantities with tildes as being $\mathcal{O}(\lambda^0)$. Hereafter, we omit the tilde for simplicity. We then find that for $\lambda \gg 1$, the YM part becomes

$$\begin{aligned}
S_{\rm YM} = &- aN_c \int d^4x dz \ \mathrm{tr}\left[\frac{\lambda}{2}F_{MN}^2 + \left(-\frac{z^2}{6}F_{ij}^2 + z^2 F_{iz}^2 - F_{0M}^2\right) + \mathcal{O}(\lambda^{-1})\right] \\
&- \frac{aN_c}{2}\int d^4x dz \left[\frac{\lambda}{2}\widehat{F}_{MN}^2 + \left(-\frac{z^2}{6}\widehat{F}_{ij}^2 + z^2\widehat{F}_{iz}^2 - \widehat{F}_{0M}^2\right) + \mathcal{O}(\lambda^{-1})\right] \ ,
\end{aligned} \tag{3·10}$$

with $i,j = 1,2,3$, while the CS term takes the same form as that given in Eq. (2·9). Here we have used Eq. (2·2). The equations of motion for the $SU(2)$ part read

$$D_M F_{0M} + \frac{1}{64\pi^2 a}\epsilon_{MNPQ}\widehat{F}_{MN}F_{PQ} + \mathcal{O}(\lambda^{-1}) = 0 \ , \tag{3·11}$$

$$D_N F_{MN} + \mathcal{O}(\lambda^{-1}) = 0 \ . \tag{3·12}$$

Also, the equations of motion for the $U(1)$ part are

$$\partial_M \widehat{F}_{0M} + \frac{1}{64\pi^2 a}\epsilon_{MNPQ}\left\{\mathrm{tr}(F_{MN}F_{PQ}) + \frac{1}{2}\widehat{F}_{MN}\widehat{F}_{PQ}\right\} + \mathcal{O}(\lambda^{-1}) = 0 \ , \tag{3·13}$$

$$\partial_N \widehat{F}_{MN} + \mathcal{O}(\lambda^{-1}) = 0 \ . \tag{3·14}$$

Now we solve the equations of motion, (3·11)–(3·14) in order to derive a static soliton solution corresponding to a baryon. First, let us consider Eq. (3·12). In this paper we expand the action about the baryon solution and keep only the terms of orders λ^1 and λ^0. For this purpose, we have only to solve the equation $D_N F_{MN} = 0$ on flat space while ignoring the $\mathcal{O}(\lambda^{-1})$ term in Eq. (3·12), because the correction to the solution from the $\mathcal{O}(\lambda^{-1})$ term in Eq. (3·12) gives only an $\mathcal{O}(\lambda^{-1})$ correction to the action. Therefore, a solution that carries a unit baryon number is given by the BPST instanton solution (3·2). Here, the parameters $(\vec{X}, Z)$ and ρ are also rescaled as in Eq. (3·9).

For the $U(1)$ part, the finite energy solution of the Maxwell equation (3·14) is given by $\widehat{F}_{MN} = 0$, which yields the trivial solution $\widehat{A}_M = 0$, up to a gauge transformation. Then the Gauss's law equation (3·11) is reduced to

$$D_M^2 A_0 = 0 \ , \tag{3·15}$$

whose solution is given in terms of a linear combination of the functions Φ_a given in Eq. (A·20) of Appendix A. We are interested in the solution that vanishes at infinity, and it is given by $A_0 = 0$.

We are thus left with Eq. (3·13) for $\widehat{A}_0$:

$$\partial_M^2 \widehat{A}_0 + \frac{3}{\pi^2 a} \frac{\rho^4}{(\xi^2+\rho^2)^4} = 0 \ . \tag{3·16}$$

This equation can easily be solved, and the regular solution that vanishes at infinity is given by

$$\widehat{A}_0 = \frac{1}{8\pi^2 a} \frac{1}{\xi^2} \left[1 - \frac{\rho^4}{(\rho^2+\xi^2)^2} \right] \ . \tag{3·17}$$

Note that we could add a constant term to Eq. (3·17) if we allow $\widehat{A}_0$ that are non-vanishing at infinity. The physical interpretation of this constant term is that it is the chemical potential μ associated with the baryon number,*) since Eq. (3·8) induces the μN_B term in the action.

Now we have obtained the configurations (3·2) and (3·17), together with $A_0 = \widehat{A}_M = 0$, which solves the leading-order equations of motion, (3·11)–(3·14). Although this solution is sufficient for calculating the $\mathcal{O}(\lambda^1)$ and $\mathcal{O}(\lambda^0)$ terms of the energy, as mentioned below Eq. (3·14), the resultant energy depends on ρ and Z, which have not yet been fixed. In fact, the soliton mass M is obtained by evaluating the action on shell, $S = -\int dt M$:

$$\begin{aligned} M &= 8\pi^2\kappa + \kappa\lambda^{-1} \int d^3x dz \left[-\frac{z^2}{6} \operatorname{tr}(F_{ij})^2 + z^2 \operatorname{tr}(F_{iz})^2 \right] \\ &\quad - \frac{1}{2}\kappa\lambda^{-1} \int d^3x dz \left[(\partial_M \widehat{A}_0)^2 + \frac{1}{32\pi^2 a} \widehat{A}_0 \, \epsilon_{MNPQ} \operatorname{tr}(F_{MN}F_{PQ}) \right] + \mathcal{O}(\lambda^{-1}) \\ &= 8\pi^2\kappa \left[1 + \lambda^{-1} \left(\frac{\rho^2}{6} + \frac{1}{320\pi^4 a^2} \frac{1}{\rho^2} + \frac{Z^2}{3} \right) + \mathcal{O}(\lambda^{-2}) \right] \ . \end{aligned} \tag{3·18}$$

The values of ρ and Z for the solution should be determined by minimizing M, which is equivalent to solving the sub-leading part of the equations of motion, (3·12) and (3·14), projected on to the space of the deformations of the solution in the ρ and Z directions.

It is worth emphasizing that the term in Eq. (3·18) proportional to ρ^{-2} results from the Coulomb interaction $\widehat{A}_0 \, \epsilon_{MNPQ} \operatorname{tr}(F_{MN}F_{PQ})$ in the CS term, while the ρ^2 and Z^2 terms are due to the warped geometry employed here. Without the Coulomb interaction, the soliton mass is minimized by the instanton with infinitesimal size, i.e. $\rho \to 0$, located at the origin, $Z = 0$, as we saw in §3.1. However, with the Coulomb interaction, the instanton is stabilized at a finite size ρ given by

$$\rho^2 = \frac{1}{8\pi^2 a} \sqrt{\frac{6}{5}} \ . \tag{3·19}$$

Going back to the original variable [see Eq. (3·9)], ρ^2 is rescaled as $\rho^2 \to \lambda^{+1}\rho^2$, ensuring that the soliton is given by an instanton with size of order $\lambda^{-1/2}$, as mentioned

*) See Refs. 41)–43) for recent developments concerning the D4/D8 model with a chemical potential.

above. Then, inserting Eq. (3·19) into Eq. (3·18), the mass of the soliton becomes

$$M \simeq 8\pi^2\kappa + \sqrt{\frac{2}{15}}N_c \ . \tag{3·20}$$

We conclude this section with a few remarks on higher-order derivative terms. The action (2·1) is obtained by omitting the higher derivative terms from the D-brane effective action. This corresponds to keeping only the leading-order terms in the $1/\lambda$ expansion. However, in our case, because the size of the soliton solution is small, the derivative of the gauge field is enhanced and may become important in the analysis. Actually, we have seen that the size of the soliton solution is of order $\lambda^{-1/2}$, which in turn implies that an infinite number of higher-derivative terms involved in the D-brane effective action are of the same order in the $1/\lambda$ expansion. To see this, recall that each derivative and gauge field is accompanied by the string length $l_s = \sqrt{\alpha'}$ in the DBI action, for example, $l_s\partial_M$, l_sA_M and $\alpha' F_{MN}$. As explained in Ref. 11), α' can be regarded as a parameter of order λ^{-1}. Therefore, after the rescaling of Eq. (3·9), $l_s\partial_M$ and l_sA_M become $\mathcal{O}(\lambda^0)$ in the rescaled variables, and hence the higher-order derivative terms can appear at the same order. Such terms may also contribute to the equations of motion, (3·11)–(3·14), and the soliton mass (3·18). On the other hand, there are some arguments indicating that, in the case of D-branes in a flat space-time, neither the BPST instanton solution nor its energy is modified, even if all the higher derivative corrections are taken into account.[44)–47)] In Appendix B, we investigate the non-Abelian DBI action and obtain some evidence that the analysis based on the Yang-Mills action given in Eq. (2·1) is not modified. It is important to carry out a more systematic analysis in order to make precise quantitative predictions. We leave this task for a future study.

§4. Lagrangian of the collective modes

The moduli space of the one-instanton solution for the $SU(2)$ Yang-Mills equation (3·12), ignoring the $\mathcal{O}(\lambda^{-1})$ terms, is given by

$$\mathcal{M} = \mathbb{R}^4 \times \mathbb{R}^4/\mathbb{Z}_2 \ . \tag{4·1}$$

The first $\mathbb{R}^4$ here corresponds to the position of the instanton parameterized by $(\vec{X}, Z)$, and $\mathbb{R}^4/\mathbb{Z}_2$ consists of the size ρ and the $SU(2)$ orientation of the instanton. (See, for example, Ref. 48) for a review.) Let us parameterize $\mathbb{R}^4/\mathbb{Z}_2$ by y_I ($I = 1,2,3,4$), which are transformed as $y_I \to -y_I$ under $\mathbb{Z}_2$. The size of the instanton corresponds to the radial coordinate, $\rho = \sqrt{y_1^2 + \cdots + y_4^2}$, and the $SU(2)$ orientation is parameterized by $a_I \equiv y_I/\rho$, with the constraint $\sum_{I=1}^4 a_I^2 = 1$.

To analyze slowly moving solitons, we adopt the moduli space approximation method.[49),50)] With this method, we treat the collective coordinates $(\vec{X}, Z, y_I)$ as time-dependent variables and consider a quantum mechanical description of a particle in the moduli space $\mathcal{M}$. The situation here is analogous to that of monopoles[50)] (See, for example, Refs. 48), 51) and 52) for a review.) and also that of the Skyrmions.[31)] In the present case, the size ρ and the position Z in the z-direction are not genuine collective coordinates, because of the ρ and Z dependent terms in the energy (3·18), which arises from the non-trivial warp factors $h(z)$ and $k(z)$. As seen at the end of this section, the excitations associated with ρ and Z are much lighter than those associated with the other massive modes around the instanton for large λ. For this reason, we treat ρ and Z as collective coordinates, along with $(\vec{X}, a_I)$.

Now we calculate the effective Lagrangian of these collective modes, presenting the derivation of Eq. (4·1) for completeness. We work in the $A_0 = 0$ gauge, which should be accompanied by the Gauss's law constraint (3·11). Also, Eq. (3·13) gives a constraint for obtaining $\widehat{A}_0$ and singles out the physical degrees of freedom.

The basic idea employed in this calculation is to approximate the slowly moving soliton by the static classical solution, with the constant moduli $X^\alpha = (\vec{X}, Z, y_I)$ promoted to the time-dependent collective coordinates $X^\alpha(t)$. Thus, the $SU(2)$ gauge field is assumed to be of the form

$$A_M(t,x) = V A_M^{\rm cl}(x; X^\alpha(t)) V^{-1} - i\, V \partial_M V^{-1} \ . \tag{4·2}$$

Here, $A_M^{\rm cl}(x; X^\alpha(t))$ is the instanton solution (3·2) with time-dependent collective coordinates $\rho(t)$, $\vec{X}(t)$ and $Z(t)$. The quantity $V = V(t,x)$ is an element of $SU(2)$ that is necessary for imposing the Gauss's law constraint (3·11) for Eq. (4·2). It also specifies the $SU(2)$ orientation and hence includes the collective coordinates $a_I(t)$. To see this, we first note that

$$F_{MN} = V F_{MN}^{\rm cl} V^{-1} \ , \quad F_{0M} = V \left(\dot{X}^\alpha \partial_\alpha A_M^{\rm cl} - D_M^{\rm cl} \varPhi \right) V^{-1} \ , \tag{4·3}$$

where $\partial_\alpha = \partial / \partial X^\alpha$, the dot denotes the time derivative ∂_0, $D_M^{\rm cl}$ is the covariant derivative with the gauge field $A_M^{\rm cl}(x; X^\alpha(t))$, and we have

$$\varPhi \equiv -i V^{-1} \dot{V} \ . \tag{4·4}$$

For a given $\varPhi$, V can be obtained as

$$V^{-1} = {\rm P} \exp \left(-i \int^t dt' \varPhi(t', x) \right) \ . \tag{4·5}$$

It then follows that Eq. (3·11) becomes

$$D_M^{\rm cl} \left(\dot{X}^N \frac{\partial}{\partial X^N} A_M^{\rm cl} + \dot{\rho} \frac{\partial}{\partial \rho} A_M^{\rm cl} - D_M^{\rm cl} \varPhi \right) = 0 \ , \tag{4·6}$$

where $X^N = (\vec{X}, Z)$, and we have used $\widehat{F}_{MN}^{\rm cl} = 0$. As shown in Appendix A, this equation is solved by choosing

$$\varPhi(t,x) = -\dot{X}^N(t) A_N^{\rm cl}(x) + \chi^a(t) \varPhi_a(x) \ , \tag{4·7}$$

where $\varPhi_a$ $(a = 1,2,3)$ are the solutions of $D_M^{\rm cl} D_M^{\rm cl} \varPhi_a = 0$ given in Eq. (A·20), and χ^a $(a = 1,2,3)$ are related to the collective coordinates a_I as

$$\chi^a = -i\, {\rm tr}(\tau^a \boldsymbol{a}^{-1} \dot{\boldsymbol{a}}) = 2(a_4 \dot{a}_a - \dot{a}_4 a_a + \epsilon_{abc} a_b \dot{a}_c) \ , \tag{4·8}$$

with

$$\boldsymbol{a} \equiv a_4 + i a_a \tau^a \ \in SU(2) \ . \tag{4·9}$$

Then, F_{0M} in Eq. (4·3) can be expressed as

$$F_{0M} = V \left(\dot{X}^N F_{MN}^{\rm cl} + \dot{\rho} \frac{\partial}{\partial \rho} A_M^{\rm cl} - \chi^a D_M^{\rm cl} \Phi_a \right) V^{-1} \ , \tag{4·10}$$

where we have used $(\partial/\partial X^N) A_M^{\rm cl} = -\partial_N A_M^{\rm cl}$.

It is also necessary to impose the condition represented by Eq. (3·13), which reads

$$-\partial_M^2 \widehat{A}_0 + \frac{1}{64\pi^2 a} \epsilon_{MNPQ} \, {\rm tr}(F_{MN}^{\rm cl} F_{PQ}^{\rm cl}) = 0 \ . \tag{4·11}$$

This shows that $\widehat{A}_0$ is again given by Eq. (3·17), except that in the present case, all the instanton moduli are time dependent.

Inserting into the action the above soliton configuration with time-dependent collective coordinates, we obtain the quantum mechanical system

$$L = \frac{m_X}{2} g_{\alpha\beta} \dot{X}^\alpha \dot{X}^\beta - U(X^\alpha) + \mathcal{O}(\lambda^{-1}) \ , \tag{4·12}$$

where $m_X \equiv 8\pi^2 \kappa \lambda^{-1} = 8\pi^2 a N_c$ and $g_{\alpha\beta}$ is the metric for the instanton moduli space (4·1), given by

$$\begin{aligned} ds^2 &= g_{\alpha\beta} \, dX^\alpha dX^\beta \\ &= d\vec{X}^2 + dZ^2 + 2(d\rho^2 + \rho^2 da_I^2) \\ &= d\vec{X}^2 + dZ^2 + 2 \, dy_I^2 \ . \end{aligned} \tag{4·13}$$

(See Appendix A for more details.) The potential $U(X^\alpha)$ is given by Eq. (3·18),

$$U(X^\alpha) = U(\rho, Z) = M_0 + m_X \left(\frac{\rho^2}{6} + \frac{1}{320\pi^4 a^2} \frac{1}{\rho^2} + \frac{Z^2}{3} \right) \ , \tag{4·14}$$

with $M_0 = 8\pi^2 \kappa$. The Lagrangian (4·12) can also be written as

$$L = L_X + L_Z + L_y + \mathcal{O}(\lambda^{-1}) \ , \tag{4·15}$$

$$\begin{aligned} L_X &= -M_0 + \frac{m_X}{2} \dot{\vec{X}}^2 \ , \\ L_Z &= \frac{m_Z}{2} \dot{Z}^2 - \frac{m_Z \omega_Z^2}{2} Z^2 \ , \\ L_y &= \frac{m_y}{2} \dot{y}_I^2 - \frac{m_y \omega_\rho^2}{2} \rho^2 - \frac{Q}{\rho^2} = \frac{m_y}{2} \left(\dot{\rho}^2 + \rho^2 \dot{a}_I^2 \right) - \frac{m_y \omega_\rho^2}{2} \rho^2 - \frac{Q}{\rho^2} \ , \end{aligned} \tag{4·16}$$

where

$$\begin{aligned} & M_0 = 8\pi^2 \kappa \ , \quad m_X = m_Z = m_y/2 = 8\pi^2 \kappa \lambda^{-1} = 8\pi^2 a N_c \ , \\ & \omega_Z^2 = \frac{2}{3} \ , \quad \omega_\rho^2 = \frac{1}{6} \ , \quad Q = \frac{N_c^2}{5 m_X} = \frac{N_c}{40\pi^2 a} \ . \end{aligned} \tag{4·17}$$

A few comments are in order. First, if we write the above Lagrangian in terms of the original variable before the rescaling described in Eq. (3·9), m_X is replaced with M_0, and then the $\dot{\vec{X}}^2$ term becomes the usual kinetic term for a particle of mass M_0. Second, note that the Lagrangian for a_I is the same as that in the case of a Skyrmion[31)] with a moment of inertia $m_y \rho^2/4$, although this moment of inertia depends on the coordinate ρ, which is promoted to an operator upon quantization. Third, as mentioned above, ρ and Z are not the collective modes in the usual sense, since they have the non-trivial potential (4·14). The reason that we focus only on ρ and Z among the infinitely many massive fluctuations about the instanton is the following. Because the Lagrangian for ρ and Z in Eq. (4·15) is of order λ^0, the energy induced by the excitation of these modes is also of order λ^0. On the other hand, the other massive fluctuations are all massive, even for a flat background, and hence the mass terms come from the $\mathcal{O}(\lambda)$ term in Eq. (3·10). This implies that their frequencies are of order $\lambda^{1/2}$. Therefore, the excitations of these modes are much heavier than the excitations of Z and ρ for $\lambda \gg 1$.

§5. Quantization

In this section, we quantize the system (4·15) in order to derive the spectra of baryons. The Hamiltonian for a baryon placed at $\vec{X} = 0$ is

$$H = M_0 + H_y + H_Z \ , \tag{5·1}$$

where

$$H_y = -\frac{1}{2m_y} \sum_{I=1}^{4} \frac{\partial^2}{\partial y_I^2} + \frac{1}{2} m_y \omega_\rho^2 \rho^2 + \frac{Q}{\rho^2} \ , \tag{5·2}$$

$$H_Z = -\frac{1}{2m_Z} \partial_Z^2 + \frac{1}{2} m_Z \omega_Z^2 Z^2 \ . \tag{5·3}$$

As argued in Appendix A, a point a_I in S^3 and its antipodal point, $-a_I$, are to be identified in the instanton moduli space. This implies that the wave function of the system must satisfy the condition

$$\psi(a_I) = \pm \psi(-a_I) \ . \tag{5·4}$$

Following Ref. 31) (see also Ref. 53)), we impose the anti-periodic boundary condition $\psi(a_I) = -\psi(-a_I)$, since we are interested in fermionic states.

5.1. *Solution to the Schrödinger equation*

As a warm-up, let us first consider H_y with $Q = 0$. Then, the system is reduced to the 4-dimensional harmonic oscillator:

$$H_y|_{Q=0} = \sum_{I=1}^{4} \left(-\frac{1}{2m_y} \frac{\partial^2}{\partial y_I^2} + \frac{1}{2} m_y \omega_\rho^2 y_I^2 \right) \ . \tag{5·5}$$

We know that the energy eigenvalues of this system are given by

$$E_y|_{Q=0} = \omega_\rho (N + 2) \ , \tag{5·6}$$

with

$$N = n_1 + n_2 + n_3 + n_4 \ , \tag{5·7}$$

where $n_I = 0, 1, 2, \cdots$ $(I = 1, 2, 3, 4)$. The degeneracy of the states with a given N is

$$d_N = \frac{1}{6}(N+3)(N+2)(N+1) \ . \tag{5·8}$$

Next, we solve this problem using polar coordinates. The Hamiltonian is then written

$$H_y|_{Q=0} = -\frac{1}{2m_y}\left(\frac{1}{\rho^3}\partial_\rho(\rho^3\partial_\rho) + \frac{1}{\rho^2}\nabla^2_{S^3}\right) + \frac{1}{2}m_y\omega_\rho^2\rho^2 \ , \tag{5·9}$$

where $\nabla^2_{S^3}$ is the Laplacian for a unit S^3. It is known that the scalar spherical harmonics for S^3 are given by

$$T^{(l)}(a_I) = C_{I_1\cdots I_l}\, a_{I_1}\cdots a_{I_l} \ , \tag{5·10}$$

where $C_{I_1\cdots I_l}$ is a traceless symmetric tensor of rank l. They satisfy

$$\nabla^2_{S^3}T^{(l)} = -l(l+2)T^{(l)} \ , \tag{5·11}$$

and the degeneracy is $(l+1)^2$. Under the isomorphism $SO(4) \simeq (SU(2)\times SU(2))/\mathbb{Z}_2$, the rank l traceless symmetric tensor representation of $SO(4)$ corresponds to the $(S_{l/2}, S_{l/2})$ representation of $(SU(2) \times SU(2))/\mathbb{Z}_2$. Here, $S_{l/2}$ denotes the spin $l/2$ representation of $SU(2)$, and its rank is $\dim S_{l/2} = l+1$. Writing the eigenfunctions of the Hamiltonian as

$$\psi(y_I) = R(\rho)\, T^{(l)}(a_I) \ , \tag{5·12}$$

$R(\rho)$ is found to satisfy

$$\mathcal{H}_l R(\rho) = E_y|_{Q=0}\, R(\rho) \ , \tag{5·13}$$

with

$$\mathcal{H}_l \equiv -\frac{1}{2m_y}\left(\frac{1}{\rho^3}\partial_\rho(\rho^3\partial_\rho) - \frac{l(l+2)}{\rho^2}\right) + \frac{1}{2}m_y\omega_\rho^2\rho^2 \ . \tag{5·14}$$

The eigenvalue equation (5·13) for $R(\rho)$ is reduced by substituting the form

$$R(\rho) = e^{-\frac{m_y\omega_\rho}{2}\rho^2}\,\rho^l\, v(m_y\omega_\rho\rho^2) \ . \tag{5·15}$$

This yields the confluent hypergeometric differential equation for $v(z)$,

$$\left\{z\partial_z^2 + (l+2-z)\partial_z + \frac{1}{2}\left(\frac{E_y|_{Q=0}}{\omega_\rho} - l - 2\right)\right\} v(z) = 0 \ . \tag{5·16}$$

A normalizable regular solution to Eq. (5·16) exists only when $(\frac{1}{2})\,(E_y|_{Q=0}/\omega_\rho - l - 2)$ $= n = 0, 1, 2, \cdots$, and it is given by

$$v(z) = F(-n, l+2; z) \ , \tag{5·17}$$

where $F(\alpha, \gamma; z)$ is the confluent hypergeometric function defined by

$$F(\alpha, \gamma; z) \equiv \sum_{k=0}^{\infty}\frac{(\alpha)_k}{(\gamma)_k}\frac{z^k}{k!} \ , \tag{5·18}$$

with $(\alpha)_k \equiv \alpha(\alpha+1)\cdots(\alpha+k-1)$. Note that $F(-n,\gamma;z)$ is a polynomial of degree n. The corresponding energy eigenvalue is

$$E_y|_{Q=0} = \omega_\rho(l+2n+2) \ , \tag{5·19}$$

which coincides with Eq. (5·6). It is easy to see that the degeneracy (5·8) is reproduced by summing $(l+1)^2$ with $l = N-2n$ over $n = 0,1,\cdots,[N/2]$.

Now we turn back to the Hamiltonian (5·2) with $Q>0$. Using polar coordinates, it is written

$$H_y = -\frac{1}{2m_y}\left(\frac{1}{\rho^3}\partial_\rho(\rho^3\partial_\rho) + \frac{1}{\rho^2}(\nabla^2_{S^3} - 2m_yQ)\right) + \frac{1}{2}m_y\omega_\rho^2\rho^2 \ . \tag{5·20}$$

Again, the wave function can be written as Eq. (5·12), and $R(\rho)$ should satisfy

$$\mathcal{H}_{\widetilde{l}}R(\rho) = E_yR(\rho) \ , \tag{5·21}$$

where $\mathcal{H}_{\widetilde{l}}$ is now given by $\mathcal{H}_l$ (Eq. (5·14)), with l replaced by $\widetilde{l}$, defined as

$$\widetilde{l} \equiv -1 + \sqrt{(l+1)^2 + 2m_yQ} \ , \tag{5·22}$$

which satisfies

$$\widetilde{l}(\widetilde{l}+2) = l(l+2) + 2m_yQ \ . \tag{5·23}$$

Therefore, the eigenfunctions and the energy eigenvalues are obtained by simply replacing l with $\widetilde{l}$ in the previous results for $Q=0$, and thus the energy spectrum becomes

$$\begin{aligned} E_y &= \omega_\rho(\widetilde{l}+2n_\rho+2) \\ &= \sqrt{\frac{(l+1)^2}{6} + \frac{2}{15}N_c^2} + \frac{2n_\rho+1}{\sqrt{6}} \ , \end{aligned} \tag{5·24}$$

with $n_\rho = 0,1,2,\cdots$ and $l = 0,1,2,\cdots$. As discussed above, the fermionic baryons correspond to the wave functions that are odd in a_I, which implies that l should be odd. We see in the next subsection that this yields baryons with half-integer spin and isospin. Finally, the quantization of Z is trivial:

$$E_Z = \omega_Z\left(n_z + \frac{1}{2}\right) = \frac{2n_z+1}{\sqrt{6}} \ , \tag{5·25}$$

with $n_z = 0,1,2,\cdots$. Adding Eqs. (5·24) and (5·25), we obtain the following baryon mass formula:

$$M = M_0 + \sqrt{\frac{(l+1)^2}{6} + \frac{2}{15}N_c^2} + \frac{2(n_\rho+n_z)+2}{\sqrt{6}} \ . \tag{5·26}$$

5.2. *Physical interpretation*

The physical interpretation of the baryon spectrum found in the previous subsection is as follows. As mentioned above, each mass eigenstate belongs to the $(S_{l/2}, S_{l/2})$ representation of $SO(4) \simeq (SU(2)_I \times SU(2)_J)/\mathbb{Z}_2$, which acts on the $SU(2)$-valued collective coordinate $\boldsymbol{a}$ defined by Eq. (4·9) as

$$\boldsymbol{a} \to g_I \, \boldsymbol{a} \, g_J \ , \quad g_{I,J} \in SU(2)_{I,J} \ . \tag{5·27}$$

This implies that $SU(2)_I$ and $SU(2)_J$ are identified with the isospin rotation and the spatial rotation, respectively, as in Ref. 31). This can be understood from the ansatz (4·2) and Eq. (A·21) in Appendix A, relating $\boldsymbol{a}$ and V: The spatial rotation of the BPST instanton configuration (3·2) gives rise to the transformation of V as

$$V \to V g_J \ , \quad g_J \in SU(2)_J \ , \tag{5·28}$$

while the isospin rotation of the gauge field (4·2) is induced by

$$V \to g_I V \ , \quad g_I \in SU(2)_I \ . \tag{5·29}$$

This transformation property, together with Eq. (A·21), implies Eq. (5·27). With this identification, we find that the spin J and isospin I of the soliton are both $l/2$. The $l = 1$ states correspond to $I = J = 1/2$ states, which include nucleons, and the $l = 3$ states correspond to $I = J = 3/2$ states, which include Δ. These are the states considered in Ref. 31).

Heavier baryons with a common spin and isospin are represented by states with non-trivial n_ρ and n_z. It is interesting that the excited states with odd n_z correspond to odd parity baryons, as the parity transformation induces $z \to -z$, as shown in Ref. 10).

For the comparison with our mass formula (5·26) to be made below, we list baryons with $I = J$ in the PDG baryon summary table,[54] along with a possible interpretation of the quantum numbers (n_ρ, n_z).

(n_ρ, n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(l=1)$	940^+	1440^+	1535^-	1655^-	1710^+, ?	2090^-_*, ?	2100^+_*, ?
$\Delta\,(l=3)$	1232^+	1600^+	1700^-	1940^-_*	1920^+, ?	?, ?	?, ?

(5·30)

The superscripts $\pm$ represent the parity. The subscript $*$ indicates that evidence of the existence of the baryon in question is poor.

5.3. *Comments on the baryon mass formula*

Let us first discuss the N_c dependence of the mass formula (5·26) in the large N_c limit. For $N_c \gg l$, the mass formula (5·26) has the following approximate expression:

$$M \simeq M_0 + \sqrt{\frac{2}{15}} N_c + \frac{1}{4}\sqrt{\frac{5}{6}} \frac{(l+1)^2}{N_c} + \frac{2(n_\rho + n_z) + 2}{\sqrt{6}} \ . \tag{5·31}$$

Note that the $\mathcal{O}(N_c)$ terms are identical to the classical formula, (3·20). It is interesting that the mass formula (5·31) is consistent with the expected N_c dependence in large N_c QCD.[31),55)] It is known that the mass splittings among the low-lying baryons with different spins are of order $1/N_c$, while those among excited baryons are of order N_c^0. This is exactly what we observe in Eq. (5·31).

The states considered in Ref. 31) correspond to the states with $n_\rho = n_z = 0$. The l-dependent term in their mass formula is proportional to $l(l+2)$, which is also reproduced in Eq. (5·31).

It is important to understand the extent to which we can trust the mass formulas (5·26) and (5·31). First, in order to approximate Eq. (5·26) with Eq. (5·31), the inequality

$$\frac{(l+1)^2}{6} < \frac{2}{15} N_c^2 \tag{5·32}$$

must be satisfied. For real QCD with $N_c = 3$, it is satisfied only for $l = 1$. For this reason, we mainly consider the formula (5·26) in the following. However, we have to keep in mind that there may be $1/N_c$ corrections to the action (2·1) that become important for large quantum numbers l, n_z and n_ρ in the mass formula (5·26).

Another uncertainty in the mass formula regards the zero-point energy. Note that the zero-point energy in Eq. (5·26) is of order N_c^0, which is the same order as possible $1/N_c$ corrections to the classical soliton mass M_0. Furthermore, an infinite number of the heavy modes around the instanton that have been ignored to this point give a divergent contribution to the total zero-point energy of order N_c^0. What we really need is the difference between the energy in the presence of a soliton and that in the vacuum, and hence the divergence in the zero-point energy in the presence of a soliton should be removed by subtracting the zero-point energy of the vacuum. In this paper, we do not attempt to analyze such contributions. Instead, we only consider the mass differences among the baryons and treat M_0 as a free parameter.

5.4. *Numerical estimates*

As suggested in §§3.2 and 5.3, we cannot fully justify the quantitative prediction for the baryon mass, especially in the case of large masses, because the contribution from higher-derivative terms, as well as the $1/N_c$ corrections, may become important. Nevertheless, here we report some numerical estimates to gain some insight from the baryon mass formula (5·26).

The difference between the masses of the $l = 3$ and $l = 1$ states is

$$M_{l=3} - M_{l=1} = \sqrt{\frac{8}{3}+\frac{6}{5}} - \sqrt{\frac{2}{3}+\frac{6}{5}} \simeq 0.600 \simeq 569\,\mathrm{MeV}\ . \tag{5·33}$$

The difference between the masses of the $(n_\rho, n_z) = (1,0)$ or $(0,1)$ state and the $(0,0)$ state with a common l is

$$M_{(1,0)/(0,1)} - M_{(0,0)} = \frac{2}{\sqrt{6}} \simeq 0.816 \simeq 774\,\mathrm{MeV}\ . \tag{5·34}$$

Here, we have used $1 = M_{\mathrm{KK}} \simeq 949$ MeV, which is consistent with the ρ meson mass.[10),11)] Unfortunately, these values are slightly too large compared with the experimental values. If M_{KK} were 500 MeV, the predicted values obtained using

Eq. (5·26) would become very close to those listed in Eq. (5·30):*)

(n_ρ, n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(l=1)$	940^+	1348^+	1348^-	1756^-	$1756^+, 1756^+$	$2164^-, 2164^-$	$2164^+, 2164^+$
$\Delta\,(l=3)$	1240^+	1648^+	1648^-	2056^-	$2056^+, 2056^+$	$2464^-, 2464^-$	$2464^+, 2464^+$

(5·35)

§6. Conclusion and discussion

In this paper, we have investigated dynamical baryons within the context of the holographic description of QCD proposed in Refs. 10) and 11). A key observation in this treatment is that the baryon number is provided with the instanton number in the five-dimensional YM-CS theory (2·1). This implies that baryons can be described as large N_c solitons, as in Refs. 20)–22), 55) and 31). We explicitly constructed a soliton solution with a unit baryon number and found that it corresponds to the BPST instanton with a size of order $\lambda^{-1/2}$. It was stressed that the Coulomb interaction in the CS term plays a crucial role in obtaining the regular solution. Although regular, the instanton is not large enough that we can employ the YM-CS theory with all the higher-order derivative terms omitted. As a first step toward the full incorporation of the infinitely many higher derivative terms, we consider the non-Abelian DBI action[56)] in Appendix B. There, we verify that the energy contribution of the static baryon configuration computed with the non-Abelian DBI action is the same as that computed with the YM action. We leave the more thorough analysis of this problem as a future work, with the goal of carrying out a precise quantitative test of the present model regarding baryon physics by properly treating all the relevant higher derivative terms.

We quantized the collective coordinates of the instanton to obtain the baryon spectrum in the hope that the model (2·1) captures some qualitative features of baryons. In fact, the N_c dependence of the baryon mass formula (5·26) is consistent with the results of the analyses of large N_c baryons in the literature. Furthermore, our model describes negative-parity baryons as the excited states of the instanton along the holographic direction z. Unfortunately, the best fit of the parameter $M_{\rm KK}$ to the experimental data for baryons is inconsistent with that found in Refs. 10) and 11), which comes from the ρ meson mass. This may be due to the fact that the higher derivative terms have not been incorporated into the YM-CS theory.

We end this paper with some comments on future directions. It is important to analyze static properties of baryons, such as the charge radii and magnetic moments, as done in Ref. 31) for the Skyrme model. Also, extension of the one-instanton solution to multi-instanton cases is quite interesting for the purpose of exploring multi-baryon systems. (See Refs. 38) and 57) for related works.) Moreover, in the present model, the role of the infinite number of (axial-)vector mesons in obtaining the soliton solution is not difficult to elucidate. It would be interesting to compare this role with the recent analysis given in Ref. 28), in which baryons are constructed

*) The nucleon mass 940 MeV is used as an input to fix M_0.

as Skyrmions in the effective action including the pion and ρ meson on the basis of the D4/D8 model.

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Appendix A
—— *Metric of the Instanton Moduli Space* ——

In this appendix, we outline the derivation of the metric of the instanton moduli space given by Eq. (4·13). (See, e.g., Ref. 48) for a review.) This metric can be read from the kinetic term of the Lagrangian of the collective coordinates, which follows from the F_{0M}^2 term in Eq. (3·10) as

$$\begin{aligned} \frac{m_X}{2} g_{\alpha\beta}\dot{X}^\alpha \dot{X}^\beta &= \kappa\lambda^{-1} \int d^3x dz \ \mathrm{tr}\, F_{0M}^2 \\ &= \kappa\lambda^{-1} \int d^3x dz \, \mathrm{tr} \left(D_M^{\mathrm{cl}} \Phi - \dot{A}_M^{\mathrm{cl}} \right)^2 \ , \end{aligned} \qquad (\mathrm{A{\cdot}1})$$

where F_{0M} is given by Eq. (4·3). We solve the Gauss's law constraint (4·6) to obtain Φ (Eq. (4·4)) for each instanton moduli and then calculate the corresponding metric using Eq. (A·1). To do this, we first decompose Φ as

$$\Phi = \Phi_X + \Phi_\rho + \Phi_{SU(2)} \ , \qquad (\mathrm{A{\cdot}2})$$

and impose the conditions

$$D_M^{\mathrm{cl}} \left(\dot{X}^N \frac{\partial}{\partial X^N} A_M^{\mathrm{cl}} - D_M^{\mathrm{cl}} \Phi_X \right) = 0 \ , \qquad (\mathrm{A{\cdot}3})$$

$$D_M^{\mathrm{cl}} \left(\dot{\rho} \frac{\partial}{\partial \rho} A_M^{\mathrm{cl}} - D_M^{\mathrm{cl}} \Phi_\rho \right) = 0 \ , \qquad (\mathrm{A{\cdot}4})$$

$$D_M^{\rm cl} D_M^{\rm cl} \Phi_{SU(2)} = 0 \ . \tag{A·5}$$

The following formula for $g(x)$ given in Eq. (3·4) is useful in the derivation below:

$$g\partial_M g^{-1} = \begin{cases} \dfrac{i}{\xi^2}\left((z-Z)\tau^i - \epsilon_{ija}(x^j - X^j)\tau^a\right) \ , & (M=i) \\ -\dfrac{i}{\xi^2}(x^a - X^a)\tau^a \ . & (M=z) \end{cases} \tag{A·6}$$

• Instanton center $X^M = (\vec{X}, Z)$

We find that the form

$$\Phi_X = -\dot{X}^N A_N^{\rm cl} \tag{A·7}$$

satisfies Eq. (A·3), since we have $(\partial/\partial X^N)A_M^{\rm cl} = -\partial_N A_M^{\rm cl}$, and hence

$$D_M^{\rm cl}\Phi_X - \dot{X}^N \frac{\partial}{\partial X^N} A_M^{\rm cl} = -\dot{X}^N F_{MN}^{\rm cl} \ . \tag{A·8}$$

The corresponding metric is

$$g_{MN} = \frac{2\kappa\lambda^{-1}}{m_X}\int d^3x\, dz\, {\rm tr}\, F_{MP}^{\rm cl} F_{NP}^{\rm cl} = \delta_{MN} \ . \tag{A·9}$$

• Instanton size ρ

Using the relation

$$\frac{\partial}{\partial\rho}A_M^{\rm cl} = -\frac{2\rho}{\xi^2+\rho^2}A_M^{\rm cl} \tag{A·10}$$

and the formula

$$\partial_M(g\,\partial_M\, g^{-1}) \propto (x_M - X_M)\, g\,\partial_M\, g^{-1} = 0 \ , \tag{A·11}$$

we find that Eq. (A·4) is satisfied by

$$\Phi_\rho = 0 \ . \tag{A·12}$$

Further, the metric is given by

$$g_{\rho\rho} = \frac{2\kappa\lambda^{-1}}{m_X}\int d^3x\, dz\, {\rm tr}\left(\frac{\partial}{\partial\rho}A_M^{\rm cl}\right)^2 = 2 \ . \tag{A·13}$$

• $SU(2)$ orientation

The $SU(2)$ rotation of the instanton solution is implemented by a global gauge transformation. To solve Eq. (A·5), it is convenient to move to the singular gauge obtained through the gauge transformation

$$\begin{aligned} \Phi_{SU(2)} &\to \overline{\Phi}_{SU(2)} \equiv g^{-1}\Phi_{SU(2)}\, g \ , \\ A_M^{\rm cl} &\to \overline{A}_M \equiv g^{-1}A_M^{\rm cl}\, g - ig^{-1}\partial_M g = -i(1-f(\xi))g^{-1}\partial_M g \ , \end{aligned} \tag{A·14}$$

where $f(\xi)$ is given by Eq. (3·3). Then, Eq. (A·5) can be recast as

$$\overline{D}_M \overline{D}_M \overline{\Phi}_{SU(2)} = 0 \ , \tag{A·15}$$

with $\overline{D}_M = \partial_M + i[\overline{A}_M\ ,\ ]$. It is not difficult to see that Eq. (A·15) is solved by

$$\overline{\Phi}_a = u(\xi)\frac{\tau^a}{2}\ , \quad (a = 1,2,3) \tag{A·16}$$

with $u(\xi)$ satisfying

$$\frac{1}{\xi^3}\partial_\xi(\xi^3\partial_\xi\, u(\xi)) = 8\frac{(1-f(\xi))^2}{\xi^2}u(\xi)\ . \tag{A·17}$$

The regular solution of this equation is

$$u(\xi) = C\,\frac{\xi^2}{\xi^2+\rho^2} = C\,f(\xi)\ , \tag{A·18}$$

with a constant C. Therefore, $\Phi_{SU(2)}$ can be written

$$\Phi_{SU(2)} = \chi^a(t)\Phi_a(x)\ , \tag{A·19}$$

with

$$\Phi_a = f(\xi)\,g\,\frac{\tau^a}{2}\,g^{-1} \tag{A·20}$$

and t-dependent real coefficients $\chi^a(t)$.

We choose the $SU(2)$-valued collective coordinate $\boldsymbol{a}(t) = a_4(t) + ia_a(t)\tau^a$ as

$$V(t,\vec{x},z) \to \boldsymbol{a}(t)\ . \quad (z\to\infty) \tag{A·21}$$

Comparing this with (A·19), we find

$$\chi^a = -i\,\mathrm{tr}\left(\tau^a\boldsymbol{a}^{-1}\dot{\boldsymbol{a}}\right) = 2\left(a_4\dot{a}_a - \dot{a}_4 a_a + \epsilon_{abc}a_b\dot{a}_c\right)\ . \tag{A·22}$$

This gives

$$(\chi^a)^2 = 4\dot{a}_I^2\ . \tag{A·23}$$

Then, the metric for a_I is obtained as

$$g_{IJ}\dot{a}_I\dot{a}_J = \frac{2\kappa\lambda^{-1}}{m_X}\int d^3x dz\,\mathrm{tr}\left(D_M^{\rm cl}\Phi_{SU(2)}\right)^2 = 2\rho^2\dot{a}_I^2\ , \tag{A·24}$$

with the constraint $a_I^2 = 1$.

It is easy to see that the off-diagonal components of $g_{\alpha\beta}$, connecting different kinds of moduli, vanish. Collecting these results, we find that the metric of the moduli space is given by

$$\begin{aligned} ds^2 &= g_{\alpha\beta}dX^\alpha dX^\beta \\ &= d\vec{X}^2 + dZ^2 + 2(d\rho^2 + \rho^2 da_I^2) \\ &= d\vec{X}^2 + dZ^2 + 2\,dy_I^2\ , \end{aligned} \tag{A·25}$$

with $y_I = \rho a_I$. Note that a_I parameterizes not S^3 but $S^3/\mathbb{Z}_2$, with $\mathbb{Z}_2$ being the center of $SU(2)$, which acts as $a_I \to -a_I$. In fact, the configuration (4·2) is unchanged under the $\mathbb{Z}_2$ transformation $V \to -V$. Hence, the one-instanton moduli space coincides with $\mathbb{R}^4 \times \mathbb{R}^4/\mathbb{Z}_2$.

Appendix B
—— Higher Derivative Terms ——

As we have seen in §3, higher derivative terms in the D-brane action also contribute to the soliton mass at the same order in the $1/\lambda$ expansion. However, it is difficult to include all the higher-order derivative terms, since the exact derivative corrections in the D-brane action are not known. (See, e.g., Ref. 58) and the references therein.) Among the various sources of higher derivative corrections in the D-brane action, here we consider the contributions from the non-Abelian DBI action[56)] as a first step toward a complete analysis.

The non-Abelian DBI action for the probe D8-branes is given by

$$S_{\rm DBI} = -\mu_8 \int d^9x\, e^{-\phi}\, {\rm str}\sqrt{-\det(g_{ab} + 2\pi\alpha'\mathcal{F}_{ab})}\ , \quad (a, b = 0, 1, \cdots, 8) \qquad ({\rm B}\cdot 1)$$

where $\mu_8 = 1/((2\pi)^8 l_s^9)$, and str denotes the symmetrized trace. Here, g_{ab} is the induced metric on the D8-brane world-volume, given by[10)]

$$ds^2_{9\,\rm dim} = \frac{\lambda l_s^2}{3}\left[\frac{4}{9}k(z)^{1/2}\,\eta_{\mu\nu}dx^\mu dx^\nu + \frac{4}{9}k(z)^{-5/6}\,dz^2 + k(z)^{1/6}\,d\Omega_4^2\right]\ , \qquad ({\rm B}\cdot 2)$$

and the dilaton on it reads

$$e^{-\phi} = \frac{3^{3/2}\pi N_c}{\lambda^{3/2}}k(z)^{-1/4}\ . \qquad ({\rm B}\cdot 3)$$

After integrating over the S^4 directions, we obtain the five-dimensional non-Abelian DBI action

$$S_{\rm DBI} = -\frac{N_c\lambda^3}{3^9\pi^5}\int d^4x dz\, k(z)^{1/12}\,{\rm str}\sqrt{-\det\left(g^{(5)}_{\hat M\hat N} + \frac{27\pi}{2\lambda}\mathcal{F}_{\hat M\hat N}\right)}\ , \qquad ({\rm B}\cdot 4)$$

where $\hat M, \hat N = 0, 1, 2, 3, z$, and the five-dimensional metric $g^{(5)}_{\hat M\hat N}$ is given by

$$ds^2_{5\,\rm dim} = k(z)^{1/2}\,\eta_{\mu\nu}dx^\mu dx^\nu + k(z)^{-5/6}\,dz^2\ . \qquad ({\rm B}\cdot 5)$$

Here, for simplicity, we have kept only the gauge potentials $A_{\hat M}$ non-zero.

The above expressions are written in terms of the original variables, before the rescaling (3·9). Upon the rescaling, it is found that the non-Abelian DBI action (B·4) can be expanded as

$$S_{\rm DBI} = -\frac{\lambda N_c}{3^9\pi^5}\int d^4x dz\left(\mathcal{L}_0 + \lambda^{-1}\mathcal{L}_1 + \mathcal{O}(\lambda^{-2})\right)\ , \qquad ({\rm B}\cdot 6)$$

where $\mathcal{L}_0$ and $\mathcal{L}_1$ are given by

$$\mathcal{L}_0 = {\rm str}\sqrt{\det\left(\mathcal{B}_{MN}\right)}\ , \qquad ({\rm B}\cdot 7)$$

$$\mathcal{L}_1 = {\rm str}\Bigg[\sqrt{\det\left(\mathcal{B}_{MN}\right)}\Bigg(\frac{2}{3}z^2 - \frac{1}{2}\left(\frac{27\pi}{2}\right)^2\mathcal{G}^{MN}\mathcal{F}_{M0}\mathcal{F}_{N0}$$

$$+\frac{z^2}{2}\left(\frac{27\pi}{2}\right)^2\left\{-\frac{1}{2}\mathcal{G}^{MN}\mathcal{F}_{Mi}\mathcal{F}_{Ni}+\frac{5}{6}\mathcal{G}^{MN}\mathcal{F}_{Mz}\mathcal{F}_{Nz}\right\}\Bigg)\Bigg] , \qquad \text{(B·8)}$$

with $M,N=1,2,3,z$ and $i=1,2,3$, and we have the definitions

$$\mathcal{B}_{MN}\equiv\delta_{MN}+\frac{27\pi}{2}\mathcal{F}_{MN} , \quad \mathcal{G}^{MN}\equiv(\mathcal{B}^{-1})^{(MN)} . \qquad \text{(B·9)}$$

From Eq. (B·7), we find that the leading-order term in the $1/\lambda$ expansion is given by the non-Abelian DBI action in a flat space-time. It is known that the BPST instanton configuration (3·2) is a solution for the non-Abelian DBI action (B·7).[45)–47)]

Inserting the BPST instanton configuration (3·2) into the action (B·4), we obtain

$$S_{\rm DBI}=-\frac{\lambda N_c}{3^9\pi^5}\int d^4x\,dz\,k^{2/3}$$
$$\times 2\left(1+2\frac{\partial}{\partial s}\right)\sqrt{1+s\left(\frac{27\pi}{4}\right)^2k^{-1}\omega^2}\sqrt{1+s\left(\frac{27\pi}{4}\right)^2k^{1/3}\omega^2}\,\Bigg|_{s=1} , \qquad \text{(B·10)}$$

where

$$\omega=\frac{4\rho^2}{(\xi^2+\rho^2)^2} , \qquad \text{(B·11)}$$

and ξ is defined as in Eq. (3·3). Here, we are using the rescaled variables, and we have $k=k(\lambda^{-1/2}z)=1+\lambda^{-1}z^2$. Hence, the energy contribution is

$$E=\frac{\lambda N_c}{3^9\pi^5}\int d^3x\,dz\,k^{2/3}$$
$$\times 2\left(1+2\frac{\partial}{\partial s}\right)\left(\sqrt{1+s\left(\frac{27\pi}{4}\right)^2k^{-1}\omega^2}\sqrt{1+s\left(\frac{27\pi}{4}\right)^2k^{1/3}\omega^2}-1\right)\Bigg|_{s=1}$$
$$=\frac{\lambda N_c}{18\pi^2}\int_{-\infty}^{\infty}dz\int_0^{\infty}dr\left[r^2\left(1+\frac{z^2}{3\lambda}\right)\omega^2+\mathcal{O}(\lambda^{-2})\right]$$
$$=8\pi^2\kappa\left[1+\lambda^{-1}\left(\frac{\rho^2}{6}+\frac{Z^2}{3}\right)+\mathcal{O}(\lambda^{-2})\right] , \qquad \text{(B·12)}$$

where we have used

$$\int_{-\infty}^{\infty}dz\int_0^{\infty}dr\,r^2\omega^2=\frac{2\pi}{3} , \quad \int_{-\infty}^{\infty}dz\int_0^{\infty}dr\,r^2z^2\omega^2=\frac{2\pi}{3}Z^2+\frac{\pi}{3}\rho^2 . \qquad \text{(B·13)}$$

The expression (B·12) is identical to (3·18), except for the contribution from the CS term. This result is highly non-trivial, since the non-Abelian DBI action (B·4) contains infinitely many higher derivative terms, while our previous analysis is based on the YM action in Eq. (2·1). This finding suggests that our previous results may not be significantly modified even if we include all of the higher derivative terms. However, because there are still infinitely many higher derivative terms that have not been included in the non-Abelian DBI action, we cannot definitively confirm the quantitative results obtained in this paper, such as the baryon mass formula (3·18). Nonetheless, we expect that these results will be useful in more systematic studies of the higher derivative terms.

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Note added: While this paper was being completed, we received the paper,[59] whose content overlaps somewhat with that of the present paper.